Dark Matter

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Standard Model explains all known experimental results in particle physics. So, the main motivation for LHC is understanding of Dark Matter.

There are 2 basic experimental facts concerning Dark Matter:

- Flat rotation curves for spiral galaxies
- WMAP measurement of DM fraction $\Omega = \rho_{DM}/\rho_c$

There are 2 kinds of experiments which try now to detect Dark Matter particles:

- Direct detection.
- Indirect detection.

Rotation curve.

Dependence of velocity of galactic gas from distance.

$$V^{ex}(R) \rightarrow Const >> V^{th}_{stars+gas}(R)$$

Velosity is measured via Doppler effect for 11cm Hydrogen line.

$$G \cdot M(R)/R^2 = V^2/R$$

$$M(R) = R \cdot V^2/G$$

$$\rho(R) = \frac{1}{4\pi R^2}\frac{dM(R)}{dR} = \frac{V^2}{4\pi G R^2}$$

In our Galaxy $V = 220km/c$, $R_{Sun} = 8kpc$, **so** $\rho(R_{Sun}) \approx 0.3GeV/cm^3$

Isothermal model

Dark Matter space distribution is related to velocity distibution caused by requarement of stability of DM density. (Vlasov equation). $\rho(R) \approx R^{-2}$ leads to isothermal model:

$$\phi(R) = V^2 \log(R)$$

$$\rho(R) = Const \int d^3v \exp^{-\frac{v^2/2+\phi(r)}{V^2/2}} =$$

So, for Dark Matter velosity one gets Maxwell distribution.

$$\rho(v) d^3v \sim exp(-v^2/V^2) d^3v$$

From the other side a cut $v < v_{max} \approx 600 km/s$ is expected.

Weighting of Univerce and Dark Matter

Friedman equation == Newton approach

$$(4/3\pi R^3\rho)\cdot G/R = v^2/2 = (dR/dt)^2/2; \quad H = R^{-1}dR/dt$$

$$\rho_c = 3(H\cdot M_p)^2/(8\pi); \quad M_p = G^{-1} = 1.22\cdot 10^{19}GeV$$

$$H = h100\frac{km}{c}/Mpc, \;\; h = 0.71 \pm 0.07 \quad => \quad \rho_c = h^2 10.54GeV/m^3$$

WMAP measurements of small angle temperature fluctuations of microwave background provides us

$$\Omega = \rho_{DM}/\rho_c = (0.09 - 0.11)h^2$$

So we have $\rho_{DM} \approx 1GeV/m^3$

Formation of Dark Matter dencity.

Stable massive particle with electroweak order of self annihilation cross section (WIMP) leads to appearance of relic density.

$n(t)$ ***-number density for the WIMP particle, then***

$$dn/dt = -3Hn - < \sigma \cdot v > (n^2(t) - n_{eq}^2(T))$$

s ***- entropy density.***

$$ds/dt = -3Hs; \quad H^2 = \frac{8\pi}{3M_p^2}\rho$$

For $Y = n/s$ ***we have***

$$dY/dt = - < \sigma \cdot v > s(Y^2 - Y_{eq}^2)$$

Because we know SM spectrum we know entropy density and matter density as given temperature

$$\rho(T) = g_{eff}(T)\frac{\pi^2 T^4}{30}; \quad s(T) = h_{eff}(T)\frac{2\pi^2 T^3}{45}$$

Entropy conservation

$$ds/dt = -3Hs; \quad H^2 = \frac{8\pi}{3M_p^2}\rho$$

allows to rewrite time evolution in terms of temperatute evolution

$$\frac{dY}{dT} = \sqrt{\frac{\pi g_*(T)}{45}} M_p < \sigma v > (Y^2(T) - Y_{eq}^2(T))$$

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At hight temperatures

$$\frac{dY}{dT} = 2A(T)Y_{eq}(T)(Y(T) - Y_{eq}(T))$$

$$Y_{eq}(T) \approx exp(-M_{DM}/T)$$

and

$$\Delta Y = Y(T) - Y_{eq}(T) \approx \frac{M_{DM}}{2A * T^2}$$

Below T_f ***where*** $Y(T) >> Y_{eq}(T)$

$$\frac{1}{Y(0)} = \frac{1}{Y(T_f)} + \int_0^{T_f} A(T)dT$$

Dark Matter and relic photons

Suppose $M_{DM} = 100 GeV$. Then WMAP tells us

$$Y_0 = R(photons/DM) = 4.5 \cdot 10^{10} = T_f M_p < v\sigma >$$

$M_p \approx 10^{19} GeV$, $< v\sigma > \approx (\alpha/M_Z)^2 \approx 10^{-8} GeV^{-2}$.

T_f should be about $M_{DM}/25$ to give $R(photons/DM) \approx 10^{10}$ at T_f.

Z_2 and co-annihilation

In order to get stable WIMP we need some discrete symmetry like Z_2. It can be R-parity in SUSY models. $x_5 \to -x_5$ in extra-dimension models.

In general $n(t)$ which appeared in equation above is a some over all odd particles. But contribution of large mass odd particles is suppressed by Boltzmann factor.

In case of small mass gap $< v\sigma >$ averaged over all odd particles is responsible for relic.

In SUSY $\tilde{\tau}$ co-annihilation helps to get Ω in WMAP region.

DM -nucleon-nucleus interactions.

Velocities of DM *have to be about rotation velocity of Sun (≈220 km/s). For typical* DM *and nucleus masses it leads to collisions with transfer momentum*

$$\Delta p \approx 100MeV \approx 1/(2fm)$$

For one side it is small enough to use $\Delta p = 0$ limit for DM-nucleon *collision, but nuclei form factors surely have to be taken into account.*

DM*-nucleon collision at rest can be presented as a sum of 2 orthogonal amplitudes,* scalar *one and* vector *one. Scalar* DM-proton *and* DM-neutron *amplitudes lead to scalar* DM-nucleus *amplitude*

$$\lambda_P Z + \lambda_N (A - Z)$$

So, the corresponding cross section is enhanced by the A^2 factor.

As for vector interaction, we know that angular momenta of protons (as well as neutrons) have a trend to compensate one other. So

$$\vec{J}_P = S_p^A \vec{J}_A / |J_A|$$

where S_p^A is about 0 - 0.5. So the vector **DM***-nucleus amplitude appears about*

$$S_P^A \xi_P + S_N^A \xi_N$$

where ξ_P and ξ_N are nucleon amplitude. In this case A-enhancement is absent. Because of periphery nucleons mainly contribute to J_N we have non-trivial form factors for protons and neutrons. At $\Delta p \neq 0$ $\vec{J}_P$ and $\vec{J}_N$ become non-collinear and the third form factor appears. So there are 3 form factors S_{00}, S_{01}, S_{11}.

DM - quark elastic scattering.

We don't restrict ourself by Majorana DM. Spin of DM particle could be 0, 1/2, 1, and it can be described as well as by neutral or complex field. In case of complex field DM-quark interactions can be separated on even and odd ones respect to $DM - \overline{DM}$ swapping.

	J_{DM}	Even operators	Odd operators
	0	$\phi_\chi \phi^*_\chi \overline{\psi}_q \psi_q$	$-i(\partial_\mu \phi_\chi \phi_\chi \chi^* - \phi_\chi \partial_\mu \phi^*_\chi)\overline{\psi}_q \gamma^\mu \psi_q$
SI	1/2	$\overline{\psi_\chi} \psi_\chi \overline{\psi}_q \psi_q$	$\overline{\psi}_\chi \gamma_\mu \psi_\chi \overline{\psi}_q \gamma^\mu \psi_q$
	1	$M_\chi A_{\chi,\mu} A^\mu_\chi \overline{\psi}_q \psi_q$	$+i(A^{*\alpha}_\chi \partial_\mu A_{\chi,\alpha} - A_\chi{}^\alpha \partial_\mu A^*_{\chi\alpha})\overline{\psi}_q \gamma_\mu \psi_q$
SD	1/2	$\overline{\psi}_\chi \gamma_\mu \gamma_5 \psi_\chi \overline{\psi}_q \gamma_\mu \gamma_5 \psi_q$	$\overline{\psi}_\chi \sigma_{\mu\nu} \psi_\chi \overline{\psi}_q \sigma^{\mu\nu} \psi_q$
	1	$\frac{\sqrt{3}}{2}(\partial_\alpha A_{\chi,\beta} A_{\chi\nu} - A_{\chi\beta} \partial_\alpha A_{\chi\nu})\epsilon^{\alpha\beta\nu\mu} \overline{\psi}_q \gamma_5 \gamma_\mu \psi_q$	$i\frac{\sqrt{3}}{2}(A_{\chi\mu} A^*_{\chi\nu} - A^*_{\chi\mu} A_{\chi\nu})\overline{\psi}_q \sigma^{\mu\nu} \psi_q$

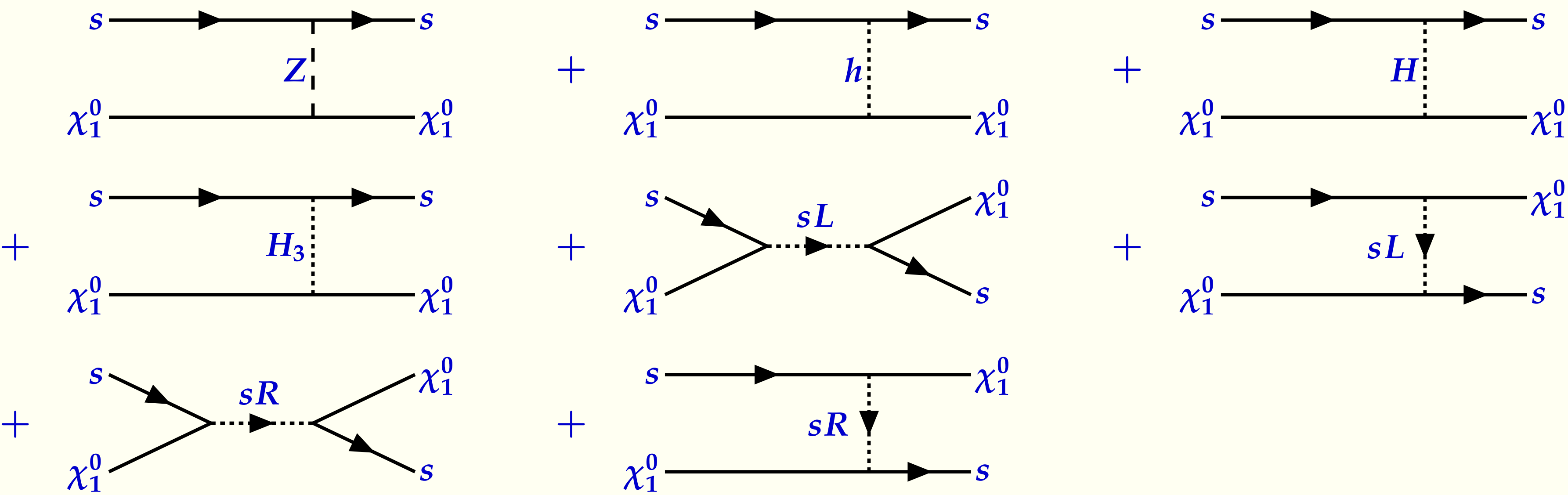
So, for DM-nucleon cross sections we have to know operator expansion of DM-quark matrix element and nucleon form factors for the following currents:

even: $\overline{\psi}_q \psi_q,\ \overline{\psi}_q \gamma_5 \gamma_\mu \psi_q$

odd : $\overline{\psi}_q \gamma_\mu \psi_q,\ \overline{\psi}_q \sigma^{\mu\nu} \psi_q$

Diagrams and operator expansion

Traditionally the coefficients at this operator are evaluated symbolically using **Fiertz** *identities.*



Nucleon form factors of light quarks

Even sector scalar form factors are known from hadron spectroscopy and πN scattering. In proton we use by default:

	d	u	s
scalar	0.033	0.023	0.26(0.12) $\pm$
vector	-0.427	0.842	-0.085

The main uncertainty comes from s-quark scalar FF.

For odd sector

	d	u	s
scalar	1	2	0
vector	-0.231	0.839	-0.046

Here scalar part is known for sure, but σ contribution comes from lattice calculations.

We have included routine setProtonFF(scalar,vector, sigma) which allows user to improve form factors.

Nucleon form factors of heavy quarks and other heavy color pa

They come from energy-momentum tensor trace anomaly:

$$M_N < N|N > = < N| \sum_{q=u,d,s} M_q \bar{\psi}_q \psi_q (1+2\gamma) + (\frac{\beta}{2\alpha^2}) \alpha G_{\mu\nu} G^{\mu\nu} |N >$$

It allows to evaluate gluon component in nucleon in LO

$$M_N < N|N > = < N| \sum_{q=u,d,s} M_q \bar{\psi}_q \psi_q - \frac{9}{8\pi} \alpha G_{\mu\nu} G^{\mu\nu} |N > \quad (1)$$

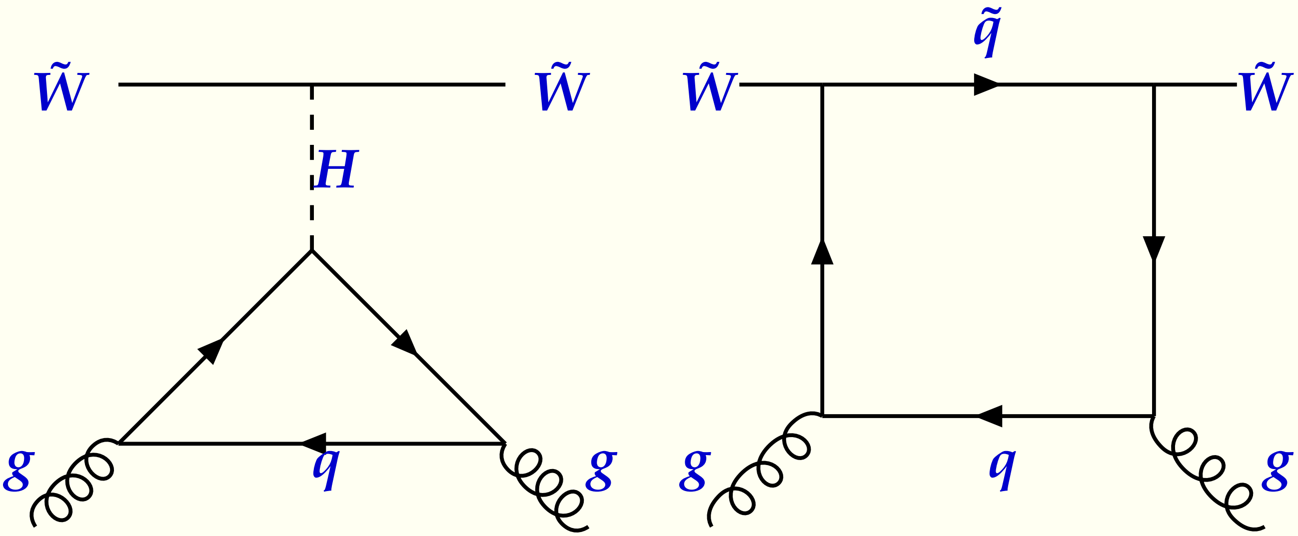
If gluon component is known one can evaluate heavy quarks condensate in nucleon. Formula (??) allows us to guess the result

$$< N| M_Q \bar{\psi}_Q \psi_Q |N > = -\frac{\Delta\beta}{2\alpha^2(1+2\gamma)} < N| \alpha G_{\mu\nu} G^{\mu\nu} |N > \quad (2)$$

Up to NLO term (QCD corrections)

$$< N| M_Q \bar{\psi}_Q \psi_Q |N > = -\frac{1}{12\pi} (1 + \frac{11\alpha(M_Q)}{4\pi}) < N| \alpha G_{\mu\nu} G^{\mu\nu} |N > \quad (3)$$

For Higgs diagrams this approach is perfect, but for s-quark exchange it has a precision about $M_Q^2/(MSQ^2 - M_\chi^2)$.



It is 100% essential for t-quark whose contribution is suppressed. And in principle it can be noticeable for b-quark. In SUGRA such correction gives only about 2%.

Twist-2 operators

They are the so-called twist-2 operators containing field derivatives which also contribute to scalar amplitude at rest. In principle each operator should be treated separately because it has its own nucleon form factor. For simplicity we do not distinguish between twist-zero and twist-2 operators since typically contribution of twist-2 operators less than 1%. A complete treatment of twist-2 operators in neutralino nucleon elastic scattering in the MSSM was first presented in Drees&Nojiri-1993.

Experimental status:

Experimental status: There is DAMA positive signal and negative signals from all other expreriments Xenon10, CDMS, COUPP ... Nutrino telescope.

Indirect detection

We expect processes of Dark Matter annihilation in our Galaxy. Final product of such annihilation are stable particles

$$\gamma, e^+, e^-, p^+, p^-, \nu$$

In Majorana particle case direct annihiolations into light particles are absent. So one has to calculate $< \sigma \cdot v >$ for reactions like

$$\chi_1^0, \chi_1^0 \rightarrow b, B \quad ; W^+, W^- \quad ; Z, h \quad ; ...$$

And after that calculate decays of these products into to stable particles.

The largest uncertainty comes from unknown $< \rho^2 >$ / $< \rho >^2$ boost factor which could be about 100 and caused by clumping.

Propagation of positrons and antiprotons.

Positrons and antiprotons propagate in galactic magnetic field.

- *parameters of propagation are not known but related via B/C rate.*
- *the same parameters are responsible for signal and background.*

The region of propagation is a disk with radius about 20 kpc and half thickness $L \approx 1$ -15 kpc. Propagation presented by diffusion equation in the medium with vertical convective velocity, energy dissipation term and Fermy reacceleration term caused by motion of scatering centers.

There are approximate analitical solutions presented by integrals and GalProp lattice numerical solution.

There are resent PAMELA result about high energy positron signal. In principle it can be explained by pulsar source.

micrOMEGAs package

micrOMEGAs is a package for evaluation of different Dark Matter properties in framework of generic model.

micrOMEGAs calculates

- relic density
- direct detection cross sections
- indirect detection rate including positrons and antiprotons propagation
- any cross sections, widths and branchings

micrOMEGAs source does not contains codes for matrix elements for numerous processes needed for calculations. Any time when some new matrix element needs micrOMEGAs launches CalcHEP for matrix element generation.

This trick allows to write a package which can be applied for generic model.

*If one would like to check **DM** in some new model:*

- `./newProject <name>`

  It creates new directory with all needed files and subdirectories.

- *Copy your model presented in CalcHEP format to*

  `work/models`

- *If your model needs auxiliary functions, put their code in* `lib\` *subdirectory and write* `Makefile` *which will collect* `lib\mLib.a`

- *Compile executable by*

  ```
  make main=main.c
  ```

  or

  ```
  make main=main.F
  ```

- *Read manual to improve* `main.c`, `main.F`

New matrix elements are generated by the command

```
numout * cc;
cc= getMEcode(Gauge,Process,excludeVirtual,excludeOut,lib);
```

For instant

```
cc=getMEcode(0,"~o1,~o1->2*x",NULL,"~o1,~o2,~o3,~o4,~seL,~seR,..."
            ,"omg_o1o1")
```

It creates shared library

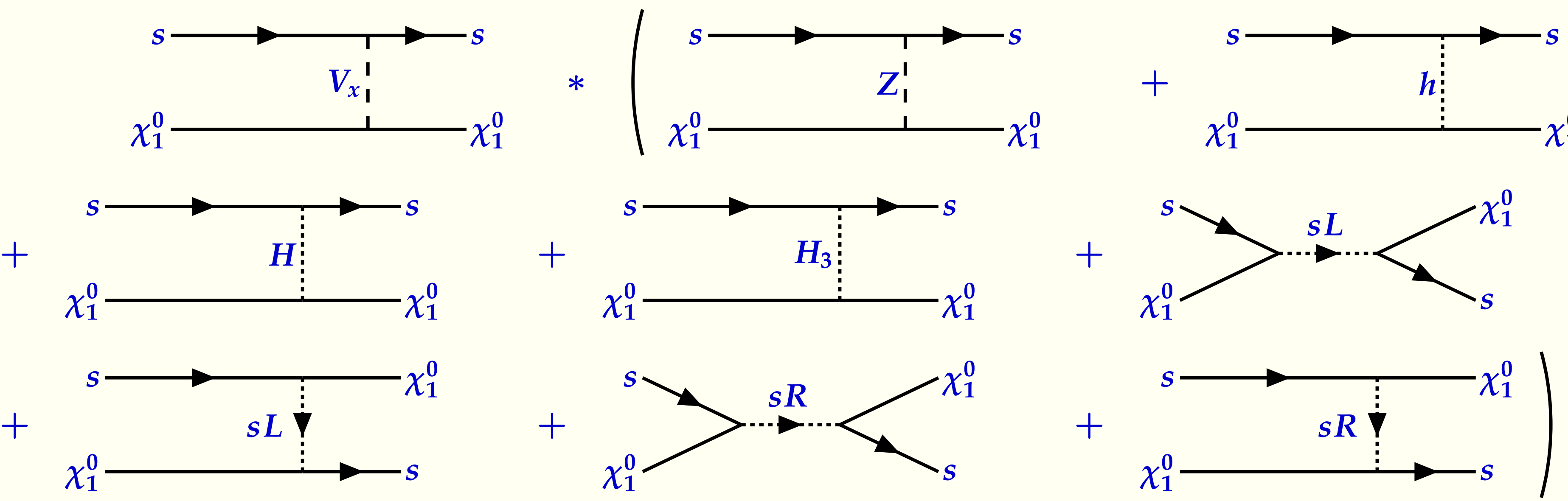
`work/so_generated/omg_o1o1.so`

where all publique names contain the `_omg_o1o1` *suffics.*

The library is generated one time and stored on the disk betweem sessions.

Operator expansion in for direct detection

Traditionally the coefficients at this operator are evaluated symbolically using Fiertz *identities. Instead* micrOMEGAs *creates an auxiliary models with addition vertices which corresponds to operators presented above. After that micrOMEAs calculates squared diagrams which contains in one side the auxiliary vertex, and in another side diagrams of physical matrix element.*



Future development

- *neutrino signal from Sun*
- *Interface with GalProp for e^+, p^- propagation.*
- *cross sections for* `~o1,~o1 -> f,f-bar, A`
- *Dirac* DM*with electric end magnetic dipole momenta.*