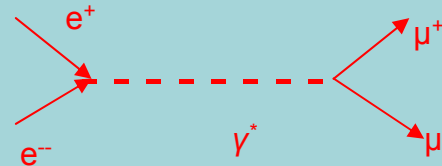


**On the spin correlations of muons and  
 $\tau$  leptons produced in the annihilation  
processes  $e^+e^- \rightarrow \mu^+\mu^-$ ,  $e^+e^- \rightarrow \tau^+\tau^-$**

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In the first approximation over the constant  $e^2/\hbar c$ , the process of conversion of the electron-positron pair into the muon one (or  $\tau^+\tau^-$ ) is described by the one-photon diagram :



The virtual photon with time-like momentum transfers angular momentum  $J=1$  and negative parity. The internal parities of  $\mu^+$  and  $\mu^-$  are opposite: the ( $\mu^+\mu^-$ ) pair is generated in triplet states (total spin  $S=1$ ), with total angular momentum  $J=1$  and negative space parity. Helicity amplitudes:

$$f_{\Lambda'\Lambda}(\theta, \phi) = R_{\Lambda'\Lambda}(E) d_{\Lambda'\Lambda}^{(1)}(\theta) \exp(i\Lambda\phi)$$

$d$ - functions

$\theta$  and  $\phi$  – polar and azimuthal angles of flight direction of  $\mu^+$  with respect to positron momentum in the c.m. frame of the reaction;  $\Lambda'$  – difference of helicities of  $\mu^+$  and  $\mu^-$ ,  $\Lambda$  - difference of helicities of  $e^+$  and  $e^-$ ;  $E$  – total energy in the c.m. frame .

Factorization:

$$R_{\Lambda'\Lambda}(E) = r_{\Lambda'}^{(\mu)}(E) r_{\Lambda}^{(\varepsilon)}(E)$$

Parity conservation:

$$r_{+1}^{(\mu)}(E) = r_{-1}^{(\mu)}(E), \quad r_{+1}^{(\varepsilon)}(E) = r_{-1}^{(\varepsilon)}(E)$$

As follows from the structure of electromagnetic current for pairs  $(e^+ e^-)$  and  $(\mu^+ \mu^-)$ :

$$r_0^{(\mu)}(E) = \frac{m_\mu}{E} r_1^{(\mu)}(E) = \sqrt{1 - \beta_\mu^2} r_1^{(\mu)}(E), \quad r_0^{(\varepsilon)}(E) = \frac{m_e}{E} r_1^{(\varepsilon)}(E)$$

$m_\mu$  and  $m_e$  - muon and electron masses,  $\beta_\mu$  - muon velocity.

Since always  $E \geq m_\mu \gg m_e$ , the contribution of states of electron and positron with antiparallel spins (equal helicities) is negligibly small:  $r_0^{(\varepsilon)}(E) \approx 0, R_{\Lambda'0}(E) \approx 0$ .

At the annihilation of electron and positron being totally polarized in the direction parallel to positron momentum in the reaction c.m. frame, the  $(\mu^+ \mu^-)$  system is generated in the triplet state:

$$|\Psi\rangle^{(+1)} = \frac{\sqrt{2}}{\sqrt{2 - \beta_\mu^2 \sin^2 \theta}} \left( \frac{1 + \cos \theta}{2} | + 1 \rangle - \sqrt{1 - \beta_\mu^2} \frac{\sin \theta}{\sqrt{2}} | 0 \rangle + \frac{1 - \cos \theta}{2} | - 1 \rangle \right)$$

$$| + 1 \rangle = | + 1/2 \rangle^{(\mu^+)} \otimes | + 1/2 \rangle^{(\mu^-)}, \quad | - 1 \rangle = | - 1/2 \rangle^{(\mu^+)} \otimes | - 1/2 \rangle^{(\mu^-)},$$

$$| 0 \rangle = \frac{1}{\sqrt{2}} \left( | + 1/2 \rangle^{(\mu^+)} \otimes | - 1/2 \rangle^{(\mu^-)} + | - 1/2 \rangle^{(\mu^+)} \otimes | + 1/2 \rangle^{(\mu^-)} \right)$$

- states with projections of total spin of  $(\mu^+ \mu^-)$  pair onto the  $\mu^+$  momentum direction in the c.m.s.   
 +1, -1 and 0

- If the electron and positron are totally polarized in the direction being **antiparallel** to the positron momentum in the c.m. frame:

$$|\Psi\rangle^{(-1)} = \frac{\sqrt{2}}{\sqrt{2 - \beta_\mu^2 \sin^2 \theta}} \left( \frac{1 - \cos \theta}{2} | + 1 \rangle + \sqrt{1 - \beta_\mu^2} \frac{\sin \theta}{\sqrt{2}} | 0 \rangle + \frac{1 + \cos \theta}{2} | - 1 \rangle \right)$$

- When the primary electron and positron are not polarized, the nonfactorizable states  $|\Psi\rangle^{(+1)}$  and  $|\Psi\rangle^{(-1)}$  are generated with equal probabilities .
- Spin states of two particles with spin  $\frac{1}{2}$  are characterized by the polarization vectors  $\vec{P}_1 = \langle \hat{\sigma}^{(1)} \rangle$  ,  $\vec{P}_2 = \langle \hat{\sigma}^{(2)} \rangle$  and correlation tensor

$$T_{ik} = \langle \hat{\sigma}_i^{(1)} \otimes \hat{\sigma}_k^{(2)} \rangle ;$$

$\hat{\vec{\sigma}} = \{ \hat{\sigma}_1, \hat{\sigma}_2, \hat{\sigma}_3 \} \equiv \{ \hat{\sigma}_x, \hat{\sigma}_y, \hat{\sigma}_z \}$  - vector Pauli operator ;

$\langle \dots \rangle$  - sign of averaging .

- In the one-photon approximation, the generated muons are unpolarized but their spins are strongly correlated .
- Correlation tensor components at the choice of axis  $z$  along the relative momentum of muons in the c.m. frame and axis  $y$  – along the normal to the reaction plane :

$$T_{zz}^{(\mu^+\mu^-)} = \frac{2 \cos^2 \theta + \beta_\mu^2 \sin^2 \theta}{2 - \beta_\mu^2 \sin^2 \theta},$$

$$T_{xx}^{(\mu^+\mu^-)} = \frac{(2 - \beta_\mu^2) \sin^2 \theta}{2 - \beta_\mu^2 \sin^2 \theta}, \quad T_{yy}^{(\mu^+\mu^-)} = -\frac{\beta_\mu^2 \sin^2 \theta}{2 - \beta_\mu^2 \sin^2 \theta},$$

$$T_{xz}^{(\mu^+\mu^-)} = T_{zx}^{(\mu^+\mu^-)} = -\frac{(1 - \beta_\mu^2)^{1/2} \sin 2\theta}{2 - \beta_\mu^2 \sin^2 \theta},$$

$$T_{xy}^{(\mu^+\mu^-)} = T_{yx}^{(\mu^+\mu^-)} = T_{yz}^{(\mu^+\mu^-)} = T_{zy}^{(\mu^+\mu^-)} = 0.$$

The trace of correlation tensor :

$$T^{(\mu^+\mu^-)} = T_{xx}^{(\mu^+\mu^-)} + T_{yy}^{(\mu^+\mu^-)} + T_{zz}^{(\mu^+\mu^-)} = 1,$$

just as it should hold for any triplet states .

The trace of correlation tensor  $T$  determines the angular correlation between the flight directions ( $\vec{n}_1$  and  $\vec{n}_2$ ) of products of decay of two unstable particles in the case when space parity is not conserved .

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- Angular distributions at parity nonconservation :

$$dW_1 = \frac{1}{4\pi} (1 + \alpha_1 \vec{P}_1 \vec{n}_1) d\Omega_1, \quad dW_2 = \frac{1}{4\pi} (1 + \alpha_2 \vec{P}_2 \vec{n}_2) d\Omega_2,$$

$\vec{P}_1, \vec{P}_2$  – polarization vectors,  $\alpha_1$  and  $\alpha_2$  – coefficients of  $P$  – odd asymmetry ;  
**decay is the spin analyzer for an unstable particle .**

Double angular distribution :

$$d^2W(\vec{n}_1, \vec{n}_2) = \frac{1}{16\pi^2} (1 + \alpha_1 \vec{P}_1 \vec{n}_1 + \alpha_2 \vec{P}_2 \vec{n}_2 + \sum_{i=1}^3 T_{ik} n_{1i} n_{2k}) d\Omega_1 d\Omega_2$$

correlation tensor components

The unit vectors  $\vec{n}_1$  ,  $\vec{n}_2$  are defined in the rest frames of the unstable particles **1** and **2** , in the coordinate axes of the c.m. frame of the particle pair .

- Angular correlations between the directions  $\vec{n}_1, \vec{n}_2$  at the decays of two unstable particles are integrated over all angles except the angle  $\beta$  between  $\vec{n}_1$  and  $\vec{n}_2$  ;

$$dN(\beta) = \frac{1}{2} \left( 1 + \frac{\alpha_1 \alpha_2}{3} T \cos \beta \right) \sin \beta d\beta$$

irrespective of the polarization vectors  $\vec{P}_1$  and  $\vec{P}_2$ , which may be equal to zero ( $\cos \beta = \vec{n}_1 \vec{n}_2$ ).

$$T = \sum_{i=1}^3 T_{ii} = \rho_t - 3\rho_s$$
 - trace of the correlation tensor ;

$\rho_s$  and  $\rho_t$  - relative fractions of the singlet and triplet states, respectively .

Coefficient of  $P$  – odd asymmetry of electron emission at the decay

$\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$ , averaged over the electron energy spectrum:  $\alpha = -1/3$  .

At the decay  $\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu$  :  $\alpha = +1/3$  .

- Angular correlations between the flight directions for the electron and positron at the decay of muon pair ( $\mu^+ \mu^-$ ) :

$$dN(\beta) = \frac{1}{2} \left( 1 - \frac{1}{27} T \cos \beta \right) \sin \beta d\beta$$

In the process  $e^+e^- \rightarrow \mu^+\mu^-$  the ( $\mu^+ \mu^-$ ) system is produced in the triplet state, thus,  $T=1$  :  $dN(\beta) = \frac{1}{2} \left( 1 - \frac{1}{27} \cos \beta \right) \sin \beta d\beta$  .

- As it was shown previously,

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**in the case of incoherent mixtures of factorizable states** of two particles with spin  $\frac{1}{2}$  the modulus of sum of any two ( and three ) diagonal components of the correlation tensor cannot exceed unity :

$$|T| = |T_{xx} + T_{yy} + T_{zz}| \leq 1 \quad |T_{xx} + T_{yy}| \leq 1 \quad |T_{xx} + T_{zz}| \leq 1 \quad |T_{yy} + T_{zz}| \leq 1$$

In the case of **non-factorizable coherent superpositions** of two-particle states , the “incoherence” inequalities may be violated .

For the **singlet** state all the inequalities are violated :

$$T_{xx} + T_{yy} = T_{xx} + T_{zz} = T_{yy} + T_{zz} = -2 , \quad T = -3 \quad .$$

In the case of the non-factorizable **triplet** state with zero projection of total spin onto the axis  $z$  , one of the restrictions is not satisfied : **instead of the inequality**  $|T_{xx} + T_{yy}| < 1$  , **the equality**  $T_{xx} + T_{yy} = 2 (>1)$  holds .

In the process of (  $e^+e^-$  ) annihilation , the (  $\mu^+\mu^-$  ) system is generated in the non-factorizable triplet states :  $|\psi\rangle^{(+1)}$  and  $|\psi\rangle^{(-1)}$ .



- One is the “incoherence” inequalities is always violated in the process  $e^+ e^- \rightarrow \mu^+ \mu^-$  at  $\theta \neq 0$  :

$$T_{xx}^{(\mu^+ \mu^-)} + T_{zz}^{(\mu^+ \mu^-)} = \frac{2}{2 - \beta_\mu^2 \sin^2 \theta} > 1.$$

- Analogous consideration -- for the process  $( m_\mu \rightarrow m_\tau, \beta_\mu \rightarrow \beta_\tau )$  .

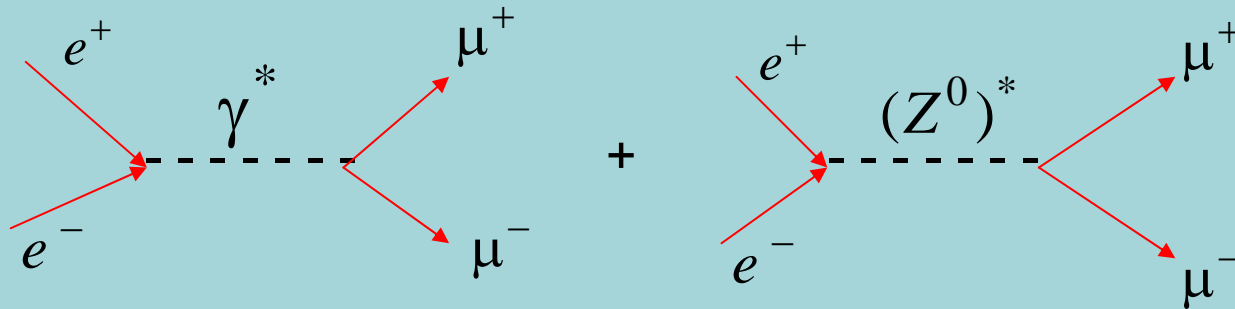
$$e^+ e^- \rightarrow \tau^+ \tau^- ,$$

- At very high energies ( $\beta_\mu \rightarrow 1, \beta_\tau \rightarrow 1$ ), the nonzero components of the correlation tensor are as follows :

$$T_{xx}^{(\mu^+ \mu^-)} = -T_{yy}^{(\mu^+ \mu^-)} = \frac{\sin^2 \theta}{1 + \cos^2 \theta}, \quad T_{zz}^{(\mu^+ \mu^-)} = 1. \quad T_{xx}^{(\mu^+ \mu^-)} + T_{zz}^{(\mu^+ \mu^-)} = \frac{2}{1 + \cos^2 \theta} > 1$$

- At high energies the annihilation processes

$e^+e^- \rightarrow \mu^+\mu^-$ ,  $e^+e^- \rightarrow \tau^+\tau^-$  are conditioned not only by electromagnetic interaction but also by the weak interaction of neutral currents through the virtual  $Z^0$  boson :



- Interference of the amplitudes of the purely electromagnetic and weak interaction  $\longrightarrow$  leads to the charge asymmetry in lepton emission and to the space parity violation.

$\mu^+\mu^-$ ,  $\tau^+\tau^- \rightarrow$  generated in the triplet states:  $^3S_1$ ,  $^3D_1$  and  $^3P_1$    
due to weak interaction

( $J = 1$ , positive  $CP$  parity)

It follows from the structure of “left” and “right” components of neutral currents that the nonzero helicity amplitudes of the processes  $e^+e^- \rightarrow \mu^+\mu^-$ ,  $e^+e^- \rightarrow \tau^+\tau^-$  in the c.m. frame take the form :

$$R_{11}(E) = \frac{e^2}{2E} \left[ 1 + x \left( \xi - \frac{1}{2} \right)^2 \right] ; \quad R_{-1-1}(E) = \frac{e^2}{2E} \left[ 1 + x \xi^2 \right] .$$

$$R_{-11}(E) = R_{1-1}(E) = \frac{e^2}{2E} \left[ 1 + x \xi \left( \xi - \frac{1}{2} \right) \right] ;$$

(  $R_{0\lambda}$  ,  $R_{\lambda'0}$   $\rightarrow$  turn practically to zero at high energies ).

Here:  $\xi = \sin^2 \theta_W$  , where  $\theta_W$  is the Weinberg angle ( angle of gauge boson mixing ) ; parameter  $x \rightarrow$  determines the relative contribution of weak interaction :

$$x = \frac{1}{\sin^2 \theta_W \cos^2 \theta_W} \frac{s}{s - \left( M_{Z^0} - i \frac{\Gamma_{Z^0}}{2} \right)^2} \quad ( s = (2E)^2 )$$

According to the standard model :

$$\frac{1}{\sin^2 \theta_W \cos^2 \theta_W} = \frac{1}{\xi(1-\xi)} \approx 6 = \frac{\sqrt{2} G_F M_{Z^0}^2}{\pi \alpha}$$

$G_F = 1.166 \cdot 10^{-5} \text{ GeV}^{-2} \rightarrow$  universal Fermi constant of weak interaction,  $\alpha = \frac{1}{137}$

Finally, performing the further analysis, we obtain, in particular, that :

- 1) Due to the weak interaction through the  $Z^0$  boson, **the final leptons**, generated at the annihilation of the unpolarized electron and positron, **acquire the longitudinal polarization** ( whereas, if the weak interaction contribution is neglected, the final leptons are correlated but unpolarized ) .

At the energies below and above the resonance energy, **the average helicities of the final leptons have different signs** :

$$E < \frac{M_{Z^0}}{2} \rightarrow x < 0, \bar{\lambda}_+ < 0, \bar{\lambda}_- > 0 ;$$

$$E > \frac{M_{Z^0}}{2} \rightarrow x > 0, \bar{\lambda}_+ > 0, \bar{\lambda}_- < 0 .$$

- 2) **Structure of the correlation tensor of the final leptons is**, on the whole, similar to that for the case of purely electromagnetic annihilation at high energies .

In doing so,  $T_{zz} = 1$  as before ;

the expression for  $T_{xx}$  changes , but, as before,  $T_{xx} = -T_{yy}$  :

$$T_{xx} = -T_{yy} = \sin^2 \theta \frac{[1 + x(\xi^2 - \frac{1}{2}\xi + \frac{1}{8})][1 + x\xi(\xi - \frac{1}{2})]}{a_+(E)(1 + \cos^2 \theta) + 2a_-(E)\cos \theta}$$

where

$$a_+(E) = 1 + \frac{1}{2}x\left(\frac{1}{2} - 2\xi\right)^2 + \frac{1}{4}x^2\left[\left(\frac{1}{2} - \xi\right)^2 + \xi^2\right]^2 ,$$

$$a_-(E) = \frac{1}{8}x + \frac{1}{4}x^2\left(\frac{1}{4} - \xi\right)^2 .$$

Again, one of the incoherence inequalities for the correlation tensor components is violated :  $T_{xx} + T_{zz} > 1$  .

## Main conclusions

- On the basis of the technique of helicity amplitudes, the process  $e^+ e^- \rightarrow \mu^+ \mu^-$  is theoretically investigated in the one-photon approximation. The structure of triplet states of the  $(\mu^+ \mu^-)$  system is found.
- It is shown that, if the primary electron and positron are not polarized, the final muons  $\mu^+$  и  $\mu^-$  are not polarized as well but their spins are strongly correlated. Explicit expressions for the components of correlation tensor of the final  $(\mu^+ \mu^-)$  system are derived.
- The formula for the angular correlation at the decays of muons  $\mu^+$  and  $\mu^-$ , generated in the annihilation process  $e^+ e^- \rightarrow \mu^+ \mu^-$ , into the channels  $\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu$  and  $\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$ , is obtained.
- It is established that in the process  $e^+ e^- \rightarrow \mu^+ \mu^-$  one of the “incoherence” inequalities for the correlation tensor components is always violated.
- The obtained results remain qualitatively valid with the account of weak interaction of neutral currents through the exchange by  $Z^0$  boson.

Thank you !