O($\alpha\alpha_s$) corrections to the Yukawa-top & Higgs-boson couplings s

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Plan

- Standard Model is Renormalizable Model
- Pole Masses of scalar/vector Bosons and Fermions: Definition
- Structure of UV counterterms in $\overline{\mathrm{MS}}$scheme
- Renormalization Group Equations in Standard Model
- Matching conditions for Coupling Constant
- Conclusion

Introduction

The Standard Model (SM) belongs to the class of *renormalizable* quantum field theories which means, in particular, that a restricted number of input parameters suffice for theoretical predictions of any process. The concrete choice of input parameters defines a specific *renormalization scheme*. The given set of independent parameters has to be extracted from an appropriate set of experimentally measured quantities. If we would be able to perform perturbative calculations to all orders all renormalization schemes would be equivalent. However, in practice, only the first few coefficients are known, so that predictions depend on the choice of the scheme. Such dependence on the truncation of the perturbative series is known as *scheme dependence*. In general, the difference between two schemes is of next higher order in the perturbation expansion. For higher order calculations those schemes are preferable for which the uncalculated higher order corrections are small. Of course, to find such a preferred scheme requires to perform calculations in different schemes. Another possibility is to find the scheme transition relations by calculating the input parameters in one scheme in terms of the input parameters of another scheme order by order in perturbation theory. For electroweak calculations a natural and generally accepted scheme is the so called *on-shell scheme* [A. Sirlin,1980] where, in addition to the fine structure constant (and/or the Fermi constant), the pole masses of particles serve as input parameters.

Introduction: 2

Quark masses require special consideration in this context, since on–shell quark masses are not accessible experimentally. Fortunately, the light quark masses in high energy processes often can be neglected (effects $O(m_q^2/M_Z^2)$) and thus can be treated as massless in practical calculations. The top quark is different. The large numerical value of the top quark mass in conjunction with the violation of the Appelquist-Carazzone theorem as a consequence of the Higgs mechanism of mass generation, implies that a class of radiative corrections are proportional to positive powers of the top-quark mass which gives rise to sizeable effects. Moreover, the concept of a pole mass of a quark is intrinsically ambiguous due to strong interaction renormalon contributions which affect seriously the convergence of the perturbation expansion. This is one of the main reasons why for quarks the $\overline{\mathrm{MS}}$-mass appears to be a better input parameter.

Pole mass of bosons.

Starting point of our consideration is the form of the renormalized propagator in the on-shell scheme:

$$\frac{1}{p^2 + m^2 - \Pi(p^2, m^2, \cdots)} = \frac{Z_2}{p^2 + s_P} \,,$$

where $\Pi(p^2, m^2, \cdots)$ is the transversal part of the one-particle irreducible self-energy, m^2 is a bare or renormalized mass in the $\overline{\mathrm{MS}}$ scheme, Z_2 is the on-shell wave-function renormalization constant and s_P is the position of the pole of the propagator of a massive boson at which the inverse of the full propagator equals zero (we use the Euclidean metric and on–shell condition is $p^2 = -m^2$). Standard parametrization of the pole is

$$s_{P,a} = M_a^2 - i M_a \Gamma_a \,,$$

where Γ_a is the width of particle.

In perturbation theory the pole equation is to be solved order by order.

Pole mass of bosons: 2

For this aim we expand the self–energy function $\tilde{\Pi}(p^2, m^2, \cdots)$ about the lowest order solution $p^2 = -m^2$:

$$\tilde{\Pi}(p^2, m^2, \cdots) = \Pi \left.\right|_{p^2=-m^2}$$
$$+\left(p^2+m^2\right) \left\{ \left[\Pi^{'}\right] + \frac{1}{2} \left(p^2+m^2\right) \left[\Pi^{''}\right] + \cdots \right\}\Bigg|_{p^2=-m^2}$$

accepting

$$m^2 = s_p + \sigma \equiv s_p + \sigma_1 + \sigma_2 + \cdots$$

so that

$$\Delta^{-1} = p^2 + m^2 - \tilde{\Pi}(p^2, m^2, \cdots)$$
$$= -\Pi(m^2, m^2)\ (p^2 + m^2) \left(1 - \Pi^{'}(m^2, m^2)\right)\Big|_{m^2=s_P+\sigma}$$
$$-\frac{1}{2}(p^2 + m^2)^2 \Pi^{''}(m^2, m^2)\Big|_{m^2=s_P+\sigma}$$
$$= (p^2 + s_p) \left(1 - \Pi^{'}(m^2, m^2) - \sigma \Pi^{''}(m^2, m^2)\right)$$
$$+\sigma \left(1 - \Pi^{'}(m^2, m^2)\right) - \Pi(m^2, m^2) + O(g^3)$$

$$\sigma \left(1 - \Pi^{'}\right) - \Pi = 0 \to \sigma_1 = \Pi_1 \ , \quad \sigma_2 = \sigma_1 \Pi_1^{'} + \Pi_2$$
$$1 - \Pi^{'} - \sigma \Pi^{''} = Z_2^{-1}$$

Pole mass of bosons: 3

Up to two-loop order the solution is

$$s_P = m^2 - \Pi^{(1)} - \Pi^{(2)} - \Pi^{(1)}\Pi^{(1)\prime}\Big|_{p^2=-m^2} \; ,$$
$$Z_2^{-1} = 1 - \Pi^{(1)\prime} - \Pi^{(2)\prime} - \Pi^{(1)}\Pi^{(1)\prime\prime} \, ,$$

where $\Pi^{(L)}$ is the bare or $\overline{\mathrm{MS}}$-renormalized L-loop contribution to Π and the the prime denotes the derivative with respect to p^2.

One of the remarkable properties of this equation that the pole mass M^2 and the width Γ is defined via self-energy diagrams and its derivatives at momentum equal to bare (or $\overline{\mathrm{MS}}$) masses which, by the construction, are the real parameters.

However, these expressions are valid only when $\Pi(p^2, m^2, \cdots)$ is analytic functions, so that Taylor expansion can be performed.

B.A. Kniehl, C.P. Palisoc and A. Sirlin, Nucl. Phys. **B591** (2000) 296;
B.A. Kniehl and A. Sirlin, Phys. Lett. **B530** (2002) 129.

Pole mass and γ-Z mixing.

The simple relation between the full propagator and the irreducible self-energy only holds if there is no mixing, like for the W-boson or Higgs. In the neutral sector, because of mixing two neutral bosons Z and γ form a 2×2 matrix propagator

$$D^{-1}(p^2) = \begin{pmatrix} p^2 - \Pi_{\gamma\gamma}(p^2) & \Pi_{\gamma Z}(p^2) \\ \Pi_{Z\gamma}(p^2) & p^2 - m_Z^2 - \Pi_{\gamma\gamma}(p^2) \end{pmatrix}$$

The equation for the pole is now modified

$$p^2 + m_Z^2 - \Pi_{ZZ}(p^2, m_Z^2, \cdots) - \frac{\Pi^2_{\gamma Z}(p^2, m_Z^2, \cdots)}{p^2 + \Pi_{\gamma\gamma}(p^2)} = 0.$$

- ♣ Mixing term $\Pi^2_{\gamma Z}$ starts contributing at two-loop
- ♠ Photon term $\Pi_{\gamma\gamma}$ only contributes beyond the two-loop

Solution (up to 2-loops):

$$s_{P,Z} = m_Z^2 - \Pi^{(1)}_{ZZ} - \Pi^{(2)}_{ZZ} - \Pi^{(1)}_{ZZ} \Pi^{(1)}_{ZZ}{}' + \frac{\Pi^{(1)2}_{\gamma Z}}{m_Z^2} .$$

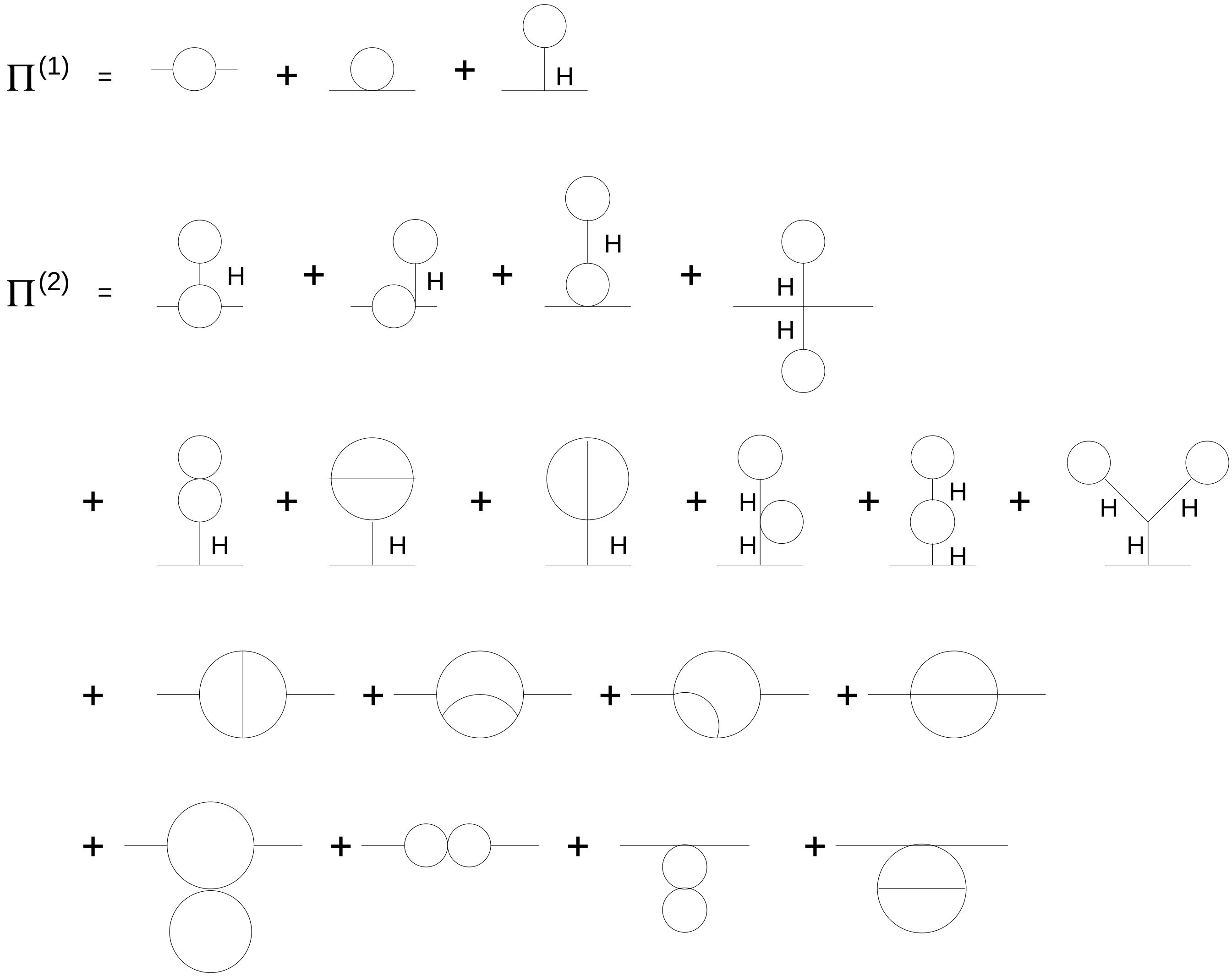

Figure 1: One- and two-loop contributions to the massive boson self-energies.

Pole mass of fermion

Starting point of fermion consideration is the tensor decomposition of the self–energy of a massive fermion $\tilde{\Sigma}(p, m, \ldots)$ which, within the SM in the limit of diagonal CKM matrix, has the form

$$\tilde{\Sigma}(p, m, \ldots) = i\hat{p}\left[\tilde{A}(p^2, m, \ldots) - \gamma_5 \tilde{C}(p^2, m, \ldots)\right] + m\tilde{B}(p^2, m, \ldots) ,$$

where $\tilde{A}$, $\tilde{B}$, $\tilde{C}$ are Lorentz scalar functions depending on all parameters of the SM.

In this case, the position of the pole $-\tilde{M}$ is defined as the formal (independently for the left- and right- handed fermions) solution for $i\hat{p}$ at which the inverse of the connected full propagator equals zero:

$$S_F^{-1} = i\hat{p} + m_0 - \tilde{\Sigma}(p, m_0, \ldots) = \frac{i\hat{p} + \tilde{M}}{Z_L} + \frac{i\hat{p} + \tilde{M}}{Z_R}$$

To two loops we then have the solution

$$\frac{\tilde{M}}{m_0} = 1 + A_1 + B_1 + A_2 + B_2 + (A_1 + B_1)\left(A_1 - 2m_0^2(\dot{A}_1 + \dot{B}_1)\right) + \frac{1}{2}C_1^2 ,$$

where X and $\dot{X}$ denote the value of function $\tilde{X}(p^2, m^2 \ldots)$ and its derivative with respect to p^2 with following putting $p^2 = -m^2$ and $A_L(B_L)$ is the bare ($m = m_0$) or $\overline{\rm MS}$–renormalized (m the $\overline{\rm MS}$–mass) L-loop contribution to the amplitudes.

The fermion pole mass (continuation)

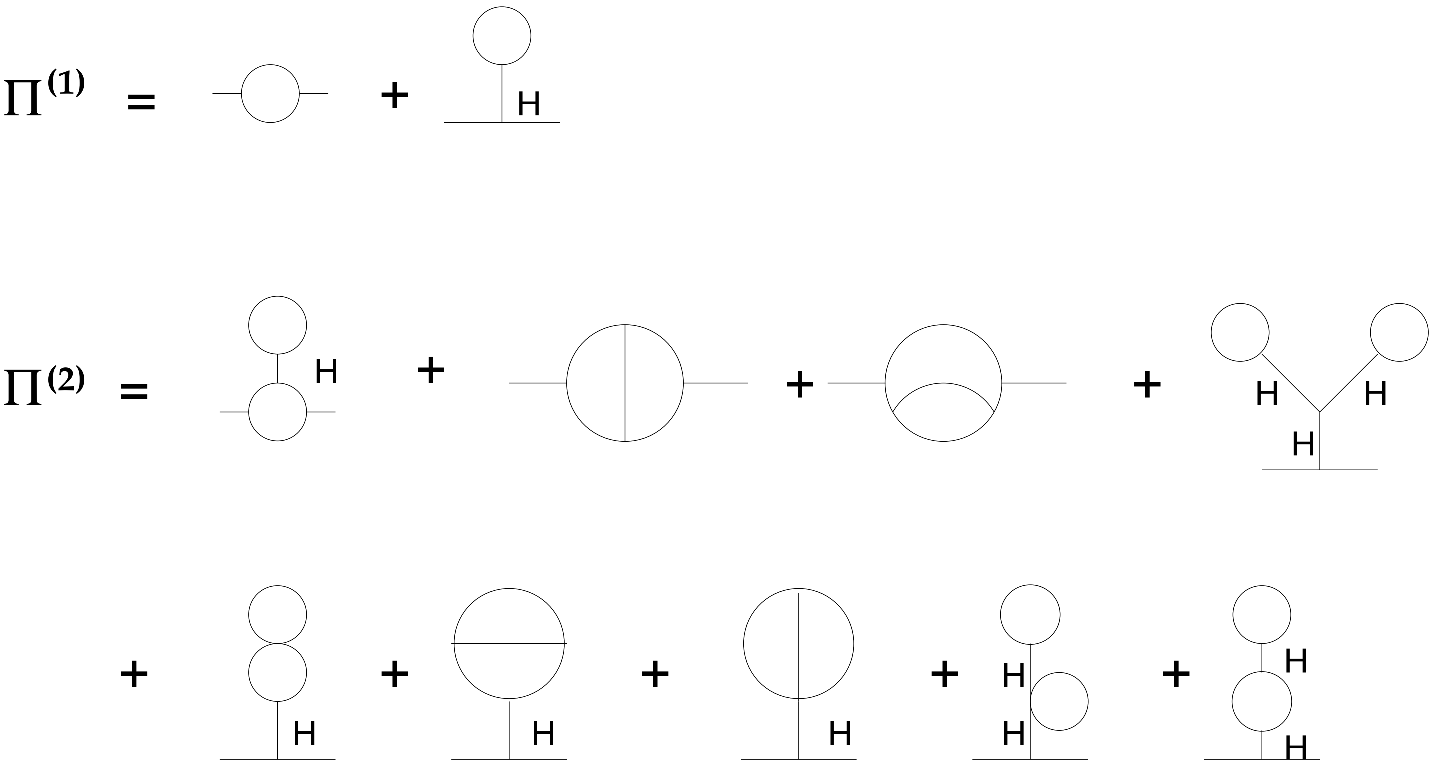

Figure 2: One- and two-loop contributions to the massive fermion self-energies.

Definition of $\overline{\mathrm{MS}}$ mass in SM

For calculations of electroweak corrections in the SM two renormalization schemes are commonly accepted: the on-shell and the $\overline{\mathrm{MS}}$ scheme. It is well understood, that in the on-shell scheme all momentum independent diagrams, in particular the tadpoles, can be omitted.

Let us first express the pole mass in terms of the bare amplitude in a manifestly gauge invariant manner. This requires to include the Higgs tadpole contribution

[J. Fleischer and F. Jegerlehner, Phys. Rev. **D23** (1981) 2001.]

Only this complete gauge invariant bare amplitude should be utilized as a starting point to set up $\overline{\mathrm{MS}}$ renormalization.

Definition of $\overline{\mathrm{MS}}$ mass in SM: 2

At the two-loop level $\overline{\mathrm{MS}}$ renormalization can be written as

$$
\begin{aligned}
s_P &= m_0^2 - \Pi_0^{(1)} - \Pi_0^{(2)} - \Pi_0^{(1)}\Pi_0^{(1)\prime} \\
&- \left[\sum_j (\delta m_{j,0}^2)^{(1)} \frac{\partial}{\partial m_{j,0}^2} + \sum_j (\delta g_{j,0})^{(1)} \frac{\partial}{\partial g_{j,0}} \right] \Pi_0^{(1)} \\
&= m_a^2 - \left\{ \Pi_a^{(1)} \right\}_{\overline{\mathrm{MS}}} - \left\{ \Pi_a^{(2)} + \Pi_a^{(1)}\Pi_a^{(1)\prime} \right\}_{\overline{\mathrm{MS}}}
\end{aligned}
$$

where the sum runs over all species of particles, $g_j = \alpha,\ g_s$, $(\delta g_{j,0})^{(1)}$ and $(\delta m_{j,0}^2)^{(1)}$ are the one-loop counterterms for the charges and physical masses in the $\overline{\mathrm{MS}}$-scheme and after differentiation we put all parameters equal to their on-shell values. The derivatives in this equation correspond to the subtraction of sub-divergencies. The genuine two-loop mass counterterm comes from the shift of the m_0^2 term. The relation between bare- and $\overline{\mathrm{MS}}$-masses has the form

$$
m_{a,0}^2 = m_a^2(\mu)\ (1 + \sum_{k=1} Z_a^{(k)} \varepsilon^{-k})\ .
$$

To renormalize the pole mass at the two-loop level requires to calculate the one-loop renormalization constants for all physical parameters (charge and masses), and the two-loop renormalization constant only for the mass itself. Not needed are the wave-function renormalization or ghost (unphysical) sector renormalization.

$\overline{\mathrm{MS}}$ mass in terms of on-shell mass.

After UV-renormalization the pole is represented in terms of finite amplitudes. Now, expression connects the pole s_P with the $\overline{\mathrm{MS}}$ parameters: masses and charges. This expression can be inverted and solved iteratively. The solution to two-loop reads

$$m_a^2 = M_a^2 + \mathrm{Re}\left\{\Pi_a^{(1)}\right\}_{\overline{\mathrm{MS}}} + \mathrm{Re}\left\{\Pi_a^{(2)} + \Pi_a^{(1)}\Pi_a^{(1)\prime}\right\}_{\overline{\mathrm{MS}}}$$
$$+ \left[(\Delta e)^{(1)}\frac{\partial}{\partial e} + \sum_j (\Delta m_j^2)^{(1)}\frac{\partial}{\partial m_j^2}\right] \mathrm{Re}\left\{\Pi_a^{(1)}\right\}_{\overline{\mathrm{MS}}}$$

where the sum runs over all species of particles $j = Z, W, H, t$, $(\Delta m_j^2)^{(1)} = \mathrm{Re}\left\{\Pi_j\right\}_{\overline{\mathrm{MS}}}$, and the transition from the $\overline{\mathrm{MS}}$to the on-shell scheme for the electric charge is also included. The mass on the l.h.s. of this expression we call the $\overline{\mathrm{MS}}$-mass of particle.

It should be noted, that in this definition the tadpole contribution does not cancel, so that higher powers of the Higgs and the top-quark mass show up at higher orders. In particular, at two-loops, the purely bosonic diagrams generate m_H^4/m_V^4 terms and the third fermion family gives rise to the appearance of $m_t^6/(m_H^2 m_V^4)$ power corrections.

Renormalization Group relations

For the $\overline{\mathrm{MS}}$-masses, defined in this way, the following properties are valid:

1. The UV counter-terms satisfy relations connecting the higher order poles with the lower order ones:

$$\left(\gamma_a + \sum_j \beta_{g_j} \frac{\partial}{\partial g_j} + \sum_i \gamma_i m_i^2 \frac{\partial}{\partial m_i^2}\right) Z_a^{(n)}$$
$$= \frac{1}{2} \sum_j g_j \frac{\partial}{\partial g_j} Z_a^{(n+1)} ,$$

where we adopt the following definitions for the RG functions: for all dimensionless coupling constants, like $g, g', g_s, e, \lambda, y_t$, the β-function is given by $\mu^2 \frac{\partial}{\partial \mu^2} g = \beta_g$ and for all mass parameters (a mass or the Higgs v.e.v. v) the anomalous dimension γ_{m^2} is given by $\mu^2 \frac{\partial}{\partial \mu^2} \ln m^2 = \gamma_{m^2}$.

2. Using the fact that s_P is RG-invariant: $\mu^2 \frac{d}{d\mu^2} s_P \equiv 0$, we are able to calculate the anomalous dimension of the masses from our finite results or from the UV counterterms

$$\gamma_a = \sum_j \frac{1}{2} g_j \frac{\partial}{\partial g_j} Z_a^{(1)} , (j = g, g_s).$$

Renormalization Group relations

3. All tree level relations between masses of any particles and parameters of the unbroken Lagrangian are RG invariant. This means, in particular, that the RG equation for the vacuum expectation value v is given by

$$\gamma_{v^2} \equiv \gamma_{m^2} - \beta_\lambda / \lambda \ ,$$

where m^2 and λ are the parameters of the symmetric scalar potential. This fact allow to get anomalous dimension of the masses via the relations

$$\gamma_W = \gamma_{m^2} - \frac{\beta_\lambda}{\lambda} + 2\frac{\beta_g}{g} \ ,$$
$$\gamma_Z = \gamma_{m^2} - \frac{\beta_\lambda}{\lambda} + 2\left(c_W \frac{\beta_g}{g} + s_W \frac{\beta_{g'}}{g'} \right) ,$$
$$\gamma_t = \gamma_{m^2} - \frac{\beta_\lambda}{\lambda} + \frac{\beta_{y_t}}{y_t} \ , \quad \gamma_H = \gamma_{m^2} \ ,$$

where $s_W (c_W)$ are the $\sin$ $(\cos)$ of the weak mixing angle and the 2-loop RG functions $\beta_g, \beta_{g'}, \beta_\lambda, \gamma_{m^2}, \beta_{y_t}$ are calculated in the unbroken phase. Thus the RG equations for the $\overline{\rm MS}$masses in the broken theory can be written as

$$m_W^2(\mu^2) = \frac{1}{4} \frac{g^2(\mu^2)}{\lambda(\mu^2)} m^2(\mu^2) \ ,$$
$$m_Z^2(\mu^2) = \frac{1}{4} \frac{g^2(\mu^2) + g'(\mu^2)}{\lambda(\mu^2)} m^2(\mu^2) \ ,$$
$$m_H^2(\mu^2) = 2m^2(\mu^2) \ ,$$
$$m_t^2(\mu^2) = \frac{1}{2} \frac{y_t^2(\mu^2)}{\lambda(\mu^2)} m^2(\mu^2) \ ,$$

where y_t is the top-quark Yukawa coupling.

RG equation for Fermi constant

Let us write now the RG equation for the effective Fermi constant G_F. G_F is usually defined as a low energy constant in one-to-one correspondence with the muon lifetime. However, if we consider physics at higher energies a parametrization in terms of low energy constants may lead to large radiative corrections. Much in the same way as the fine structure constant α often is replaced by the effective running fine structure constant $\alpha(\mu)$ we expect that G_F should be replaced by an effective version of it at higher energies. Unlike in the case of α, however, because of the smallness of the light fermion Yukawa couplings, G_F starts to run effectively only at scales beyond the W–pair production threshold

In the broken phase, we define $\overline{MS}$ Fermi constant as

$$G_F(\mu^2) \equiv \frac{\sqrt{2}\ e^2(\mu^2)}{8m_W^2(\mu^2)\sin^2\theta_W(\mu^2)} = \frac{1}{\sqrt{2}v^2(\mu^2)}$$

It satisfies the following RG equation

$$\mu^2 \frac{\partial}{\partial \mu^2} \ln G_F(\mu^2) = \frac{\beta_\lambda}{\lambda} - \gamma_{m^2} \ .$$

The equation is written in $\overline{\mathrm{MS}}$ scheme. As usual in this scheme, in solving the renormalization group equation the decoupling of the heavy particles has to be performed "by hand". This means that below the W mass, the effective Fermi constant does practically not change with scale. Obviously, the running of G_F only starts at about $\mu \sim m_Z$, when the scale of a process exceeds the masses of the bosons.

Our RG equations for the v.e.v. v and the particle masses m are different from the ones obtained in the effective potential approach. A comparison of predictions based on these two approaches have been recently performed

P. Kielanowski and S. R. Juarez W., hep-ph/0310122.

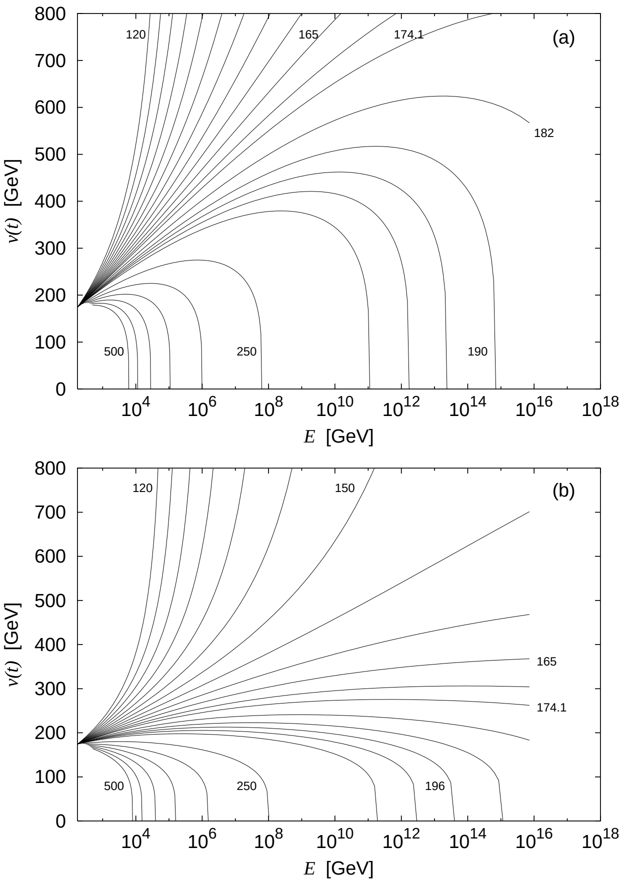

Figure 3: Running of the vacuum expectation value of the Higgs field $v(t)$. Figure (a) corresponds to the case of the effective potential and figure (b) to the tree-level relation for the Higgs mass. The function $v(t)$ depends on the initial Higgs mass. The numbers at the ends of some plots correspond to the initial Higgs mass in GeV. In case (a) for the Higgs masses $m_H < 174$ GeV the $v(t)$ has a singularity at the UV cut off energy. For the Higgs masses $m_H > 190$ GeV the function $v(t)$ for large values of t is decreasing and at the UV cut off it vanishes. In case (b) for the Higgs masses $m_H < 164$ GeV the $v(t)$ has a singularity at the UV cut off energy. For the Higgs masses $m_H > 178$ GeV the function $v(t)$ for large values of t is decreasing and at the UV cut off it vanishes. The behavior of $v(t)$ in both cases is significantly different.

Running $\sin^2\theta_W^{\overline{MS}}$

We can analyze the Higgs mass dependence of $\sin^2\theta_W$. The relation between the $\overline{\rm MS}$ weak mixing parameter and its version in terms of the pole masses reads

$$
\begin{aligned}
\sin^2\theta_W &= 1-\frac{m_W^2}{m_Z^2}=\frac{g'^2}{g^2+g'^2}\\
&= 1-\frac{M_W^2}{M_Z^2}\left(\frac{1+\delta_W^{(1)}+\delta_W^{(2)}}{1+\delta_Z^{(1)}+\delta_Z^{(2)}}\right)\\
&= \left(1-\frac{M_W^2}{M_Z^2}\right)-\frac{M_W^2}{M_Z^2}\left[(\delta_W^{(1)}-\delta_Z^{(1)})(1-\delta_Z^{(1)})+\delta_W^{(2)}-\delta_Z^{(2)}\right]
\end{aligned}
$$

where we adopted the notation, $m_V^2/M_V^2 = 1+\delta_V^{(1)}+\delta_V^{(2)}$

Unphysical terms, proportional to m_H^4 and m_t^6 drop out in $\sin^2\theta_W$

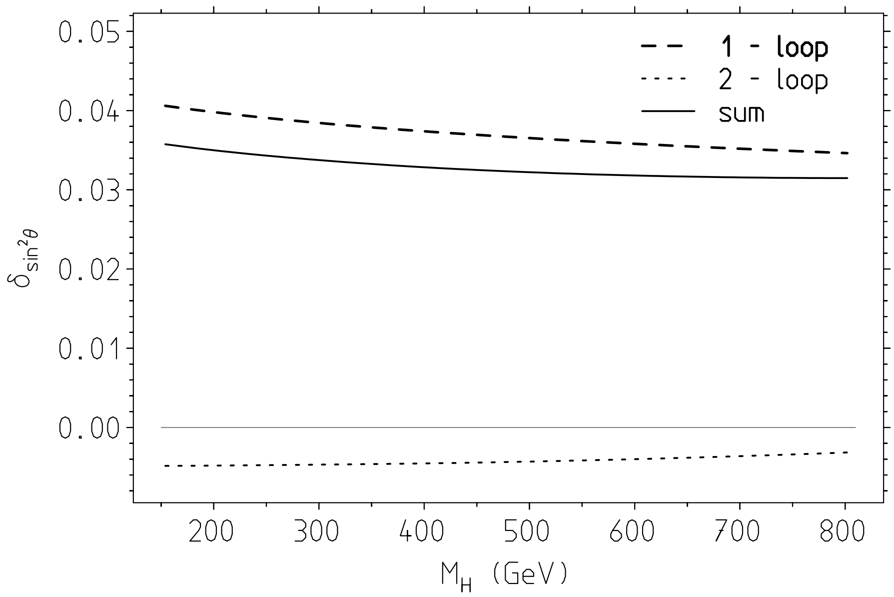

Figure 4: One- and two-loop corrections to $\delta_{\sin^2\Theta} = \sin^2\theta_W^{\overline{MS}}/\sin^2\theta_W^{\rm OS}-1$ as a function of the Higgs mass m_H $(\mu = M_Z)$.

Pole masses I

Crucial point of our definition of the $\overline{\mathrm{MS}}$-mass is the gauge invariant construction for the pole in terms of the unrenormalized, bare diagrams. It can be done only after inclusion of the Higgs tadpole contribution.

[J. Fleischer and F. Jegerlehner, Phys. Rev. **D23** (1981) 2001.]

Another important ingredient are the Ward identities.

A. The inclusion of the tadpoles is necessary to ensure, that the physical Higgs field has zero vacuum expectation value in each order of the loop expansion.

B. It is well know, that in order to preserve the Ward identities for the longitudinal part of the gauge boson propagator it is necessary to add the tadpole contribution, which is equal to the propagator of the would-be-Goldstone bosons at zero momentum transfer. In particular, at the two-loop level, the photon would acquire a mass if the tadpole contribution would be omitted.

Pole masses II

By an explicit 2-loop calculation we have shown that (to this order):

1. The position of the complex pole s_p of a the gauge-boson (Z, W) propagator is a gauge invariant quantity after inclusion of the Higgs tadpole contributions.
Stuart '91, Sirlin'91, $\cdots$ Kniehl & Sirlin '98

2. The renormalized on-shell self-energies are infrared finite. This derives from the fact that within dimensional regularization, which allows to regularize UV and IR singularities by the same ε $(\varepsilon = (4-d)/2 \to 0)$ parameter, the singular $1/\varepsilon$ terms are absent after UV renormalization.

Gambino & Grassi '00

3. The inclusion of the tadpoles is important for the renormalization group invariance and for the gauge invariance of the parameter renormalization.

4. By our calculation we have proven that the $\overline{MS}$ renormalization scheme is self consistent and works properly in case of unstable particles.

Pole masses (Properties of MS-masses)

1. The UV counter-terms satisfy relations connecting the higher order poles with the lower order ones:

$$\left(\gamma_a + \sum_j \beta_{g_j} \frac{\partial}{\partial g_j} + \sum_i \gamma_i m_i^2 \frac{\partial}{\partial m_i^2}\right) Z_a^{(n)} = \frac{1}{2} \sum_j g_j \frac{\partial}{\partial g_j} Z_a^{(n+1)} \ ,$$

where we adopt the following definitions for the RG functions: for all dimensionless coupling constants, like $g, g', g_s, e, \lambda, y_t$, the β-function is given by $\mu^2 \frac{\partial}{\partial \mu^2} g = \beta_g$ and for all mass parameters (a mass or the Higgs v.e.v. v) the anomalous dimension γ_{m^2} is given by $\mu^2 \frac{\partial}{\partial \mu^2} \ln m^2 = \gamma_{m^2}$.

2. All tree level relations between masses of any particles and parameters of the unbroken Lagrangian are RG invariant. This means, in particular, that the RG equation for the vacuum expectation value v is given by

$$\gamma_{v^2} \equiv \gamma_{m^2} - \beta_\lambda / \lambda \ ,$$

where m^2 and λ are the parameters of the symmetric scalar potential.

top-Yukawa and Higgs-boson couplings:

The relation between top-Yukawa (Higgs) coupling and Fermi constant G_F is defined in terms of inverse of the renormalized constants (follows from structure of RG equations)

$$\frac{h_{\mathsf{Sirlin}}(\mu^2)}{\sqrt{2}G_F M_H^2} = \frac{m_H^2(\mu^2)}{M_H^2}\frac{G_F(\mu^2)}{G_F} ,$$

$$\frac{y_t^2(\mu^2)}{2\sqrt{2}G_F M_t^2} = \frac{m_t^2(\mu^2)}{M_t^2}\frac{G_F(\mu^2)}{G_F} ,$$

where in r.h.s. all masses and coupling constants are taken in the $\overline{\mathrm{MS}}$-renormalized scheme.

$$\frac{m_t(\mu^2)}{M_t} = 1 + \sigma_\alpha + \sigma_{\alpha_s} + \sigma_{\alpha_s^2} + \sigma_{\alpha_s^3} + \sigma_{\alpha\alpha_s} + \cdots ,$$

$$M_H^2 = m_H^2 + \Delta_{m_H^2,\alpha} + \Delta_{m_H^2,\alpha\alpha_s} ,$$

$$\frac{G_F}{\sqrt{2}} = \frac{G_F(\mu^2)}{\sqrt{2}}(1 + \Delta_{G_F,\alpha}(\mu^2) + \Delta_{G_F,\alpha\alpha_s}(\mu^2) + \cdots) .$$

and

$$\gamma_{G_F} \equiv \mu^2\frac{\partial}{\partial\mu^2}\ln G_F(\mu^2) = \frac{\beta_\lambda}{\lambda} - \gamma_{m^2} = 2\frac{\beta_g}{g} - \gamma_W .$$

top-Yukawa coupling:

The $O(\alpha\alpha_s)$ solution for the top-Yukawa coupling reads

$$\sqrt{\frac{y_t^2(\mu^2)}{2\sqrt{2}G_F M_t^2}} - 1 = (1+\sigma_\alpha+\sigma_{\alpha_s}+\sigma_{\alpha\alpha_s})$$

$$\times \left(1-\Delta_{G_F,\alpha}-\Delta_{G_F,\alpha\alpha_s}-\sum_f \left[m_f^2-M_f^2\right]_{\alpha_s} \frac{\partial}{\partial m_f^2}\Delta_{G_F,\alpha}\right)^{\frac{1}{2}}\Bigg|_{m_j^2=M}$$

$$-1$$

$$= \left(\sigma_\alpha-\frac{1}{2}\Delta_{G_F,\alpha}+\sigma_{\alpha_s}\right)\Bigg|_{m_j^2=M_J^2.e^2=e_{OS}^2}$$

$$+\Bigg(\sigma_{\alpha\alpha_s} - \frac{1}{2}\Delta_{G_F,\alpha\alpha_s} - \frac{1}{2}\sigma_{\alpha_s}\Delta_{G_F,\alpha}$$

$$-\frac{1}{2}\sum_f \left[m_f^2-M_f^2\right]_{\alpha_s} \frac{\partial}{\partial m_f^2}\Delta_{G_F,\alpha}(m_t^2)\Bigg)\Bigg|_{m_j^2=M_J^2.e^2=e_{OS}^2}$$

Higgs coupling:

The $O(\alpha\alpha_s)$ solution for the Higgs coupling is

$$\frac{h_{\text{Sirlin}}(\mu^2)}{\sqrt{2}G_F M_H^2} - 1 = + \left(-\Delta_{G_F,\alpha} - \frac{\Delta_{m_H^2,\alpha}}{M_H^2} \right) \Bigg|_{m_j^2 = M_J^2 . e^2 = e^2_{OS}}$$

$$+ \Bigg(-\Delta_{G_F,\alpha\alpha_s} - \frac{\Delta_{m_H^2,\alpha\alpha_s}}{M_H^2}$$

$$- \left[m_t^2 - M_t^2 \right]_{\alpha_s} \frac{\partial}{\partial m_t^2} \left[\Delta_{G_F,\alpha} + \frac{\Delta_{m_H^2,\alpha}}{M_H^2} \right] \Bigg) \Bigg|_{m_j^2 = M_J^2 . e^2 = e^2_{OS}} ,$$

where

$$\left[m_f^2 - M_f^2 \right]_{\alpha_s} = -2 M_f^2 C_f \frac{g_s^2}{16\pi^2} \left(4 - 3 \ln \frac{M_f^2}{\mu^2} \right) ,$$

and the sum runs over all quarks.

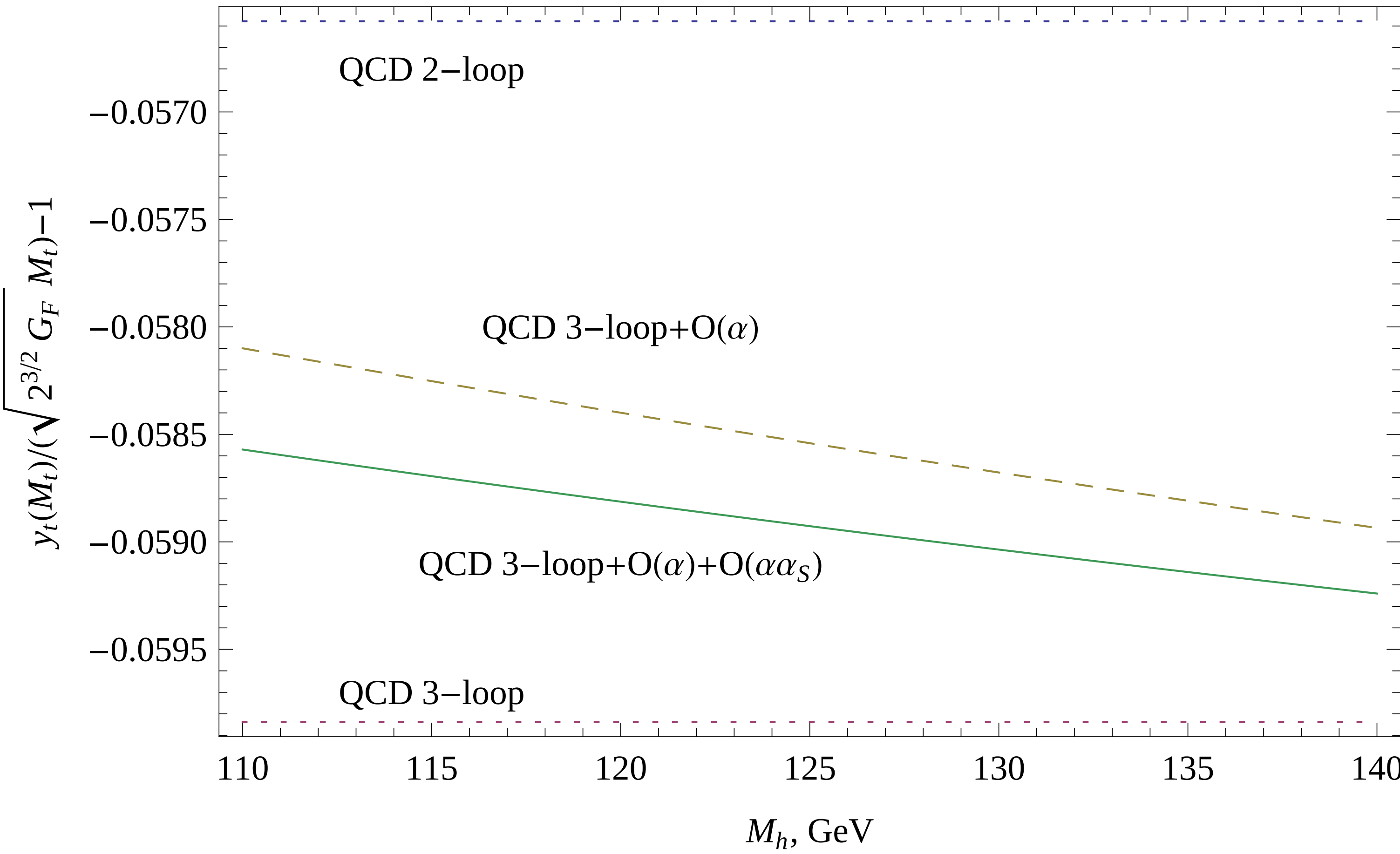

Figure 5: Contribution to the top-Yukawa constant from QCD up to 2 loops, up to 3 loops, QCD and 1 loop EW corrections O(α) and QCD with O(α)+O$(\alpha\alpha_s)$ corrections.

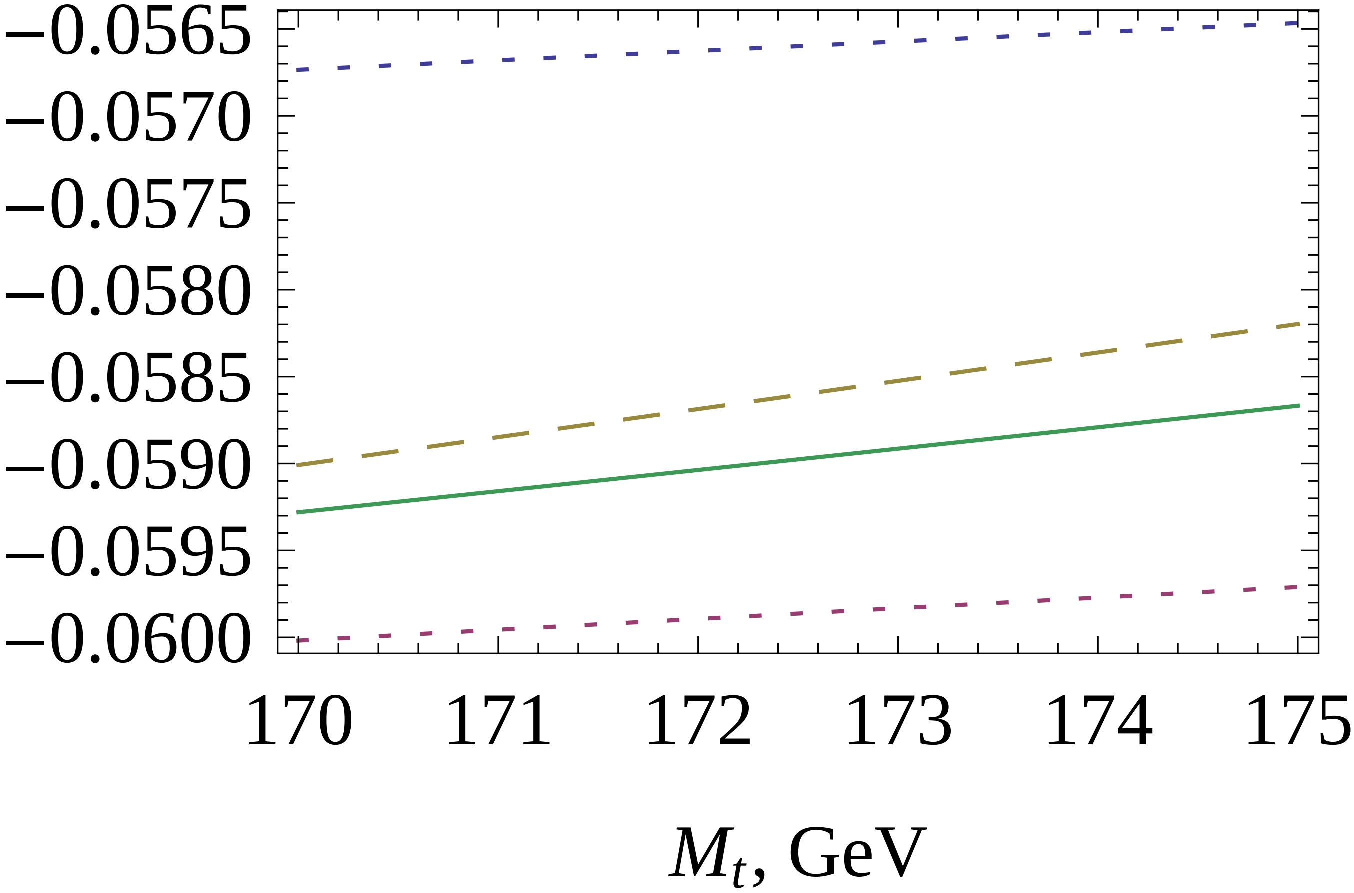

Figure 6: Contribution to the top-Yukawa constant from QCD up to 2 loops, up to 3 loops, QCD and 1 loop EW corrections $O(\alpha)$ and QCD with $O(\alpha)+O(\alpha\alpha_s)$ corrections.

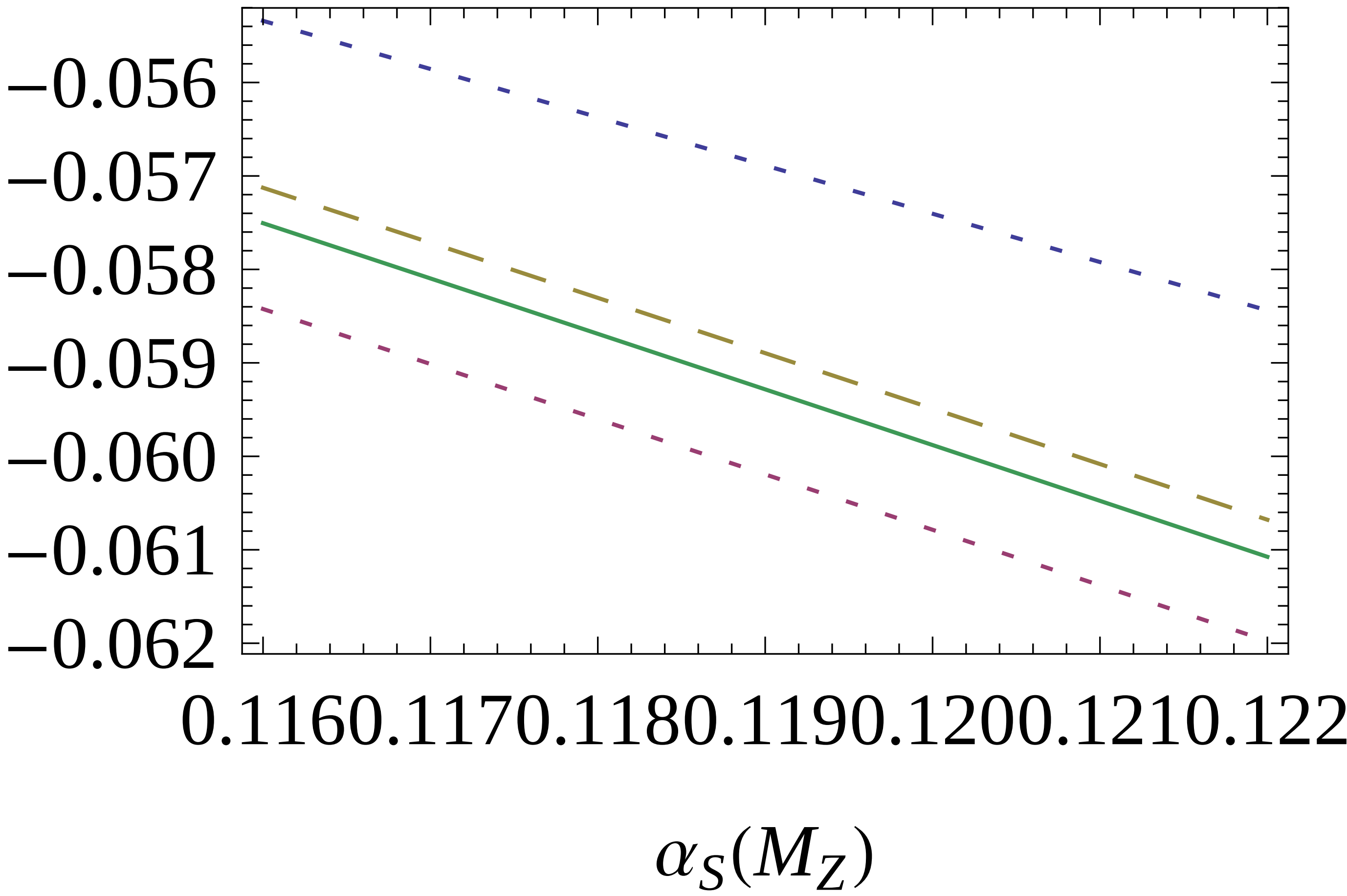

Figure 7: Contribution to the top-Yukawa constant from QCD up to 2 loops, up to 3 loops, QCD and 1 loop EW corrections O(α) and QCD with O(α)+O($\alpha\alpha_s$) corrections.

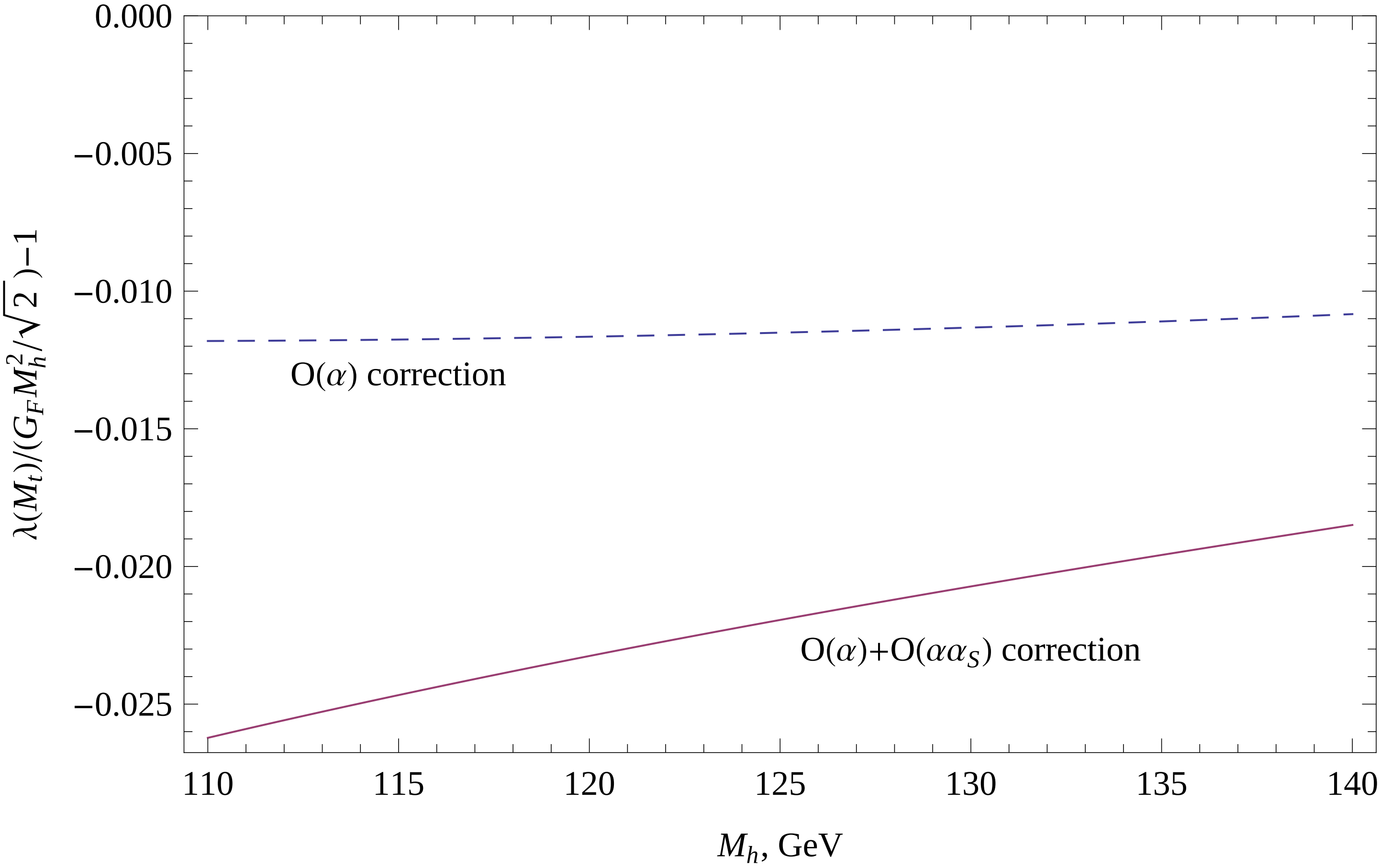

Figure 8: Contribution to the Higgs self-coupling constant of order O(α) and QCD with O(α)+O$(\alpha\alpha_s)$.

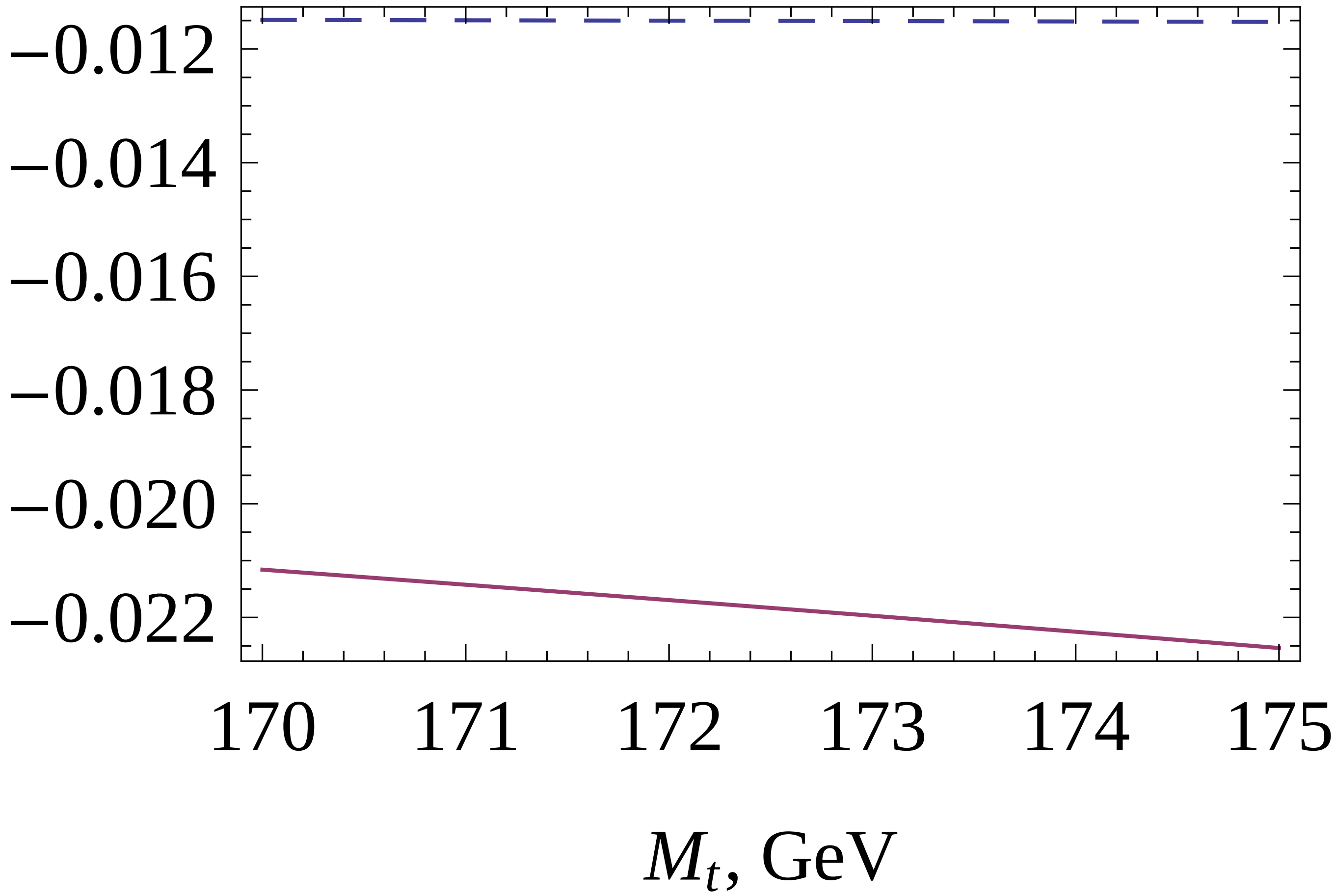

Figure 9: Contribution to the Higgs self-coupling constant of order O(α) and QCD with O(α)+O($\alpha\alpha_s$).

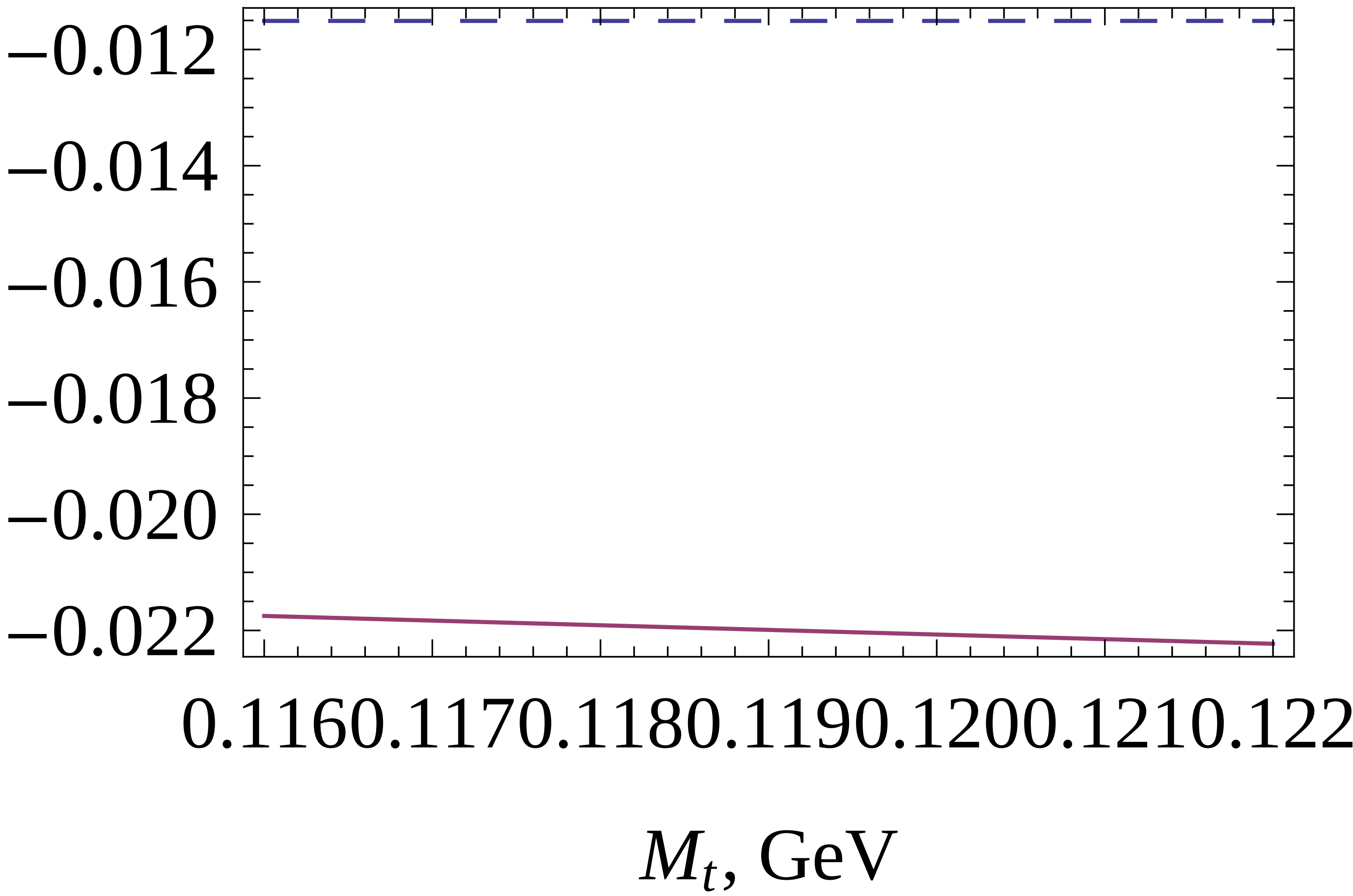

Figure 10: Contribution to the Higgs self-coupling constant of order O(α) and QCD with O(α)+O($\alpha\alpha_s$).

The stability bound [arXiv:1205.2893]:

by Fedor Bezrukov & Mikhail Shaposhnikov analysis

From the conditions

$$\lambda(\mu_0) = 0, \quad \beta_\lambda(\lambda(\mu_0)) = 0,$$

$$M_{\min} = \left[128.95 + \frac{M_t - 172.9\text{GeV}}{1.1\text{GeV}} \times 2.2 - \frac{\alpha_s - 0.1184}{0.0007} \times 0.56\right] \text{GeV}$$

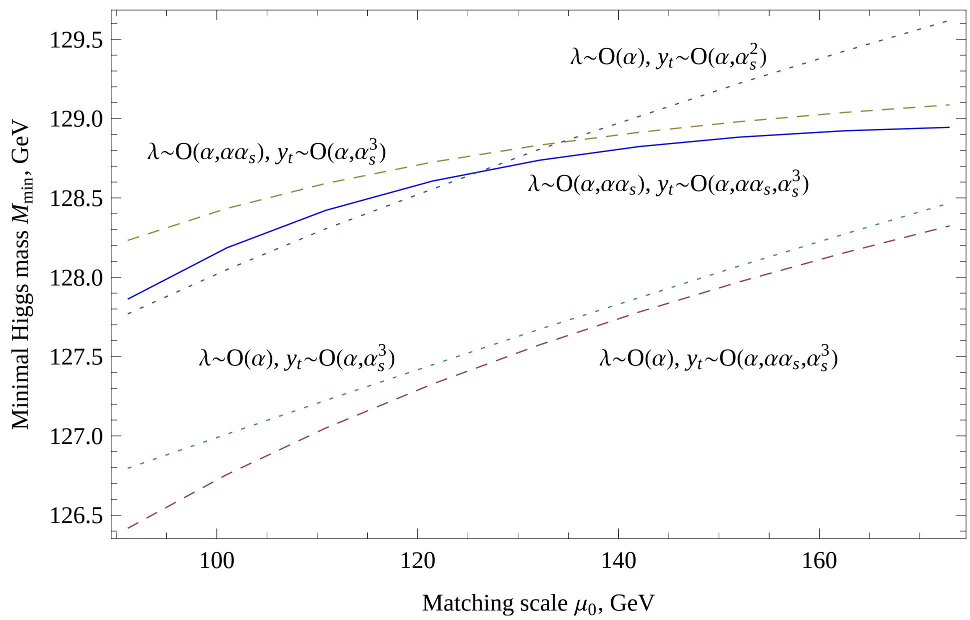

Figure 11: The dependence of the reference Higgs boson mass $M_{\min}$ on the matching scale μ_0 (the $\overline{\text{MS}}$ constants are obtained by matching formulas at scale μ_0 and then used for the solution of the equations (**??**)). The solid line corresponds to the full matching formulas $\lambda \sim O(\alpha, \alpha\alpha_s)$, $y_t \sim O(\alpha_s^3, \alpha, \alpha\alpha_s)$; the dashed and dotted lines correspond to using matching formulas of lower order. Here $M_t = 172.9\text{GeV}$ and $\alpha_s = 0.1184$.

Conclusion

In perfect agreement with a new experimental results:

$$M_H = 125.3 \pm 0.4(\text{stat}) \pm 0.5(\text{syst})\text{Gev}$$

In full agreement with analysis of two another groups:

"Higgs mass and vacuum stability in the Standard Model at NNLO"
Giuseppe Degrassi, Stefano Di Vita, Joan Elias-Miro, Jose R. Espinosa, Gian F. Giudice, Gino Isidori, Alessandro Strumia [arXiv:1205.6497]

"The top quark and Higgs boson masses and the stability of the electroweak vacuum",
S. Alekhin, A. Djouadi, S. Moch. [arXiv:1207.0980]