Kaon Light-Cone Distribution Amplitude in QCD: NNLO Radiative Corrections

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Outline

Introduction

Light-cone distribution amplitudes

Correlation function at NNLO

Numeric analysis with QCD sum rules

Conclusion

Colliders $\rightarrow$ expensive tools for studying particle interactions

Particle interactions at colliders = SM at present

SM= ElectroWeak (EW) + strong (QCD)

EW = QED (photon γ) + proper EW (W, Z, H) = $SU(2) \otimes U(1)$: weak coupling regime (coupling constants α and α_W are small and PT theory applies)

QCD - interaction of hadrons that are composite states of quarks and gluons that are fundamental fields of the theory $SU_c(3)$: theory is in strong coupling regime (the scale is about $1\ \mathrm{GeV}$ with $\alpha_s(m_\tau = 1.73\ \mathrm{GeV}) = 0.35$ is large) – PT in general does not work

QCD: transition matrix elements between hadrons are not directly calculable in PT that is still the main working tool.

Instead correlation functions of some operators are computed with factorization in some ways (OPE).

For a general amplitude with hadrons $Amp(Q, \Lambda_{\rm QCD})$

$$Amp(Q, \Lambda_{\rm QCD}) = \int d\xi\, C(Q, \mu, \alpha_s(\mu), \xi) \otimes \Phi(\mu, \Lambda_{\rm QCD}, \xi)$$

$C(Q, \mu, \alpha_s(\mu), \xi)$ – hard coefficient computable in PT;
$\Phi(\mu, \Lambda_{\rm QCD}, \xi)$ – soft part characterizing hadrons that is nonPT quantity;
μ is an auxiliary scale (factorization), ξ just indicates convolution of hard and soft parts

In general $\Phi(\mu, \Lambda_{\rm QCD}, \xi)$ has the form

$$\Phi(\mu, \Lambda_{\rm QCD}, \xi) = \langle H(\vec{p})|\bar{\psi}(x)...G^a_{\mu\nu}(y)...\psi(z)|H'(\vec{p}')\rangle$$

Simplest are vacuum matrix elements of local operators

$$\langle vac|\bar{\psi}(0)\psi(0)|vac\rangle, \quad \langle vac|G^a_{\mu\nu}G^a_{\mu\nu}|vac\rangle,$$

$$\langle vac|\bar{\psi}(0)|G^a_{\mu\nu}t^a\psi(0)|vac\rangle$$

More complicated example: Parton Distribution Functions (in electron-proton DIS) determined by

$$\langle p(\vec{p})|\bar{\psi}(z)\psi(0)|p(\vec{p})\rangle$$

with $z^2 = 0$ which is light-cone separation

Light-cone distribution amplitudes (LCDA)

LCDAs form a wide class of (soft, nonPT) quantities describing the relation between hadrons and quark-gluon operators in QCD. They enter factorization formulae for exclusive processes in QCD:

- Pion EM form factor at large Q^2
- Light-cone sum rules for form factors of heavy hadrons ($B \to K$, $D \to \pi$)
- QCD factorization in B-meson decays (B-physics at BABAR and BELLE). ($B \to K\gamma$, $B \to \pi\pi$)

Soft-collinear effective theory and HQET as formal framework

Twist-2 LCDA of the kaon $\varphi_K(u,\mu)$

$$\langle K^-(q)|\bar{s}(z)\gamma_\mu\gamma_5\,[z,-z]\,u(-z)|0\rangle_{z^2=0}$$

$$= -iq_\mu f_K \int_0^1 du\; e^{iuq\cdot z - i\bar{u}q\cdot z}\varphi_K(u,\mu)$$

s- and $\bar{u}$ carry the momentum fractions u and $\bar{u} = 1-u$;

$$[x_1,x_2] = P\exp(i\int_0^1 dv(x_1-x_2)_\rho A^\rho(vx_1+\bar{v}x_2))$$

μ – the normalization scale.

Gegenbauer polynomials $C_n^{3/2}(x)$ expansion

$$\varphi_K(u,\mu) = 6u\bar{u}\left(1 + \sum_{n=1}^{\infty} a_n^K(\mu) C_n^{3/2}(u-\bar{u})\right)$$

$a_n^K(\mu)$ - Gegenbauer moments.

a_1^K is related to the difference between the longitudinal momenta of the strange and nonstrange quarks in the kaon.

Determination of a numerical value of asymmetry parameter $a_1^K(\mu)$ at a low scale $\mu \sim 1\ \mathrm{GeV}$ with NNLO accuracy (K. G. Chetyrkin, A. Khodjamirian, AAP (2008)).

The method is based on QCD sum rules. a_1^K reduces to the vacuum-to-kaon matrix element of a local operator

$$\langle K^-(q)|\bar{s}\gamma_\nu\gamma_5 i \overleftrightarrow{D}_\lambda u|0\rangle = -iq_\nu q_\lambda f_K \frac{3}{5} a_1^K$$

Previous results (average)

$$a_1^K(1\ \mathrm{GeV}) = 0.06 \pm 0.03$$

Correlation function

In QCD sum rules one choses a correlation function for a_1^K

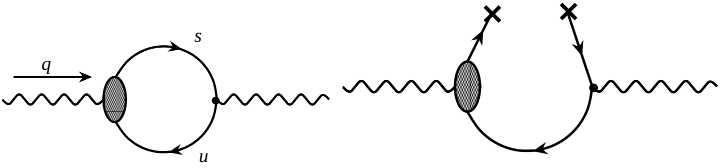
$$\Pi_{\mu\nu\lambda}(q) = q_\mu q_\nu q_\lambda \Pi(q^2) + \dots$$

$$= i \int d^4x\, e^{iq\cdot x} \langle T\left\{\bar{u}(x)\gamma_\mu\gamma_5 s(x), \bar{s}(0)\gamma_\nu\gamma_5 i \overleftrightarrow{D}_\lambda u(0)\right\}\rangle$$

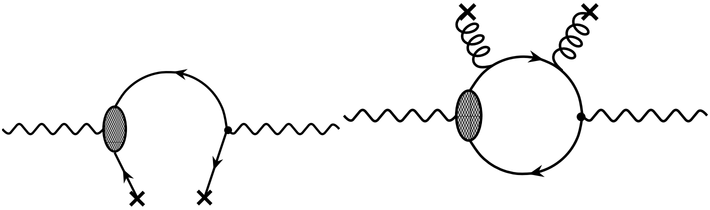
with

$$\overleftrightarrow{D}_\lambda = \overrightarrow{D}_\lambda - \overleftarrow{D}_\lambda, \quad D_\lambda = \partial_\lambda - i g_s A_\lambda$$

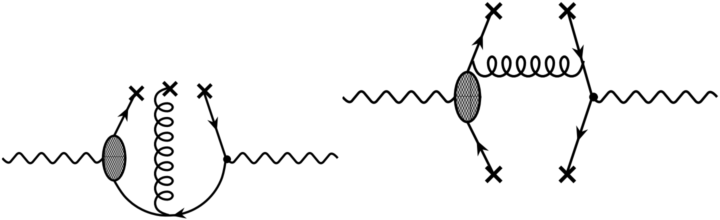
and computes the OPE in PT



Diagrams for OPE at LO:
PT loop and quark-condensate



Diagrams for OPE at LO:
quark-condensate and gluon-condensate



Diagrams for OPE at LO:
quark-gluon- and four-quark condensate diagrams.

General structure of the OPE

OPE gives an expansion for $\Pi(q^2)$

$$\Pi(Q^2,\mu) = \frac{\mathcal{A}_2(Q^2,\mu)}{Q^2} + \frac{\mathcal{A}_4(Q^2,\mu)}{Q^4} + \frac{\mathcal{A}_6(Q^2,\mu)}{Q^6} + \ldots$$

$\mathcal{A}_j$ has a double expansion in α_s and m_s^2 (u, d-quark masses are neglected)

$$\mathcal{A}_d = a_d^{(0,0)} + \left(\frac{\alpha_s}{\pi}\right) a_d^{(1,0)} + \left(\frac{\alpha_s}{\pi}\right)^2 a_d^{(2,0)} + \left(\frac{m_s^2}{Q^2}\right) a_d^{(0,1)}$$

$$+ \left(\frac{m_s^2}{Q^2}\right)^2 a_d^{(0,2)} + \left(\frac{\alpha_s}{\pi}\right)\left(\frac{m_s^2}{Q^2}\right) a_d^{(1,1)} +$$

Numerical role of small parameters at $Q^2 \simeq 1~\mathrm{GeV}^2$: for $\alpha_s(1~\mathrm{GeV}) = 0.47$ and $m_s(1~\mathrm{GeV}) < 150~\mathrm{MeV}$, one has $m_s^2/Q^2 \leq 0.02 \ll \alpha_s/\pi \simeq 0.15$.

Only the $O(\alpha_s)$ correction to the quark-condensate contribution $\mathcal{A}_4$ was previously known

For the largest $d = 2, 4$ terms of the OPE the NNLO accuracy in α_s is now achieved

The techniques of loop calculations are employed (programs QGRAF, FORM and MINCER).

Here are the results. Dimension $d = 2$ term is proportional to the mass squared ("pure" PT term)

$$\mathcal{A}_2(Q^2, \mu) = \frac{m_s^2}{4\pi^2}\left(1 + \frac{\alpha_s}{\pi}\left[\frac{26}{9} + \frac{10}{9} l_Q\right] + \left(\frac{\alpha_s}{\pi}\right)^2 \left[\frac{366659}{11664} - \frac{29}{9}\zeta(3) + \frac{14449}{972} l_Q + \frac{605}{324} l_Q^2\right] + 3\frac{m_s^2}{Q^2}\left(\frac{5}{2} + l_Q\right)\right);$$

$$l_Q = \ln(\mu^2/Q^2)$$

Dimension four term $d = 4$ is proportional to quark condensate and strange quark mass

$$\mathcal{A}_4(Q^2,\mu) = -m_s\langle\bar{s}s\rangle\left(1 - \frac{\alpha_s}{\pi}\left[\frac{112}{27} + \frac{8}{9}l_Q\right]\right.$$

$$\left. - \left(\frac{\alpha_s}{\pi}\right)^2\left[\frac{28135}{1458} - 4\zeta(3) + \frac{218}{27}l_Q + \frac{49}{81}l_Q^2\right] + 2\frac{m_s^2}{Q^2}\right)$$

$$-m_s\langle\bar{u}u\rangle\left(\frac{4\alpha_s}{9\pi} + \left(\frac{\alpha_s}{\pi}\right)^2\left[\frac{59}{54} + \frac{49}{81}l_Q\right]\right);$$

(light quark u mass m_u set to zero)

Dimension $d = 6$ term contains v.e.v. of more complicated operators

$$\mathcal{A}_6(Q^2, \mu) = \frac{2}{3} m_s \langle \bar{s} G s \rangle + \frac{1}{3} m_s^2 \langle G^2 \rangle (1 + l_Q)$$

$$- \frac{32}{27} \pi \alpha_s \left(\langle \bar{s} s \rangle^2 - \langle \bar{u} u \rangle^2 \right)$$

$$l_Q = \ln(\mu^2 / Q^2)$$

To relate OPE to physics and extract a_1^K one uses the dispersion relation

$$\Pi(q^2) = \frac{\frac{3}{5} a_1^K f_K^2}{m_K^2 - q^2} + \int\limits_{s_h}^{\infty} ds \frac{\rho^h(s)}{s - q^2}$$

$\rho^h(s)$ includes contributions of $K\pi\pi$, $K^*\pi$, $K\rho$, $K_1(1270)$, $K_1(1400)$,... The lower limit of integration is $s_h = (m_K + 2m_\pi)^2$. To approximate $\rho^h(s)$, we employ the quark-hadron duality

$$\rho^h(s)\Theta(s - s_0^h) = \rho^{OPE}(s)\Theta(s - s_0^K)\,,$$

where s_0^K is the effective threshold

Sum rule for a_1^K

Finally, one applies Borel transfrorm

$$\mathcal{B}(Q,M)\left(Q^{-2\nu}\right) = M^{-2\nu}/\Gamma(\nu)$$

and obtains the sum rule for a_1^K

$$a_1^K = \frac{5}{3f_K^2} e^{m_K^2/M^2} \left(\Pi(M^2) - \int\limits_{s_0^K}^{\infty} ds \rho^{OPE}(s) e^{-s/M^2} \right)$$

f_K is the kaon leptonic decay constant.

Input parameters:

- kaon mass $m_K^{\pm} = 493.58\ \mathrm{MeV}$
- kaon decay constant $f_K = 159.8 \pm 1.4 \pm 0.44\ \mathrm{MeV}$
- strange quark mass $m_s(2\ \mathrm{GeV}) = 98 \pm 16\ \mathrm{MeV}$ ($m_s(1\ \mathrm{GeV}) = 128 \pm 21\ \mathrm{MeV}$)
- $\alpha_s(m_Z) = 0.1176 \pm 0.002$ ($\alpha_s(1\ \mathrm{GeV})/\pi = 0.15 \pm 0.01$)
- $\langle \bar{q}q(2\ \mathrm{GeV})\rangle = -(0.264^{+0.031}_{-0.020}\ \mathrm{GeV})^3$
- $\langle \bar{s}s\rangle / \langle \bar{q}q\rangle = 0.8 \pm 0.3$
- $\langle \bar{s}Gs\rangle = m_0^2 \langle \bar{s}s\rangle(1\ \mathrm{GeV})$ with $m_0^2 = 0.8 \pm 0.2\ \mathrm{GeV}^2$
- $\langle G^2\rangle = 0.012 \pm 0.012\ \mathrm{GeV}^4$

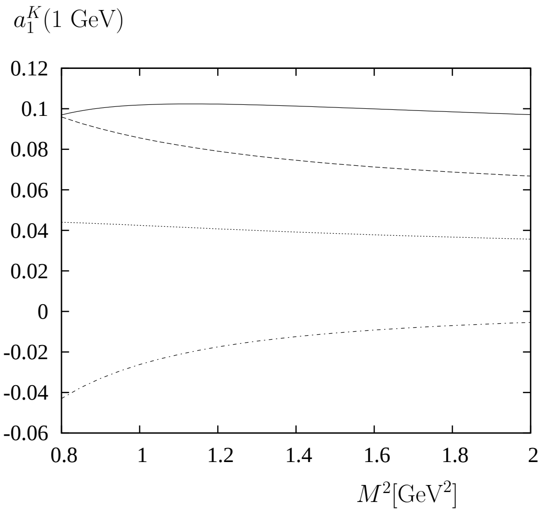
The sum rule analysis was done in the Borel parameter interval $M^2 = 1.0 - 2.0\ \mathrm{GeV}^2$

Threshold parameter $s_0^K = 1.05\ \mathrm{GeV}^2$ (sum rule for f_K).

s_0^K-dependence is weak

μ-dependence is weak

$a_1^K(1\ \text{GeV})$ as a function of the Borel parameter



a_1^K

d=2

d=4

d=6

.

Numerical prediction of the sum rule is

$$a_1^K(1\ \mathrm{GeV}) = 0.100$$

$$\pm 0.003|_{\mathrm{SR}} \pm 0.003|_{\alpha_s} \pm 0.035|_{m_s} \pm 0.022|_{m_q} \pm 0.013|_{cond}$$

Adding the individual uncertainties in quadrature we obtain

$$a_1^K(1\ \mathrm{GeV}) = 0.10 \pm 0.04$$

Now discuss the structure of the perturbative series as it appears in numerical analysis.

PT corrections strongly enhance $d = 2$, $O(m_s^2)$ term

$$\Pi^{(m_s^2)} = \frac{m_s^2}{4\pi^2}\left[1 + 3.53\left(\frac{\alpha_s}{\pi}\right) + 33.7\left(\frac{\alpha_s}{\pi}\right)^2\right]$$

For quark condensate contribution ($d = 4$) corrections are smaller

$$\Pi^{(m_s\langle\bar{s}s\rangle)} = m_s\langle\bar{s}s\rangle\left(1 - 3.77\left(\frac{\alpha_s}{\pi}\right) - 10.8\left(\frac{\alpha_s}{\pi}\right)^2\right) ,$$

Thus, at NNLO the numerical pattern of the sum rule for a_1^K changes: relative weight of $d = 2$ term becomes larger
Bad convergence of m_s^2-part and determination of m_s:
the Borel-transformed pseudoscalar correlation function is

$$\Pi^{(5)''(m_s^2)} = \frac{3m_s^2}{8\pi^2}(1 + 4.82\left(\frac{\alpha_s}{\pi}\right) + 22.0\left(\frac{\alpha_s}{\pi}\right)^2)$$

Conclusion

The NNLO PT corrections to the QCD sum rule for a_1^K are numerically important, they change the relative magnitude of the $d = 2$ (loop diagrams) and $d = 4, 6$ (condensate) terms in the OPE and give

$$a_1^K(1 \text{ GeV}) = 0.10 \pm 0.04$$

while previous result (average)

$$a_1^K(1 \text{ GeV}) = 0.06 \pm 0.03$$

The uncertainty of a_1^K is still large due mainly to the poor precision of the light quark masses: m_s directly entering the sum rule and $m_{u,d}$ determining the quark-condensate densities via Gell-Mann-Oakes-Renner relation.

Our result for a_1^K is larger than lattice determinations:

$$a_1^K(2\ \mathrm{GeV}) = 0.0453 \pm 0.0009 \pm 0.0029$$

V. M. Braun *et al.*, [QCDSF/UKQCD Collaboration] Phys. Rev. D **74**, 074501 (2006) and

$$a_1^K(2\ \mathrm{GeV}) = 0.048 \pm 0.003$$

M. A. Donnellan *et al.*, "Lattice Results for Vector Meson Couplings and Parton Distribution Amplitudes," arXiv:0710.0869 [hep-lat];
P. A. Boyle, M. A. Donnellan, J. M. Flynn, A. Juttner, J. Noaki, C. T. Sachrajda and R. J. Tweedie [UKQCD Collaboration], "A lattice computation of the first moment of the kaon's distribution amplitude," Phys. Lett. B **641**, 67 (2006).

By evolving our result to the scale $2\ \mathrm{GeV}$ we find

$$a_1^K(2\ \mathrm{GeV}) = 0.08 \pm 0.04$$