# Essentials of the Muon $g-2$


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# Introductory Remarks

- The electron and muon anomalous magnetic moments ($a_\ell = (g_\ell - 2)/2$) belong to the most precisely measured quantities in particle physics.
Actual precision: $e$: .24ppb, $\mu$: .54 ppm

- They are pure relativistic quantum correction effects (vanishing at tree level) and hence test the concept of relativistic quantum field theory in general and the Standard Model (SM) of elementary particle physics in particular with highest sensitivity (up to the leading 5-loop effects)

- The high precision is an extraordinary challenge both for theory and experiment

- The last muon $g - 2$ experiment (BNL 2004) has reached a precision at which non-perturbative hadronic effects have to be known with high precision. Hadronic vacuum polarization (HVP) about 11 SD's, hadronic light-by-light scattering (HLBL) about 2 SD's

- Experiments in design/progress will improve the accuracy by a factor 5 which represents a tremendous challenge to theory in the coming years. Most important are improvements in the calculation of the hadronic effects, a particular challenge for lattice QCD. Real progress recently.

Outline of lecture:

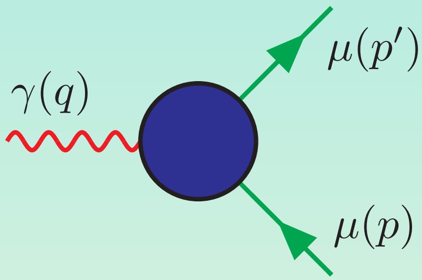
- $g-2$ introduction, history, muon properties, lepton moments
- $g-2$ experimental principles, the Muon $g-2$ experiments
- Standard Model Prediction for $a_\mu$
- Evaluation of $a_\mu^{\rm had}$
- About the hadronic light-by-light scattering contribution
- Theory vs Experiment; do we see New Physics?
- Summary and Outlook

# Muon $g-2$ introduction, history, muon properties, lepton moments

Particle with spin $\vec{s} \Rightarrow$ magnetic moment $\vec{\mu}$ (internal current circulating)

$$\vec{\mu} = g_\mu \frac{e\hbar}{2m_\mu c}\,\vec{s}\;;\quad g_\mu = 2\,(1+a_\mu)$$

Dirac: $g_\mu = 2 \quad , \quad a_\mu = \frac{\alpha}{2\pi} + \cdots$ muon anomaly



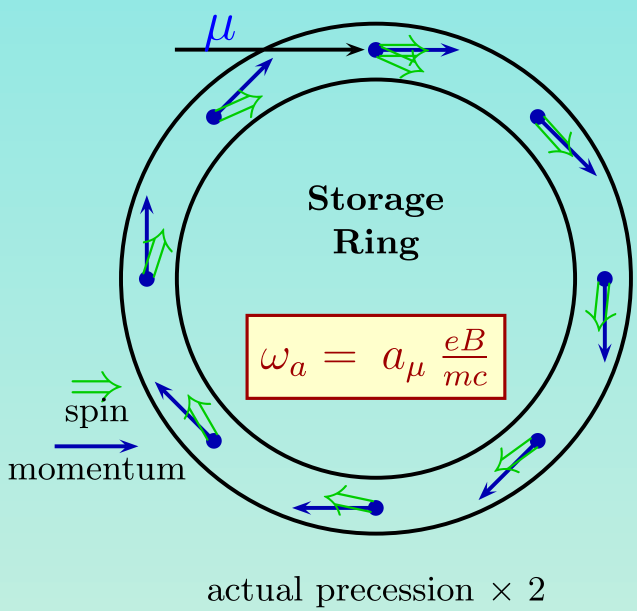

Electromagnetic Lepton Vertex

$$= (-\mathrm{i}e)\,\bar{u}(p')\left[\gamma^\mu F_1(q^2) + \mathrm{i}\,\frac{\sigma^{\mu\nu}q_\nu}{2m_\mu}F_2(q^2)\right]u(p)$$

$$F_1(0) = 1\;;\quad F_2(0) = a_\mu$$

$a_\mu$ responsible for the Larmor precession

Larmor precession $\vec{\omega}$ of beam of spin particles in a homogeneous magnetic field $\vec{B}$




Spin precession in the $g-2$ ring
($\sim 12'$/circle)

Magic Energy: $\vec{\omega}$ is directly proportional to $\vec{B}$ at magic energy ~ 3.1 GeV

$$\vec{\omega}_a = \frac{e}{m}\left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2-1}\right)\vec{\beta}\times\vec{E}\right]_{\text{at "magic } \gamma\text{"}}^{E\sim 3.1\,\text{GeV}} \simeq \frac{e}{m}\left[a_\mu \vec{B}\right]$$

CERN, BNL g-2 experiments

Stern, Gerlach 22: $g_e = 2$; Kusch, Foley 48: $g_e = 2\,(1.00119 \pm 0.00005)$

Basic principle of experiment: measure Larmor precession of highly polarized muons circulating in a ring

$a_\mu = 0$ would mean no rotation of spin relative to muon momentum!

Lorentz factor $\gamma = 1/\sqrt{1-v^2/c^2} = E/mc^2$, $\gamma_{\rm mag} = \sqrt{1+1/a_\mu} \simeq 29.3 \Rightarrow$ muon lifetime $\tau_\mu = 2.19711\,\mu{\rm s}$ at rest $\rightarrow \tau_\mu = 64.435\,\mu{\rm s}$ in motion.

For the measurement of the anomalous magnetic moment we need to look at the

❒ equation of motion of a charged Dirac particle in an external field $A^{\rm ext}_\mu(x)$:

$$\left(\mathrm{i}\hbar\gamma^\mu\partial_\mu + Q_\ell \tfrac{e}{c}\gamma^\mu(A_\mu(x) + A^{\rm ext}_\mu(x)) - m_\ell c\right)\,\psi_\ell(x) = 0$$
$$\left(\Box g^{\mu\nu} - \left(1-\xi^{-1}\right)\partial^\mu\partial^\nu\right)A_\nu(x) = -Q_\ell e\,\bar{\psi}_\ell(x)\gamma^\mu\psi_\ell(x)\;.$$

Neglecting the radiation field (2nd eq.) in a first step: Dirac equation (1st eq.) as a relativistic one–particle problem

$$\mathrm{i}\hbar\frac{\partial\psi}{\partial t} = \boldsymbol{H}\psi\;,\quad \boldsymbol{H} = c\,\vec{\alpha}\left(\vec{p} - \frac{e}{c}\vec{A}\right) + \beta\,mc^2 + e\,\Phi$$

with

$$\beta = \gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \;, \quad \vec{\alpha} \; = \gamma^0 \vec{\gamma} \; = \begin{pmatrix} 0 & \vec{\sigma} \\ \vec{\sigma} & 0 \end{pmatrix} .$$

Interpretation:

1. Non–relativistic limit

Dipole moments (static): orbiting particle with electric charge $e$ and mass $m$ exhibits a magnetic dipole moment

$$\vec{\mu}_L = \frac{e}{2m} \vec{L}$$

where $\vec{L} = m \, \vec{r} \times \vec{v}$ is the orbital angular momentum ($\vec{r}$ position, $\vec{v}$ velocity). An electrical dipole moment can exist due to relative displacements of the centers of positive and negative electrical charge distributions. Magnetic and electric moments contribute to the electromagnetic interaction Hamiltonian with magnetic

$\vec{B}$ and electric $\vec{E}$ fields

$$\mathcal{H} = -\vec{\mu}_m \cdot \vec{B} - \vec{d}_e \cdot \vec{E}$$

where $\vec{\mu}_m$ and $\vec{d}_e$ the magnetic and electric dipole moment operators.

In the absence of an external field spin is a conserved quantity in the rest frame, i.e. the Dirac equation must be equivalent to the Pauli equation via a unitary transformation (Foldy-Wouthuysen):

$$\psi' = \boldsymbol{U}\,\psi\;, \quad \boldsymbol{H}' = \boldsymbol{U}\left(\boldsymbol{H} - \mathrm{i}\hbar\frac{\partial}{\partial t}\right)\boldsymbol{U}^{-1} = \boldsymbol{U}\boldsymbol{H}\boldsymbol{U}^{-1}$$

where the time–independence of $\boldsymbol{U}$ has been used, and we obtain

$$\mathrm{i}\hbar\frac{\partial\psi'}{\partial t} = \boldsymbol{H}'\psi'\;;\;\; \psi' = \begin{pmatrix} \varphi' \\ 0 \end{pmatrix}\,,$$

where $\varphi'$ is the Pauli spinor. In fact $\boldsymbol{U}$ is a Lorentz boost matrix

$$\boldsymbol{U} = \mathbf{1}\,\cosh\theta + \vec{n}\,\vec{\gamma}\,\sinh\theta = \mathrm{e}^{\theta\vec{n}\vec{\gamma}}$$

with

$$\vec{n} = \frac{\vec{p}}{|\vec{p}|}\;,\quad \theta = \frac{1}{2}\mathrm{arccosh}\frac{p^0}{mc} = \mathrm{arcsinh}\frac{|\vec{p}|}{mc}$$

and we obtain, with $p^0 = \sqrt{\vec{p}^2 + m^2c^2}$,

$$\boldsymbol{H}' = cp^0\beta\;;\quad [\boldsymbol{H}', \vec{\Sigma}] = 0\;,\quad \vec{\Sigma} = \vec{\alpha}\,\gamma_5 = \begin{pmatrix} \vec{\sigma} & \mathbf{0} \\ \mathbf{0} & \vec{\sigma} \end{pmatrix}$$

where $\vec{\Sigma}$ is the spin operator. The $v/c$–expansion simply follows by expanding the matrix $\boldsymbol{U}$:

$$\boldsymbol{U}(\vec{p}\,) = \exp\theta\frac{\vec{p}}{|\vec{p}|}\vec{\gamma} = \exp\theta\frac{\vec{p}\vec{\gamma}}{2mc}\;;\quad \theta = \sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1}\left(\frac{\vec{p}^2}{m^2c^2}\right)^n\;.$$

2. Non–relativistic lepton with $A_\mu^{\rm ext} \neq 0$

To get non–relativistic representation for small velocities we have to split off the phase of the Dirac field due to the rest energy of the lepton $\psi = \hat{\psi}\,{\rm e}^{-{\rm i}\,\frac{mc^2}{\hbar}\,t}$. Consequently, the Dirac equation takes the form

$$
{\rm i}\hbar\frac{\partial\hat{\psi}}{\partial t} = \left(\boldsymbol{H} - mc^2\right)\hat{\psi}\;;\quad \hat{\psi} = \left(\begin{array}{c}\hat{\varphi}\\ \hat{\chi}\end{array}\right)\,,
$$

and describes the coupled system of equations

$$
\begin{aligned}
\left({\rm i}\hbar\frac{\partial}{\partial t} - e\,\Phi\right)\hat{\varphi} &= c\,\vec{\sigma}\,(\vec{p} - \frac{e}{c}\vec{A}\,)\,\hat{\chi}\\
\left({\rm i}\hbar\frac{\partial}{\partial t} - e\,\Phi + 2mc^2\right)\hat{\chi} &= c\,\vec{\sigma}\,(\vec{p} - \frac{e}{c}\vec{A}\,)\,\hat{\varphi}\,.
\end{aligned}
$$

For $c \to \infty$ we obtain

$$\hat{\chi} \simeq \frac{1}{2mc}\vec{\sigma}\,(\vec{p} - \frac{e}{c}\vec{A}\,)\,\hat{\varphi} + O(v^2/c^2)$$

and hence

$$\left(\mathrm{i}\hbar\frac{\partial}{\partial t} - e\,\Phi\right)\hat{\varphi} \quad \simeq \quad \frac{1}{2m}\left(\vec{\sigma}\,(\vec{p} - \frac{e}{c}\vec{A}\,)\right)^2\,\hat{\varphi}\;.$$

As $\vec{p}$ does not commute with $\vec{A}$, we may use the relation

$$(\vec{\sigma}\vec{a})(\vec{\sigma}\vec{b}) = \vec{a}\vec{b} + \mathrm{i}\vec{\sigma}\,(\vec{a} \times \vec{b})$$

to obtain

$$\left(\vec{\sigma}\,(\vec{p} - \frac{e}{c}\vec{A}\,)\right)^2 = (\vec{p} - \frac{e}{c}\vec{A}\,)^2 - \frac{e\hbar}{c}\,\vec{\sigma}\cdot\vec{B}\;;\quad \vec{B} = \mathrm{rot}\vec{A}\;.$$

This leads us to the *Pauli equation* (W. Pauli 1927)

$$\mathrm{i}\hbar\frac{\partial\hat{\varphi}}{\partial t} = \hat{\boldsymbol{H}}\,\hat{\varphi} = \left(\frac{1}{2m}(\vec{p} - \frac{e}{c}\vec{A}\,)^2 + e\,\Phi - \frac{e\hbar}{2mc}\,\vec{\sigma}\cdot\vec{B}\right)\hat{\varphi}$$

which up to the spin term is nothing but the non–relativistic Schrödinger equation. The last term is the one this lecture is about: it has the form of a potential energy of a magnetic dipole in an external field. In leading order in $v/c$ the lepton behaves as a particle which has besides a charge also a magnetic moment

$$\vec{\mu} = \frac{e\hbar}{2mc}\,\vec{\sigma} = \frac{e}{mc}\,\vec{s}\,;\quad \vec{s} = \hbar\frac{\vec{\sigma}}{2}$$

with $\vec{s}$ the angular momentum. For comparison: the orbital angular momentum reads

$$\vec{\mu}_{\mathrm{orbital}} = \frac{Q}{2m}\,\vec{L} = g_l\,\frac{Q}{2m}\,\vec{L}\,;\quad \vec{L} = \vec{r}\times\vec{p} = -\mathrm{i}\hbar\,\vec{r}\times\vec{\nabla} = \hbar\vec{l}$$

and thus the total magnetic moment is

$$\vec{\mu}_{\rm total} = \frac{Q}{2m}\left(g_l\,\vec{L} + g_s\,\vec{s}\right) = Q\,\frac{m_e}{m}\,\mu_{\rm B}\,\left(g_l\,\vec{l} + g_s\,\vec{s}\right)$$

where

$$\mu_{\rm B} = \frac{e\hbar}{2m_e c}$$

is Bohr's magneton. As a result for the electron $m = m_e$:

$$g_l = 1 \quad \text{and} \quad \boxed{g_s = 2}\;.$$

The last remarkable result is due to Dirac (1928) and tells us that the gyromagnetic ratio ($\frac{e}{mc}$) is twice as large as the one from the orbital motion.

The Foldy-Wouthuysen transformation for arbitrary $A_\mu$ cannot be performed in closed analytic form. However, the expansion in $v/c$ can be done in a systematic way (see e.g Landau-Lifschitz, Bjorken-Drell) and yields the effective Hamiltonian

$$
\begin{aligned}
\boldsymbol{H'} &= \beta \left( mc^2 + \frac{(\vec{p} - \frac{e}{c}\vec{A}\,)^2}{2m} - \frac{\vec{p}^4}{8m^3c^2} \right) + e\,\Phi - \beta\,\frac{e\hbar}{2mc}\,\vec{\sigma}\cdot\vec{B} \\
&\quad - \frac{e\hbar^2}{8m^2c^2}\,\mathrm{div}\vec{E} - \frac{e\hbar}{4m^2c^2}\,\vec{\sigma}\cdot\left[ (\vec{E}\times\vec{p} + \frac{\mathrm{i}}{2}\mathrm{rot}\vec{E}) \right] + O(v^3/c^3)\;.
\end{aligned}
$$

Origin of additional terms:

- ❖ $\frac{\vec{p}^4}{8m^3c^2}$ leading relativistic correction,
- ❖ $\mathrm{div}\vec{E}$ Darwin term - fluctuations of the electrons position
- ❖ $\vec{\sigma}\cdot\left[ (\vec{E}\times\vec{p} + \frac{\mathrm{i}}{2}\mathrm{rot}\vec{E}) \right]$ spin–orbit interaction

● experimental setup $\mathrm{div}\vec{E} = 0\;;\;\; \mathrm{rot}\vec{E} = 0\;.$

● besides a homogeneous magnetic field an electric quadrupole field is required for focusing the beam

For the magnetic term $\propto \vec{\sigma}$ we then have

$$\boldsymbol{H}_{\text{mag}} = -\vec{\mu}\cdot\left\{\vec{B} + \underbrace{\frac{1}{2}}_{g_l/g_s}\frac{\vec{E}\times\vec{v}}{c^2}\right\}\;;\quad \vec{\mu} = \frac{e\hbar}{2mc}\vec{\sigma} = \frac{e}{m}\vec{s} = \frac{e}{2m}g_s\,\vec{s}$$

- in fact full relativistic kinematics is required (tuning to magic energy)

The correct relativistic formula [$g_2 = 2 \to 2\,(1+a_\mu)$ and appropriate $\gamma$ factors] for the spin precession in transversal fields is

$$\frac{\mathrm{d}\vec{P}}{\mathrm{d}t} = \vec{\omega}_s \times \vec{P}\;;\quad \vec{\omega}_s = -\frac{e}{\gamma m}\left\{(1+\gamma a)\,\vec{B} + \gamma\left(a + \frac{1}{\gamma+1}\right)\frac{\vec{E}\times\vec{v}}{c^2}\right\},$$

where $a = g/2 - 1$ is the anomaly term. While the cyclotron motion

$$\frac{\mathrm{d}\vec{v}}{\mathrm{d}t} = \vec{\omega}_c \times \vec{v}\,,\quad \vec{\omega}_c = -\frac{e}{\gamma m}\left(\vec{B} + \frac{\gamma^2}{\gamma^2-1}\frac{\vec{E}\times\vec{v}}{c^2}\right).$$

The velocity $\vec{v}$ thus rotates, without change of magnitude, with the relativistic cyclotron frequency $\vec{\omega}_c$. The precession of the polarization $\vec{P}$=muon spin $\vec{s}_\mu$ , transversal fields is

$$\vec{\omega}_a = \vec{\omega}_s - \vec{\omega}_c = -\frac{e}{m}\left\{a\,\vec{B} + \left(a - \frac{1}{\gamma^2-1}\right)\frac{\vec{E}\times\vec{v}}{c^2}\right\}\,.$$

This establishes the key formula for measuring $a_\mu$. The motion is simple only for the magic energy $a - \frac{1}{\gamma_{\mathrm{mag}}^2-1} = 0$.

Future:

- Fermilab E969 follow up experiment of BNL E821, traditional, working at magic energy

- New measurements of muon g-2 and EDM with ultra-cold muon beam at J-PARC (works with $\vec{E} = 0$) new concept, vastly different kinematics region (slow muons) providing important cross check

# The role of $a_\mu$ in precision physics

Precision measurement of $a_\mu$ provides most sensitive test of magnetic helicity flip transition

$$\bar{\psi}_L \, \sigma_{\mu\nu} \, F^{\mu\nu} \, \psi_R \qquad (\text{ dim 5 operator })$$

such a term must be absent for any fermion in any renormalizable theory at tree level (no adjustable parameter)

⇓

$a_\mu$ is a pure “quantum correction” effect:
a finite model-specific prediction in any renormalizable quantum field theory (QFT)

- test of quantum structure
- monitor for new physics

Most fascinating aspect highly complex mathematics meets reality !

Note that in higher orders the form factors in general acquire an imaginary part. One may write therefore an effective dipole moment Lagrangian with complex "coupling"

$$\mathcal{L}_{\rm eff}^{\rm DM} = -\frac{1}{2}\left\{\bar{\psi}\,\sigma^{\mu\nu}\left[D_\mu\,\frac{1+\gamma_5}{2} + D_\mu^*\,\frac{1-\gamma_5}{2}\right]\psi\right\}\,F_{\mu\nu}$$

with $\psi$ the muon field and

$$\mathrm{Re}\,D_\mu = a_\mu\,\frac{e}{2m_\mu}\ \ ,\quad \mathrm{Im}\,D_\mu = d_\mu = \frac{\eta_\mu}{2}\frac{e}{2m_\mu}\ .$$

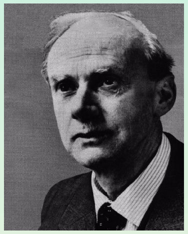
Thus the imaginary part of $F_{\rm M}(0)$ corresponds to an electric dipole moment. The latter is non–vanishing only if we have $T$ violation. Highly suppressed in the SM.

# Some $g-2$ history

It started with atomic spectra in magnetic fields!

The electron:

- 1924 Stern-Gerlach see level splitting due to electron spin,
- 1925 Gouldsmit-Uhlenbeck postulate electron spin $\frac{1}{2}\hbar$ and spin angular momentum implying a magnetic moment $e\hbar/2m_e$ = Bohr magneton,
- 1927 Pauli QM of spin,
- 1928 Dirac relativistic QM Dirac electron, surprisingly $g_e = 2$, twice the value known from orbital angular momentum



- 1934 Kinster & Houston supports strongly $g_e \simeq 2$
- 1936 Anderson & Neddermeyer discovery of the muon in cosmic rays. Rabi: "Who ordered that?"
- 1948 Tomonaga, Schwinger, Feynman renormalization of QED [Nobel Prize 1965] (curing the notorious infinities)

$\Rightarrow$ Feynman rules, Feynman diagrams and all that

- 1947 Nafe et al., Nagle et al. HFS of H and D differ by $2 \times 10^{-3}$ from Fermi Theory; Breit maybe $g \neq 2$.
- 1947 Kusch, Foley atomic precession in a constant magnetic field $\Rightarrow$ first precision determination of the magnetic moment of the electron $g_e = 2 \times [1.00119(5)]$.

Anomaly $a_e = \frac{g_e - 2}{2}$, $a_e \neq 0 \rightarrow$ structure of object!

- 1948 Schwinger unambiguous prediction of a higher order effects, leading (one–loop diagram) contribution to the anomalous magnetic moment $a_\ell^{\rm QED(1)} = \frac{\alpha}{2\pi} \simeq 0.00116$ (which accounts for 99 % of the anomaly), contribution is due to quantum fluctuations via virtual electron photon interactions [universal ($\ell = e, \mu, \tau$)]

  Together with Schwinger's result the first tests of the virtual quantum corrections, predicted by a relativistic quantum field theory [together with (Lamb–shift)].

A triumph which established
QFT is the basic structure of elementary particle theory

- 1987 Dehmelt et al. [U. of Washin.] $a_e^{\rm exp} = 1.159\,652\,1883(42) \times 10^{-3}$ [3.62 ppb]
  Penning Trap

- 2007 Gabrielse et al. [Harvard Univ.] $a_e^{\rm exp} = 1.159\,652\,180\,85(76) \times 10^{-3}$ [.66 ppb]
  Quantum Cyclotron

❖ 2008 Gabrielse et al. [Harvard Univ.] $a_e^{\rm exp} = 1.159\,652\,180\,73(76) \times 10^{-3}$ [.24 ppb]

The muon:

❖ 1956 Berestetskii et al.

$$\delta a_\ell \propto \frac{\alpha}{\pi} \frac{m_\ell^2}{M^2} \qquad (M \gg m_\ell) \,,$$

where $M$ may be

- the mass of a heavier SM particle, or
- the mass of a hypothetical heavy state beyond the SM, or
- an energy scale or an ultraviolet cut–off where the SM ceases to be valid.

$\Rightarrow$ muon much better monitor for heavy physics! enhanced by factor $(m_\mu/m_e)^2 \sim 43000$

## But how to measure $a_\mu$?

- 1957 Lee & Yang parity violation in weak transitions $\Rightarrow$ polarized muons!
- 1957 Garwin, Lederman & Weinrich determined $g_\mu = 2.00$ within 10%

  Friedman & Telegdi point out CP conserved with high accuracy, while P and C are maximally violated
- 1960 Columbia precession experiment $a_\mu = 0.00122(8)$ at a precision of about 5%
- 1961 first CERN cyclotron muon $g - 2$ experiment $\rightarrow$ nothing special was observed within the 0.4% level of accuracy of the experiment $\Rightarrow$ first real evidence the muon is just a heavy electron!
- 1962 1st CERN muon storage ring, $\mu^+$ and $\mu^-$ at the same machine, CPT test!

- 1969 2nd CERN muon storage ring, precision of 7 ppm reached
- 2001 BNL E821 experiment 20 years later
- 2004 BNL $g-2$ experiment closed, precision of 0.54 ppm reached (14–fold improvement)

The begin of E821 in 1984:
G. Danby, J. Field, F. Farley,
E. Picasso, F. Krienen, J. Bailey,
V. Hughes, F. Combley

## Lepton properties:

- most puzzling replica of identical particles
- 3 families required to get CP violation via CKM flavor mixing

❖ Leptons $\ell = e, \mu, \tau$ in SM interact via gauge bosons $\gamma$ electromagnetically and $Z, W$ weakly

❖ Masses: $m_e = 0.511$ MeV, $m_\mu = 105.658$ MeV and $m_\tau = 1776.99$ MeV mass patterns are a big puzzle!

As masses differ by orders of magnitude the leptons show very different behavior, the most striking being the very different lifetimes.

❖ Lifetimes: $\tau_e = \infty$, $\tau_\mu = 2.197 \times 10^{-6}$ sec, $\tau_\tau = 2.906 \times 10^{-13}$ sec

❖ Decays:

- $\mu$ decays very close to 100% in $e\bar{\nu}_e\nu_\mu$
- $\tau$ decays to about 65% into hadronic states $\pi^-\nu_\tau\ ,\quad \pi^-\pi^0\nu_\tau\ ,\quad ,\cdots$
  17.36% $\mu^-\bar{\nu}_\mu\nu_\tau$ and 17.85% $e^-\bar{\nu}_e\nu_\tau$

The **intrinsic** magnetic moment of a particle is proportional to the *spin operator*

$$\vec{L} \to \vec{s} = \frac{\hbar\vec{\sigma}}{2}$$

$\Rightarrow$ defines gyromagnetic ratio $g$ ($g$-factor $\Rightarrow$ Zeeman effect) and its electric pendant $\eta$

$$\vec{\mu}_m = g\,Q\,\mu_0\,\frac{\vec{\sigma}}{2}\quad,\quad \vec{d}_e = \eta\,Q\,\mu_0\,\frac{\vec{\sigma}}{2}$$

$\mu_0 = e\hbar/2m$, $\sigma_i$ ($i = 1,2,3$) are the Pauli spin matrices, $Q$ is the electrical charge in units of $e$, $Q = -1$ for the leptons $Q = +1$ for the antileptons and $m$ the mass.

Anomalous magnetic moment $a_\ell \equiv \frac{g_\ell - 2}{2}$

## Production and Decay of Muons

- Muon $g-2$ experiment requires polarized muons
- Maximum P violating weak decays (no right–handed neutrinos can be produced) allows to do this easily from pion decay
- Pions are produced by shooting protons on a target [at Brookhaven the 24 GeV proton beam extracted from the AGS with $60 \times 10^{12}$ protons per AGS cycle of 2.5 s impinges on a Nickel target of one interaction length]
- Pions are momentum selected in forward direction

Relevant decay chain in muon $g-2$ experiment: $\pi \rightarrow \mu + \nu_\mu$

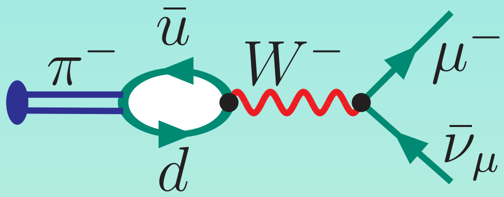
$\hookrightarrow e + \nu_e + \nu_\mu$

producing polarized muons which decay into electrons which carry along their direction of motion the knowledge of the muon's polarization


Illustration B. Touchek

## 1) Pion decay:

The $\pi^-$ is a pseudoscalar bound state $\pi^- = (\bar{u}\gamma_5 d)$ of a $d$ quark and a $u$ antiquark $\bar{u}$. The main decay channel is via the diagram:




π–decay

Two–body decay of the charged spin zero pseudoscalar meson → lepton energy is fixed (monochromatic) $E_\ell = \sqrt{m_\ell^2 + p_\ell^2} = \frac{m_\pi^2 + m_\ell^2}{2m_\pi}\,,\; p_\ell = \frac{m_\pi^2 - m_\ell^2}{2m_\pi}\,.$

Fermi type effective Lagrangian:

$$\mathcal{L}_{\mathrm{eff,int}} = -\frac{G_\mu}{\sqrt{2}} V_{ud} \left(\bar{\mu}\gamma^\alpha \left(1-\gamma_5\right) \nu_\mu\right) \left(\bar{u}\gamma_\alpha \left(1-\gamma_5\right) d\right) + \mathrm{h.c.}$$

$G_\mu$ Fermi constant, $V_{ud} \sim 1$ CKM matrix element

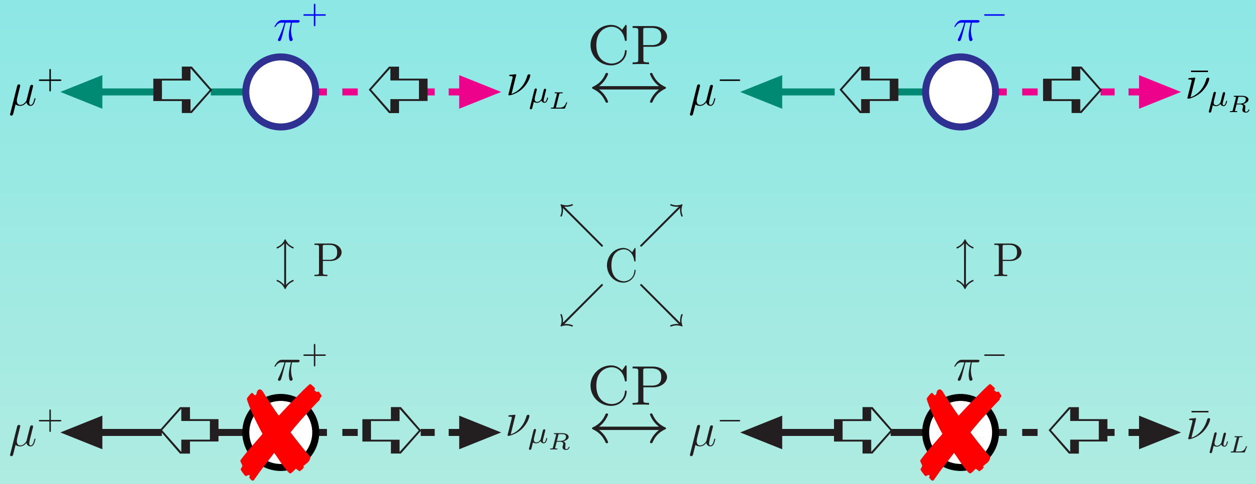
Transition matrix–element:

$$T \quad = \quad {}_{\rm out}< \mu^-, \bar{\nu}_\mu | \pi^- >_{\rm in} = -{\rm i} \frac{G_\mu}{\sqrt{2}} \, V_{ud} \, F_\pi \left( \bar{u}_\mu \gamma^\alpha \, (1 - \gamma_5) \, v_{\nu_\mu} \right) \, p_\alpha$$

hadronic matrix–element $\left\langle 0 \right| \bar{d} \, \gamma_\mu \gamma_5 \, u \, \left| \pi(p) \right\rangle \doteq {\rm i} F_\pi p_\mu$, $F_\pi$ pion decay constant. As $\pi$ pseudoscalar $\rightarrow$ only $A$ of weak charged $V - A$ current couples to the pion.

Pion decay rate [$\delta_{\rm QED}$ = electromagnetic correction]

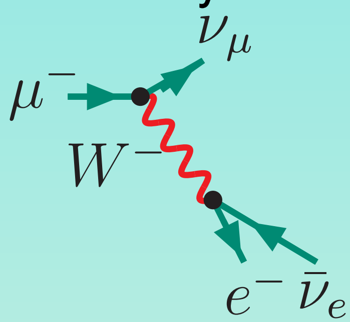
$$\Gamma_{\pi^- \to \mu^- \bar{\nu}_\mu} = \frac{G_\mu^2}{8\pi} \, |V_{ud}|^2 F_\pi^2 \, m_\pi \, m_\mu^2 \, \left( 1 - \frac{m_\mu^2}{m_\pi^2} \right)^2 \times (1 + \delta_{\rm QED}) \ ,$$




Pion decay is a parity violating weak decay where leptons of definite handedness are produced depending on the given charge. CP is conserved while P and C are violated maximally (unique handedness). $\mu^-$ [$\mu^+$] is produced with positive [negative] helicity $h = \vec{s} \cdot \vec{p}/|\vec{p}|$. The existing $\mu^-$ and $\mu^+$ decays are related by a CP transformation. The decays obtained by C or P alone are inexistent in nature.

## 2) Muon decay:

Muon decay $\mu^- \to e^- \bar{\nu}_e \nu_\mu$ is a three body decay



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$\mu$–decay

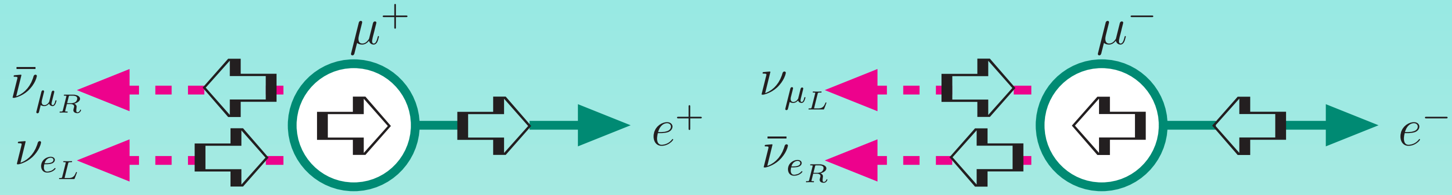
Effective Lagrangian:

$$\mathcal{L}_{\rm eff,int} = -\frac{G_\mu}{\sqrt{2}} \left(\bar{e}\gamma^\alpha \left(1-\gamma_5\right) \nu_e\right) \left(\bar{\nu}_\mu \gamma_\alpha \left(1-\gamma_5\right) \mu\right) + \text{h.c.}$$

and

$$T \quad = \quad {}_{\rm out}< e^-, \bar{\nu}_e \nu_\mu | \mu^- >_{\rm in} = \frac{G_\mu}{\sqrt{2}} \left(\bar{u}_e \gamma^\alpha \left(1-\gamma_5\right) v_{\nu_e}\right) \left(\bar{u}_{\nu_\mu} \gamma_\alpha \left(1-\gamma_5\right) u_\mu\right)$$

$\Rightarrow$ $\mu^-$ and the $e^-$ have both the same left–handed helicity [the corresponding anti–particles are right–handed] in the massless approximation:




In $\mu^-$ [$\mu^+$] decay the produced $e^-$ [$e^+$] has negative [positive] helicity, respectively

The electrons are thus emitted in the direction of the muon spin, i.e. measuring the direction of the electron momentum provides the direction of the muon spin.

After integrating out the two unobservable neutrinos, the differential decay probability to find an $e^\pm$ with reduced energy between $x$ and $x + dx$ emitted at an angle between $\theta$ and $\theta + d\theta$ reads

$$\frac{d^2\Gamma^\pm}{dx\, d\cos\theta} = \frac{G_\mu^2 m_\mu^5}{192\pi^3}\, x^2\, \left(3 - 2x \pm P_\mu\, \cos\theta\, (2x - 1)\right)$$

and typically is strongly peaked at small angles. The reduced $e^\pm$ energy is

$x = E_e/W_{\mu e}$ with $W_{\mu e} = \max E_e = (m_\mu^2 + m_e^2)/2m_\mu$, the $e^\pm$ emission angle $\theta$ is the angle between the $e$ momentum $\vec{p}_e$ and the muon polarization vector $\vec{P}_\mu$. The result above holds in the approximation $x_0 = m_e/W_{e\mu} \sim 9.67 \times 10^{-3} \simeq 0$.

Result: since parity is violated maximally in this weak decay there is a strong correlation between the muon spin direction and the direction of emission of the positrons. The differential decay rate for the muon in the rest frame is given by and

$$d\Gamma/\Gamma = N(E_e) \left(1 + \frac{1 - 2\,x_e}{3 - 2\,x_e}\,\cos\theta\right) d\Omega\;,$$

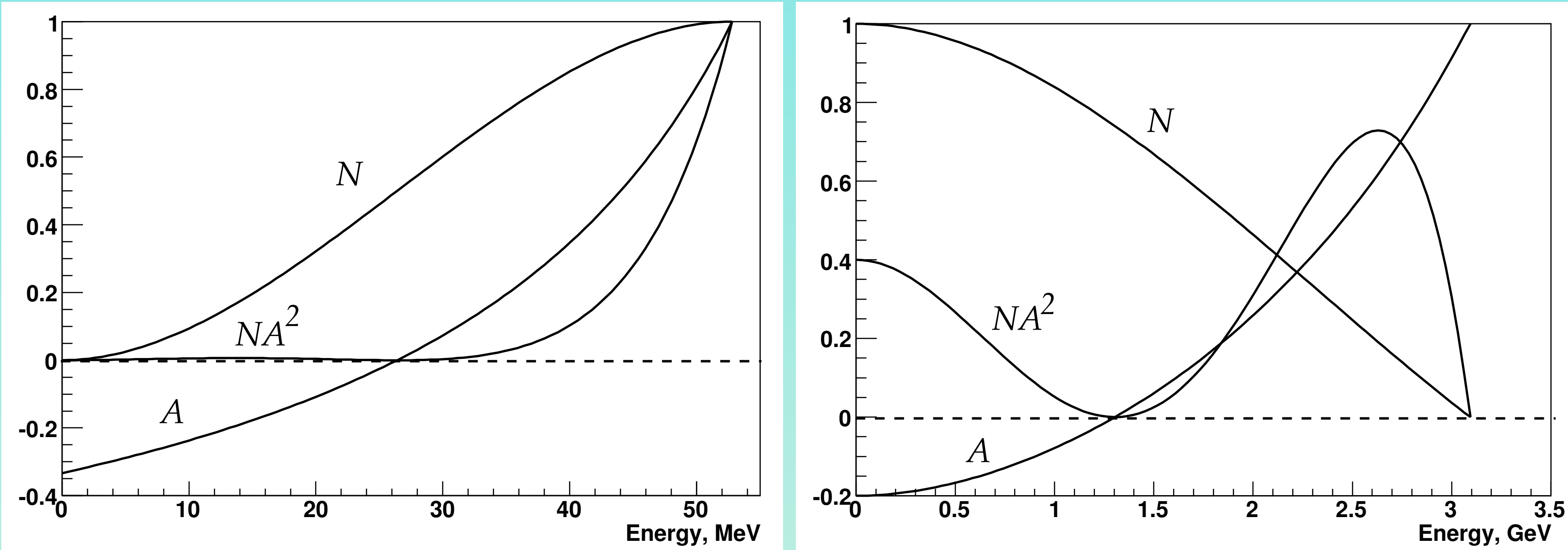
in which $E_e$ is the positron energy, $x_e$ is $E_e$ in units of the maximum energy $m_\mu/2$, $N(E_e)$ is a normalization factor

$$N(E_e) = 2\,x_e^2\,(3 - 2\,x_e)$$

and $\theta$ the angle between the positron momentum in the muon rest frame and the muon spin direction. The $\mu^+$ decay spectrum is peaked strongly for small $\theta$ due to the non–vanishing coefficient of $\cos\theta$

$$A(E_e) \doteq \frac{1-2\,x_e}{3-2\,x_e}\;,$$

which is called asymmetry factor and reflects the *parity violation*




[Muon rest frame (left), laboratory frame (right)]

Number of decay electrons per unit energy, $N$ (arbitrary units), value of the asymmetry $A$, and relative figure of merit $NA^2$ (arbitrary units) as a function of electron energy. The polarization is unity. For the third CERN experiment and E821, $E_{max} \approx 3.1$ GeV ($p_\mu = 3.094$ GeV/c) in the laboratory frame

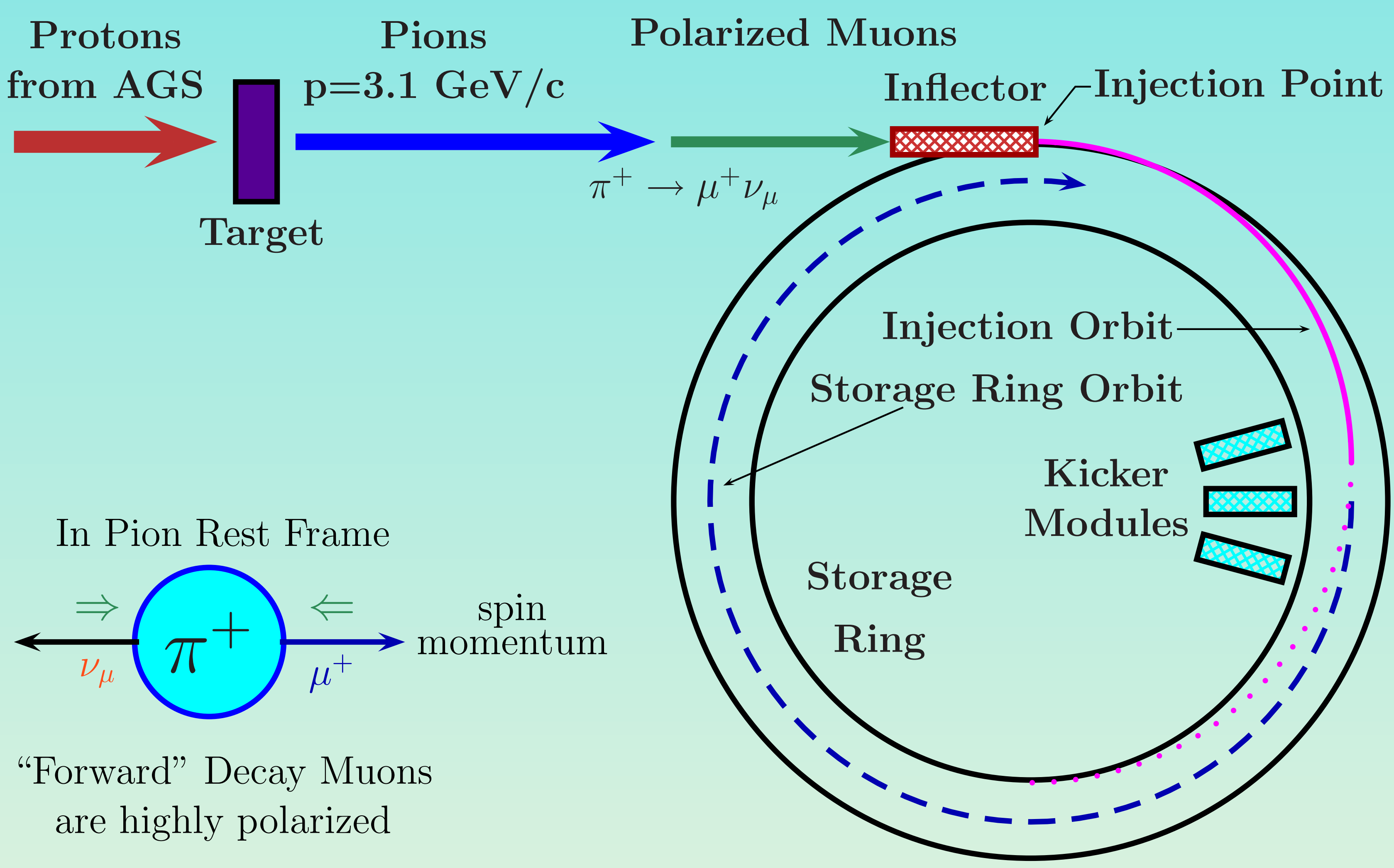
## $g-2$ experimental principles, the Muon $g-2$ experiments

Principle of CERN and BNL muon $g-2$ experiment:

Polarized muons circulating at magic energy in a storage ring

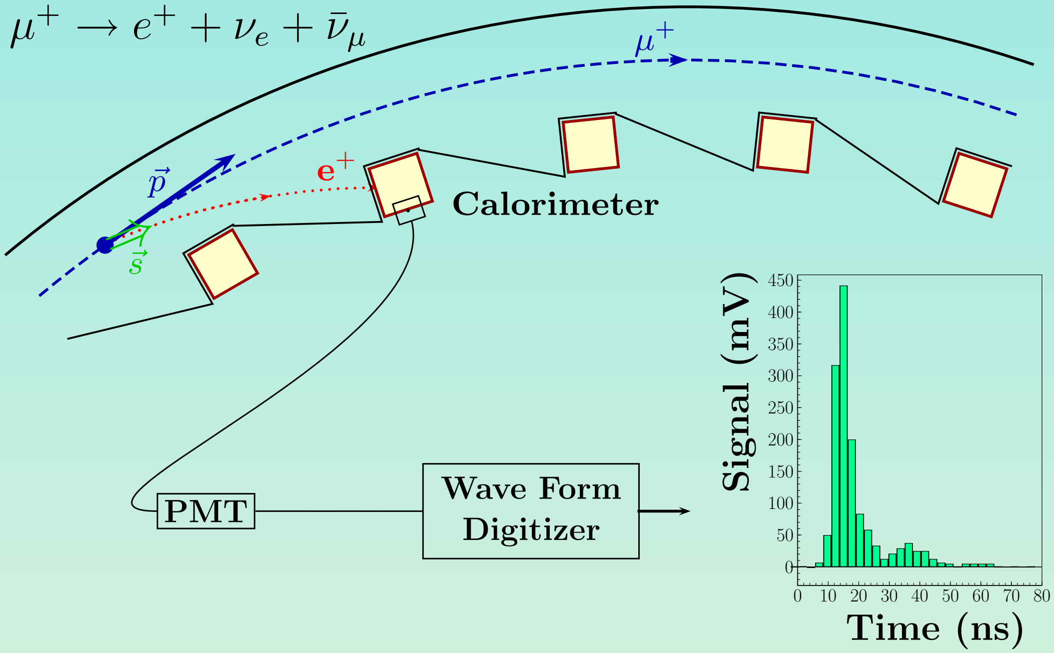
- improvements with E821
  - very high intensity of the primary proton beam from the proton storage ring AGS (Alternating Gradient Synchrotron) $\to$ much higher statistics
  - the injection of muons instead of pions into the storage ring $\to$ much less background
  - a super–ferric storage ring magnet $\to$ improved homogeneous magnetic field


BNL muon storage ring: r= 7.112 meters, aperture of the beam pipe 90 mm, field 1.45 Tesla, momentum of the muon $p_\mu = 3.094$ GeV/c (see `http://www.g-2.bnl.gov/`)




The schematics of muon injection and storage in the $g-2$ ring

Muons are circling in the ring many times before they decay into a positron plus two neutrinos: $\mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu$. Maximal parity violation impies that the positron is emitted along the spin axis of the muon.




Decay of $\mu^+$ and detection of the emitted $e^+$ (PMT=Photomultiplier)

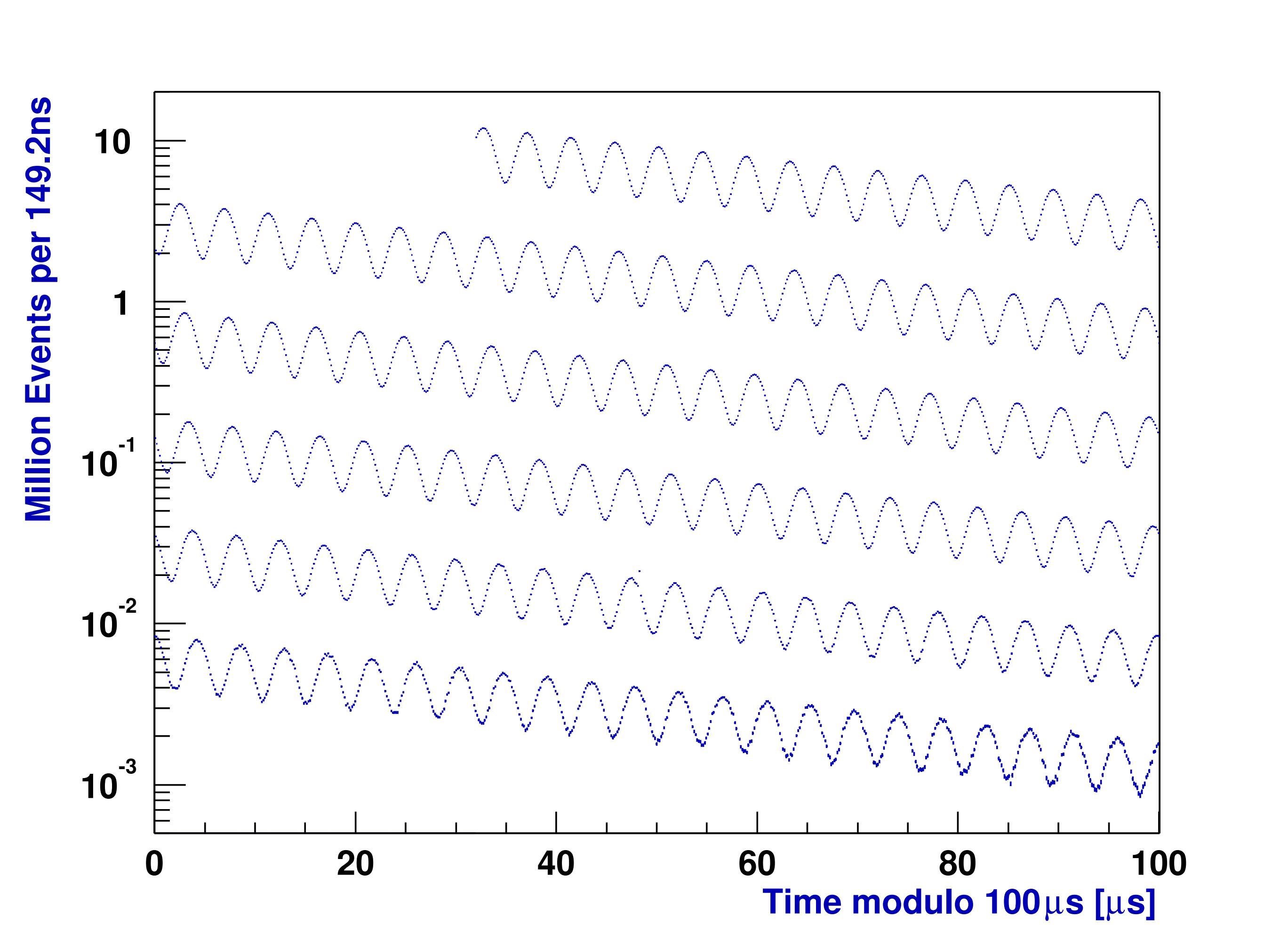
The decay positrons detected by 24 lead/scintillating fiber calorimeters inside the muon storage ring and the measured positron energy provides the direction of the muon spin.

The number of decay positrons with energy greater than $E$ emitted at time $t$ after muons are injected into the storage ring is

$$N(t) = N_0(E)\ \exp\left(-t/\gamma\tau_\mu\right)\ [1 + A(E)\ \sin(\omega_a t + \phi(E))]\ ,$$

– $N_0(E)$ is a normalization factor, – $\tau_\mu$ the muon life time, – $A(E)$ is the asymmetry factor for positrons of energy greater than $E$.

- exponential decay modulated by the $g-2$ angular frequency
- angular frequency $\omega_a$ neatly determined from the time distribution of the decay positrons observed with the electromagnetic calorimeters




Distribution of counts versus time for the 3.6 billion decays in the 2001 negative muon data–taking period

The magnetic field is measured by *Nuclear Magnetic Resonance* (NMR) using a standard probe of $H_2O$. This standard can be related to the magnetic moment of a free proton by

$$B = \frac{\hbar\omega_p}{2\mu_p} \,,$$

where $\omega_p$ is the Larmor spin precession angular velocity of a proton in water. Using this, the frequency $\omega_a$ and $\mu_\mu = (1 + a_\mu)\, e\hbar/(2m_\mu c)$, one obtains

$$a_\mu = \frac{R}{\lambda - R}$$

where

$$R = \omega_a/\omega_p \;\; \text{and} \quad \lambda = \mu_\mu/\mu_p \;.$$

The quantity $\lambda$ appears because the value of the muon mass $m_\mu$ is needed, and also because the $B$ field measurement involves the proton mass $m_p$.

Measurements of the microwave spectrum of ground state muonium ($\mu^+ e^-$) at LAMPF at Los Alamos, in combination with the theoretical prediction of the Muonium hyperfine splitting $\Delta\nu$ (and references therein), have provided the precise value CODATA 2011: [raXiv:1203.5425v1]

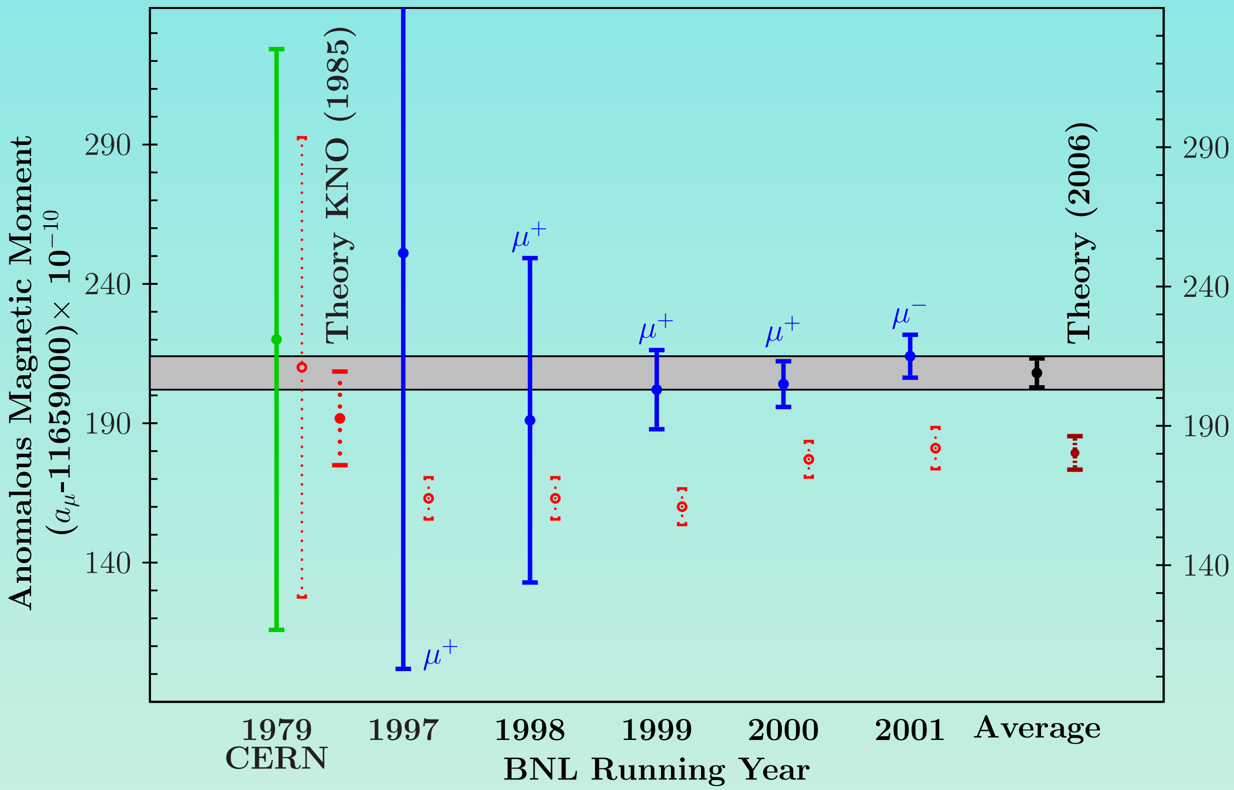
$$\frac{\mu_\mu}{\mu_p} = \lambda = 3.183\,345\,107(84)\ (25\ \mathrm{ppb})\ ,$$

Since the spin precession frequency can be measured very well, the precision at which $g-2$ can be measured is essentially determined by the possibility to manufacture a constant homogeneous magnetic field $\vec{B}$ and to determine its value very precisely.

Final BNL determined $R = 0.0037072063(20)$, which yields new world average value

$$\mathbf{a}_\mu \quad = \quad \mathbf{11659209.1(5.4)(3.3)[6.3] \times 10^{-10}}\ ,$$

with a relative uncertainty of 0.54 ppm.




Results of individual E821 measurements, together with last CERN result and theory values quoted by the experiments

# Standard Model Prediction for $a_\mu$

## What is new?

- new CODATA values for lepton mass ratios $m_\mu/m_e$, $m_\mu/m_\tau$
- spectacular progress by Aoyama, Hayakawa, Kinoshita and Nio on 5–loop QED calculation (as well as improved 4–loop results) a number of leading terms checked analytically by Kataev!
  - $O(\alpha^5)$ electron $g-2$, substantially more precise $\alpha(a_e)$
  - Complete $O(\alpha^5)$ muon $g-2$, settles better the QED part

### QED Contribution

The QED contribution to $a_\mu$ has been computed through 5 loops

Growing coefficients in the $\alpha/\pi$ expansion reflect the presence of large $\ln \frac{m_\mu}{m_e} \simeq 5.3$ terms coming from electron loops. Input:

$a_e^{\rm exp} = 0.001\,159\,652\,180\,73(28)$ Gabrielse et al. 2008

$\alpha^{-1}(a_e) = 137.0359990842(331)(120)(370)(20)[0.37\text{ppb}]$ Gabrielse et al 2007

$\alpha^{-1}(a_e) = 137.0359991657(331)(068)(046)(24)[0.25\text{ppb}]$ Aoyama et al 2012

New: includes the universal 5-loop QED result for the first time!
Errors: from $a_e$ input, $\alpha^4$, $\alpha^5$, hadronic
Used is SM prediction:

$$a_e^{\rm SM} = a_e^{\rm QED} + 1.691(13) \times 10^{-12} \text{ (hadronic \& weak)}\,.$$

dominated by LO hadronic: $a_e^{\rm had} = 1.652(13) \times 10^{-12}$, $a_e^{\rm weak} = 0.039 \times 10^{-12}$

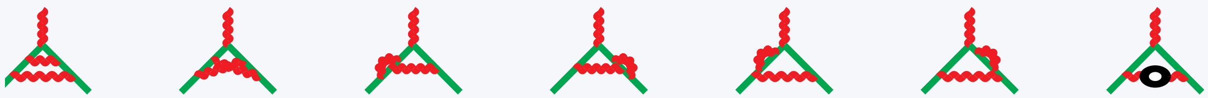
$$a_\mu^{\rm QED} = 116\,584\,718.851\ \underbrace{(0.029)}_{\alpha_{\rm inp}}\ \underbrace{(0.009)}_{m_e/m_\mu}\underbrace{(0.018)}_{\alpha^4}\underbrace{(0.007)}_{\alpha^5}[0.36] \times 10^{-11}$$

The current uncertainty is well below the $\pm 60 \times 10^{-11}$ experimental error from E821

| # n of loops | $C_i$ $[(\alpha/\pi)^n]$ | $a_\mu^{\rm QED} \times 10^{11}$ |
|---|---|---|
| 1 | +0.5 | 116140973.289 (43) |
| 2 | +0.765 857 426(16) | 413217.628 (9) |
| 3 | +24.050 509 88(32) | 30141.9023 (4) |
| 4 | +130.8796(63) | 381.008 (18) |
| 5 | +753.290(1.04) | 5.094 (7) |
| tot | | 116584718.851 (0.036) |

❶ **1 diagram** Schwinger 1948

❷ **7 diagrams** Peterman 1957, Sommerfield 1957



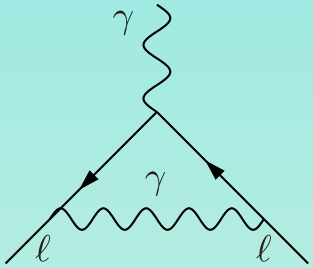
❸ **72 diagrams** Lautrup, Peterman, de Rafael 1974, Laporta, Remiddi 1996

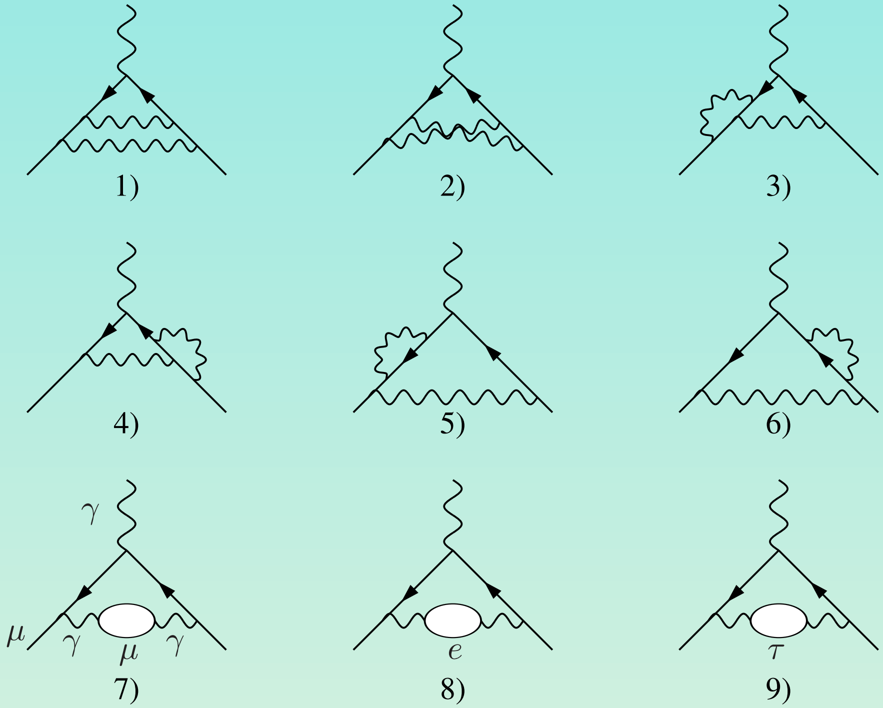
❹ **about 1000 diagrams** Kinoshita 1999, Kinoshita, Nio 2004, Ayoama et al. 2009/2012

❺ **estimates of leading terms** Karshenboim 93, Czarnecki, Marciano 00, Kinoshita, Nio 05

❏ **all 12672 diagrams (fully automated numerical)** Ayoama et al. 2012

## Universal contributions: $a_\mu$ internal muons loops only




$$a^{(2)}_{\ell\,\mathrm{universal}} = \frac{1}{2}\left(\frac{\alpha}{\pi}\right)$$

Schwinger 1948

$$a^{(4)}_{\ell\,\mathrm{universal}}[1-7] = \left[\frac{197}{144} + \frac{\pi^2}{12} - \frac{\pi^2}{2}\ln 2 + \frac{3}{4}\zeta(3)\right]\left(\frac{\alpha}{\pi}\right)^2$$

Peterman 57, Sommerfield 57

compact dispersive calculation by Terentev 1962

# Universal 3–loop contribution:

(Remiddi et al., Remiddi, Laporta 1996 [27 years for 72 diagrams])

Result turned out to be surprisingly compact

$$
\begin{aligned}
a_{\ell\,\text{universal}}^{(6)} = & \Big[\frac{28259}{5184} + \frac{17101}{810}\pi^2 - \frac{298}{9}\pi^2\ln 2 + \frac{139}{18}\zeta(3) \\
& + \frac{100}{3}\left\{\mathrm{Li}_4(\frac{1}{2}) + \frac{1}{24}\ln^4 2 - \frac{1}{24}\pi^2\ln^2 2\right\} \\
& - \frac{239}{2160}\pi^4 + \frac{83}{72}\pi^2\zeta(3) - \frac{215}{24}\zeta(5)\Big]\left(\frac{\alpha}{\pi}\right)^3
\end{aligned}
$$

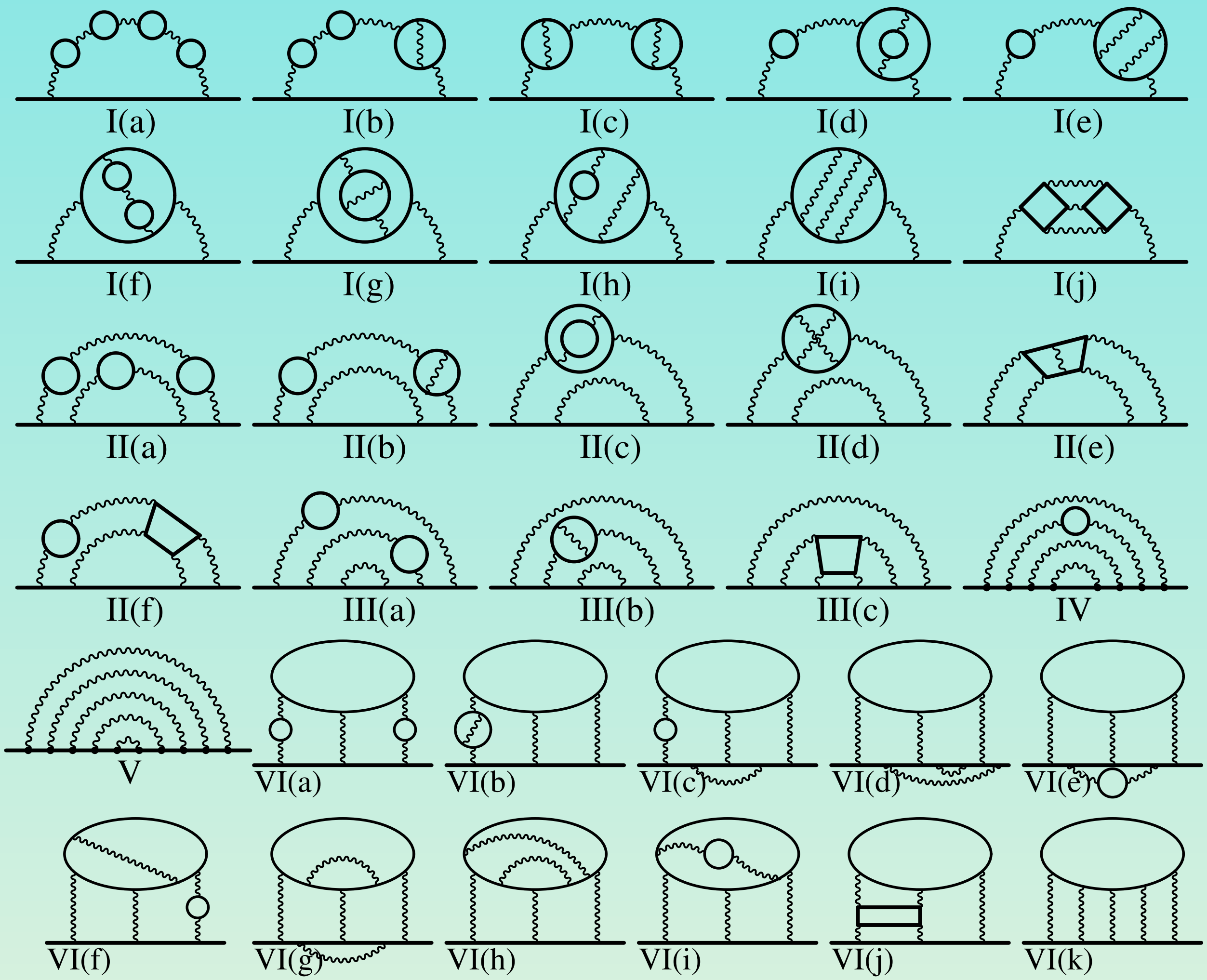
Laporta & Remiddi 96

a monument!

Note on 4–loop contribution: (Kinoshita et al., Aoyama et al. 2007)


4-loop Group V diagrams. 47 self-energy-like diagrams of $M_{01} - M_{47}$ represent 518 vertex diagrams [by inserting the external photon vertex on the virtual muon lines in all possible ways].

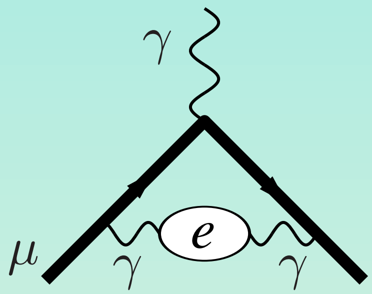
30 years of heroic effort and succesful improvements!.




First complete 5-loop calculation! (Aoyama et al. 2012)

## Mass dependent contributions:

electron and tau loops bringing in mass ratios $me/m_\mu$ and $m_\mu/m_\tau$
– LIGHT internal masses $\Rightarrow$ large logarithms [of mass ratios] singular in the limit $m_{\text{light}} \to 0$




$$a_\mu^{(4)}(\text{vap}, e) = \left[\frac{1}{3}\ln\frac{m_\mu}{m_e} - \frac{25}{36} + O\left(\frac{m_e}{m_\mu}\right)\right]\left(\frac{\alpha}{\pi}\right)^2 .$$

note large log $\ln\frac{m_\mu}{m_e} \simeq 5.3$
exact two–loop result [errors due to uncertainty in mass ratio $(m_e/m_\mu)$]

$$a_\mu^{(4)}(\text{vap}, e) \simeq 1.094\,258\,3111(84)\left(\frac{\alpha}{\pi}\right)^2 = 5.90406007(5)\times 10^{-6} .$$

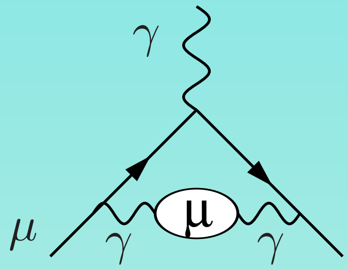
LL UV log; $m_\mu$ serves as UV cut–off, electron mass as IR cut–off, relevant integral

$$\int_{m_e}^{m_\mu} \frac{\mathrm{d}E}{E} = \ln \frac{m_\mu}{m_e}$$

may be obtained by renormalization group replace in one–loop result $\alpha \to \alpha(m_\mu)$

$$a_\mu = \frac{1}{2}\frac{\alpha}{\pi}\,(1 + \frac{2}{3}\frac{\alpha}{\pi}\;\ln\frac{m_\mu}{m_e})$$

– EQUAL internal masses yields pure number

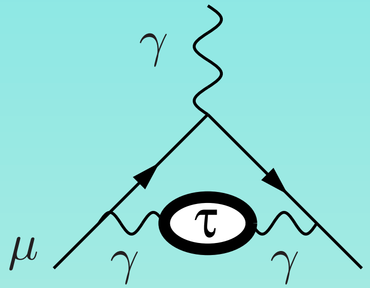



$$a_\mu^{(4)}(\mathrm{vap},\mu) = \left[\frac{119}{36} - \frac{\pi^2}{3}\right] \left(\frac{\alpha}{\pi}\right)^2 ,$$

large cancellation between rational $[3.3055...]$ and transcendental $\pi^2$ term $[3.2899...]$, result 0.5% of individual terms:

$$a_\mu^{(4)}(\mathrm{vap},\mu) \simeq 0.015\,687\,4219 \left(\frac{\alpha}{\pi}\right)^2 = 8.464\,1332 \times 10^{-8} .$$

– HEAVY internal masses decouple in the limit $m_{\mathrm{heavy}} \to \infty$, small power correction



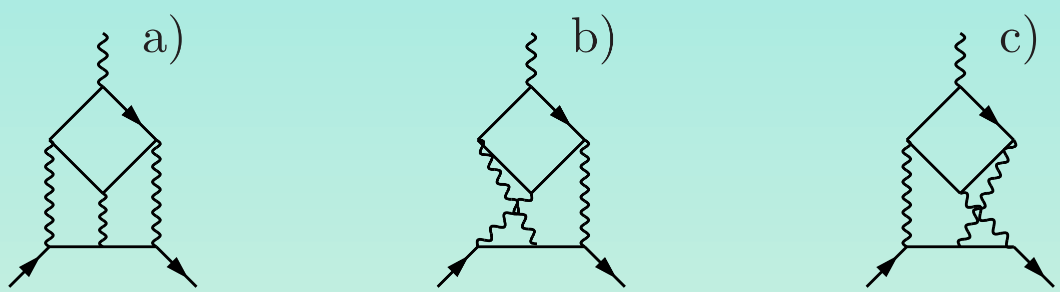

$$a_\mu^{(4)}(\mathrm{vap},\tau) = \left[\frac{1}{45}\left(\frac{m_\mu}{m_\tau}\right)^2 + O\left(\frac{m_\mu^4}{m_\tau^4}\ln\frac{m_\tau}{m_\mu}\right)\right]\left(\frac{\alpha}{\pi}\right)^2 .$$

Note "heavy physics" contributions, from mass scales $M \gg m_\mu$, typically are proportional to $m_\mu^2/M^2$. This means that besides the order in $\alpha$ there is an extra suppression factor, e.g. $O(\alpha^2) \to Q(\alpha^2 \frac{m_\mu^2}{M^2})$ in our case. To unveil new heavy states thus requires a corresponding high precision in theory and experiment. $\tau$ contribution tiny

$$a_\mu^{(4)}(\mathrm{vap},\tau) \simeq 0.000\,078\,064(25)\left(\frac{\alpha}{\pi}\right)^2 = 4.211\,935\,34(87)\times 10^{-10} ,$$

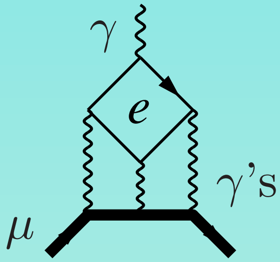
## Light-by-Light scattering contribution to $g-2$

6 diagrams related by permutation of photon lines attached to muon:




Again, different regimes:

– LIGHT internal masses also in this case give rise to potentially large logarithms of mass ratios which get singular in the limit $m_{\text{light}} \to 0$

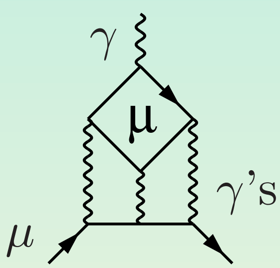



$$a_\mu^{(6)}(\mathrm{lbl}, e) = \left[\frac{2}{3}\pi^2 \ln\frac{m_\mu}{m_e} + \frac{59}{270}\pi^4 - 3\,\zeta(3) - \frac{10}{3}\pi^2 + \frac{2}{3} + O\left(\frac{m_e}{m_\mu}\ln\frac{m_\mu}{m_e}\right)\right]\left(\frac{\alpha}{\pi}\right)^3 .$$

Again a light loop which yields a unexpectedly large contribution

$$a_\mu^{(6)}(\mathrm{lbl}, e) \simeq 20.947\,924\,89(16)\left(\frac{\alpha}{\pi}\right)^3 = 2.625\,351\,02(2)\times 10^{-7} .$$

– EQUAL internal masses case which yields a pure number which is usually included in the $a_\ell^{(6)}$ universal part:



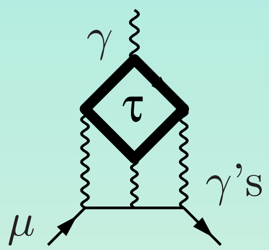

$$a_\mu^{(6)}(\mathrm{lbl}, \mu) = \left[\frac{5}{6}\,\zeta(5) - \frac{5}{18}\,\pi^2\,\zeta(3) - \frac{41}{540}\pi^4 - \frac{2}{3}\pi^2\ln^2 2 + \frac{2}{3}\ln^4 2 + 16 a_4 - \frac{4}{3}\,\zeta(3) - 24\pi^2\ln 2 + \frac{931}{54}\pi^2 + \frac{5}{9}\right]\left(\frac{\alpha}{\pi}\right)^3 ,$$

where $a_4$ is a known constant. The single scale QED contribution is much smaller

$$a_\mu^{(6)}(\mathrm{lbl},\mu) \simeq 0.371005293 \left(\frac{\alpha}{\pi}\right)^3 = 4.64971651 \times 10^{-9}$$

but is still a substantial contributions at the required level of accuracy.

– HEAVY internal masses again decouple in the limit $m_{\mathrm{heavy}} \to \infty$ and thus only yield small power correction

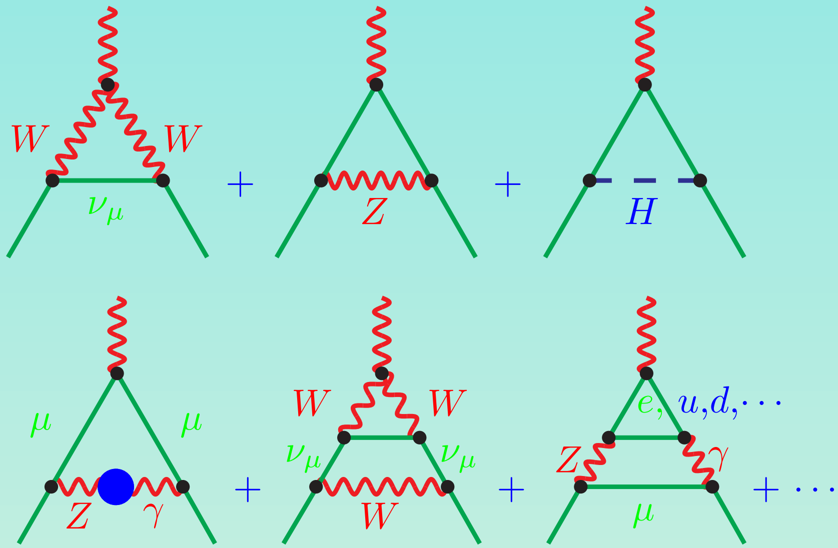



$$a_\mu^{(6)}(\mathrm{lbl},\tau) = \left[\left[\frac{3}{2}\,\zeta(3) - \frac{19}{16}\right]\left(\frac{m_\mu}{m_\tau}\right)^2 + O\left(\frac{m_\mu^4}{m_\tau^4}\ln^2\frac{m_\tau}{m_\mu}\right)\right]\left(\frac{\alpha}{\pi}\right)^3 .$$

As expected this heavy contribution is power suppressed yielding

$$a_\mu^{(6)}(\mathrm{lbl},\tau) \simeq 0.002\,142\,90(69) \left(\frac{\alpha}{\pi}\right)^3 = 2.685\,65(86) \times 10^{-11} .$$

# Weak contributions




Brodsky, Sullivan 67, ...,
Bardeen, Gastmans, Lautrup 72
Higgs contribution tiny!

$a_\mu^{\mathrm{weak}(1)} = (194.82 \pm 0.02) \times 10^{-11}$

Kukhto et al 92
potentially large terms $\sim G_F m_\mu^2 \frac{\alpha}{\pi} \ln \frac{M_Z}{m_\mu}$
Peris, Perrottet, de Rafael 95
quark-lepton (triangle anomaly) cancellation
Czarnecki, Krause, Marciano 96

Heinemeyer, Stöckinger, Weiglein 04, Gribouk, Czarnecki 05 full 2–loop result

Most recent evaluations: improved hadronic part (beyond QPM)

$$a_\mu^{\mathrm{weak}} = (153.2 \pm 1.0[\mathrm{had}] \pm 1.5[\mathrm{m_H, m_t}, 3 - \mathrm{loop}]) \times 10^{-11}$$

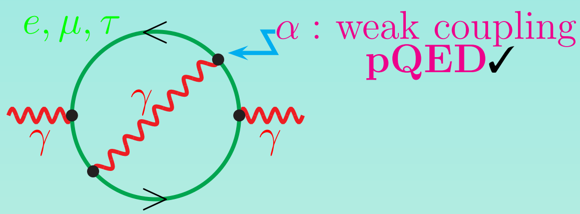
(Knecht, Peris, Perrottet, de Rafael 02, Czarnecki, Marciano, Vainshtein 02)

# ❒ Hadronic contributions

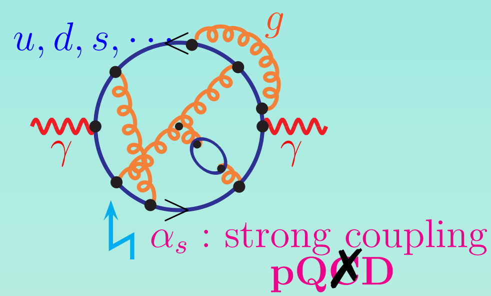
General problem in electroweak precision physics:

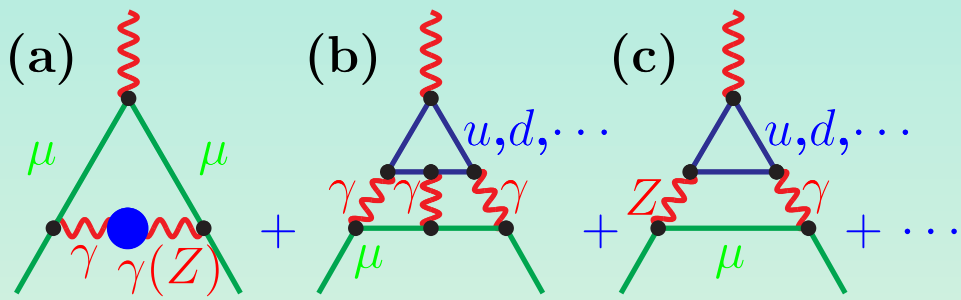
contributions from hadrons (quark loops) at low energy scales

Leptons




Quarks




(a) Hadronic vacuum polarization $O(\alpha^2), O(\alpha^3)$

(b) Hadronic light-by-light scattering $O(\alpha^3)$

(c) Hadronic effects in 2-loop EWRC $O(\alpha G_F m_\mu^2)$

Light quark loops

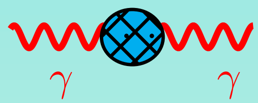
↓

Hadronic “blobs”

## Hadronic vacuum polarization effects in $g-2$ [quark loops]

Role of hadronic two point correlator (non-perturbative):

- key object $\langle 0 | T\, j^{\mu\,\mathrm{had}}_{\mathrm{em}}(x)\; j^{\nu\,\mathrm{had}}_{\mathrm{em}}(0) | 0 \rangle$
- hadronic electromagnetic current




$$j^{\mu\,\mathrm{had}}_{\mathrm{em}} = \sum_c \left( \frac{2}{3} \bar{u}_c \gamma^\mu u_c - \frac{1}{3} \bar{d}_c \gamma^\mu d_c - \frac{1}{3} \bar{s}_c \gamma^\mu s_c + \frac{2}{3} \bar{c}_c \gamma^\mu c_c - \frac{1}{3} \bar{b}_c \gamma^\mu b_c + \frac{2}{3} \bar{t}_c \gamma^\mu t_c \right) \;,$$

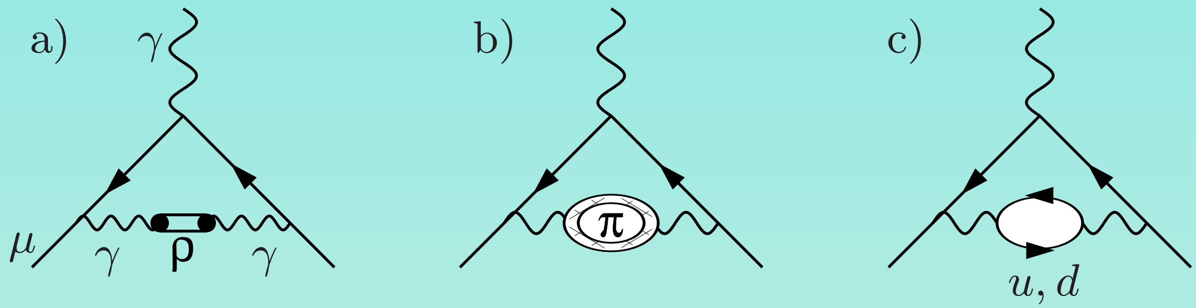
- hadronic part on photon self-energy $\Pi'^{\,\mathrm{had}}_{\gamma}(s) \Leftrightarrow \langle 0 | j^{\mu\,\mathrm{had}}_{\mathrm{em}}(x)\; j^{\nu\,\mathrm{had}}_{\mathrm{em}}(0) | 0 \rangle$

- hadronic vacuum polarization due to the 5 “light” quarks $q = u, d, s, c, b$

- top quark [mass $m_t \simeq 173$ GeV] pQCD applies [$\alpha_s(m_t)$ small]

- in fact $t$ is irrelevant by decoupling theorem [heavy particles decouple in QED/QCD], $t$ like $\tau$ VP loop extra factor $N_c Q_t^2 = 4/3$:

Low energy effective theory: e.g. CHPT here equivalently scalar QED of pions




Low energy effective graphs a) [$\rho$-exchange] and b) [$\pi$-loop] and high energy graph c) [quark-loops]

Low energy effective estimates of the leading VP effects $a_\mu^{(4)}(\mathrm{vap}) \times 10^8$
For comparison: $5.8420$ for $\mu$–loop, $590.41$ for $e$–loop

| data [280,810] MeV | $\rho^0$-exchange | $\pi^\pm$-loop | $(u,d)$-loops |
|---|---|---|---|
| 4.2666 | 4.2099 | 1.4154 | 2.2511[449.25]* |

* current quarks: $m_u \sim 3$MeV, $m_d \sim 8$MeV

Often resorting to QPM using effective “constituent quark masses” [concept not well-defined] e.g. $m_u \sim m_d \sim 300$ MeV (about 1/3 of the proton mass) one gets $2.2511 \times 10^{-8}$ (ambiguous)

Quark and pion loops fail: missing is the pronounced $\rho^0$ spin 1 resonance $e^+e^- \to \rho^0 \to \pi^+\pi^-$ almost saturates the result based on dispersion relation and $e^+e^-$-data.

Lesson:

- pQCD fails; QPM result arbitrary (quark masses)
- ChPT (only knows pions) fails; reason only converge for $p \lesssim 400$ MeV
- dominating is spin 1 resonance $\rho^0$ at $\simeq 775$ MeV (VDM); cries for large $N_c$ QCD
- lattice QCD now on the way to solve the problem once one can simulate at physical quark masses
- resort on sum rule type semi-phenomenological approach Dispersion Relations (DR) and experimental data.

# Dispersion relations and VP insertions in $g-2$

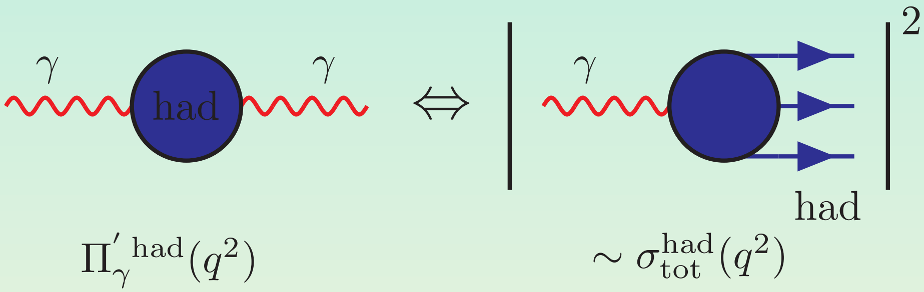
Starting point:

❒ *Optical Theorem* (unitarity) for the photon propagator

$$\mathrm{Im}\Pi'_\gamma(s) = \frac{s}{4\pi\alpha}\,\sigma_{\mathrm{tot}}(e^+e^- \to \mathrm{anything})$$

❒ *Analyticity* (causality), may be expressed in form of a so–called (subtracted) dispersion relation

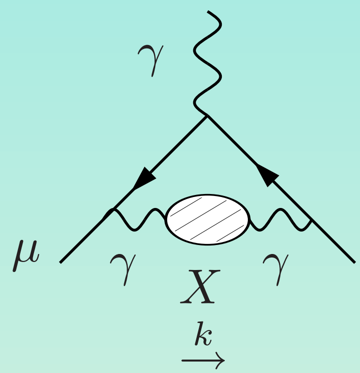
$$\Pi'_\gamma(k^2) - \Pi'_\gamma(0) = \frac{k^2}{\pi}\int_0^\infty \mathrm{d}s \frac{\mathrm{Im}\Pi'_\gamma(s)}{s\,(s-k^2-\mathrm{i}\varepsilon)} \quad .$$



– based on general principles

– holds beyond perturbation theory

Use of DRs in g-2 calculations, prototype example: diagram of the type




"blob" = full photon propagator $g^{\mu\nu}$ term of the full photon propagator, carrying loop momentum $k$, reads

$$\frac{-\mathrm{i}g^{\mu\nu}}{k^2\,(1+\Pi'_\gamma(k^2))} \simeq \frac{-\mathrm{i}g^{\mu\nu}}{k^2}\left(1-\Pi'_\gamma(k^2)+\left(\Pi'_\gamma(k^2)\right)^2-\cdots\right)$$

and the renormalized photon self–energy may be written as

$$-\frac{\Pi'_{\gamma\,\mathrm{ren}}(k^2)}{k^2} = \int_0^\infty \frac{\mathrm{d}s}{s}\frac{1}{\pi}\mathrm{Im}\,\Pi'_\gamma(s)\,\frac{1}{k^2-s}\;.$$

– $k$ dependence under the convolution integral shows up in free propagator only

– free photon propagator in next higher order is replace by

$$-\mathrm{i}g_{\mu\nu}/k^2 \to -\mathrm{i}g_{\mu\nu}/(k^2-s)$$

= exchange of a "massive photon" of mass square $s$.

– afterwards convoluted with imaginary part of the photon vacuum polarization

– calculate the contributions from the massive photon analytically

– this is possible to 3 loops in QED

The leading order result is

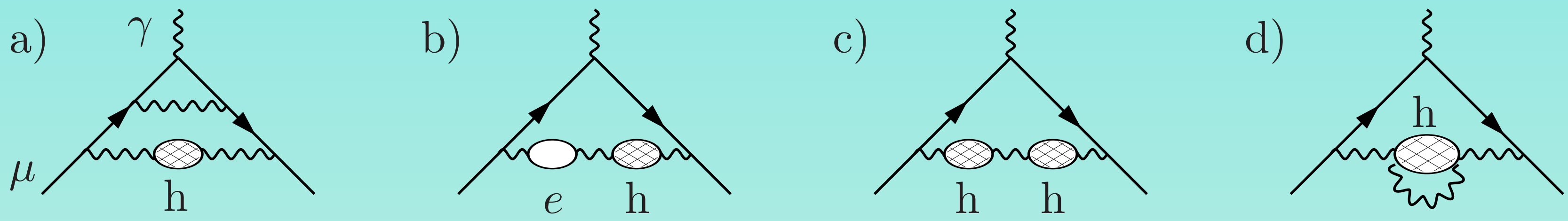
$$K_\mu^{(2)}(s) \equiv a_\mu^{(2)\,\mathrm{heavy}\,\gamma} = \frac{\alpha}{\pi} \int\limits_0^1 \mathrm{d}x\, \frac{x^2\,(1-x)}{x^2 + (s/m_\mu^2)(1-x)}$$

second order contribution to $a_\mu$ from an exchange of a photon with square mass $s$ ($s = 0$ Schwinger result).

The contribution from the "blob" to $g-2$ then reads

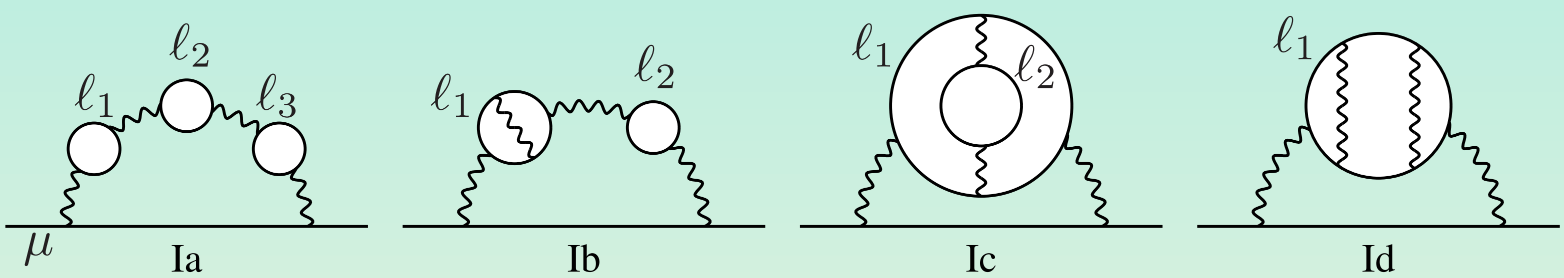
$$a_\mu^{(X)} = \frac{1}{\pi} \int\limits_0^\infty \frac{\mathrm{d}s}{s}\, \mathrm{Im}\, \Pi_\gamma^{\prime(X)}(s)\, K_\mu^{(2)}(s)\ .$$

“Trick” applies to **higher order hadronic VP contributions**




Kinoshita, Nizic, Okamoto 1985, Krause 1996, ...

as well as to **analytic calculations of higher order diagrams** like

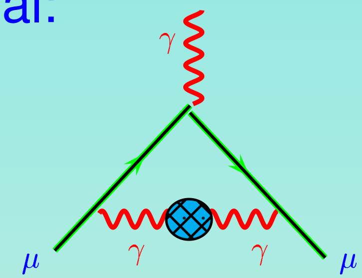



3–loop: Hoang et al 95, 4–loop: Broadhurst, Kataev, Tarasov 93, Kinoshita et al

# Evaluation of $a_\mu^{\rm had}$

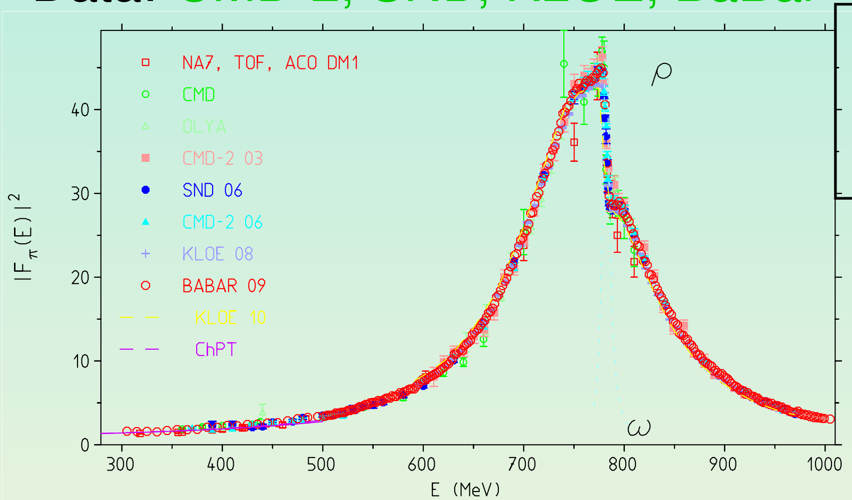
Leading non-perturbative hadronic contributions $a_\mu^{\rm had}$ can be obtained in terms of $R_\gamma(s) \equiv \sigma^{(0)}(e^+e^- \to \gamma^* \to \text{hadrons})/\frac{4\pi\alpha^2}{3s}$ data via dispersion integral:

$$a_\mu^{\rm had} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \left( \int\limits_{4m_\pi^2}^{E_{\rm cut}^2} ds \frac{R_\gamma^{\rm data}(s)\ \hat{K}(s)}{s^2} + \int\limits_{E_{\rm cut}^2}^{\infty} ds \frac{R_\gamma^{\rm pQCD}(s)\ \hat{K}(s)}{s^2} \right)$$



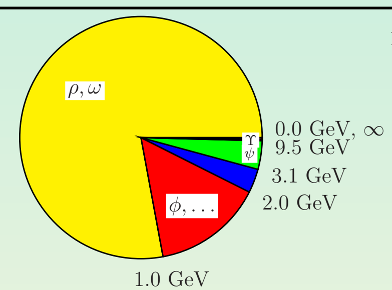

- Experimental error implies theoretical uncertainty!
- Low energy contributions enhanced: $\sim 75\%$ come from region $4m_\pi^2 < m_{\pi\pi}^2 < M_\Phi^2$

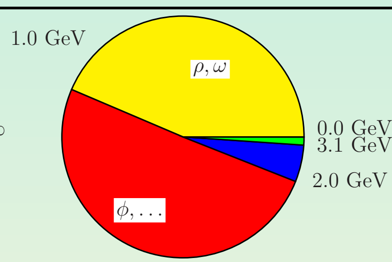
Data: CMD-2, SND, KLOE, BaBar



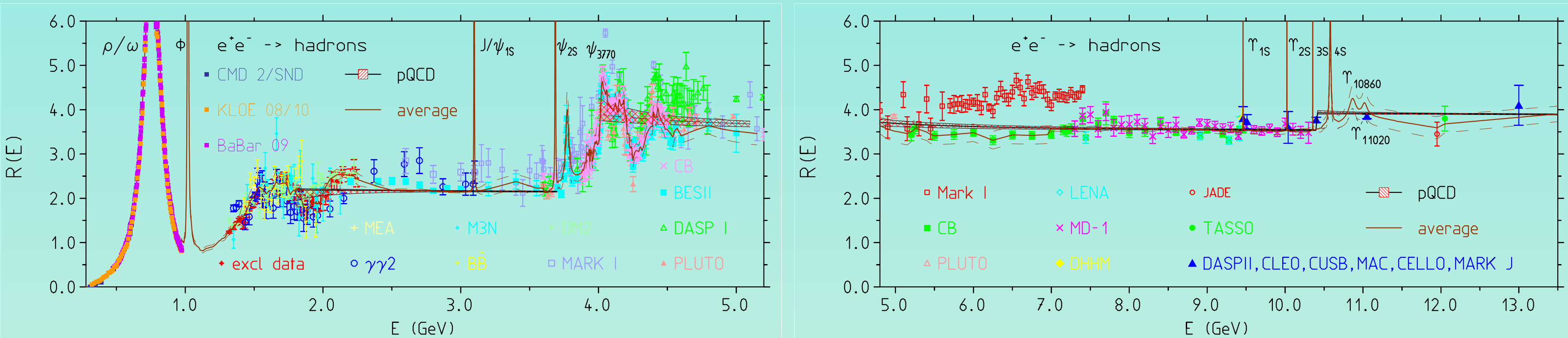

$$a_\mu^{\rm had(1)} = (690.7 \pm 4.7)[695.5 \pm 4.1]\ 10^{-10}$$

$e^+e^-$–data based [incl. BaBar MD09]



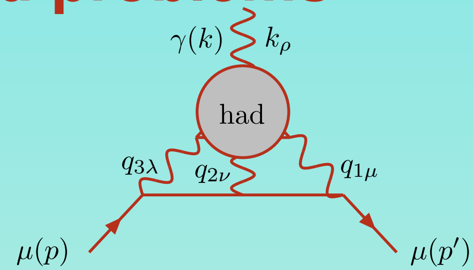
The dominating low energy tail is given by the channel $e^+e^- \to \pi^+\pi^-$ which forms the $\rho$–resonance. The $\rho - \omega$ mixing caused by isospin breaking $(m_u - m_d \neq 0)$ is distorting the ideal Breit-Wigner resonance shape of the $\rho$




Experimental results for $R_\gamma^{\text{had}}(s)$ in the range 1 GeV $< E = \sqrt{s} <$ 13 GeV, obtained at the $e^+e^-$ storage rings. The perturbative quark–antiquark pair–production cross–section is also displayed (pQCD). Parameters: $\alpha_s(M_Z) = 0.118 \pm 0.003$, $M_c = 1.6 \pm 0.15$ GeV, $M_b = 4.75 \pm 0.2$ GeV and $\mu \in (\frac{\sqrt{s}}{2}, 2\sqrt{s})$

# The hadronic LbL: setup and problems

Hadrons in $\langle 0|T\{A^\mu(x_1)A^\nu(x_2)A^\rho(x_3)A^\sigma(x_4)\}|0\rangle$



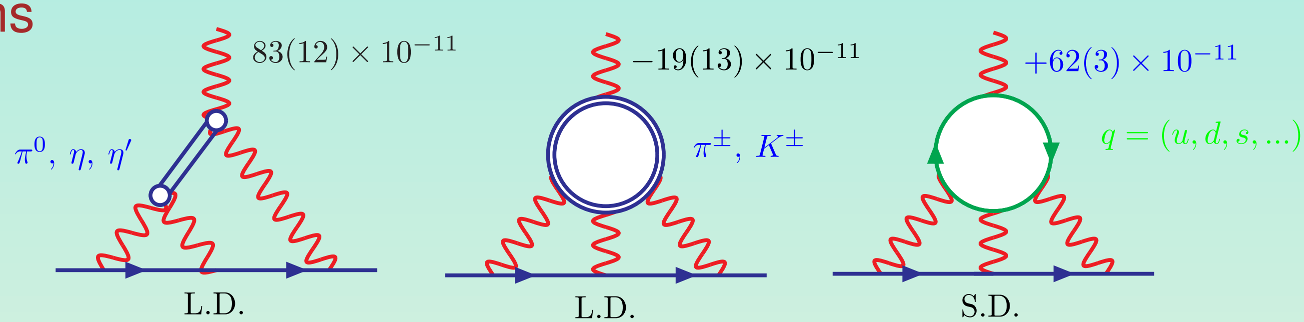

Key object full rank-four hadronic vacuum polarization tensor

$$\Pi_{\mu\nu\lambda\rho}(q_1,q_2,q_3) = \int \mathrm{d}^4x_1\,\mathrm{d}^4x_2\,\mathrm{d}^4x_3\,\mathrm{e}^{\mathrm{i}\,(q_1x_1+q_2x_2+q_3x_3)} \times \langle\,0\,|\,T\{j_\mu(x_1)\,j_\nu(x_2)\,j_\lambda(x_3)\,j_\rho(0)\}\,|\,0\,\rangle\;.$$

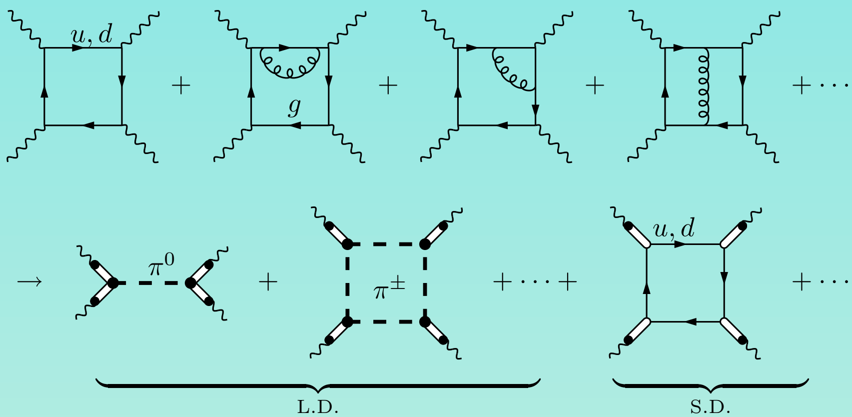
- non-perturbative physics
- general covariant decomposition involves 138 Lorentz structures of which
- 32 can contribute to $g-2$
- fortunately, dominated by the pseudoscalar exchanges $\pi^0,\eta,\eta',...$ described by the effective Wess-Zumino Lagrangian

- generally, pQCD useful to evaluate the short distance (S.D.) tail
- the dominant long distance (L.D.) part must be evaluated using some low energy effective model which includes the pseudoscalar Goldstone bosons as well as the vector mesons which play a dominant role (vector meson dominance mechanism); HLS, ENJL, general RLA, large $N_c$ inspired ansätze, and others

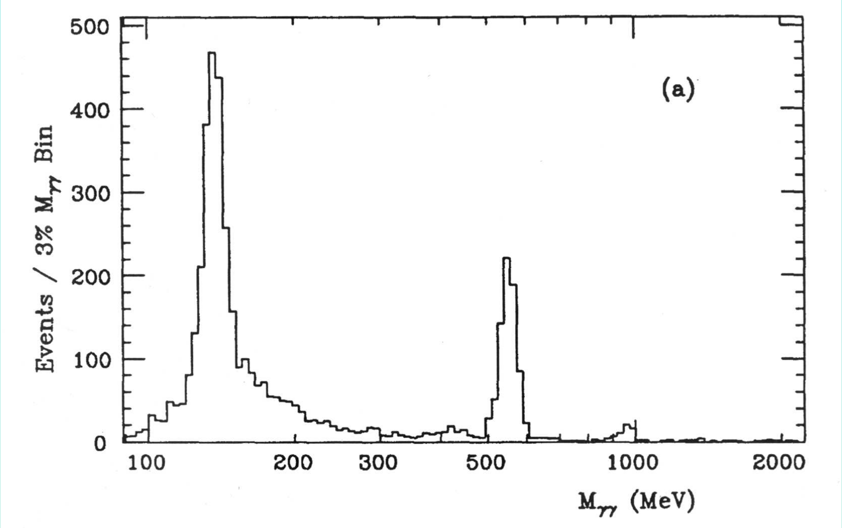
Need appropriate low energy effective theory ⇒amount to calculate the following type diagrams




LD contribution requires low energy effective hadronic models: simplest case $\pi^0\gamma\gamma$ vertex




Crystal Ball 1988




Data show almost background free spikes of the PS mesons! Substantial background form quark loop is absent (seems to contradict large quark-loop contribution as obtained in Schwinger-Dyson approach (SDA) Darmstadt group). Clear message from data: fully non-perturbative, evidence for PS dominance. However, no information about axial mesons (Landau-Yang theorem). Illustrates how data can tell us where we are.

Low energy expansion in terms of hadronic components: theoretical models vs experimental data ➠ KLOE, KEDR, BES, BaBar, Belle, ?

## A new representation for single particle exchange in LbL

• $a_\mu$ does not depend on direction of muon momentum $p \Rightarrow$ may average in Euclidean space over the directions $\hat{P}$:

$$\langle \cdots \rangle = \frac{1}{2\pi^2} \int \mathrm{d}\Omega(\hat{P}) \, \cdots$$

Hadronic single particle exchange amplitudes independent of $p \Rightarrow$ 2 integrations may be done analytically: amplitudes $T_i$, propagators $(4) \equiv (P + Q_1)^2 + m_\mu^2$ and $(5) \equiv (P - Q_2)^2 + m_\mu^2$ with $P^2 = -m_\mu^2$

$$\begin{aligned} \langle \frac{1}{(4)} \frac{1}{(5)} \rangle &= \frac{1}{m_\mu^2 R_{12}} \arctan\left(\frac{zx}{1 - zt}\right) \\ \langle (P \cdot Q_1) \frac{1}{(5)} \rangle &= -(Q_1 \cdot Q_2) \frac{(1 - R_{m2})^2}{8 m_\mu^2} \,, \end{aligned}$$

$$
\begin{aligned}
\langle (P \cdot Q_2) \frac{1}{(4)} \rangle &= (Q_1 \cdot Q_2) \, \frac{(1 - R_{m1})^2}{8 m_\mu^2} \\
\langle \frac{1}{(4)} \rangle &= - \frac{1 - R_{m1}}{2 m_\mu^2} \\
\langle \frac{1}{(5)} \rangle &= - \frac{1 - R_{m2}}{2 m_\mu^2}
\end{aligned}
$$

$R_{mi} = \sqrt{1 + 4 m_\mu^2 / Q_i^2}$, $(Q_1 \cdot Q_2) = Q_1 \, Q_2 \, t$, $t = \cos\theta$, $\theta$= angle between $Q_1$ and $Q_2$.
Denoting $x = \sqrt{1 - t^2}$, we have $R_{12} = Q_1 \, Q_2 \, x$ and

$$
z = \frac{Q_1 Q_2}{4 m_\mu^2} \, (1 - R_{m1}) \, (1 - R_{m2}) \ \ .
$$

- For any hadronic form-factor end up with 3–dimensional integral over $Q_1 = |Q_1|$,

$Q_2 = |Q_2|$ and $t = \cos\theta$:

$$a_\mu(\mathrm{LbL};\pi^0) \;=\; -\frac{2\alpha^3}{3\pi^2}\int_0^\infty \mathrm{d}Q_1\mathrm{d}Q_2 \int_{-1}^{+1} \mathrm{d}t\;\sqrt{1-t^2}\,Q_1^3\,Q_2^3 \times \left(F_1\,P_6\,I_1(Q_1,Q_2,t) + F_2\,P_7\,I_2(Q_1,Q_2,t)\right)$$

where $P_6 = 1/(Q_2^2+m_\pi^2)$, and $P_7 = 1/(Q_3^2+m_\pi^2)$ denote the Euclidean single particle exchange propagators. $I_1$ and $I_2$ known integration kernels. The non-perturbative factors are

$$\begin{aligned} F_1 &= \mathcal{F}_{\pi^{0*}\gamma^*\gamma^*}(q_2^2,q_1^2,q_3^2)\,\mathcal{F}_{\pi^{0*}\gamma^*\gamma}(q_2^2,q_2^2,0)\;, \\ F_2 &= \mathcal{F}_{\pi^{0*}\gamma^*\gamma^*}(q_3^2,q_1^2,q_2^2)\,\mathcal{F}_{\pi^{0*}\gamma^*\gamma}(q_3^2,q_3^2,0)\;. \end{aligned}$$

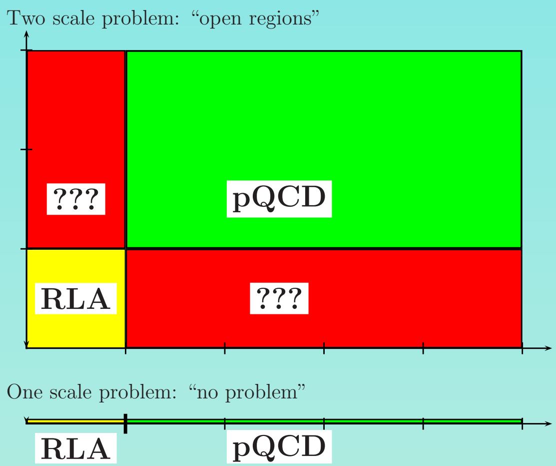
Note: SU(3) flavor decomposition of em current → weight factors

$$W^{(a)} = \frac{\left(\mathrm{Tr}\,[\lambda_a \hat{Q}^2]\right)^2}{\mathrm{Tr}\,[\lambda_a^2]\mathrm{Tr}\,[\hat{Q}^4]}\;; \quad W^{(3)} = \frac{1}{4}\,, \quad W^{(8)} = \frac{1}{12}\,, \quad W^{(0)} = \frac{2}{3}\,.$$

where $\mathrm{Tr}\,[\hat{Q}^4] = 2/9$ is the overall normalization such that $\sum_a W^{(a)} = 1$. Note $(W^{(8)} + W^{(0)})/W^{(3)} = 3$, higher states enhanced in coupling by factor 3! [Melnikov&Vainshtein] overlooked by previous analyzes [HKS,HK,BPP].

Such representations I worked out for axial exchanges as well as for scalar ones. Missing is tensor state, could play similar role as $\rho$ exchange vs scalar QED $\pi\pi$ contribution.

Basic problem: $(s, s_1, s_2)$–domain of $\mathcal{F}_{\pi^{0*}\gamma^*\gamma^*}(s, s_1, s_2)$; here $(0, s_1, s_2)$–plane




??? – Data, OPE,
– QCD factorization,
– Brodsky-Lepage approach

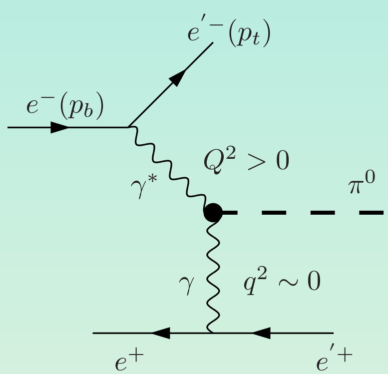
Novel approach: refer to quark–hadron duality of large-$N_c$ QCD, hadron spectrum known, infinite series of narrow spin 1 resonances 't Hooft 79 ⇒no matching problem (resonance representation has to match quark level representation) De Rafael 94, Knecht, Nyffeler 02

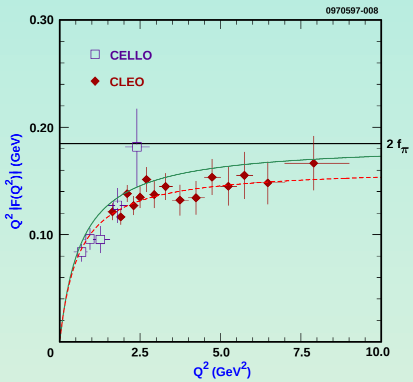
Constraints for on-shell pions (pion pole approximation)

- General form–factor $\mathcal{F}_{\pi^{0*}\gamma^*\gamma^*}(s, s_1, s_2)$ is largely unknown

- The constant $e^2\,\mathcal{F}_{\pi^0\gamma\gamma}(m_\pi^2,0,0) = \frac{e^2 N_c}{12\pi^2 f_\pi} = \frac{\alpha}{\pi f_\pi} \approx 0.025$ GeV$^{-1}$ well determined by $\pi^0 \to \gamma\gamma$ decay rate (from Wess-Zumino Lagrangian); experimental improvement needed!

- Information on $\mathcal{F}_{\pi^0\gamma^*\gamma}(m_\pi^2,-Q^2,0)$ from $e^+e^- \to e^+e^-\pi^0$ experiments




**CELLO and CLEO measurement of the $\pi^0$ form factor $\mathcal{F}_{\pi^0\gamma^*\gamma}(m_\pi^2,-Q^2,0)$ at high space–like $Q^2$. outdated now by BaBar?**

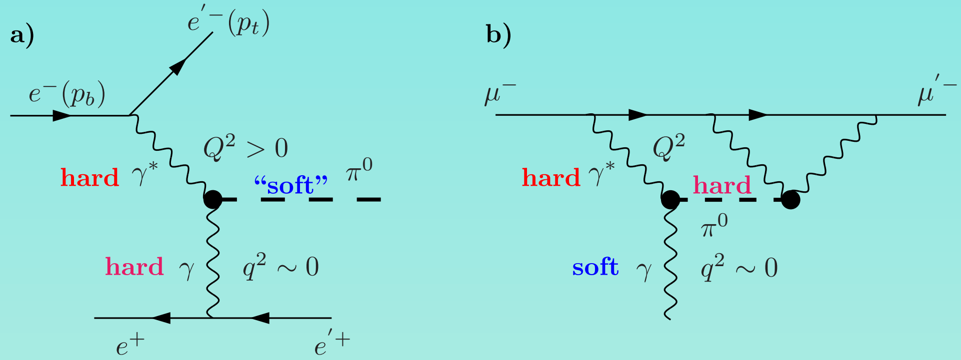
Brodsky–Lepage interpolating formula gives an acceptable fit.

$$\mathcal{F}_{\pi^0\gamma^*\gamma}(m_\pi^2,-Q^2,0) \simeq \frac{1}{4\pi^2 f_\pi}\frac{1}{1+(Q^2/8\pi^2 f_\pi^2)} \sim \frac{2 f_\pi}{Q^2}$$

Inspired by pion pole dominance idea this FF has been used mostly (HKS,BPP,KN) in the past, but has been criticized recently (MV and FJ07).

❒ Melnikov, Vainshtein: in chiral limit vertex with external photon must be non-dressed! i.e. use $\mathcal{F}_{\pi^0\gamma^*\gamma}(0,0,0)$, which avoids eventual kinematic inconsistency, thus no VMD damping ⇒result increases by 30% !

❒ In $g-2$ external photon at zero momentum ⇒ only $\mathcal{F}_{\pi^{0*}\gamma^*\gamma}(-Q^2,-Q^2,0)$ not $\mathcal{F}_{\pi^0\gamma^*\gamma}(m_\pi^2,-Q^2,0)$ is consistent with kinematics. Unfortunately, this off–shell form factor is not known and in fact not measurable and CELLO/CLEO constraint does not apply!. Obsolete far off-shell pion (in space-like region).




**Measured is $\mathcal{F}_{\pi^0\gamma^*\gamma}(m_\pi^2, -Q^2, 0)$ at high space–like $Q^2$, needed at external vertex is $\mathcal{F}_{\pi^{0*}\gamma^*\gamma}(-Q^2, -Q^2, 0)$.**

- I still claim using $\mathcal{F}_{\pi^{0*}\gamma^*\gamma}(0, 0, 0)$ in this case is not a reliable approximation!

**Need realistic "model" for off–shell form–factor $\mathcal{F}_{\pi^{0*}\gamma^*\gamma}(-Q^2, -Q^2, 0)$!**

**Is it really to be identified with $\mathcal{F}_{\pi^{0*}\gamma^*\gamma}(0, 0, 0)$?**

Can we check such questions experimentally or in lattice QCD?

## Evaluation of $a_\mu^{\rm LbL}$ in the large-$N_c$ framework

- Knecht & Nyffeler and Melnikov & Vainshtein were using pion-pole approximation together with large-$N_c$ $\pi^0\gamma\gamma$–form-factor

- FJ & A. Nyffeler: relax from pole approximation, using KN off-shell LDM+V form-factor

$$
\begin{aligned}
\mathcal{F}_{\pi^{0*}\gamma^*\gamma^*}(p_\pi^2,q_1^2,q_2^2) &= \frac{F_\pi}{3}\,\frac{\mathcal{P}(q_1^2,q_2^2,p_\pi^2)}{\mathcal{Q}(q_1^2,q_2^2)} \\
\mathcal{P}(q_1^2,q_2^2,p_\pi^2) &= h_7 + h_6\,p_\pi^2 + h_5\,(q_2^2+q_1^2) + h_4\,p_\pi^4 + h_3\,(q_2^2+q_1^2)\,p_\pi^2 \\
&\quad + h_2\,q_1^2\,q_2^2 + h_1\,(q_2^2+q_1^2)^2 + q_1^2\,q_2^2\,(p_\pi^2+q_2^2+q_1^2)) \\
\mathcal{Q}(q_1^2,q_2^2) &= (q_1^2-M_1^2)\,(q_1^2-M_2^2)\,(q_2^2-M_1^2)\,(q_2^2-M_2^2)
\end{aligned}
$$

all constants are constraint by SD expansion (OPE). Again, need data to fix parameters!

Note: Need at lest two VMD states $\rho$ and $\rho'$, mix with both photons $\rightarrow$ four denominators. Numerator polynomial in all variables of degree, such that FF remains unitary (bounded by constant). OPE in the different channels must satisfy QCD constraints.

Looking for new ideas to get ride of model dependence

- ❒ Need better constrained effective resonance Lagrangian (e.g. HSL and ENJL models vs. RLA of Ecker et al.). "Global effort" needed!
  recent: HLS global fit available Benayoun et al 2010

- ❒ Lattice QCD will provide an answer [take time ("yellow" region only?)]!

- ❒ Try exploiting possible new experimental constraints:

# Pseudoscalar exchanges: $\pi^0, \eta, \eta'$

Leading LbL contribution from PS mesons:

$$a_\mu[\pi^0, \eta, \eta'] \sim (93.91 \pm 12.40) \times 10^{-11}$$

## ❐ $\pi^0\gamma\gamma$ form-factor: experimental facts and possibilities

- relation between the off-shell (needed for $a_\mu$) and the on-shell (measured) from-factor is all but obvious

Note: $\mathcal{F}_{\pi^{0*}\gamma^*\gamma}(-Q^2, -Q^2, 0)$ is a one-scale problem. Self-energy type of problem $\Rightarrow$ can get it via dispersion relation from appropriate data

Existing data for $F(m_\pi^2, Q^2, 0)$: $e^+e^- \to e^+e^-\pi^0$ single tag data $\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2}$

➠ CELLO: $0.5\ \mathrm{GeV}^2 < Q^2 < 2.17\ \mathrm{GeV}^2$ [Z. Phys. C49 (1991) 401]

➠ CLEO: $1.5\ \mathrm{GeV}^2 < Q^2 < 9\ \mathrm{GeV}^2$ [Phys. Rev. D57 (1998) 33]

➠ BaBar: $4\ \mathrm{GeV}^2 < t_2 < 40\ \mathrm{GeV}^2$ [Phys. Rev. D80 (2009) 052002]

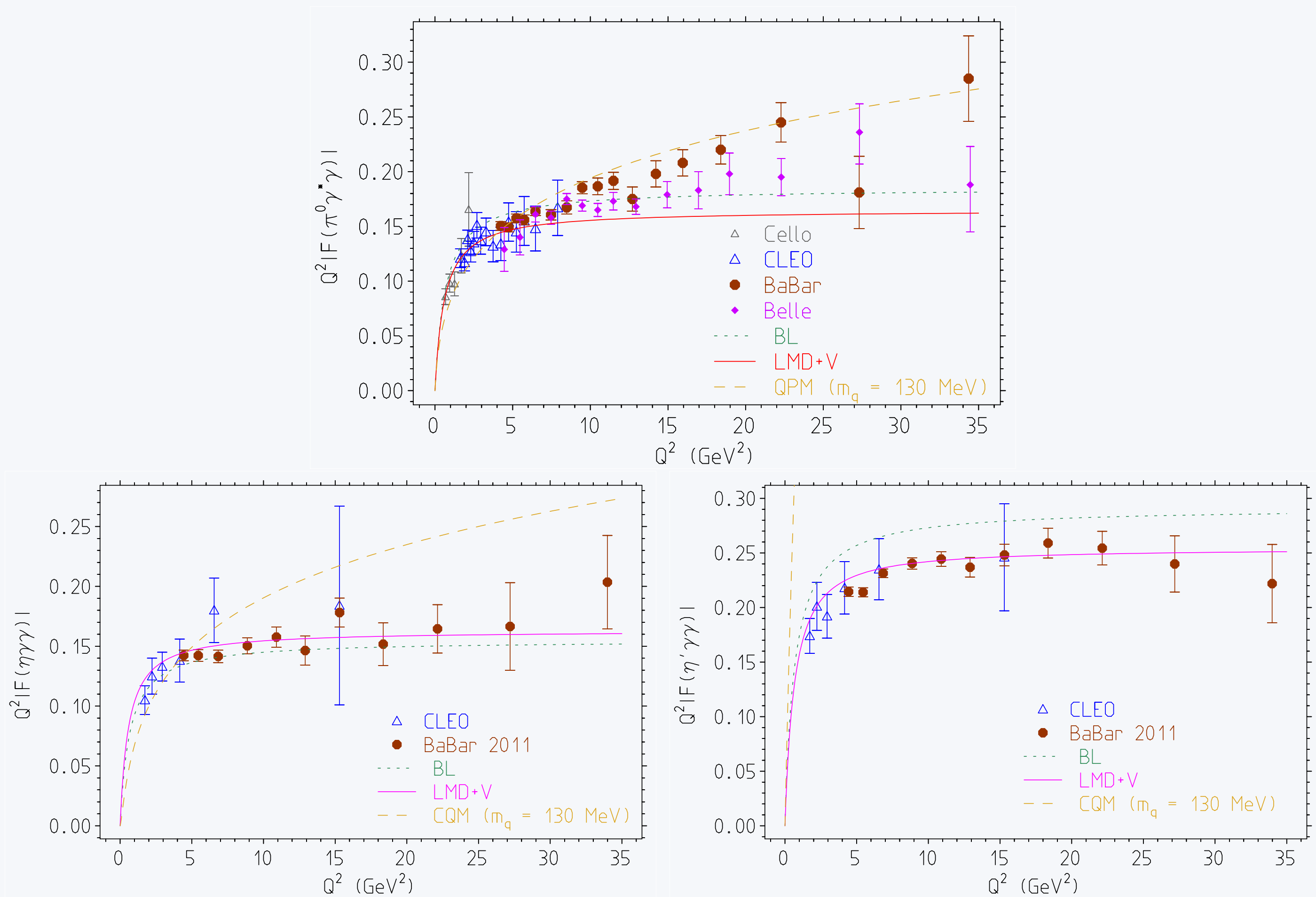
➠ Belle: $4\ \mathrm{GeV}^2 < t_2 < 40\ \mathrm{GeV}^2$ [arXiv:1205.3249 [hep-ex]]

➠ new quest for theory

- BaBar seems to violate $Q^2\, F(m_\pi^2, Q^2, 0) \to 2 f_{\pi^0}\ \ \text{(constant)}$ in $\pi^0$ channel
- BaBar: $\pi^0$, $\eta$ and $\eta'$ seem to show different behavior

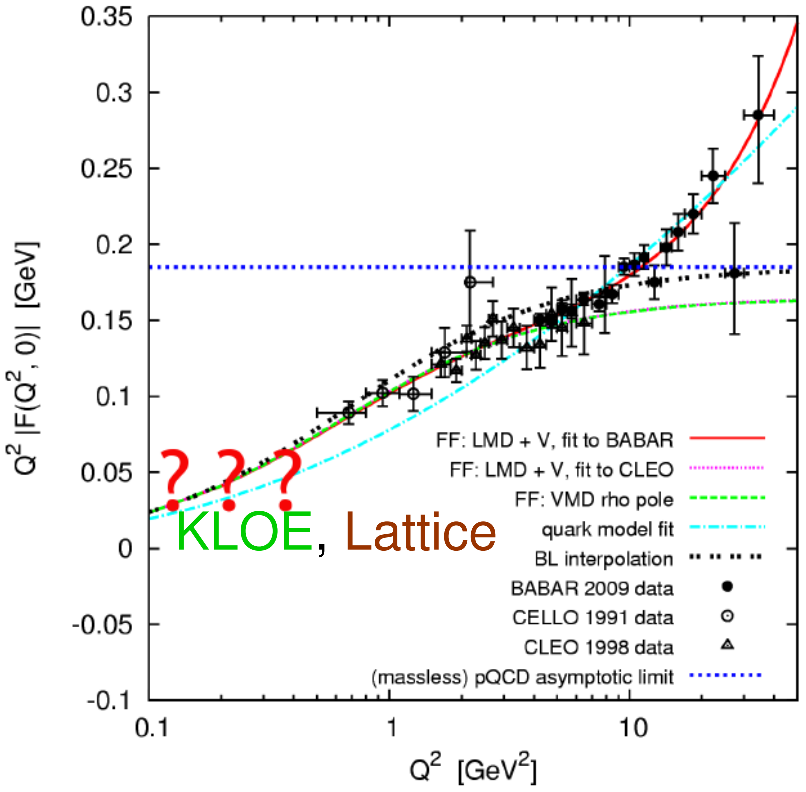
➠ theory: Brodsky-Lepage (BL) behavior $\sim 1/Q^2$ for all pseudoscalars

Different approaches/models Mikhailov et al, Dorokhov, Teryaev et al. and others
no coherent theory picture!




- asymptotic behavior ? data consistent ? BaBar conflict relaxed by Belle

## $Q^2 F_{\pi^0\gamma^*\gamma}(m_\pi^2, Q^2, 0)$




Theory:

[ A. Nyffeler, 0912.1441 ]

[ M. Knecht and A. Nyffeler, Phys. Rev. D**65**, 073034 (2002) ]

[ ibid. ]

[ A. E. Dorokhov, 0905.4577 ]

[ G. P. Lepage and S. J. Brodsky, Phys. Rev. D **22**, 2157 (1980) ]

No data at $0.02\ \mathrm{GeV}^2 < Q^2 < 0.4\ \mathrm{GeV}^2$

Sergiy IVASHYN (Katowice, Kharkov) $\pi^0\gamma\gamma$ 21 / VI / 2010 @ Mainz 22 / 66

Cross check of BaBar by *Belle* (anomalous increase not seen), BESIII middle

# **Axial exchanges:** $a_1, f_1', f_1$

Axial exchanges

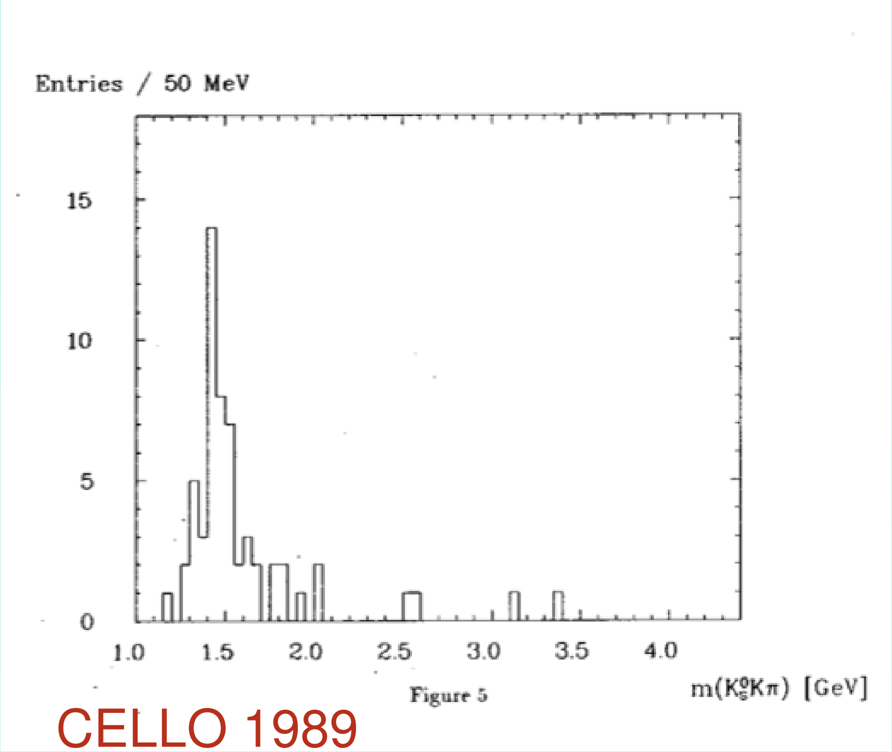
Landau-Yang Theorem: $\mathcal{A}$ (axial meson $\to \gamma\gamma$)=0

e.g. $Z^0 \not\to \gamma\gamma$, while $Z^0 \to \gamma e^+ e^-$ ✓

Why $a_\mu[a_1, f_1', f_1] \;\sim\; 25 \times 10^{-11}$ so large?

- untagged $\gamma\gamma \to f()$ no signal!
- single-tag $\gamma^*\gamma \to f()$ strong peak is $Q^2 \gg m_f^2$

$\sigma(\gamma^*\gamma \to f_1 \to K_s^0 K\pi)$



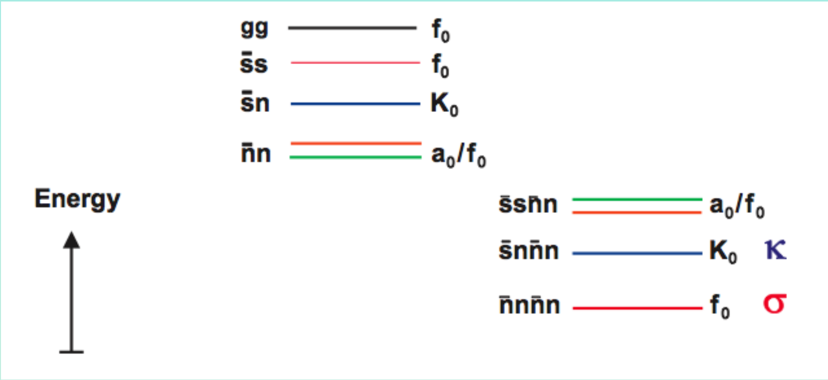

Sparse data so far, new measurements important; in particular momentum dependent $\Gamma(a_1 \to \gamma\gamma^*)$ etc.

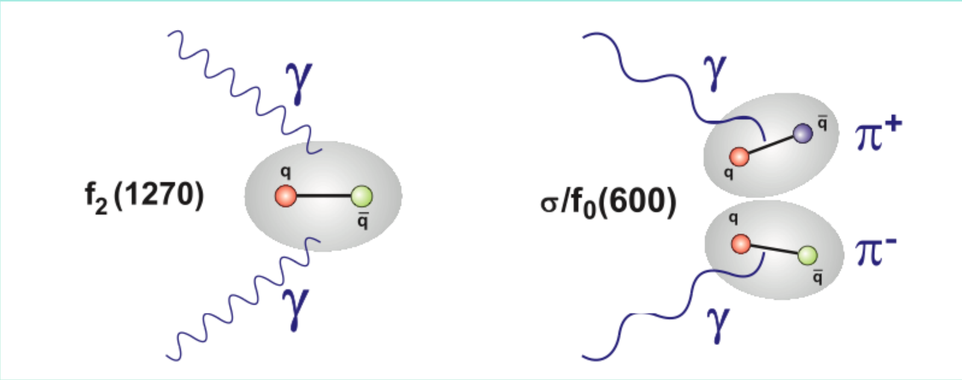
Expected contribution from axial mesons:

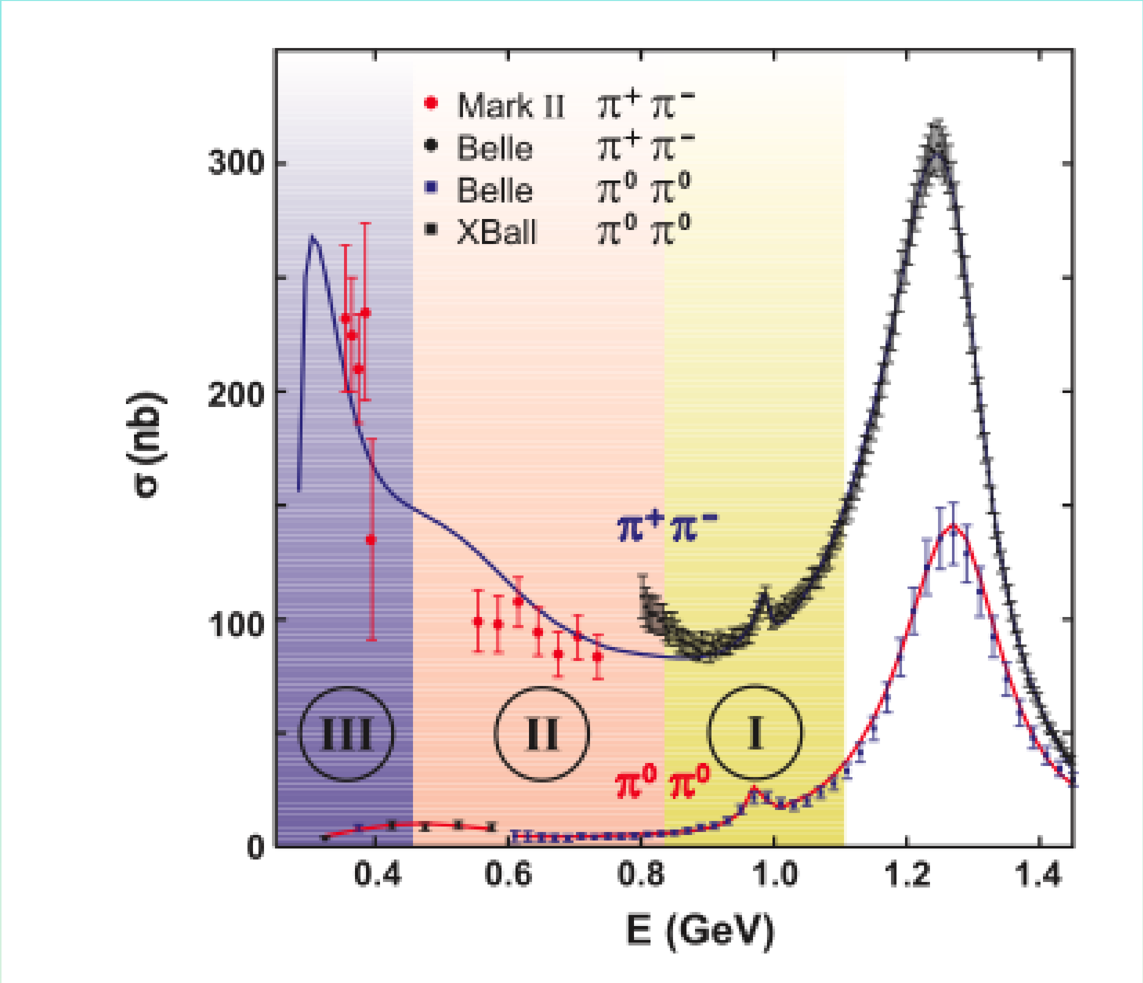
$$a_\mu[a_1, f_1', f_1] \sim (28.13 \pm 5.63) \times 10^{-11}$$

## Scalar exchanges: $a_0, f_0', f_0, \cdots$

Mesons: $M(q\bar{q})$, $M(qq\bar{q}\bar{q})$, glueballs mixing
Experimental: Crystal Ball, Mark II, Belle!
Theory: Mennessier, Pennington et al., Mousallam et al., Achasov et al., ...






Strong tensor meson resonance in $\pi\pi$ channel $f_2(1270)$

So: expect usual pion-loop in HLbL plays role like pion-loop in VP. i.e. like missing the $\rho$.
➠ Need to explicitly include tensor mesons

The di-pion amplitude $M_{\text{res}}^{\text{direct}}(\gamma\gamma \to \pi^+\pi^-; s)$ gets contribution caused by mixed $\sigma(600)$ and $f_0(980)$ resonances with the direct coupling constants of the $\sigma(600)$ and $f_0(980)$ to photons, $g_{\sigma\gamma\gamma}^{(0)}$ and $g_{f_0\gamma\gamma}^{(0)}$,

$$M_{\text{res}}^{\text{direct}}(\gamma\gamma \to \pi^+\pi^-; s) = s\, e^{i\delta_B^{\pi\pi}(s)}$$

$$\times \frac{g_{\sigma\gamma\gamma}^{(0)}[D_{f_0}(s)g_{\sigma\pi^+\pi^-} + \Pi_{f_0\sigma}(s)g_{f_0\pi^+\pi^-}] + g_{f_0\gamma\gamma}^{(0)}[D_\sigma(s)g_{f_0\pi^+\pi^-} + \Pi_{f_0\sigma}(s)g_{\sigma\pi^+\pi^-}]}{D_\sigma(s)D_{f_0}(s) - \Pi_{f_0\sigma}^2(s)}\,.$$

For $\sqrt{s} < 2m_K$, the phase coincides with the I=0, $S$ wave $\pi\pi$ phase shift $\delta_0^0(s) = \delta_B^{\pi\pi}(s) + \delta_{\text{res}}(s)$.

Scalars everywhere. Many scalars many small contributions may sum up to substantial effect!

Expected contribution from $q\bar{q}$ scalars:

$$a_\mu[a_0, f_0', f_0] \sim (-5.98 \pm 1.20) \times 10^{-11}$$

So far nobody has evaluated $qq\bar{q}\bar{q}$ in $SU(3)$ sector $[u, d, s]$ many possible states, which individually are expected rather small

# LbL: Present

JN09 based on Nyffeler 09:

$$a_\mu^{\mathrm{LbL;had}} = (116 \pm 39) \times 10^{-11}$$

Summary of results

| Contribution | BPP | HKS | KN | MV | PdRV | N/JN |
|---|---|---|---|---|---|---|
| $\pi^0, \eta, \eta'$ | 85±13 | 82.7±6.4 | 83±12 | 114±10 | 114±13 | 99±16 |
| $\pi, K$ loops | −19±13 | −4.5±8.1 | – | 0±10 | −19±19 | −19±13 |
| axial vectors | 2.5±1.0 | 1.7±1.7 | – | 22± 5 | 15±10 | 22± 5 |
| scalars | −6.8±2.0 | – | – | – | −7± 7 | −7± 2 |
| quark loops | 21± 3 | 9.7±11.1 | – | – | 2.3 | 21± 3 |
| total | 83±32 | 89.6±15.4 | 80±40 | 136±25 | 105±26 | 116±39 |

Is this the final answer? How to improve? A limitation to more precise $g-2$ tests?

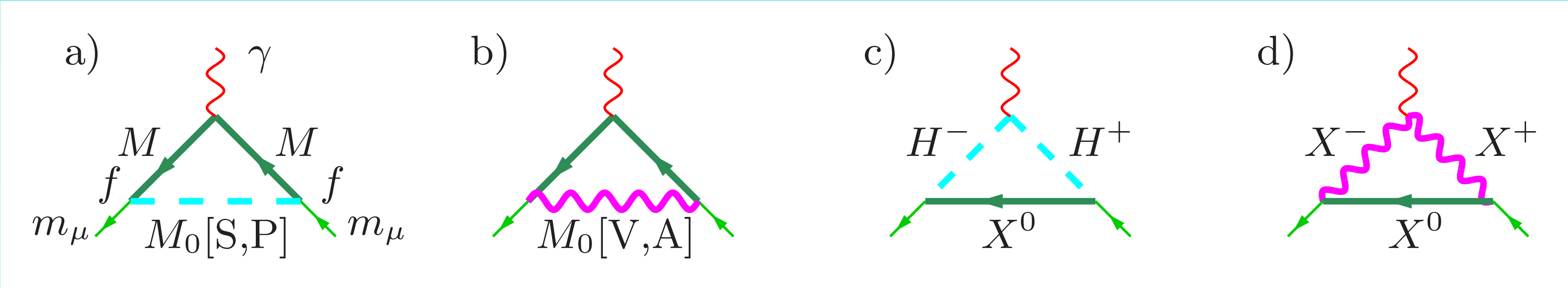
Looking for new ideas to get ride of model dependence

# Theory vs experiment: do we see New Physics?

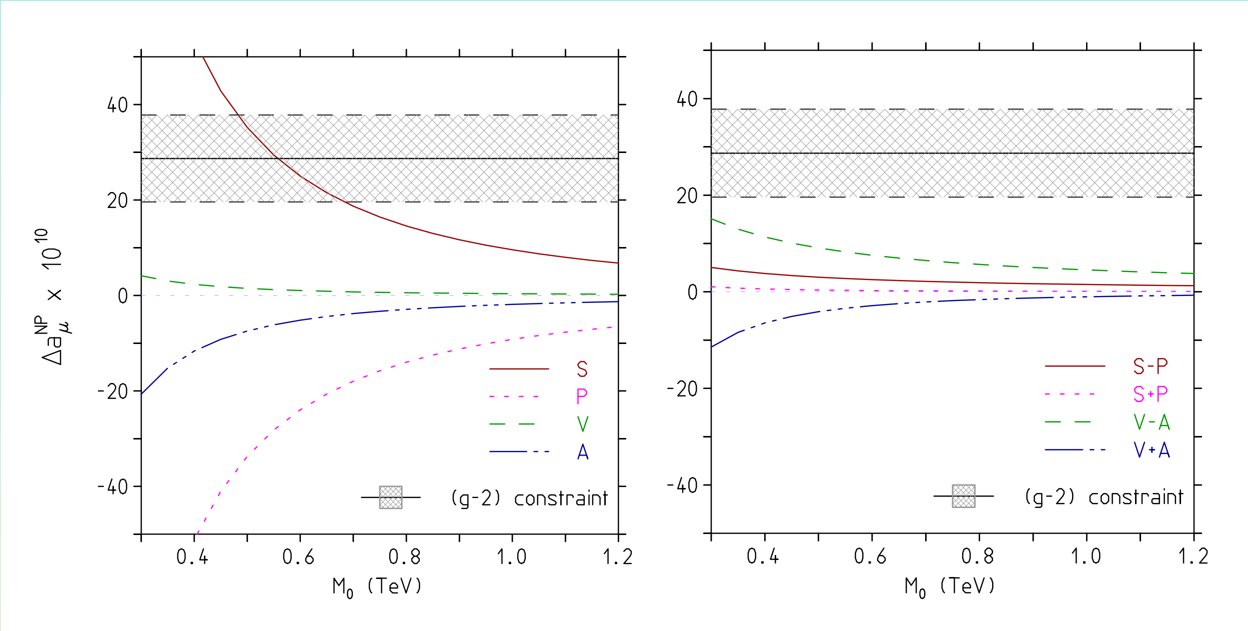
| Contribution | Value | Error | Reference |
|---|---|---|---|
| QED incl. 4-loops+5-loops | 11 658 471.885 | 0.04 | Remiddi, Kinoshita ... |
| Leading hadronic vac. pol. | 693.2 | 3.7 | 2011 update |
| Subleading hadronic vac. pol. | -10.0 | 0.1 | 2011 update |
| Hadronic light–by–light | 11.6 | 3.9 | evaluation (J&N 09) |
| Weak incl. 2-loops | 15.4 | 0.1 | CMV06 |
| Theory | 11 659 181.8 | 5.3 | – |
| Experiment | 11 659 209.1 | 6.3 | BNL Updated |
| Exp.- The. 3.3 standard deviations | 27.3 | 8.2 | – |

Standard model theory and experiment comparison [in units $10^{-10}$]. What represents the 3.4 $\sigma$ deviation: ❒ new physics? ❒ a statistical fluctuation? ❒ underestimating uncertainties (experimental, theoretical)? ❖ do experiments measure what theoreticians calculate?

## Most natural New Physics contributions: (examples)




neutral boson exchange: a) scalar or pseudoscalar and c) vector or axialvector, flavor changing or not, new charged bosons: b) scalars or pseudoscalars, d) vector or axialvector




Left: $m_\mu = M \ll M_0$ Right: $m_\mu \ll M_0 = M$

In general:

$$\Delta a_\mu^{\rm NP} = \alpha^{\rm NP}\,\frac{m_\mu^2}{M_{\rm NP}^2}$$

NP searches (LEP, Tevatron, LHC): typically $M_{\rm NP} >> M_W$, then $\Delta a_\mu^{\rm exp-the} = \Delta a_\mu^{\rm NP}$ requires $\alpha^{\rm NP} \sim 1$ spoiling perturbative arguments. Exception: 2HDM, SUSY $\tan\beta$ enhanced coupling!

**Most promising New Physics scenario: SUSY (MSSM: two for one SM!**

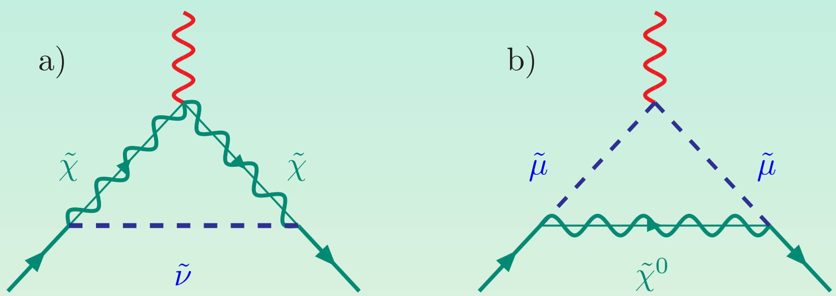
- muon $g-2$ in contrast requires moderately light SUSY masses and in the pre-LHC era fitted rather well with expectations from SUSY
- a particular role is played by the mass of the light Higgs

At tree level in the MSSM $m_h \leq M_Z$. This bound receives large radiative corrections from the $t/\tilde{t}$ sector, which changes the upper bound to (Haber & Hempfling 1990)

$$m_h^2 \sim M_Z^2 \cos^2 2\beta + \frac{3\sqrt{2} G_\mu m_t^4}{2\pi^2 \sin^2\beta} \ln\left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2}\right) + \cdots$$

which in any case is well below 200 GeV. A given value of $m_h$ fixes the value of $m_{1/2}$ represented by $\{m_{\tilde{t}_1}, m_{\tilde{t}_2}\}$

- Higgs found at 125 GeV (CERN "observed") we must have $m_{1/2} > 800$ GeV or higher! More specifically: heavy stop!
- if universal sfermion masses: all sfermion masses go up!



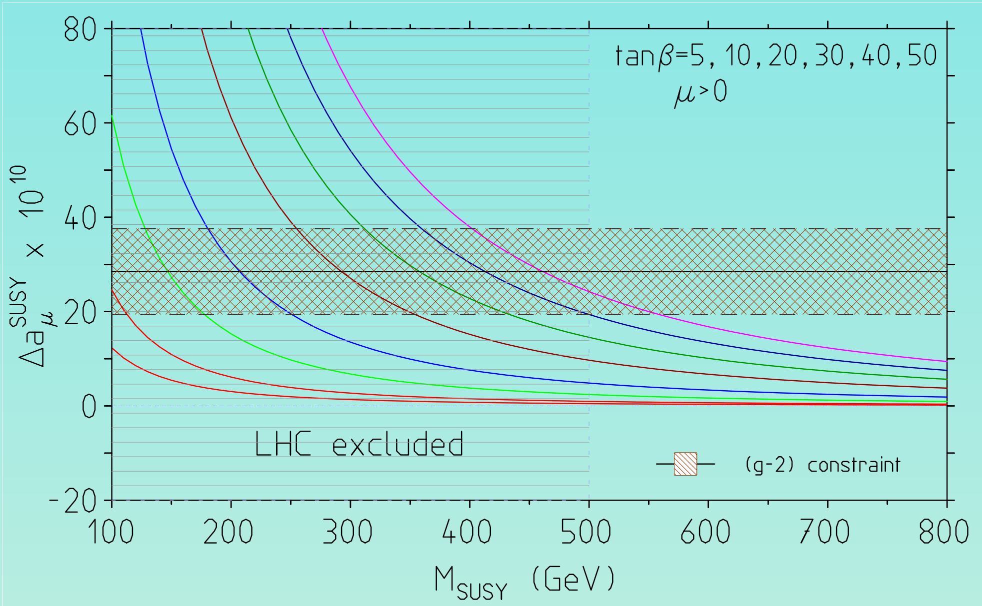

Leading SUSY contributions to $g-2$ in supersymmetric extension of the SM.

- $\tilde{m}$ lightest SUSY particle; SUSY requires two Higgs doublets

- $\tan\beta = \frac{v_1}{v_2}$, $v_i =< H_i >$ ; $i = 1, 2$ ; $\tan\beta \sim m_t/m_b \sim 40$ $[4-40]$

- $$a_\mu^{\rm SUSY} \simeq \frac{\mathrm{sign}(\mu M_2)\,\alpha(M_Z)}{8\pi \sin^2\Theta_W}\,\frac{\left(5+\tan^2\Theta_W\right)}{6}\,\frac{m_\mu^2}{M_{\rm SUSY}^2}\,\tan\beta\left(1-\frac{4\alpha}{\pi}\ln\frac{M_{\rm SUSY}}{m_\mu}\right)$$

  with $M_{\rm SUSY}$ a typical SUSY loop mass and the sign is determined by the Higgsino mass term $\mu$, RG improved.




Constraint on large $\tan\beta$ SUSY contributions as a function of $M_{\rm SUSY}$. The horizontal band shows $\Delta a_\mu^{\rm NP} = \delta a_\mu$. The region left of $M_{\rm SUSY} \sim$ 500 GeV is excluded by LHC searches. If $m_h \sim 125 \pm 1.5$ GeV actually $M_{\rm SUSY} > 800$ GeV depending on details of the stop sector ($\{\tilde{t}_1, \tilde{t}_2\}$ mixing and mass splitting) and weakly on $\tan\beta$ .

To be precise: $a_\mu$ depends on masses of sneutrino, chargino, smuon and neutralino, only direct constraints on them are unambiguous!

There are a lot of “SUSY’s”

- General MSSM has > 100 free parameters

- CMSSM – “constrained” and, related but even more constrained MSUGRA, and others

  - These models assume many degeneracies of masses and couplings in order to restrict the number of parameters
  - Typically, $m_0, m_{1/2}, \mathrm{sign}(\mu), \tan\beta, A$ (or even more)

- Then there is R–parity – sparticle number conserved (dark matter candidate!)?

- And, many ways to describe EW symmetry breaking

Role for LHC searches:
3 $\sigma$ deviation in muon g-2 (if real) requires $\mathrm{sign}(\mu)$ positive and $\tan\beta$ preferably large.

## Other strong constraints

❒ Data on the penguin loop induced $B \to X_s\gamma$ transition

SM prediction $\mathcal{B}(b \to s\gamma)_{\rm NNLL} = (3.15 \pm 0.23) \times 10^{-4}$ is consistent within 1.2 $\sigma$ with the experimental result (HFAG) $\mathcal{B}(b \to s\gamma) = (3.55 \pm 0.24 \pm 0.09) \times 10^{-4}$.

● it implies that SUSY requires heavier $m_{1/2}$ and/or $m_0$ in order not to spoil the good agreement.

❒ Data on dark matter relict density $\Omega_{\rm CDM}h^2 = 0.1126 \pm 0.0081$

SUSY+R-parity scenarios represent a tough constraint for the relic density of neutralinos produced in the early universe.

● A DM neutralino is a WIMP DM candidate. The density predicted is

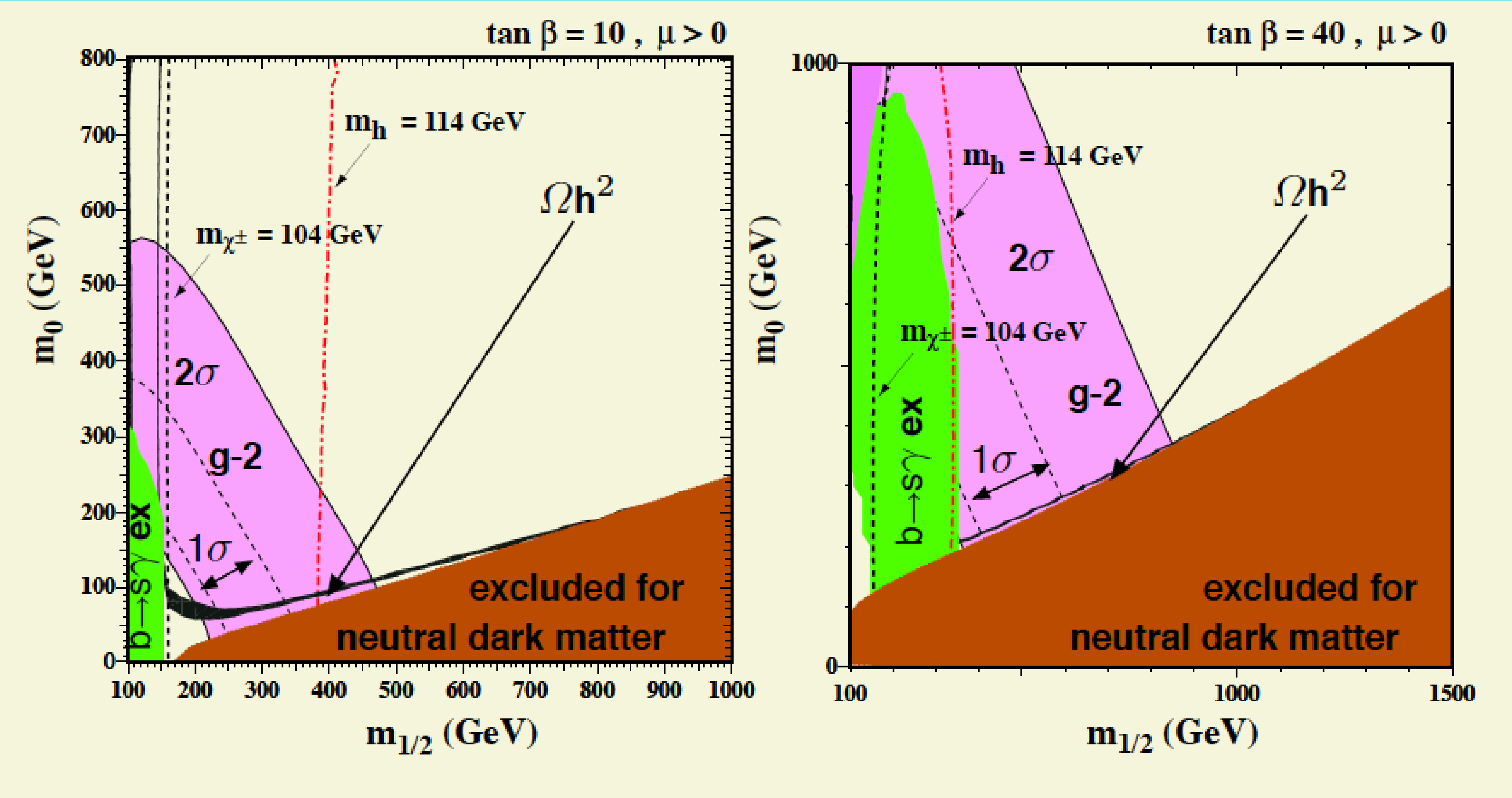
$$\Omega h^2 \sim \frac{0.1\,\mathrm{pb}}{\langle \sigma v \rangle} \sim 0.1 \left( \frac{M_{\rm WIMP}}{100\mathrm{GeV}} \right)^2 \;,$$

where $\langle \sigma v \rangle$ is the relativistic thermally averaged annihilation cross-section.

- in most scenarios the dominating neutralino annihilation process is $\chi + \chi \to A \to b\bar{b}$ and the observed relict density requires the cross section to be tuned to $\langle \sigma v \rangle \sim 2 \times 10^{-26}\ \mathrm{cm}^3/\mathrm{s}$. The cross section is of the form

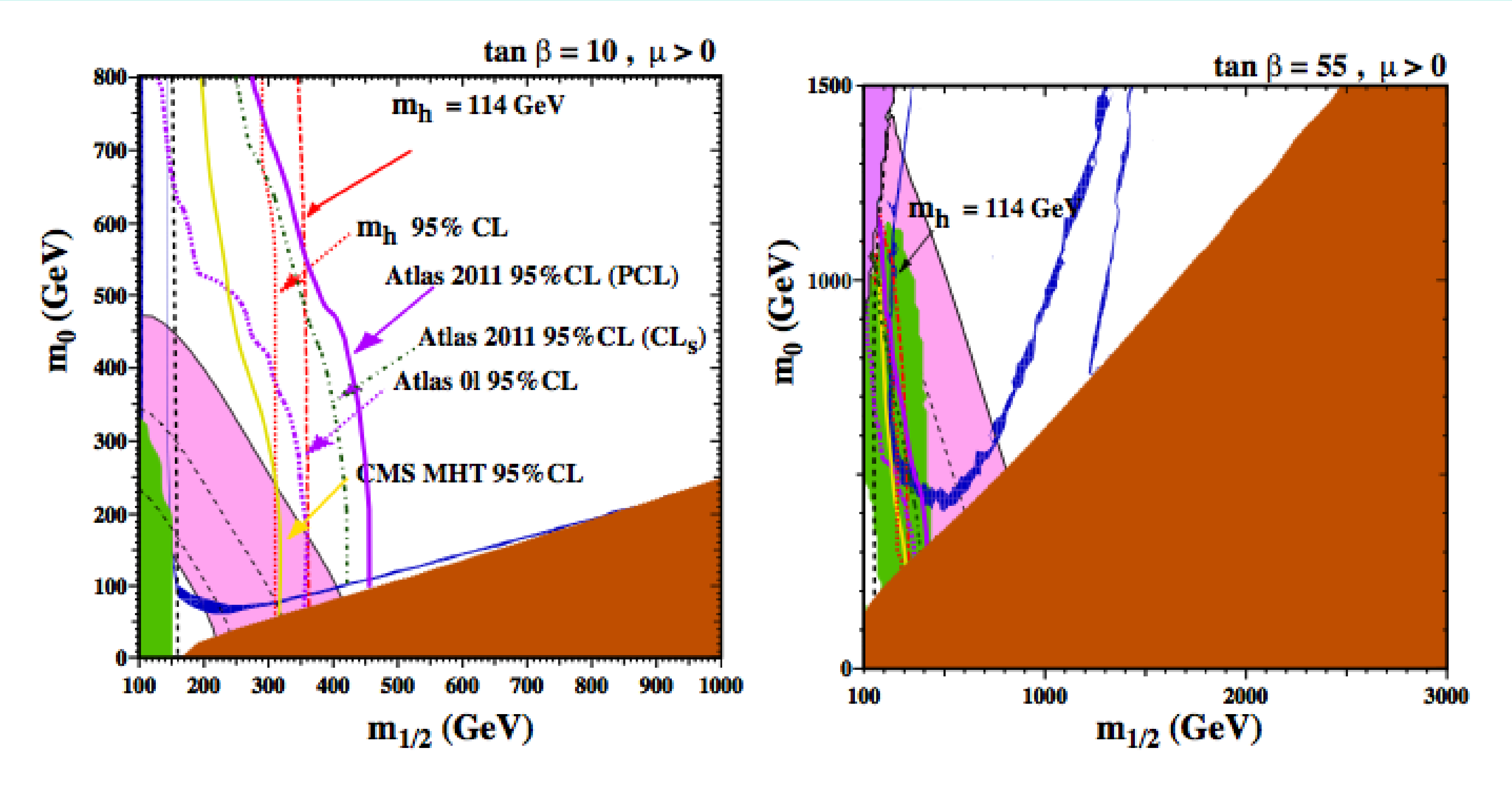
$$\langle \sigma v \rangle \propto \tan^2 \beta \, \frac{m_b^2}{M_Z^2} \, \frac{M_\chi^4}{(4M_\chi^2 - M_A^2)^2 + M_A^2 \Gamma_A^2}$$

and has to be adjusted to $M_\chi \approx 1.8\, M_A$ to $2.2\, M_A$. On resonance the cross section would be too big, too far off resonance too small. Note that except from $\Omega_{\mathrm{CDM}}$ all observables prefer heavier SUSY masses such that effects are small by decoupling. See recent study by Kazakov et al.

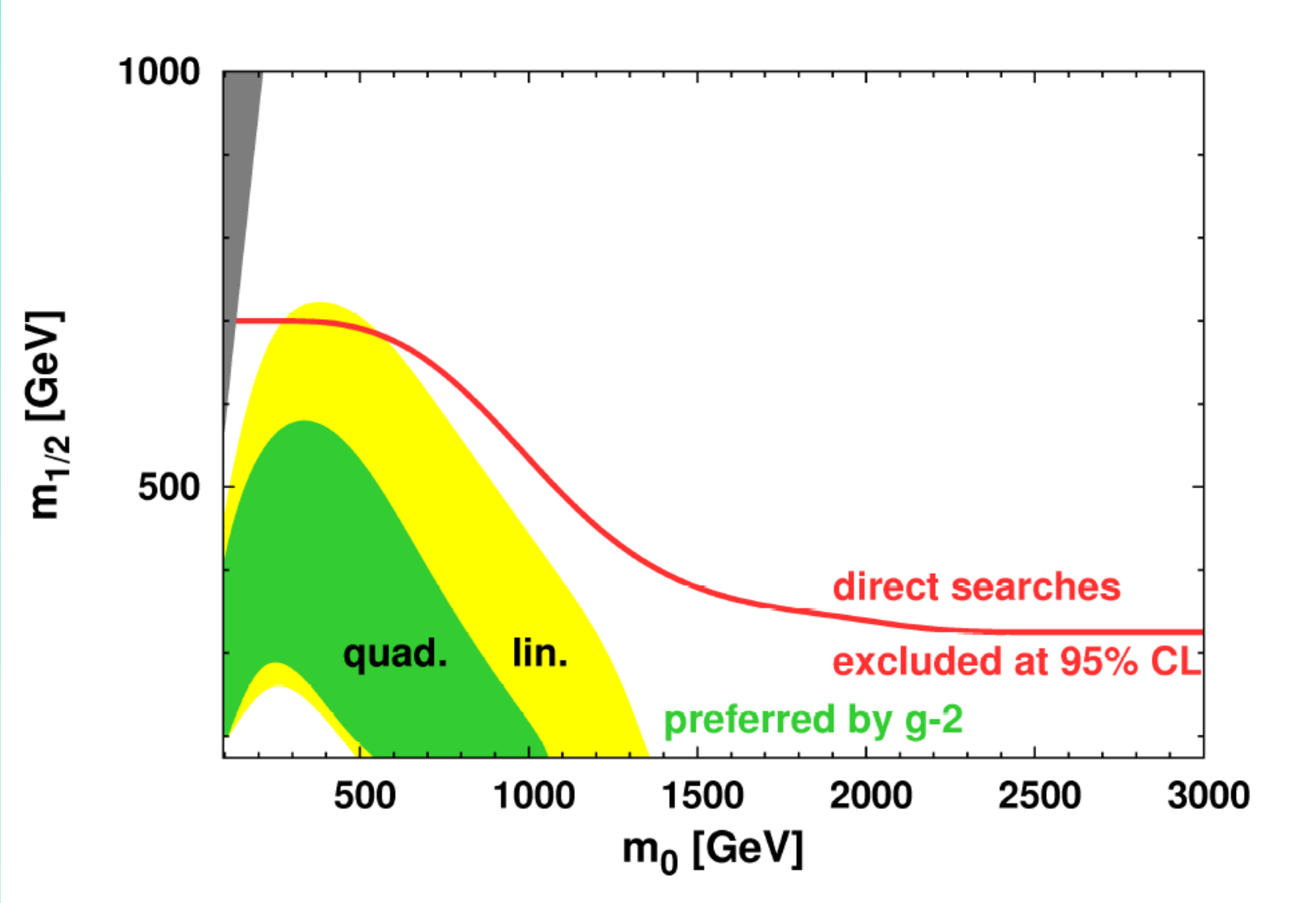



constraints from LEP, B-physics, g-2, cosmic relict density [plots Olive 09].

$\mathbf{m_0}$ scalar mass $\mathbf{m_{1/2}}$ gaugino mass



tan β = 10 , μ > 0
m_h = 114 GeV
m_h 95% CL
Atlas 2011 95%CL (PCL)
Atlas 2011 95%CL (CL_s)
Atlas 0l 95%CL
CMS MHT 95%CL
m_0 (GeV)
m_1/2 (GeV)
tan β = 55 , μ > 0
m_h = 114 GeV

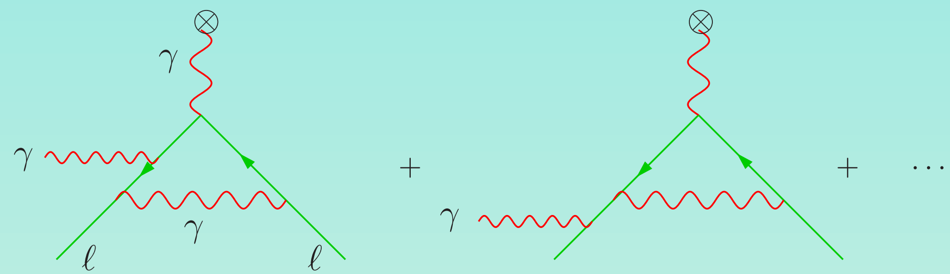



Kazakov et al. very recent analysis July 2012

- with the Higgs found at 125 GeV muon $g-2$ looks to me in possible trouble.

If $\delta a_\mu$ is not SUSY, what else? most other NP scenarios give likely even smaller contributions!

❖ most likely for me we could have been missing some electromagnetic radiation effects in the relation between observed and calculated quantity!




Does real radiation not affect $g-2$ measurement? Could yield IR finite correction to helicity flip amplitude?

❖ the other obstacle: hadronic light-by-light

❖ progress in evaluating HVP: more data (BaBar, Belle, VEPP 2000, BESIII,...), Lattice QCD in progress, effective field theories etc.

The big challenge: two <u>complementary</u> experiments: Fermilab with ultra hot muons and KEK with ultra cold muons (very different radiation profile) to come

Provided deviation is real $3\sigma \to 9\sigma$ possible? Provided theory and needed cross section data improves the same as the muon $g-2$ experiments!

Results to be improved: **Summary hadronic stuff:**

Hadronic vacuum polarization based on $e^+e^-$–annihilation data:

| | |
|---|---|
| $(694.4 \pm 3.7) \times 10^{-10}$ | [Hagiwara et al. $ee$] |
| $(691.0 \pm 4.7) \times 10^{-10}$ | [FJ&Szafron update $ee$] |
| $(692.3 \pm 4.2) \times 10^{-10}$ | [Davier et al. $ee$] |
| $(693.2 \pm 3.7) \times 10^{-10}$ | [Davier et al. $ee + \tau^*$ ] |

$^*$ combined by FJ after correction for $\rho - \gamma$ mixing

Differences between experiments [in common range] (examples):
- 4.8 between KLOE ’08 and SND ’06
- 8.5 between KLOE ’08 and BABAR ’09

Recent results for hadronic LbL:

| | |
|---|---|
| $(10.5 \pm 2.6) \times 10^{-10}$ | PdeRV |
| $(11.6 \pm 3.9) \times 10^{-10}$ | JN |

Muon $g-2$ remains a key monitor for NP
and
a great challenge for theorists as well as for experimenters!

An interesting frontier of digging for deeper understanding
of the SM and its limitations and extensions

# Electroweak fits

New physics sensible observable is the $W$ mass given by

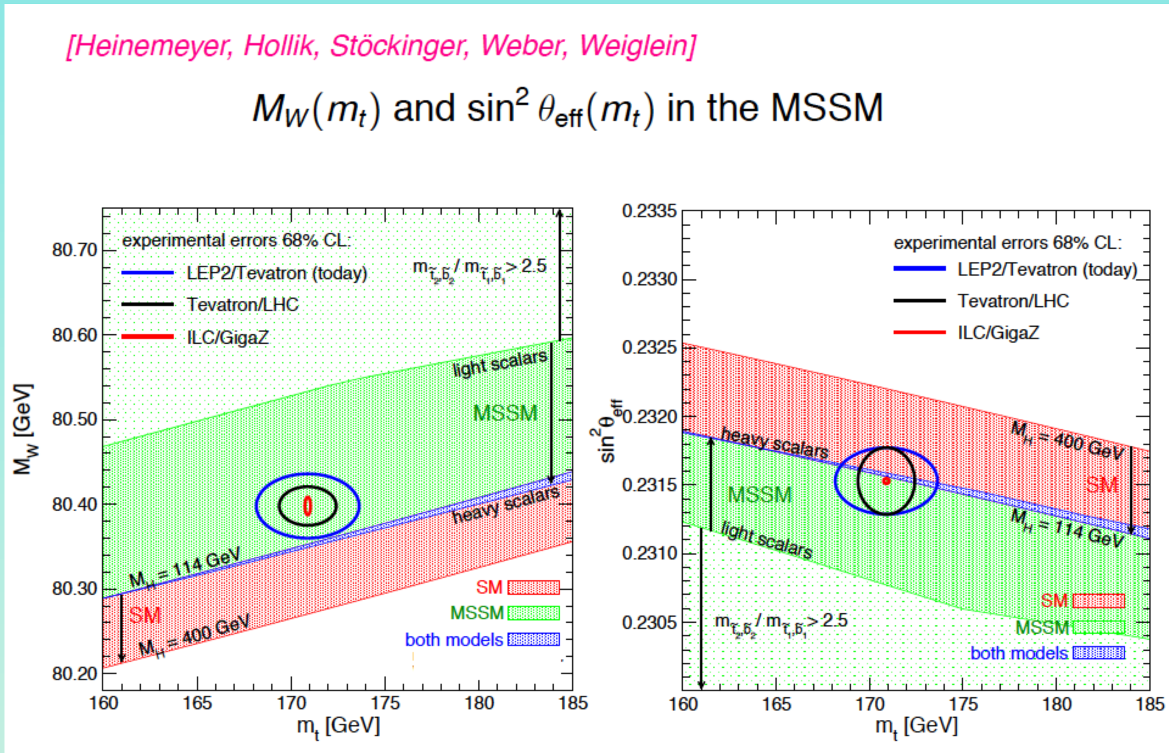
$$M_W^2\left(1-\frac{M_W^2}{M_Z^2}\right)=\frac{\pi\alpha}{\sqrt{2}G_F}\,(1+\Delta r)\ ;\quad \Delta r=f(\alpha,G_F,M_Z,m_t,\cdots)$$

- $\Delta r$ model-dependent radiative corrections
- in SUSY models $M_W$ is sensitive to the top/stop sector parameters

while

$$\sin^2\Theta_{\rm eff}=\frac{1}{4}\left(1-\mathrm{Re}\,\frac{v_{\rm eff}}{a_{\rm eff}}\right)$$

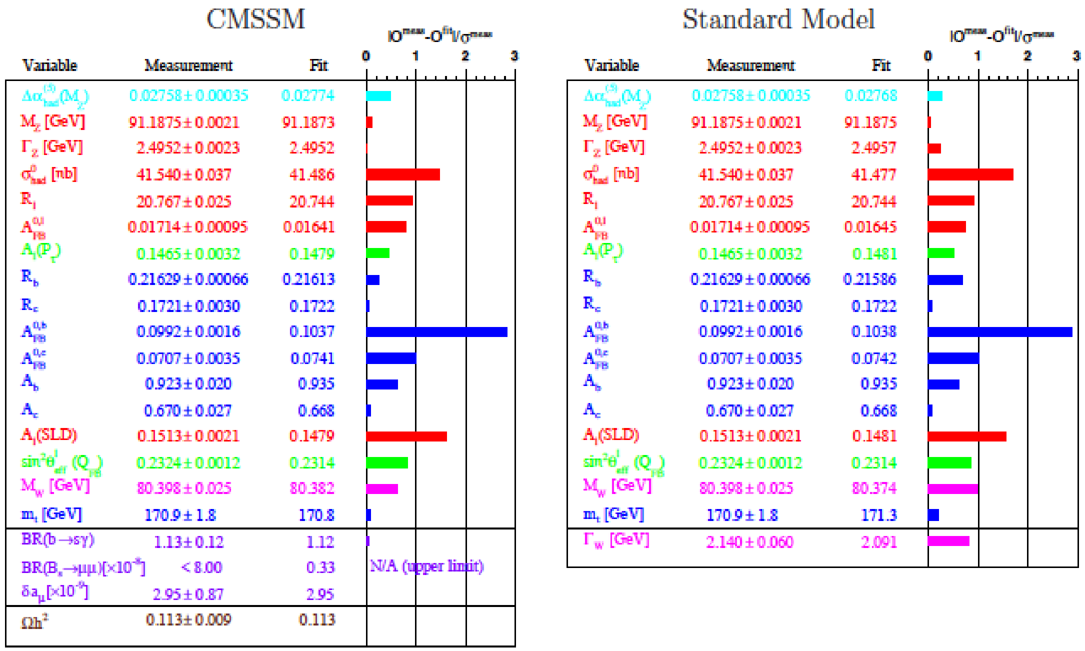
remains much less affected Buchmueller et al., Heinemeyer et al.




❐ for $M_W$ SUSY looks favored by data!

● sensitivity to top/stop sector strongly enhances in $M_W$

● General: MSSM results merge into SM results for larger SUSY masses, as decoupling is at work.



**CMSSM**

| Variable | Measurement | Fit | $\vert O^{meas}-O^{fit}\vert/\sigma^{meas}$ |
|---|---|---|---|
| $\Delta\alpha_{had}^{(5)}(M_Z)$ | $0.02758 \pm 0.00035$ | 0.02774 | |
| $M_Z$ [GeV] | $91.1875 \pm 0.0021$ | 91.1873 | |
| $\Gamma_Z$ [GeV] | $2.4952 \pm 0.0023$ | 2.4952 | |
| $\sigma_{had}^0$ [nb] | $41.540 \pm 0.037$ | 41.486 | |
| $R_l$ | $20.767 \pm 0.025$ | 20.744 | |
| $A_{FB}^{0,l}$ | $0.01714 \pm 0.00095$ | 0.01641 | |
| $A_l(P_\tau)$ | $0.1465 \pm 0.0032$ | 0.1479 | |
| $R_b$ | $0.21629 \pm 0.00066$ | 0.21613 | |
| $R_c$ | $0.1721 \pm 0.0030$ | 0.1722 | |
| $A_{FB}^{0,b}$ | $0.0992 \pm 0.0016$ | 0.1037 | |
| $A_{FB}^{0,c}$ | $0.0707 \pm 0.0035$ | 0.0741 | |
| $A_b$ | $0.923 \pm 0.020$ | 0.935 | |
| $A_c$ | $0.670 \pm 0.027$ | 0.668 | |
| $A_l$(SLD) | $0.1513 \pm 0.0021$ | 0.1479 | |
| $\sin^2\theta_{eff}^{l}$ $(Q_{FB})$ | $0.2324 \pm 0.0012$ | 0.2314 | |
| $M_W$ [GeV] | $80.398 \pm 0.025$ | 80.382 | |
| $m_t$ [GeV] | $170.9 \pm 1.8$ | 170.8 | |
| $BR(b \to s\gamma)$ | $1.13 \pm 0.12$ | 1.12 | |
| $BR(B_s \to \mu\mu)[\times 10^{-8}]$ | $< 8.00$ | 0.33 | N/A (upper limit) |
| $\delta a_\mu[\times 10^{-9}]$ | $2.95 \pm 0.87$ | 2.95 | |
| $\Omega h^2$ | $0.113 \pm 0.009$ | 0.113 | |

**Standard Model**

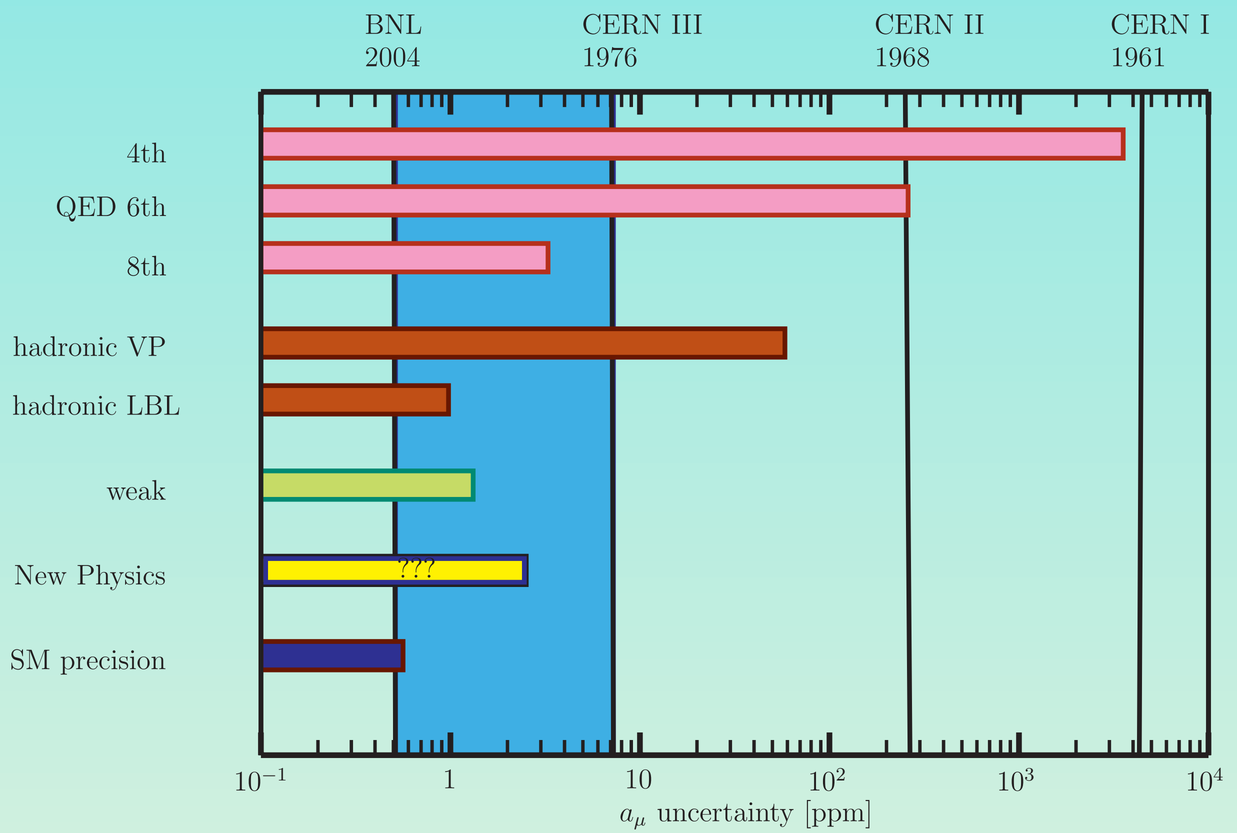
| Variable | Measurement | Fit |
|---|---|---|
| $\Delta\alpha_{had}^{(5)}(M_Z)$ | $0.02758 \pm 0.00035$ | 0.02768 |
| $M_Z$ [GeV] | $91.1875 \pm 0.0021$ | 91.1875 |
| $\Gamma_Z$ [GeV] | $2.4952 \pm 0.0023$ | 2.4957 |
| $\sigma_{had}^0$ [nb] | $41.540 \pm 0.037$ | 41.477 |
| $R_l$ | $20.767 \pm 0.025$ | 20.744 |
| $A_{FB}^{0,l}$ | $0.01714 \pm 0.00095$ | 0.01645 |
| $A_l(P_\tau)$ | $0.1465 \pm 0.0032$ | 0.1481 |
| $R_b$ | $0.21629 \pm 0.00066$ | 0.21586 |
| $R_c$ | $0.1721 \pm 0.0030$ | 0.1722 |
| $A_{FB}^{0,b}$ | $0.0992 \pm 0.0016$ | 0.1038 |
| $A_{FB}^{0,c}$ | $0.0707 \pm 0.0035$ | 0.0742 |
| $A_b$ | $0.923 \pm 0.020$ | 0.935 |
| $A_c$ | $0.670 \pm 0.027$ | 0.668 |
| $A_l$(SLD) | $0.1513 \pm 0.0021$ | 0.1481 |
| $\sin^2\theta_{eff}^{l}$ $(Q_{FB})$ | $0.2324 \pm 0.0012$ | 0.2314 |
| $M_W$ [GeV] | $80.398 \pm 0.025$ | 80.374 |
| $m_t$ [GeV] | $170.9 \pm 1.8$ | 171.3 |
| $\Gamma_W$ [GeV] | $2.140 \pm 0.060$ | 2.091 |

global fit in the constrained MSSM including data from $g-2$, $B$ physics, and cosmic relic density

*[O. Buchmueller, . . . , Weber, Weiglein, arXiv:0707.3447]*

- SUSY does not yield better global fit! Means SUSY effects on precision observables are small, looks like heavier SUSY spectrum favored (decoupling at work)! Higgs at 125 GeV also looks to piont in this direction. And the muon $g-2$ deviation? A puzzle yet to be solved!

And here we are:




Sensitivity of $g-2$ experiments to various contributions. The increase in precision with the BNL $g-2$ experiment is shown as a gray vertical band. New Physics is illustrated by the deviation $(a_\mu^{\rm exp} - a_\mu^{\rm the})/a_\mu^{\rm exp}$

# Upcoming Experiments

**Fermilab E989: Approved January 2011**

- Re-locate the $(g - 2)$ storage ring to Fermilab
- Use the many proton storage rings to form the ideal proton beam
- Use one of the antiproton rings as a 900 m decay line to produce a pure muon beam
- Accumulate 21 times the statistics
- Improve the systematic errors
- Final goal: At least a factor of 4 more precise over E821

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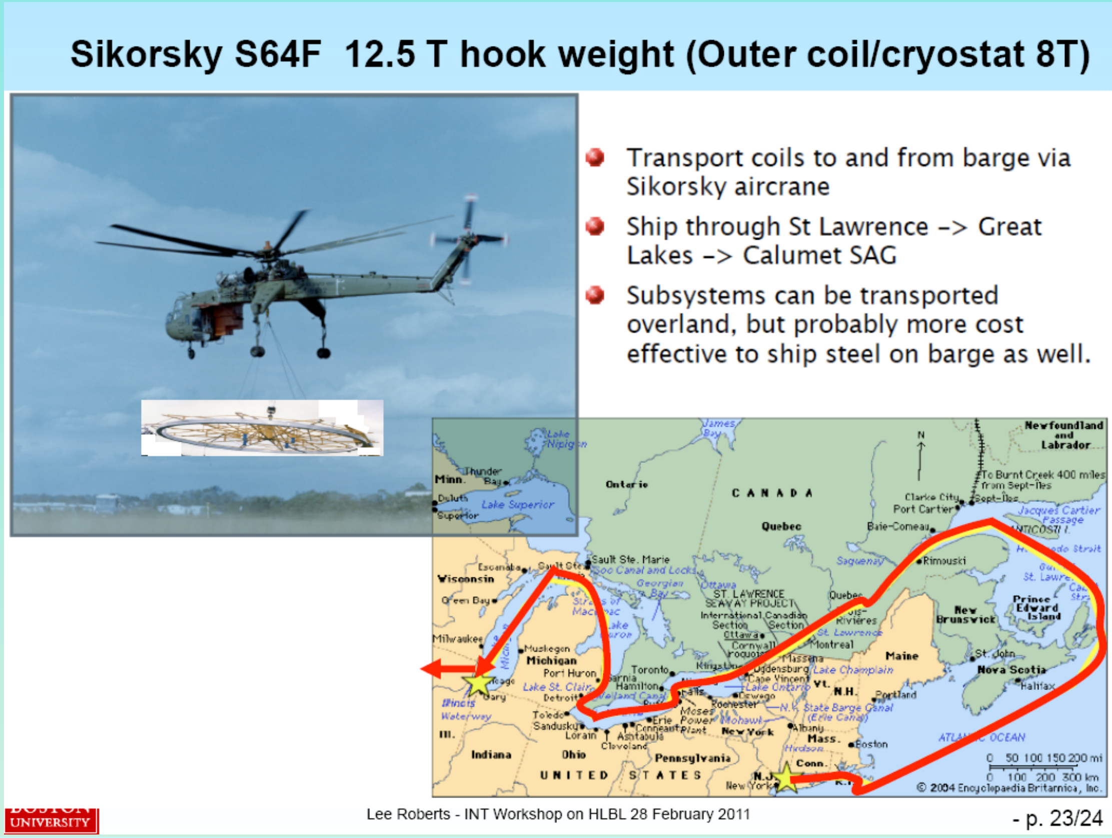
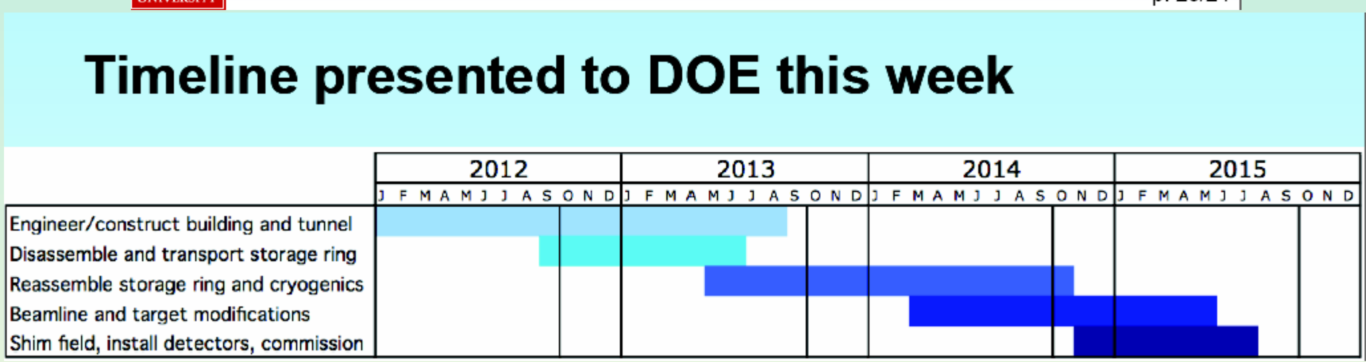
# The adventure:



## On this timescale it's essential that the theory improve

- Lowest-order hadronic
  - BaBar and Belled have additional unanalyzed data
    - especially important for multihadron channels
  - VEPP2000 at Novosibirsk
    - CMD3
    - SND
- HLBL
  - Agreement among theorists and additional work
  - KLOE 2 photon physics
  - BES, Mainz

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# The new muon $g-2$: Fermilab E989

- $\delta a_\mu = 16 \times 10^{-11}$ by 2015
- Magnetic field: $\frac{\delta \langle B \rangle_\mu}{\langle B \rangle_\mu} \leq 2 \times 10^{-8}$
- Requires 10% error on HLbL
- HLbL white paper in progress

Present:

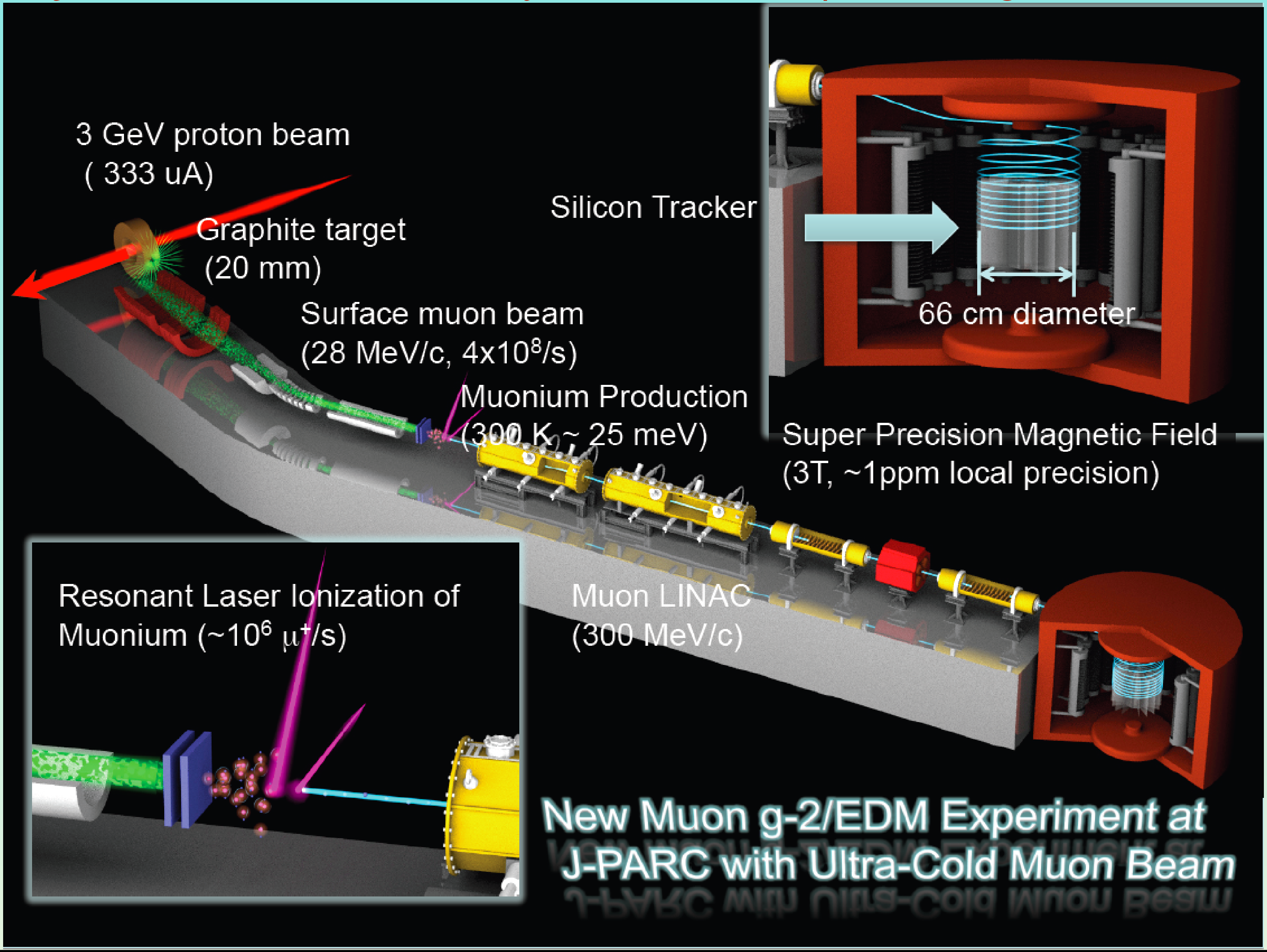
- $a_\mu^{\mathrm{exp}} = 116\,592\,089(63) \times 10^{-11}$ ; $a_\mu^{\mathrm{SM}} = 116\,591\,793 \pm 51 \times 10^{-11}$

E989: statistics $21\times$; total error factor $4$ more precise

$$\left.\begin{array}{l} \sigma_{\mathrm{stat}} = 0.1 \text{ ppm} \\ \sigma_{\mathrm{syst}} = 0.1 \text{ ppm} \end{array}\right\} \sigma_{\mathrm{tot}} = 0.14 \text{ ppm}$$

- $a_\mu^{\mathrm{exp}} = 116\,59x\,xxx(16) \times 10^{-11}$

## Muon $g-2$/EDM at J-PARC: very different concept, working with slow muons



From: N. Saito KEK

# BNL, FNAL, and J-PARC

- **complimentary**

| | BNL-E821 | Fermilab | J-PARC |
|---|---|---|---|
| Muon momentum | 3.09 GeV/c | | 0.3 GeV/c |
| gamma | 29.3 | | 3 |
| Storage field | B=1.45 T | | 3.0 T |
| Focusing field | Electric quad | | None |
| # of detected $\mu$+ decays | 5.0E9 | 1.8E11 | 1.5E12 |
| # of detected $\mu$- decays | 3.6E9 | - | - |
| Precision (stat) | 0.46 ppm | 0.1 ppm | 0.1 ppm |

# Outlook

Precision experiments remain an important complement to LHC:

$a_\mu$ still a great challenge!

Time horizon for next step in improvement: 5 years

Will provide important information on Physics Beyond the SM scenarios!

Provided deviation is real $3\sigma \to 9\sigma$ possible?

If SUSY:

$$\delta a_\mu \leftrightarrow \text{sign}(\mu) \text{ and } \tan\beta$$

If not SUSY or 2HDM may be even more interesting!

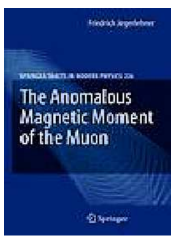
In any case
establishing a new theory replacing SM
likely is a long way to go and requires
efforts on very different levels

Complementarity crucial: LHC, ILC, Super-B, g-2/EDM, MEG, DM search and all that!

This was

**Muon $g-2$ in a Nutshell**

Further reading:
F. Jegerlehner, A. Nyffeler, Phys. Rept. **477**:1-110,2009, arXiv:0902.3360 [hep-ph]

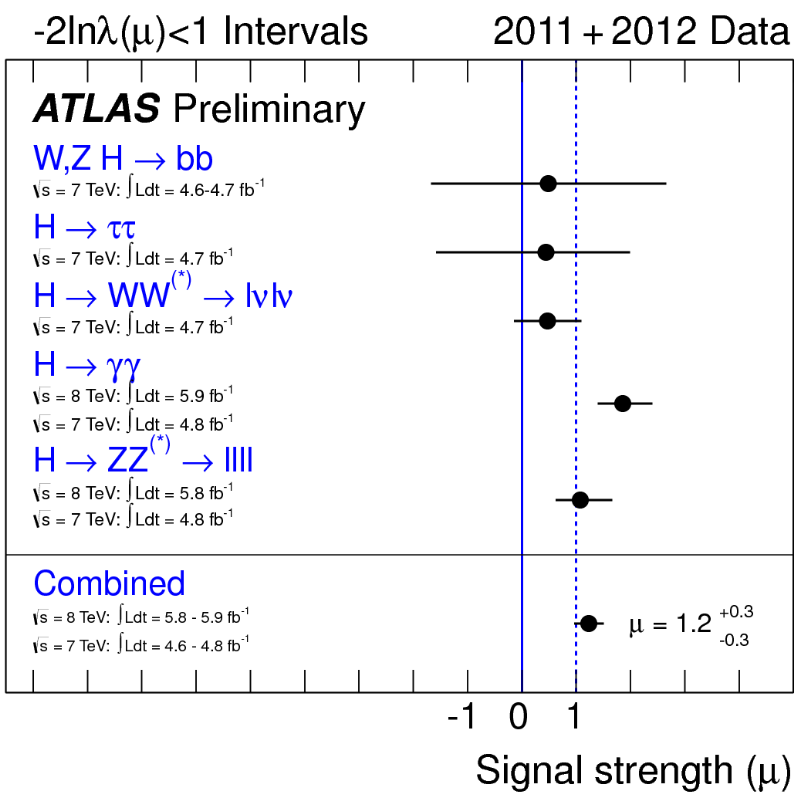


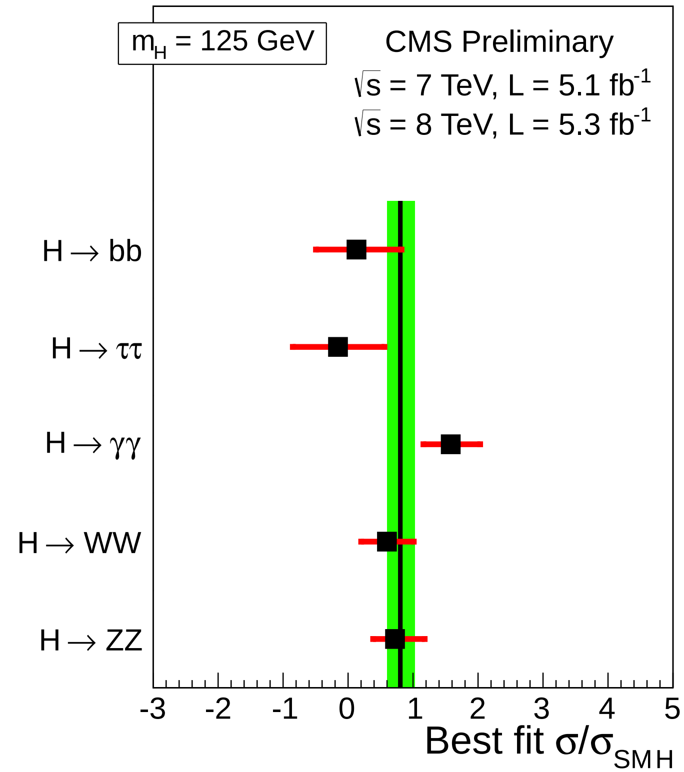

Book: F. Jegerlehner,
The Anomalous Magnetic Moment of the Muon,
Springer Tracts in Modern Physics,
Vol. 226, November 2007

Thank you for your attention!

# Backup Slides

# Is SUSY found?



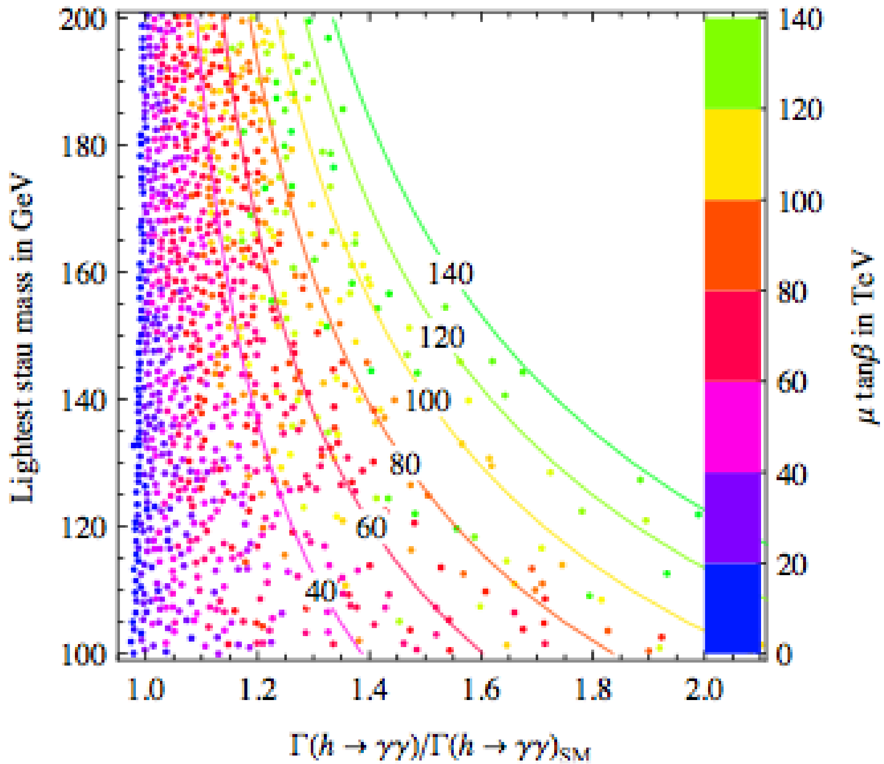

Data/SM excess of $H \to \gamma\gamma$ while $ZZ^*, WW^*$ in accord with SM at both ALTAS and CMS

Last weeks paper by Guidice, Paradisi, Strumia arXiv:1207.6393v1 [hep-ph]

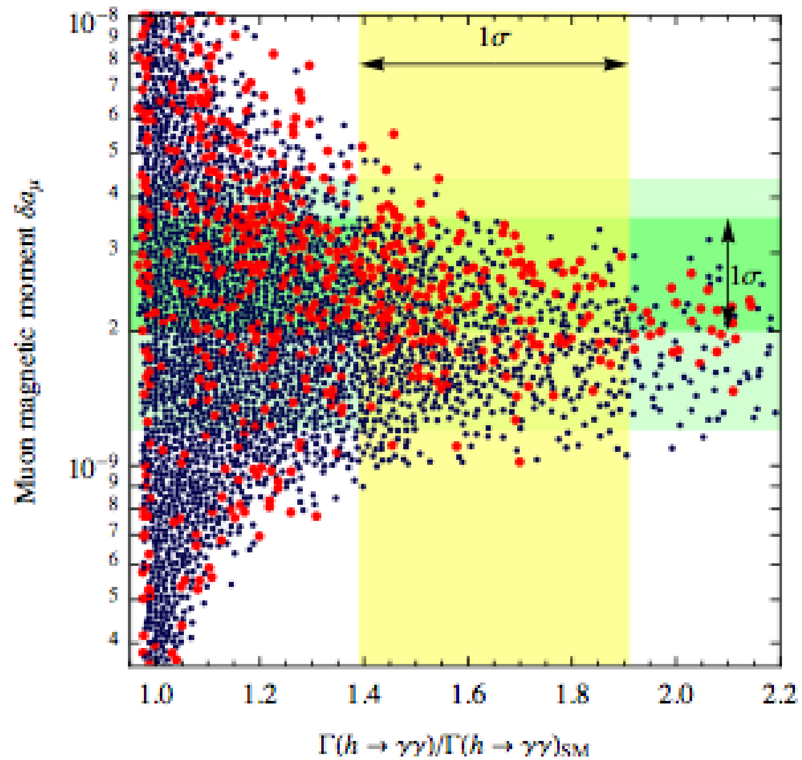
❒ light maximally mixed stau-loop can accomodate for it (stau in range 100-200 GeV)

❒ peculiar technically "unnatural" choice of parameters allows to explain $\delta a_\mu$ by SUSY.

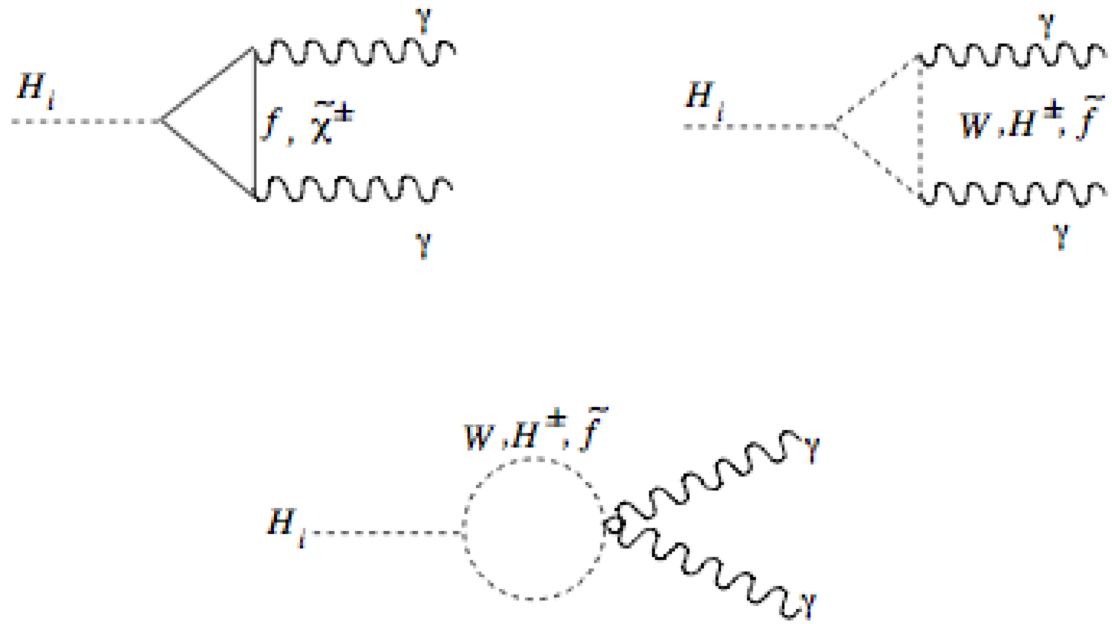
❒ requires higginos above 1 TeV and a light bino as the LSP



Lightest stau mass in GeV
$\mu \tan\beta$ in TeV
$\Gamma(h \to \gamma\gamma)/\Gamma(h \to \gamma\gamma)_{SM}$

$1\sigma$
Muon magnetic moment $\delta a_\mu$
$\Gamma(h \to \gamma\gamma)/\Gamma(h \to \gamma\gamma)_{SM}$




Higgs decay in the MSSM: $f = t, b, \tau$ and $\tilde{f} = \tilde{t}_{1,2}, \tilde{b}_{1,2}, \tilde{\tau}_{1,2}$

$$g_{h\tilde{\tau}_1\tilde{\tau}_1} \;=\; T_3^\tau \cos^2\theta_{\tilde{\tau}} - Q_\tau \sin^2\theta_W \cos 2\theta_{\tilde{\tau}} - \frac{m_t au^2}{M_Z^2} - \frac{m_\tau (A_\ell - \mu \tan\beta)}{2M_Z^2} \sin 2\theta_{\tilde{\tau}}$$

$$g_{h\tilde{\tau}_2\tilde{\tau}_2} \;=\; T_3^\tau \sin^2\theta_{\tilde{\tau}} + Q_\tau \sin^2\theta_W \cos 2\theta_{\tilde{\tau}} - \frac{m_t au^2}{M_Z^2} + \frac{m_\tau (A_\ell - \mu \tan\beta)}{2M_Z^2} \sin 2\theta_{\tilde{\tau}}$$

For $m_{\tilde{\tau}_2} \gg m_{\tilde{\tau}_1}$ and large $\tan\beta$

$$\cos 2\theta_{\tilde{\tau}} = \frac{m_L^2 - m_R^2}{m_{\tilde{\tau}_1}^2 - m_{\tilde{\tau}_2}^2} \;; \quad \sin 2\theta_{\tilde{\tau}} = \frac{2m_\tau (A_\ell - \mu \tan\beta)}{m_{\tilde{\tau}_1}^2 - m_{\tilde{\tau}_2}^2}$$

$$m_{\tilde{\tau}_{1,2}}^2 = \frac{1}{2}\left[m_L^2 + m_R^2 \mp \sqrt{(m_L^2 - m_R^2) + 4m_\tau^2\, (A_\ell - \mu \tan\beta)^2}\right]$$

$$\frac{\Gamma(h \to \gamma\gamma)_{\rm MSSM}}{\Gamma(h \to \gamma\gamma)_{\rm SM}} \approx \left(1 + 0.025\,\frac{|m_\tau \mu \tan\beta \sin 2\theta_{\tilde{tau}}|}{m^2_{\tilde{\tau}_1}}\right)^2$$

$\mu \gg m_{L,R}, M_{1,2}$ common slepton/gaugino mass $\tilde{m} = m_{L,R} = M_{1,2}$

$$\delta a_\mu \approx 2.8 \times 10^{-9}\,\frac{\tan\beta}{20}\left(\frac{300\ \text{GeV}}{\tilde{m}}\right)^2\left[\frac{1}{8}\,\frac{10}{\mu/\tilde{m}} + \frac{\mu/\tilde{m}}{10}\right]$$

Stability bound of Higgs potential:

$$M_{\rm min} = \left[128.95 + \frac{M_t - 172.9\ \text{GeV}}{1.1\ \text{GeV}\ \times 2.2 - \frac{\alpha_s - 0.1184}{0.0007}\ \times 0.56}\right]\ \text{GeV}$$