Pion transition form factor: Factorization, LCSR and higher-order corrections vs experimental data

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Plan of Presentation

$\gamma^*(q_1)\gamma^*(q_2) \to \pi^0(p)$, factorization, structure of $\mathrm{F}^{\gamma^*\gamma^*\pi}$

- Meson Distribution Amplitudes (DA)
- NLO and NNLO hard-scattering amplitudes

Light Cone Sum Rules (LCSR)

- Why Light Cone Sum Rules (LCSR)?
- Dispersion relations for $\mathrm{F}^{\gamma^*\gamma\pi}$ at $q_2^2 \downarrow 0$
- NLO Spectral density
- β_0-part of NNLO density

$\mathrm{F}^{\gamma^*\gamma\pi}_{\mathrm{LCSR}}$ analysis vs CELLO and CLEO data

- CLEO data on $\mathrm{F}^{\gamma^*\gamma\pi}_{\mathrm{LCSR}} \Rightarrow$ endpoint-suppressed Pion DA
- Recent Lattice data confirm CLEO LCSR analysis and endpoint-suppressed Pion DA obtained from NLO QCD SR
- Independent estimate of Pion DA from inverse moment also conform

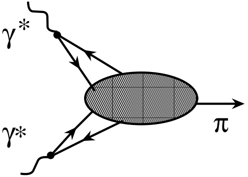
Conclusions

Is new BaBar data a "pion puzzle"?

$$\gamma^*(q_1)\gamma^*(q_2) \to \pi^0(p),$$

factorization, and structure of $F^{\gamma^*\gamma^*\pi}$

$\gamma^*(q_1)\gamma^*(q_2) \to \pi^0(p)$ in pQCD



$$\int d^4x e^{-iq_1\cdot z}\langle\pi^0(p) \mid T\{j_\mu(z)j_\nu(0)\} \mid 0\rangle =$$

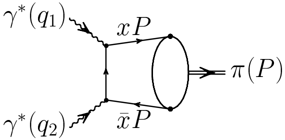
$$i\epsilon_{\mu\nu\alpha\beta}q_1^\alpha q_2^\beta \cdot F^{\gamma^*\gamma^*\pi}(Q^2, q^2),$$

$$-q_1^2 = Q^2 > 0, \quad -q_2^2 = q^2 \geq 0$$

Collinear factorization at $Q^2, q^2 \gg$ (hadron scale$\sim m_\rho)^2$

$$F^{\gamma^*\gamma^*\pi}(Q^2, q^2) = T(Q^2, q^2, \mu_F^2; x) \otimes \varphi_\pi^{(2)}(x; \mu_F^2) + O(\frac{1}{Q^4})$$

For leading twist 2 and at parton level



$$F^{\gamma^*\gamma^*\pi} = \frac{\sqrt{2}}{3}f_\pi \int_0^1 dx \frac{1}{Q^2x + q^2\bar{x}} \varphi_\pi^{(2)}(x)$$

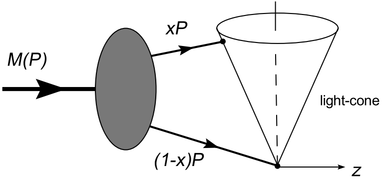
$$Q^2F^{\gamma^*\gamma^*\pi}(Q^2, 0) = \frac{\sqrt{2}}{3}f_\pi \int_0^1 \frac{dx}{x}\varphi_\pi^{(2)}(x)$$

$$\equiv \frac{\sqrt{2}}{3}f_\pi\langle x^{-1}\rangle_\pi$$

Distribution amplitudes in exclusive reactions

$$< 0|\bar{q}(z)\gamma_\mu\gamma_5 E(z,0)q(0)|\pi(P) > \Big|_{z^2=0} = iP_\mu \int dx e^{ix(zp)} \boldsymbol{\varphi_\pi^{(2)}(x,\mu_F^2)}$$

$$E(z,0) = P\exp(ig\int_0^z A_\mu(\tau)d\tau^\mu)$$



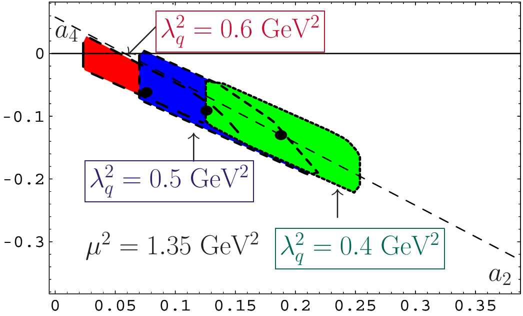
Distribution amplitudes can be obtained in:

- QCD SR [CZ 1984],
  NLC QCD SR [M&Radyushkin 1986–92; Bakulev&M&Stefanis 1998, 2001–04]
- Instanton vacuum approaches,
  [Polyakov et al. 1998, 2009; Dorokhov et al. 2000]
- Lattice QCD, [Braun et al. 2006; Donnellan et al. 2007]
- from experimental data [Schmedding&Yakovlev 2000, BMS 2003–2006]
- not within pQCD, but with μ_F^2-evolution according to ERBL [79–80] in pQCD

Pion distribution amplitudes in NLC QCD SR

$$\varphi_\pi^{(2)}(x;\mu_F^2) = \psi_0(x) + a_2(\mu_F^2)\ \psi_2(x) + a_4(\mu_F^2)\ \psi_4(x) + \ldots$$

$$\varphi_\pi^{(2)} \Leftrightarrow \{a_n\}; \qquad \psi_n(x) = 6x\bar{x}\ C_n^{(3/2)}(x-\bar{x})$$ –Gegenbauer harmonics



λ_q^2 – average virtuality of vacuum quarks – the single parameter of NLC approach
BMS [PLB 508(2001)279]
BMS [Ann.Phys.13(2004)629]

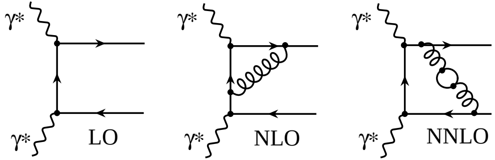
- Green rectangle forms the BMS bunch of DA, $\psi_0 + a_2\psi_2 + a_4\psi_4$
- ψ_0 – Asymptotic (As) DA
- Chernyak-Zhitnitsky (CZ) DA, QCD SR, $\psi_0 + a_2\psi_2$, $a_2(1) \approx 0.56$

NLO and NNLO amplitudes.

Collinear factorization is Theorem [Efremov&Radyushkin 1978]

$$F^{\gamma^*\gamma^*\pi} \quad \sim \quad \Big(T_0(Q^2,q^2;x) + \ a_s^1 \ T_1(Q^2,q^2;\mu_F^2;x) + a_s^2 \ T_2(Q^2,q^2;\mu_F^2;\mu_{\rm R}^2;x) + \ldots\Big) \otimes \ \boldsymbol{\varphi_\pi^{(2)}(x;\mu_F^2)} - \boldsymbol{\delta^2_{tw4}} \cdot T_0^2(Q^2,q^2;x) \otimes \ \boldsymbol{\varphi_\pi^{(4)}(x)}$$

— calculable in pQCD, $a_s(\mu_{\rm R}^2) = \alpha_s/(4\pi)$. Usually $\mu_{\rm R}^2 = \mu_F^2 = \langle Q^2 \rangle$ to simplify and to minimize rad. corrections.
$\boldsymbol{\delta^2_{tw4} = (0.19 \pm 0.02)}$ GeV^2 – twist-4 scale parameter.



LO: $$T_0(Q^2,q^2;x) = \frac{1}{x\,Q^2 + \bar{x}\,q^2}$$

NLO amplitudes and NLO evolution

NLO (last editions):
[Bakulev&MS&Stefanis(2003)],
[Melić&Müller&Passek-Kumerički(2003)]

$$T_1\left(x;Q^2,q^2\right)\otimes\varphi(x)=T_0(Q^2,q^2;y)\otimes\left\{C_{\rm F}\mathcal{T}^{(1)}(y,x)+\mathbf{L}(y)\cdot V^{(0)}(y,x)\right\}\otimes\varphi(x)$$

$$\mathcal{T}^{(1)}=\left[-3V^b+g\right](x,y)_+-3\delta(x-y),\qquad \mathbf{L}(y)\equiv\ln\left[\left(Q^2y+q^2\bar{y}\right)/\mu_{\rm F}^2\right]$$

$\varphi(x;\mu^2)\longrightarrow\varphi(x;Q^2)$ evolves according to NLO ERBL [79-80] equation:

$$\mu^2\frac{d}{d\mu^2}\varphi(x;\mu^2)\quad=\quad\left(a_s\,V^{(0)}(x,y)+a_s^2\,V^{(1)}(x,y)\right)\otimes\varphi(y;\mu^2)$$

$$V^{(0)}\otimes\psi_n=v(n)\cdot\psi_n$$

NNLO amplitude and coefficient functions

β_0-part of NNLO: $T_2 \otimes \varphi \to \beta_0 \cdot T_\beta \otimes \varphi$
[Melić&Müller&Passek-Kumerički(2003)]

$$T_\beta \otimes \varphi = \ln\left(\frac{\mu_R^2}{\mu_F^2}\right) T_1 \otimes \varphi + T_0 \otimes \Big\{ C_F \mathcal{T}_\beta^{(2)} - C_F \mathbf{L}(y) \cdot \mathcal{T}^{(1)} + \mathbf{L}(y) \cdot \left(V_\beta^{(1)}\right)_+ - \frac{1}{2}\mathbf{L}^2(y) \cdot V^{(0)} \Big\} \otimes \varphi \,.$$

The origins of these terms:
$\sim V^{(0)}$ – 1-loop ERBL together with a_s RG evolution, while $\sim \left(V_\beta^{(1)}\right)_+$ – as the β_0-part of 2-loop ERBL kernel; $\mathcal{T}^{(1)}$ – from a_s RG evolution.
These terms together form the exponential RG-solution:

$$\exp\left\{ \int^{\mathbf{L}} V(a_s(L)) dL \right\}$$

$\mathcal{T}_\beta^{(2)}$ – the coefficient function – original, the most cumbersome part

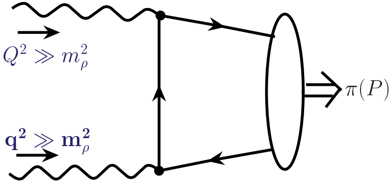
This contribution gives the sign and size of NNLO effect following to BLM prescription

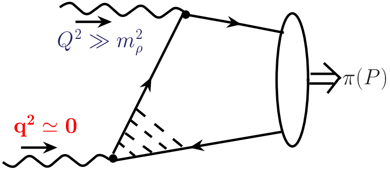
Light Cone Sum Rules (LCSR)

Why Light Cone Sum Rules (LCSR)?

The experimental conditions require $q^2 \to 0$

For $Q^2 \gg m_\rho^2, \quad q^2 \ll m_\rho^2$ pQCD factorization valid only in leading twist and higher twists are of importance [Radyushkin–Ruskov, NPB (1996)].
Reason: if $q^2 \longrightarrow 0$ one needs to take into account interaction of real photon at long distances of order of $O(1/\sqrt{q^2})$





pQCD is OK

photons behaves like a hadron

[Khodjamirian, EJPC (1999)]: LCSR effectively accounts for long-distances effects of real photon using dispersion relation in variable q^2 and quark-hadron duality in vector channel.

Dispersion relation for $F^{\gamma^*\gamma\pi}$

The main further goal – spectral density ρ

$$F^{\gamma^*\gamma\pi}(Q^2,q^2) = \int\limits_0^\infty ds\, \frac{\rho^{\rm ph}(Q^2,s)}{s+q^2}$$

$$\rho^{\rm ph} = \theta(s_0-s)\,\rho^{\rm phen}(Q^2,s) + \theta(s-s_0)\,\rho^{\rm PT}(Q^2,s)$$

$$\rho^{\rm PT}(Q^2,s) = \frac{\rm Im}{\pi}\left[F^{\gamma^*\gamma^*\pi}(Q^2,-s)\right]$$

$$\rho^{\rm phen}(Q^2,s) = \sqrt{2} f_\rho F^{\gamma^* V\pi}(Q^2)\cdot\delta(s-m_V^2)\Big|_{V=\rho,\omega}$$

using quark-hadron duality in vector channel for $F^{\gamma^* V\pi}$ [Khodjamirian 1999]:

$$F^{\gamma\gamma^*\pi}(Q^2,q^2\to 0) = \frac{1}{\pi}\int_{s_0}^{\infty}\frac{{\rm Im}F^{\gamma^*\gamma^*\pi}(Q^2,-s)}{s}\,ds + \frac{1}{\pi}\int_0^{s_0}\frac{{\rm Im}F^{\gamma^*\gamma^*\pi}(Q^2,-s)}{m_\rho^2}\,e^{(m_\rho^2-s)/M^2}ds$$

$s_0 \simeq 1.5$ GeV^2 – effective threshold in vector channel,
M^2 – Borel parameter ($0.5 - 0.9$ GeV^2).

NLO Spectral density $\rho^{(1)}$

$$\rho^{(1)}(Q^2,s) = \frac{\mathbf{Im}}{\pi}\left[(T_1 \otimes \varphi_\pi)(Q^2,-s)\right]\ ,\ s \geq 0$$

$\rho_n^{(1)}(x,\mu_{\rm F}^2)$ for Gegenbauer harmonic ψ_n, $\boxed{x = Q^2/(s+Q^2)}$

The general case [M&Stefanis NPB(2009)] in press:

$$\begin{aligned}\bar{\rho}_n^{(1)}\left(x;\mu_{\rm F}^2\right) \quad &= \quad C_{\rm F}\left\{-3\left[1 - v^a(n)\right] + \frac{\pi^2}{3} - \ln^2\left(\frac{\bar{x}}{x}\right) + 2v(n)\ln\left(\frac{\bar{x}}{x}\frac{Q^2}{\mu_{\rm F}^2}\right)\right\}\psi_n(x) \\ &\quad -C_{\rm F}2\left[\sum_{l=0,2,\ldots}^{n}\left(G_{nl} + v(n)\cdot b_{nl}\right)\psi_l(x)\right]\end{aligned}$$

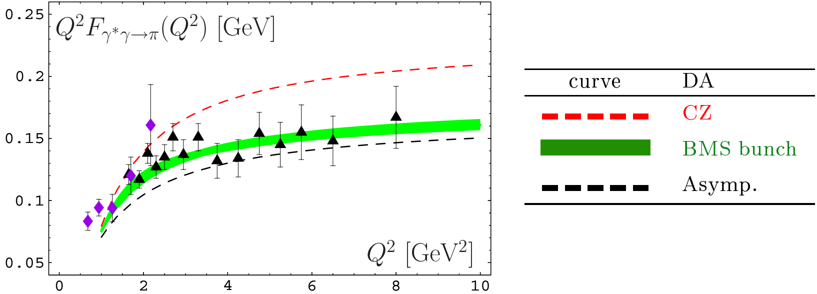
G_{nl} (originates from g), b_{nl} – calculable triangular matrices

The partial case $\bar{\rho}_0^{(1)}$ [Schmedding&Yakovlev (2000)]:

$$\bar{\rho}_0^{(1)}(x) = C_{\rm F}\left[-5 + \frac{\pi^2}{3} - \ln^2\left(\frac{\bar{x}}{x}\right)\right]\psi_0(x)$$

Conclusion: The spectral density and $F^{\gamma\gamma^*\pi}$ can be obtained for any numbers of Gegenbauer harmonics

NLO LCSR vs. CELLO (♦) & CLEO (▲) data



Radiative corrections contribute up to **−17%** at low/moderate Q^2

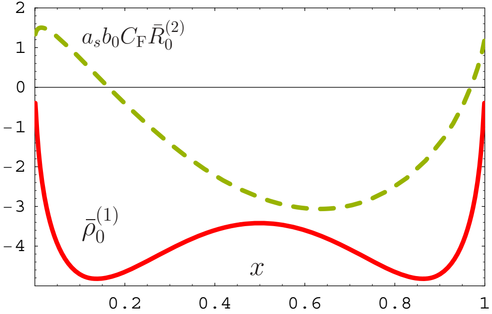
- BMS bunch describes rather well all data above $\mathbf{Q^2 \gtrsim 1.5}$ GeV^2;
- Low-Q^2 CELLO data (only statistical errors shown) excludes As DA and high-Q^2 CLEO data excludes CZ DA .

NNLO$_{\beta_0}$ Spectral density

$$\rho^{(2)}(Q^2,s) = \frac{\mathbf{Im}}{\pi}\left[(T_2 \otimes \varphi_\pi)(Q^2,-s)\right]\ ,\ s \geq 0$$

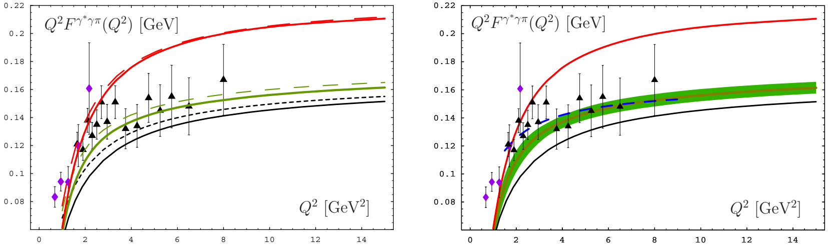
$$\boxed{x = Q^2/(s+Q^2)},\quad \text{put } \boldsymbol{\mu_R^2 = \mu_F^2}$$

$$\bar{\rho}_n^{(2)} \quad \rightarrow \quad b_0 C_F \left[\bar{R}_n^{(2)}\left(x; \frac{\bar{x}}{x}\frac{Q^2}{\mu_F^2}\right)\right] .$$



The dashed green line shows $a_s(\mu_F^2)\ \bar{\rho}_0^{(2)} = a_s(\mu_F^2)\ b_0 C_F \bar{R}_0^{(2)}(x, \bar{x}/x)$ at the typical CLEO reference scale $Q^2 = \mu_F^2 = \mu_{SY}^2 = (2.4\ \text{GeV})^2$,
whereas the solid red line represents $\bar{\rho}_0^{(1)}(x)$

NNLO$_{\beta_0}$ LCSR vs. NLO LCSR



Dashed lines – NLO form factor
Solid lines – [NNLO$_{\beta_0}$] $\oplus$ [Improved Breit-Wigner ρ^{phen} model]

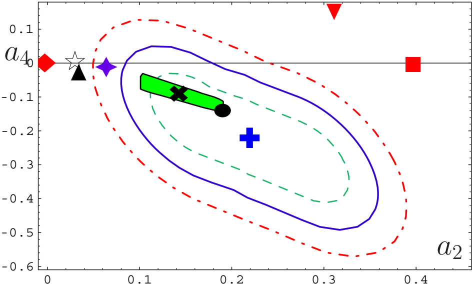
- Combined effect: -7% at $Q^2 \simeq 2$ GeV^2, -2% at $Q^2 \geq 6$ GeV^2
- Effect of BLM prescription can be stronger $\Rightarrow$ we expect the size of the total NNLO contribution is less -10%

$F_{\rm LCSR}^{\gamma^*\gamma\pi}$, CLEO data, and inverse problem for $\varphi_\pi^{(2)}$

CLEO data on $F^{\gamma^*\gamma\pi}_{\text{LCSR}} \Rightarrow$ Pion DA in plane (a_2, a_4)

NLO Light-Cone SR $\oplus$ Twist-4 $\oplus(\mu^2 = Q^2)$
with 20% uncertainty of $\delta^2_{\text{Tw-4}}$: $\delta^2_{\text{Tw-4}} = 0.19$ (4) GeV^2
BMS [PLB 578 (2004) 91]: $\lambda^2_q = 0.4$ GeV^2

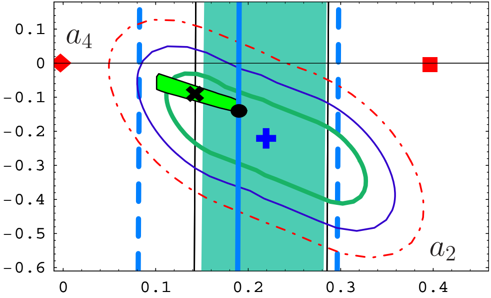
1σ– green line; 2σ– blue line; 3σ–dashed-dotted red line.



at $\mu^2_{SY} = 5.76$ GeV^2
✚= best-fit BMS, ●=SY points
◆ = Asymptotic DA
■ = CZ DA, ▼= Braun-Filyanov DA
✖ = BMS model

Even with 20% uncertainty in twist-4
CZ, BF DA excluded <u>at least</u> at 4σ-level!
As DA — at 3σ-level.
BMS DA and most of BMS bunch — inside 1σ-domain.

Recent Lattice data vs. CLEO analysis and Pion DA



Comparison of the CLEO-data constraints on $F^{\gamma^*\gamma\pi}(Q^2)$ in the (a_2, a_4) plane at the scale $\mu^2_{SY} = 5.76$ GeV^2 in terms of error regions around the BMS best-fit point ✚

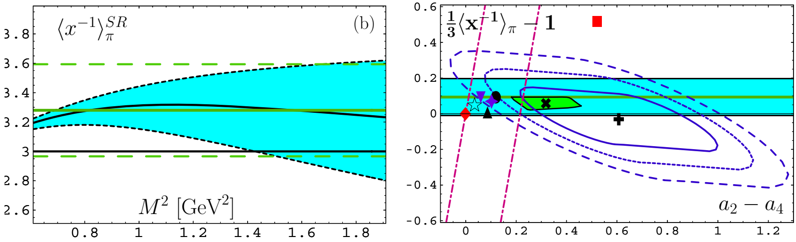
1σ– green line; 2σ– blue line; 3σ–dashed-dotted red line.

Recent lattice simulations, denoted by vertical dashed lines [LAT2006] and solid ones [PoS LAT2007] are also shown together with predictions obtained from NLC QCD SR (slanted green rectangle)

Independent estimate of Pion DA inverse moment $\langle x^{-1} \rangle_\pi$

BMS [Ann. Phys.(Leipzig)13(2004) 629]:

processing of CLEO data is evolved to normalization scale $\mu^2 \simeq 1$ GeV^2



$$\lambda_q^2 = 0.4 \text{ GeV}^2, \quad \frac{1}{3}\langle x^{-1} \rangle_\pi^{\text{SR}} - 1 = 0.1 \pm 0.1$$

Good agreement of a theoretical “tool” of different origin with CLEO data

Conclusion

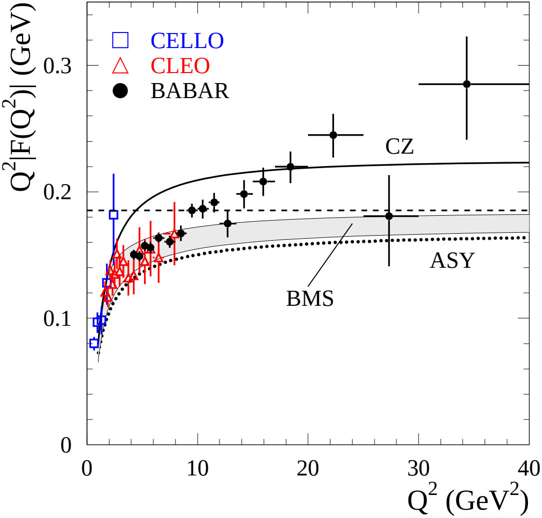
- NLO LCSR produces constraints on pion DA parameters (a_2, a_4) in conjunction with CLEO data. The same can be done for any number of parameters $\{a_n\}$.
- QCD SR with NLC for pion DA gives admissible sets of DAs and, independently, the inverse moment $\langle x^{-1}\rangle_\pi^{SR}$
- This NLO LCSR constraints agree well with: the bunch of pion DAs and $\langle x^{-1}\rangle_\pi^{SR}$, and recent lattice data.
- NNLO LCSR correction does no change $F^{\gamma^*\gamma\pi}(Q^2)$ significantly. The common effect of NNLO_{β_0} and improved hadron model leads: from -7% at $Q^2 \simeq 2$ GeV^2, to -2% at $Q^2 \geq 6$ GeV^2

We thank A. P. Bakulev for collaboration and help in the preparation of this report.

Do the new BaBar data pose a “pion puzzle”?

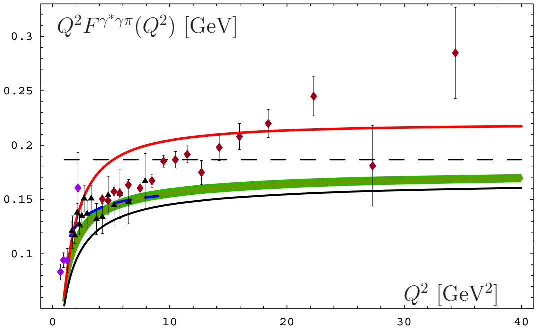
New data on transition form factor

by BaBar Collaboration [B. Aubert et al., arXiv:0905.4778[hep-exp]]:



"BaBar data contradicts most models for the pion DA ..."

Is the new BaBar data a "pion puzzle"?



Pion DAs	CLEO and BaBar $\bar{\chi}^2$	BaBar $\bar{\chi}^2$	$\bar{\chi}^2$ BaBar (pt > 10 GeV^2)
Asy	**11.5**	**19.2**	**19.8**
BMS	**4.4**	**7.8**	**11.9**
CZ	**20.9**	**36.0**	**6.0**

The high-Q^2 BaBar data points—growing with Q^2—contradict all pion DA models that have a finite projection on Gegenbauer polynomials, i.e., vanish at the endpoints $0, 1$.

Is the new BaBar data a "pion puzzle"?

- Moreover, they contradict the
  collinear factorization formula per se.
- Even more, they contradict the "counting rules" – the most reliable method up to now.
- Provided the BaBar date are correct, they constitute a challenge for QCD

First attempts to overcome the "pion puzzle"

A possible scenario to explain the BaBar data was proposed by [Dorokhov arXiv:0905.3577], [Radyushkin arXiv:0906.0323] and [Polyakov arXiv:0906.0538]: "practically flat DA" that generates the logarithmic growth in Q^2 and transfers the start of the collinear factorization far from the BaBar region

	Cello& CLEO	BaBar
Cello& CLEO	**1.22**	**15.8**
BaBar	**3.5**	**1.8**

$\boldsymbol{\sigma}, \boldsymbol{m^2}$ – adjusted parameters

$$Q^2 F^{\gamma^*\gamma\pi} = \frac{\sqrt{2}}{3} f_\pi \int_0^1 \frac{\mathbf{1}}{x}\left[1 - \exp\left(-\frac{xQ^2}{\bar{x}2\boldsymbol{\sigma}}\right)\right] dx$$

$$\text{or} \quad = \frac{\sqrt{2}}{3} f_\pi \int_0^1 \frac{\mathbf{1}}{x + \boldsymbol{m^2}/Q^2} dx$$

$$\sim \log(1 + Q^2/\boldsymbol{m^2})$$

	Cello& CLEO	BaBar
Cello& CLEO	**0.48**	**7.8**
BaBar	**10.8**	**1.8**

'QCD like' Dipole fit, $\boldsymbol{\Lambda}, \boldsymbol{b}$ – adjusted parameters

$$Q^2 F^{\gamma^*\gamma\pi} = \frac{Q^2}{2\sqrt{2} f_\pi}\left[\frac{\boldsymbol{\Lambda^2}}{\boldsymbol{\Lambda^2} + Q^2} + \boldsymbol{b}\left(\frac{\boldsymbol{\Lambda^2}}{\boldsymbol{\Lambda^2} + Q^2}\right)^2\right]$$

Preliminary conclusions about BaBar dats

- This is a hint that the "old data" – Cello and CLEO and the "new data" – BaBar cannot be fitted simultaneously.
- From the theoretical side, it is not possible to describe these data with the inclusion of higher radiative corrections–as we have shown at the NNLO level.