Drell–Yan production in hadron-hadron collisions

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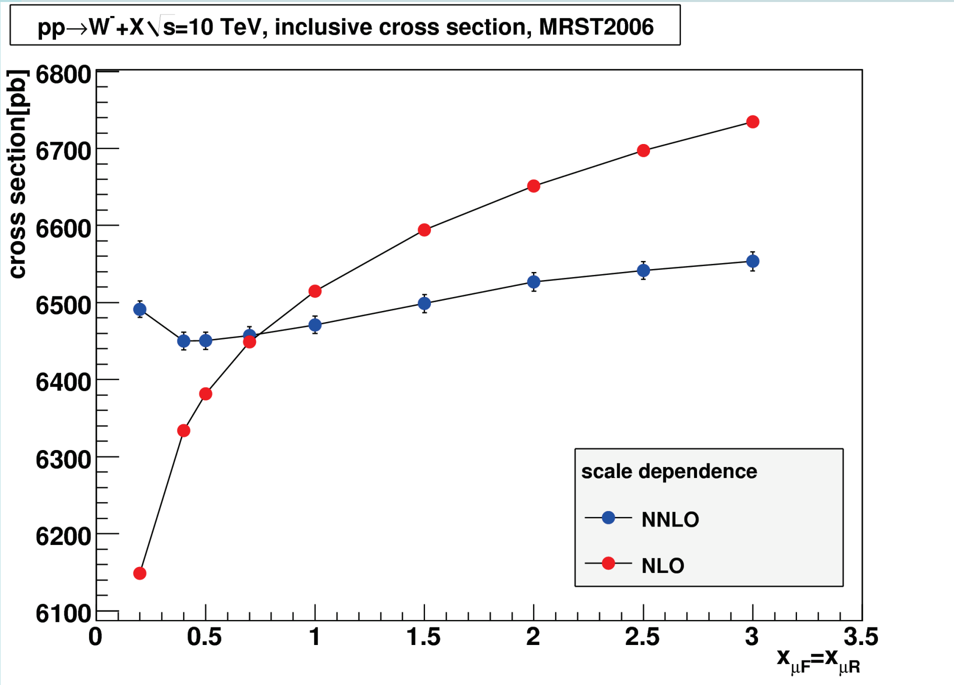
- introduction
- calculation
- cancelation of divergencies
- conclusions

single W and Z production

- one of the main processes at LHC and the background to many processes
- access to the parameters ot weak bosons
- gauge the parton distribution functions
- luminosity measerement
  - using W/Z counting: needs theoretical cross-section with cuts
- both differntial and total cross-section are required
- a lot already has been done!

Calculations at NNLO:FEWZ

- FEWZ code (splitted in W and Z branches) from K. Melnikov and F. Petriello, Phys.Rev.Lett 96(2006) 231803, Phys.Rev. D74(2006) 114017
- fully differential calculation of differential cross sections at NNLO
- PDF interface for CTEQ6.6 and MRST2006 NNLO
- implement ATLAS cuts
- check scale dependence, estimate PDF uncertainty
- requires heavy CPU (NAF)



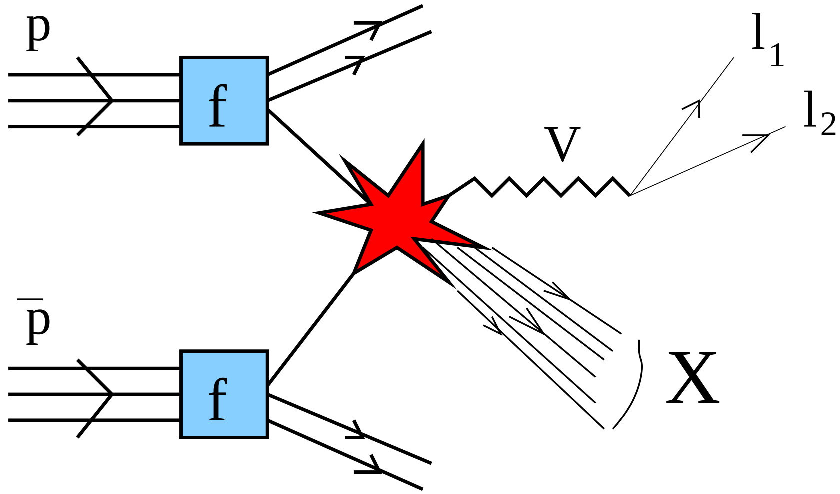
Example of total W^- cross section at 10 TeV scale dependence as estimate of missing higher orders.

simulatneous variation of renormalisation and factorisation scale

NNLO error < 1%
NLO error = 2.5%

Drell–Yan process

$$p(P_1)+\bar{p}(P_2)\to V(Q)+X$$
$$V=Z,W,\gamma^*$$



<u>kinematics:</u>

$$S=(P_1+P_2)^2,\quad T=(P_1-Q)^2,$$
$$U=(P_2-Q)^2,\quad S_2=(P_1+P_2-Q)^2$$

$$Y=\frac{1}{2}\ln\frac{Q^2-T}{Q^2-U},$$
$$Q_T^2=\frac{UT-Q^2S_2}{S}$$

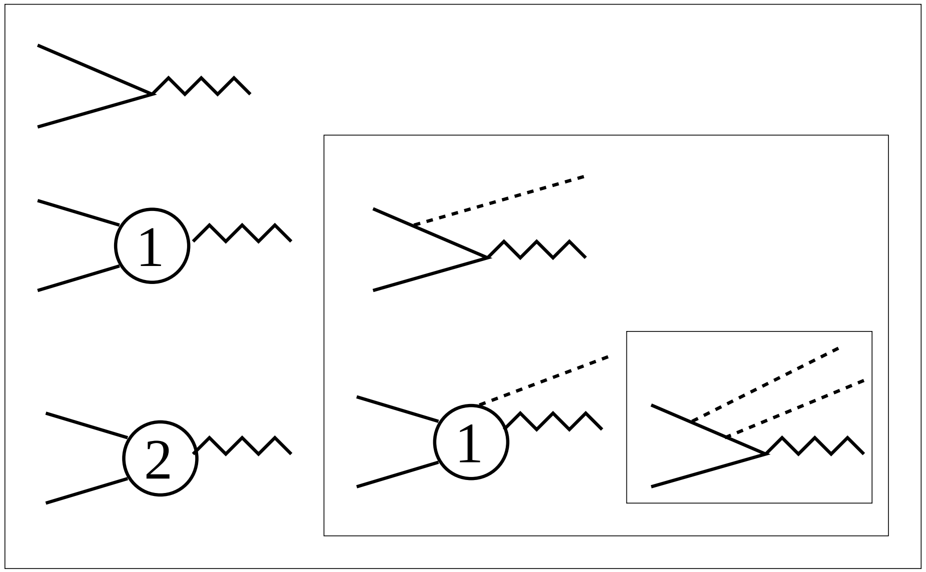
$$\frac{d\sigma_{p\bar{p}}}{dQ_T^2\,dY}=\sum_{\text{part. } i,j}\int dx_1\,dx_2\,f_i^{(p)}(x_1)\,\frac{s\,d\hat{\sigma}_{ij}}{dt\,du}(x_1P_1,x_2P_2)\,f_j^{(\bar{p})}(x_2)$$

theoretical uncertainites

- uncertainties in PDF $f_j(x, \mu_F) \longrightarrow$ few %
- narrow width approximation
  - stable on-shell W, Z
  - background (?): direct production, WW-, etc.
- perturbative corrections to hard processes $\hat{\sigma}_{ij}$
  - NLO and NNLO corrections (QCD) — Gonsalves, Pawlowski, Wai; van Neerven, Matsuura, Anastasiou, Melnikov, Petrielo
  - electroweak (EW) corrections — Hollik et al.
  - mixed EW/QCD corrections — Kühn, Kulesza, Pozzorini, Schulze
  - resummations of low p_T and/or EW Sudakov logaritms

orders

$$p + \bar{p} \longrightarrow Z, W, \gamma^*$$



- UV and IR divergencies

  $\longrightarrow$ use dimensional regularization

- cancellation of singularities

  – soft and (final) collinear singularities (Lee, Kinoshita)

  – UV renormalization $\longrightarrow \mu_R$

  – collinear factorization $\longrightarrow \mu_F$

DY: partonic subprocesses

possible initial states:

$$\bar{q}' + q, \quad \bar{q} + q \qquad g + g, \qquad \bar{q} + g, \qquad q + g,$$

$$q' + q, \qquad q + q, \qquad \bar{q}' + \bar{q}, \qquad \bar{q} + \bar{q}$$

possible final states (to NNLO + EW/QCD):

$$V, \qquad V + g, \qquad V + \gamma, \qquad V + q, \qquad V + \bar{q},$$

$$V + g + g, \qquad V + g + \gamma, \qquad V + g + q, \qquad V + g + \bar{q},$$

$$V + q + \bar{q}, \qquad V + q + q', \qquad V + \bar{q} + \bar{q}'$$

with V being Z, W or γ^*

evaluation

- generation of diagrams (integrands) $\longrightarrow$ DIANA — Tentyukov
- reduction of tensor integrals to scalar $\longrightarrow$ AIR — Anastasiou, Lazolopoulos
- evaluation of phase-space integrals $\longrightarrow$ FORM — Vermaseren
- ultraviolet renormalization
- cancelation of IR poles
- convolution with PDF's

integration by parts

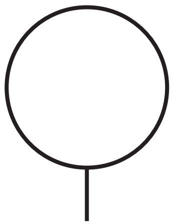
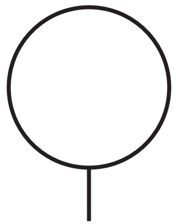
L-loop integral in d-dimensions

$$I^{(L)}_{a_1,a_2,\ldots,a_n} = \int \frac{d^dk_1\ldots d^dk_L}{D^{a_1}D^{a_2}\ldots D^{a_n}}$$

partial integration

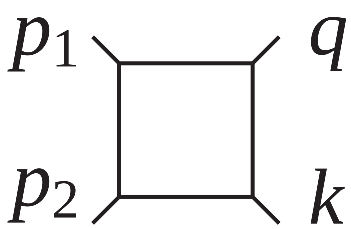
$$0 = \int \frac{\partial}{\partial k_\mu} p_\mu f(k,\ldots,p,\ldots) d^dk$$

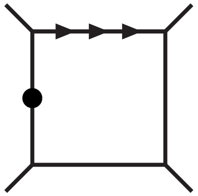
$\longrightarrow$ algebraic relation between I's with different indices a_i

<u>example:</u> $2m^2a_1$  a_1+1 $=$ $(d-2a_1)$  a_1

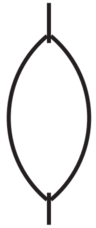
IBP: 1-loop expamples

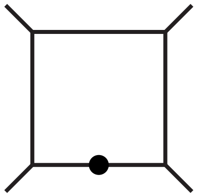
$$s=(p_1+p_2)^2, \quad t=(p_1-q)^2, \quad u=(p_2-q)^2$$
$$p_1^2=0, \quad p_2^2=0, \quad k^2=0, \quad q^2=Q^2$$



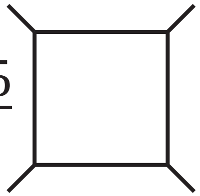


$$= \frac{d(u+s)-4s-2t-6u}{(d-3)(d-4)t^2}$$

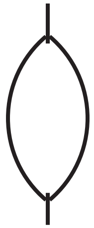




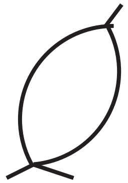
$$= -\frac{d-5}{s}$$



$$-\frac{4}{(d-3)(d-5)(d-6)\,s\,t\,(s+u)}$$

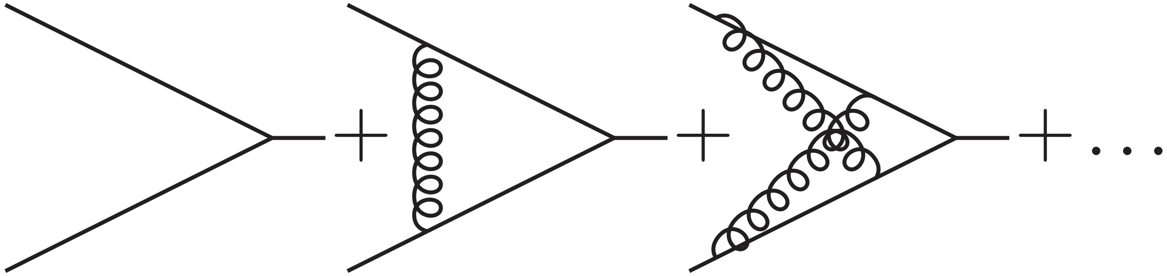


$$+\frac{4}{(d-3)(d-5)(d-6)\,s\,(s+u)\,(s+t+u)}$$



2 loop virtual contribution

QCD quark formfactor: $ig\,\bar{u}\gamma_\mu \frac{\lambda_a}{2} F\, u$



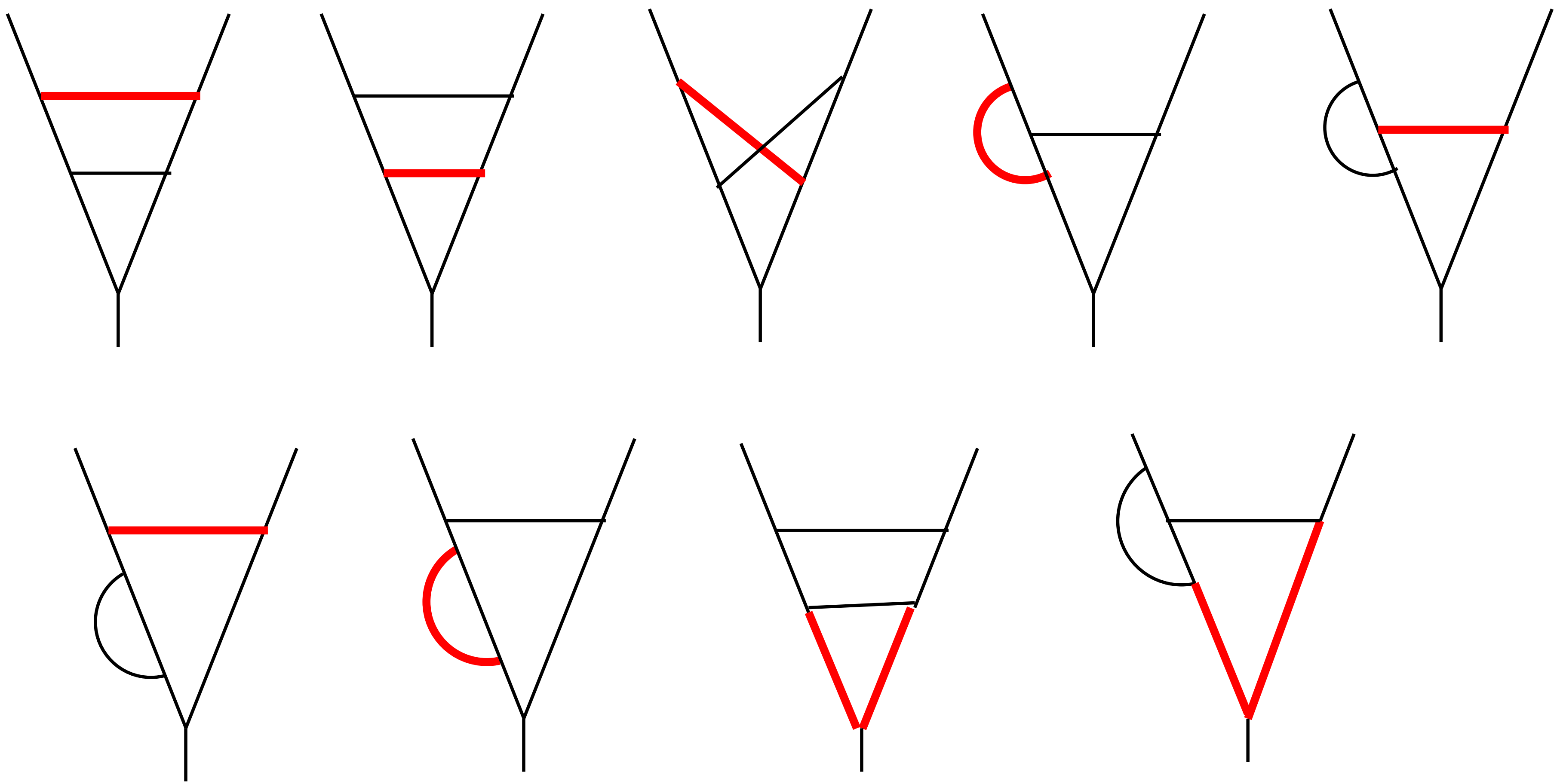
Matsuura, van Neerven 1988

$$F(Q^2) = 1 + \frac{g^2}{(4\pi)^2}(-Q^2)^{\varepsilon/2} C_F \left(-\frac{8}{\varepsilon^2} + \frac{6}{\varepsilon} + \ldots \right)$$

$$+ \left(\frac{g^2}{(4\pi)^2}(-Q^2)^{\varepsilon/2} \right)^2 \left\{ C_F^2 \left(\frac{32}{\varepsilon^4} - \frac{48}{\varepsilon^3} + \ldots \right) + C_F C_A \ldots \right\}$$

- mixed QCD/EW correction $\longrightarrow$

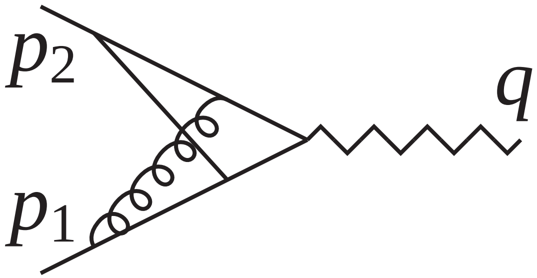
2-loop virtual corrections (EW/QCD contributions)



red lines — massive
black lines — massless

Kotikov, Kühn, O.V., 2007

expansions and differential equations



$$q = p_1 + p_2$$

$$\begin{cases} p_1^2 = 0 \\ p_2^2 = 0 \\ z = s/M^2 \end{cases}$$

- use large mass expansion in parameter $z = s/M^2$ or
- use IBP to write the differential equation in z
- solve one of the above with the anzatz

$$\text{Diagram}(z) = x_1\phi_1(z) + x_2\phi_2(z) + \ldots + x_N\phi_N(z)$$

where

$x_1, x_2, \ldots, x_N$ are *unknown numbers*

$\phi_1, \phi_2, \ldots, \phi_N$ are *known functions*

Harmonic basis up to weight 4 $(z = q^2/M_V^2)$

1	2	3	4
$\log(1-z)$	$\log^2(1-z)$	$\log^3(1-z)$	$\log^4(1-z)$
	$\mathrm{Li}_2(z)$	$\mathrm{Li}_2(z)\log(1-z)$	$\mathrm{Li}_2(z)\log^2(1-z)$
		$\mathrm{Li}_3(z)$	$\mathrm{Li}_3(z)\log(1-z)$
		$S_{12}(z)$	$S_{12}(z)\log(1-z)$
			$\mathrm{Li}_2^2(z)$
			$\mathrm{Li}_4(z)$
			$S_{13}(z)$
			$S_{22}(z)$

with

$$S_{n,p}(z) = \frac{(-1)^{n+p-1}}{n!(p-1)!}\int_0^1 \log^{p-1}t \, \log^n(1-zt)\,\frac{dt}{t}$$

more generally...

$$H_{a,b,\ldots,c}(z) = \int_0^z \frac{dx_1}{x_1 - a} \int_0^{x_1} \frac{dx_2}{x_2 - b} \cdots\cdots \int_0^{x_k} \frac{dx_{k-1}}{x_{k-1} - c}$$

- in case $a, b, \ldots, c = +1, -0, 1$ harmonic polylogarithms
- *nonabelian* diagrams require also functions with $a, b, \ldots, c = +e^{i\pi/3}, -e^{i\pi/3}$

  or equviv. factors $\int_0^{x_j} dx_j / \sqrt{x_j(4 + x_j)}$ (4 new functions)
- general $a, b, \ldots, c$ — Lappo–Danilewski polylogarithms

relations to polylogarithms

can express these functions in terms of polylogarithms of new nonlinear argument (at weight 3)

$$y = \frac{1-\sqrt{z/(z-4)}}{1+\sqrt{z/(z-4)}}$$

e.g.

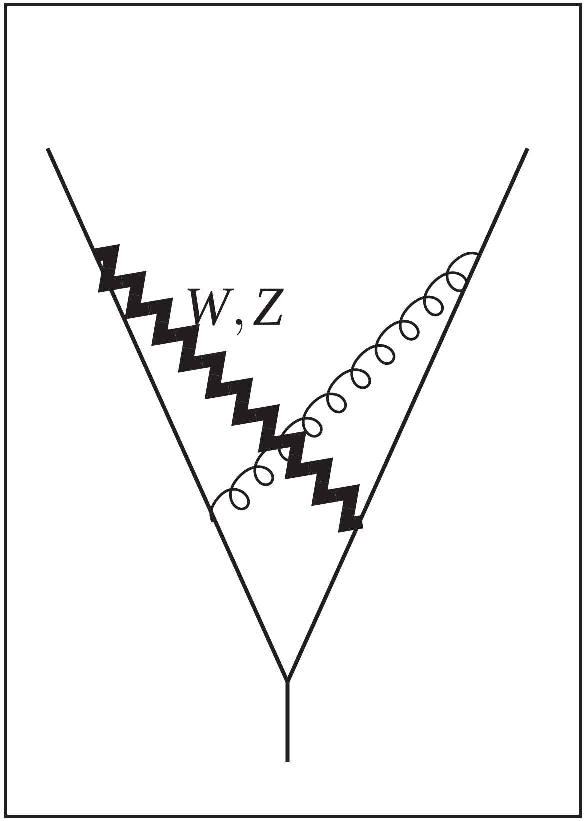
$$\int_0^{-z} \frac{dx_1}{\sqrt{x_1(x_1+4)}} \int_0^{x_1} \frac{dx_2}{\sqrt{x_2(x_2+4)}} \int_0^{x_2} \frac{dx_3}{1+x_3}$$

$$= -\frac{1}{6}\ln^3 y + \frac{1}{2}\zeta(2)\ln y + \frac{2}{3}\zeta(3) + \mathrm{Li}_3(-y) - \frac{1}{9}\mathrm{Li}_3(-y^3)$$

at weight 4 $\longrightarrow$ 1 H-function remains

Solution. Generated code for FORM (example).

```
id N5(k2.p2,1,1,1,1,1,0) =
+ 1 * 1/z^0 * ( 2*LOG-3*zt2+3/2-3/4*LOG^2 )
+ 1 * 1/z^0 * (
    + (2*zt2-3+1/2*LOG^2) * 1 * log(1-z)
    + (LOG-2) * 1 * Li2(z)
    + 1 * 1 * log(1-z)*Li2(z)
    + (-1) * 1 * Li3(z)
    + 2 * 1 * S12(z)
    + (-2*zt2+3-1/2*LOG^2) * 1/z * log(1-z)
    + (-LOG+zt2-1+1/4*LOG^2) * 1/z * Li2(z)
    + (-1) * 1/z * log(1-z)*Li2(z)
    + (-LOG+1) * 1/z * Li3(z)
    + (-2) * 1/z * S12(z)
    + 1/4 * 1/z * Li2(z)^2
    + 3/2 * 1/z * Li4(z)
    + (-1) * 1/z * S22(z)
         );
```



INFRARED SINGULARITIES

- soft
- collinear
  - final
  - initial

subtruction (I)

problem to combain

$$d\sigma \sim \int_n d\sigma^{(n)} + \int_{n+1} d\sigma^{(n+1)} + \int_{n+2} d\sigma^{(n+1)} + \dots$$

$$\int_n d\sigma^{(n)} + \int_{n+1} [d\sigma^{(n+1)} - dA^{(n+1)} + dA^{(n+1)}] = \underbrace{\int_n [d\sigma^{(n)} - \int_1 dA^{(n+1)}]}_{\text{finite}} + \underbrace{\int_{n+1} [d\sigma^{(n+1)} - dA^{(n+1)}]}_{\text{finite}}$$

We should choose $dA^{(n+1)}$ such that:

- it has the same *infrared* structure as $d\sigma^{(n+1)}$
- it should be simple enough to integrate $\int_1 dA^{(n+1)}$

Different choises, different methods

- space slicing subtruction
- ”subtruction” subtruction
- dipole/antenna subtruction

structure of the cross-sections and singularities

single V: $$\frac{d\hat{\sigma}_{ij}}{dt\,du} \sim \delta(t)\delta(u)\delta(s_2)$$

V + jet: $$\frac{d\hat{\sigma}_{ij}}{dt\,du} \sim \delta(s_2)$$

V + 2 jets: $$\frac{d\hat{\sigma}_{ij}}{dt\,du} \sim \int \sin^{d-3}\theta_1 \sin^{d-4}\theta_2\, d\theta_1\, d\theta_2$$

- $d\Gamma_3 = d\Gamma_3'\, d\theta_1,\, d\theta_2$
- analytical integration over $d\theta_1,\, d\theta_2$
- $d\Gamma_1 = d\Gamma_1 \int \delta(t)\delta(u)\, dt\, du$

structure of the cross-sections and singularities

$$d\Gamma_3' \sim (tu - Q^2 s_2)^{-\varepsilon} (s_2)^{-\varepsilon}$$

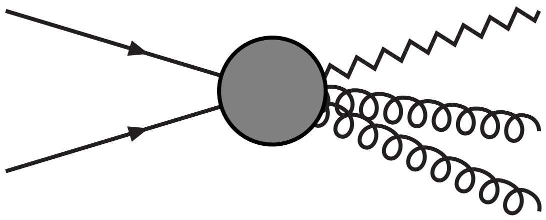
$$\longrightarrow d\Gamma_2 \sim (tu)^{-\varepsilon} \delta(s_2)$$

$$\longrightarrow d\Gamma_1 \sim \delta(t)\delta(u)\delta(s_2)$$

always

$$s + t + u = Q^2 + s_2$$

soft + collinear (1)



- $s_2 \longrightarrow 0$ if k_1 and/or k_2 soft, or $k_1 || k_2$

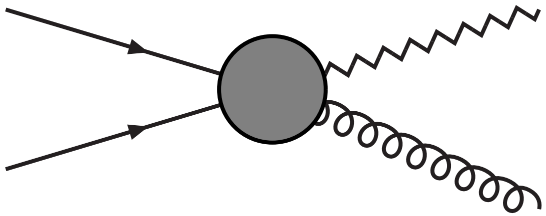
- $\frac{1}{s_2^{1+\varepsilon}} = -\frac{1}{\varepsilon}\delta(s_2)(1 - \varepsilon \ln A) + \frac{1}{(s_2)_A} + O(\varepsilon)$

  with

$$\int_0^A \frac{1}{(s_2)_A} f(s_2) ds_2 = \int_0^A \frac{1}{(s_2)_A} (f(s_2) - f(0)) ds_2$$

- cancellation of $1/\varepsilon$ with the 1-loop virtual part

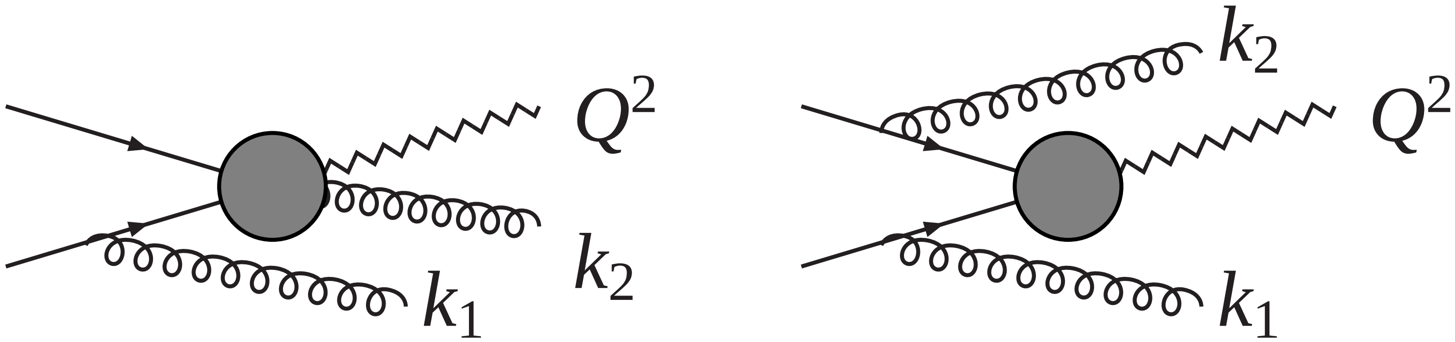
soft + collinear (2)



further reduction:

- $t + u \rightarrow 0$ if k soft, or $k||p_1$ or $k||p_2$
- $\frac{1}{t^\alpha u^\beta} = \frac{1+\delta(t)\delta(u)}{t^\alpha u^\beta} - \delta(t)\delta(u)(-a)^{2-\alpha-\beta} B_{\alpha\beta}$
- $\frac{1}{(t+u)^2 t^\alpha u^\beta} = \frac{1-\delta(t)\delta(u)}{(t+u)^2 t^\alpha u^\beta} + \delta(t)\delta(u) C_{\alpha,\beta}$,
- cancellation of $1/\varepsilon$ with the 2-loop virtual part

collinear factorization (1)



- $(p_1 - k_1)^2 \longrightarrow 0$ if $k_1 || p_1$

- "renormalization" of structure function

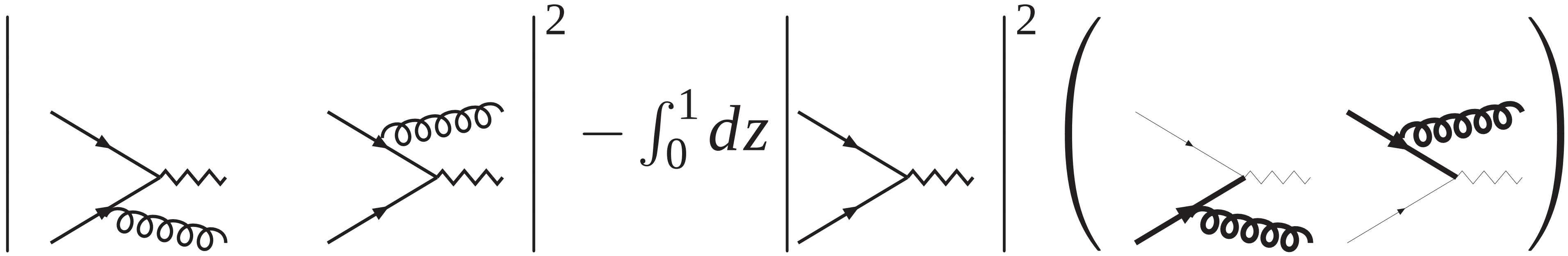
$$f_i^{(2)}(x,\mu_F) = \int_x^1 \frac{dz}{z}\left[\delta_{ji}\delta(1-z) - \frac{\alpha_s}{2\pi}\frac{1}{\varepsilon}P_{ji}^{(1)}(z,\mu_f)\right] f_j^{(1)}(x/z)$$

with splitting function $P_{ji}(z,\mu_F)$

- for partonic cross-section $\longrightarrow$ inverse procedure

$$\hat{\sigma}_{ij}^{(2)} = \hat{\sigma}_{ij}^{(2),0} + \int_0^1 dz\, \frac{\alpha_s}{2\pi}\frac{1}{\varepsilon}P_{ki}^{(1)}(z,\mu_f)\, \hat{\sigma}_{kj}^{(1),0}\Big|_{p_1 \to z p_1} + (1 \leftrightarrow 2)$$

initial collinear (2)



$$P_{qq}(z,\mu_F) \;=\; (\mu_F^2)^{-\varepsilon} C_F \left[\frac{1}{\varepsilon}\left\{\frac{1+z^2}{(1-z)_+} + \frac{3}{2}\delta(1-z)\right] + C(z)\right\} + \ldots$$

- pole part is unique
- $C(z)$ is scheme dependent

conclusions

- DY single vector boson production plays an important role in at the LHC experiment and the test of the Standard Model
- we discussed the framework of calculation of the differential distribution in W/Z production

  $$\frac{d\sigma}{dQ_T^2\, dY}$$

  the integration to the total cross-section also is possible
- analytical evaluation ot 1- and 2-loop diagrams for the mixed $O(\alpha\alpha_s)$ corrections are discussed
- the complete $O(\alpha\alpha_s)$ analysis is still missing

  combine all together ...