Lepton Flavour Violation & Signals in Colliders

S. Lola

Dubna, CALC 2009

STRUCTURE OF LECTURES

- **LECTURE 1:**
  Motivation for LVF and generic aspects

- **LECTURE 2:**
  LFV searches in SUSY with R-parity violation *(unstable LSP)*
  [flavour violations from new Yukawa couplings]

- **LECTURE 3:**
  LFV searches in SUSY with R-parity conservation *(stable LSP)*
  [flavour violations from non-universalities & RGE effects]

<u>OBVIOUS LIMITATIONS OF STANDARD MODEL (SM)</u>

- Arbitrary values <u>for masses</u> and <u>interaction constants</u>

- No explanation of correlations in particle charges

  Why Q(e) =Q(p), for two particles that are so very different?

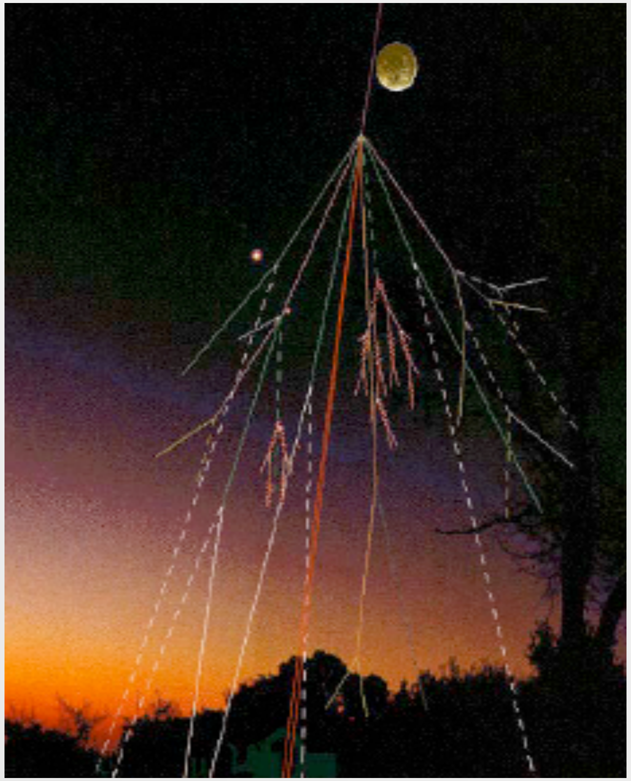
- The SM <u>does not contain interactions that violate</u> L & B

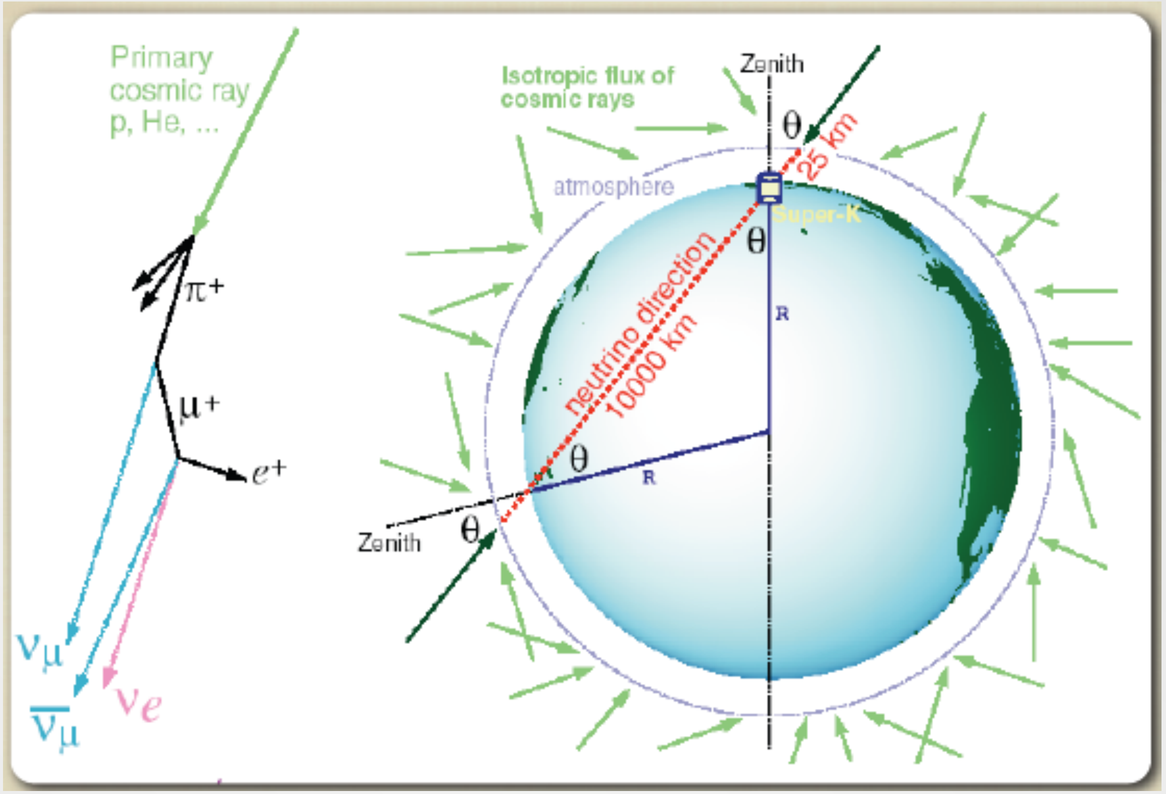
  How is then ΔB / ΔL arising in the Universe ?

- *The symmetries of the SM do not allow neutrino masses*

  In contrast to the experimental evidence of recent years!

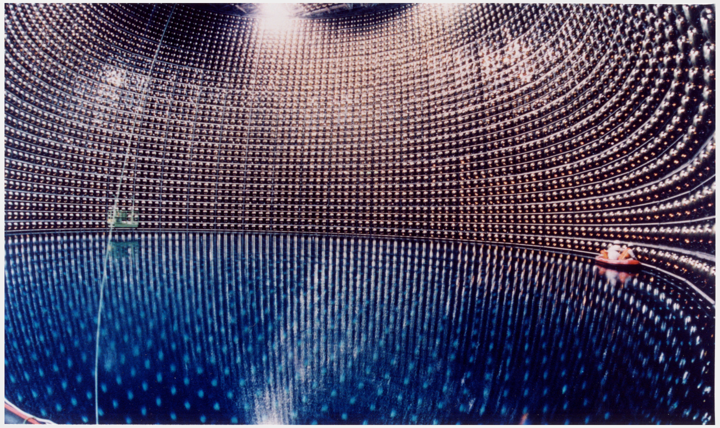
Atmospheric Neutrinos (fewer νμ)

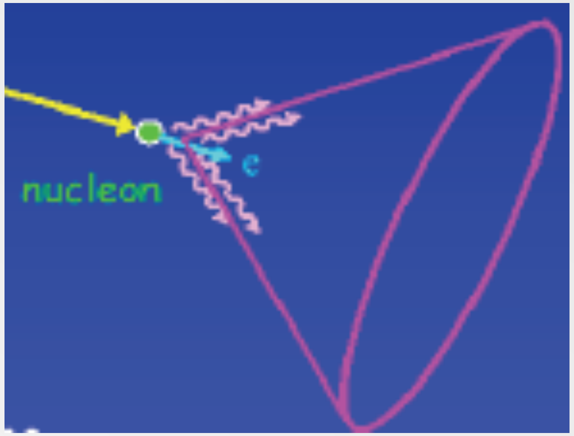




$$p + X \rightarrow \pi + Y$$
$$\hookrightarrow \mu \nu_\mu$$
$$\hookrightarrow e \nu_e \nu_\mu$$

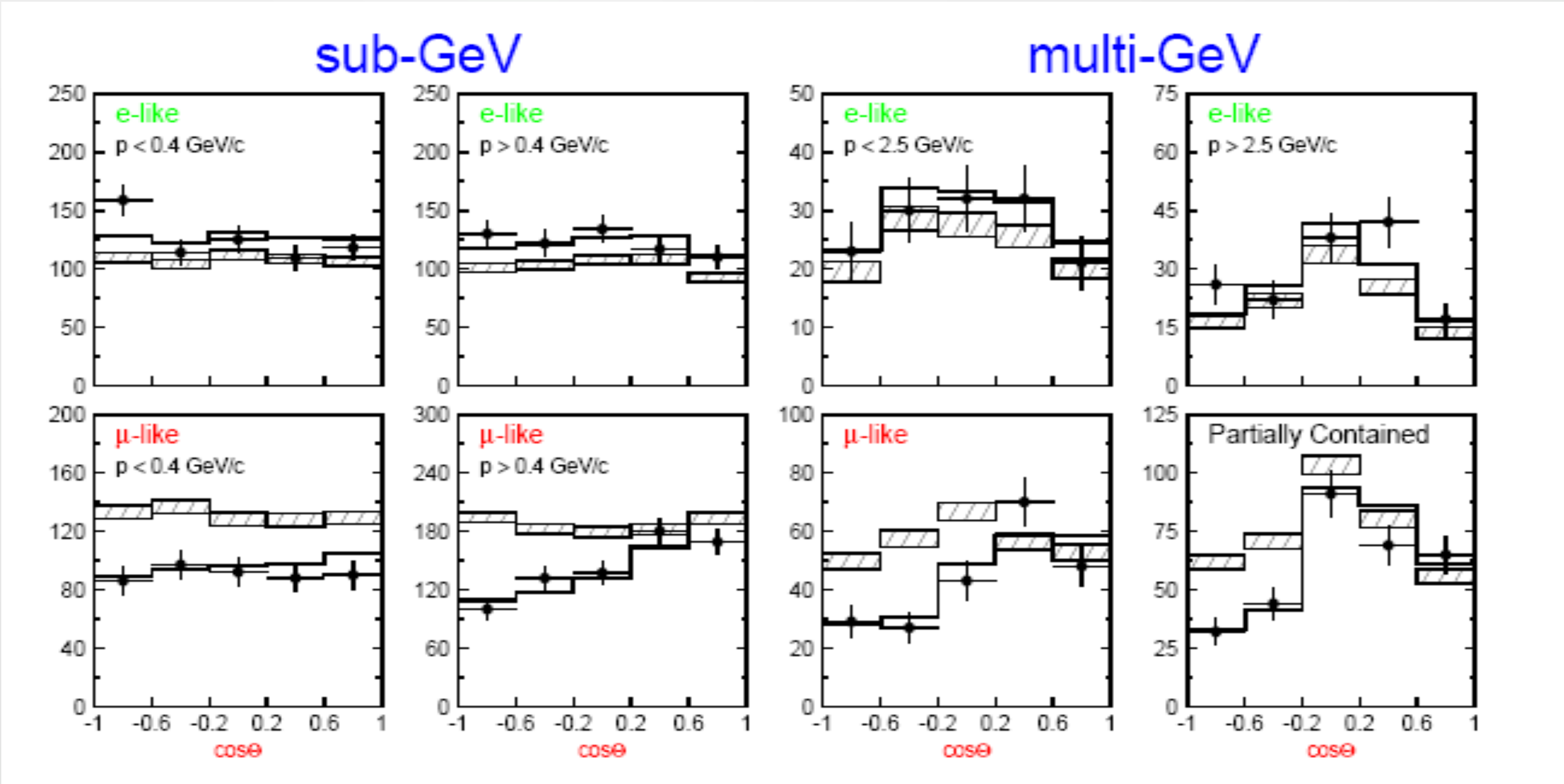
- Expected νμ/νe ratio: **2**
- Observed: **1.2**





Super-Kamiokande

$\nu_\mu \rightarrow \nu_\tau$?



SOLAR NEUTRINOS at SNO

Compare flows:

- Interactions of solar neutrinos with deuterium at SNO

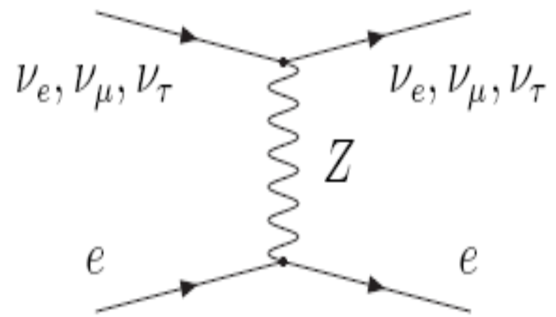
- only the ν_e interact

$$\nu_e + d \rightarrow p + p + e \qquad (1)$$

- $\nu - e$ scattering at Super-Kamiokande

where all $\nu_{e,\mu,\tau}$ interact

$$\nu_{e,\mu,\tau} + e \rightarrow \nu_{e,\mu,\tau} + e \qquad (2)$$

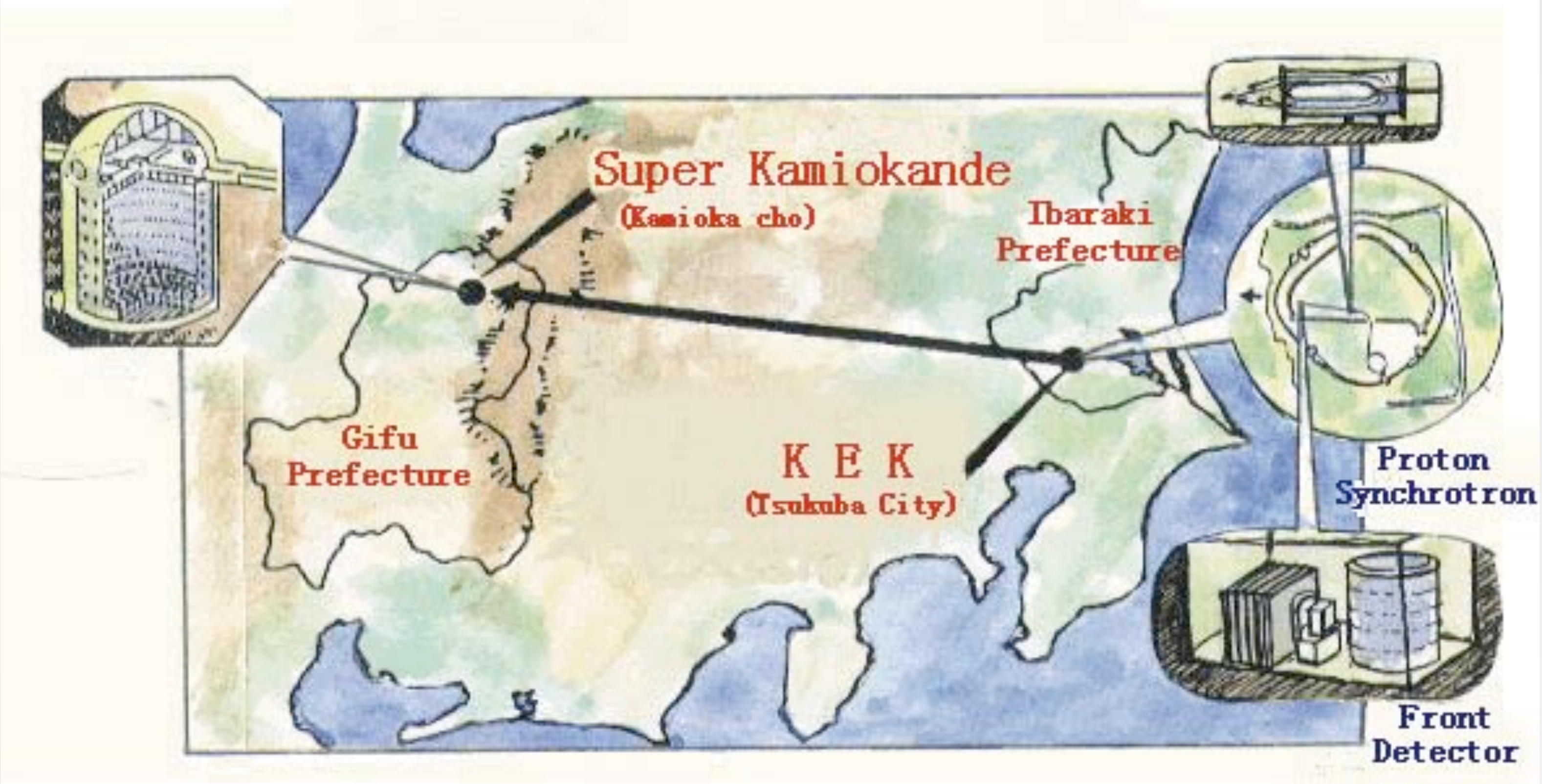


The 25% is due to ν_e *from the sun oscillating away - still counted in (2) but not in (1)*

νe → νμ ?

Kamland: Measurements of $\bar{\nu}_e + p \rightarrow e^+ + n$

K2K *Neutrino Oscillations:* Beam travels for 250 km and fewer events than expected are detected



MINOS - the beam travels for 750 km

What did we learn for neutrino masses/mixings

	lower limit	best value	upper limit
$\Delta m^2_{sun}(10^{-5}\ eV^2)$	5.4	6.9	9.5
$\Delta m^2_{atmo}(10^{-3}\ eV^2)$	1.4	2.6	3.57
$\sin^2\theta_{12}$	0.23	0.30	0.39
$\sin^2\theta_{23}$	0.31	0.52	0.72
$\sin^2\theta_{13}$	0	0.006	0.1

-Nearly maximal 2-3 (atmospheric) mixing

-Large 1-2 (solar) mixing

-Small 1-3 mixing *(CHOOZ exp)*

-Very small neutrino mass differences

- **In absence of light ν_s:**

1: Sol.+Atmo+$\mathcal{O}$ (eV) mass $\Rightarrow m_{\nu_e} \approx m_{\nu_\mu} \approx m_{\nu_\tau}$ $\mathcal{O}$(eV)

2: Only Sol. + Atmo: large mass hierarchies

m_{ν_3}

Δm_{atm} $\qquad$ $m_{\nu_3} \approx \Delta m_{atmo}$

m_{ν_2}

Δm_{sol} $\qquad$ $m_{\nu_2} \approx \Delta m_{sol}$

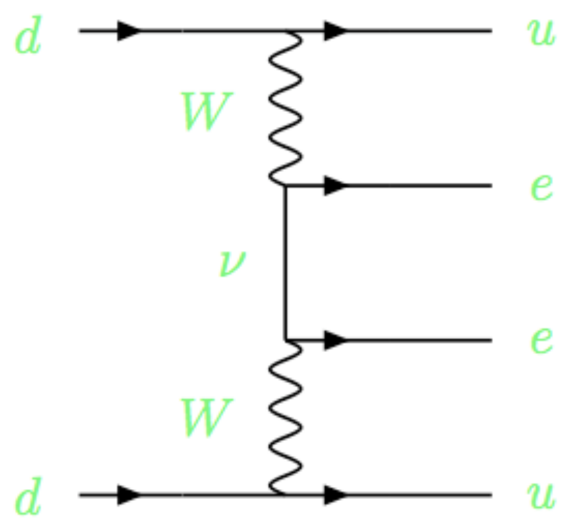
m_{ν_1}

(also possibility for inverse hierarchies)

Direct experimental bounds

- $m_{\nu_e} \leq 2.8\ \text{eV}$ by searches in β decays
- $m_{\nu_\mu} \leq 170\,\text{keV}$, $\pi^+ \to \mu^+ + \nu_\mu$
- $m_{\nu_\tau} \leq 18\ \text{MeV}$, by hadronic τ-decays
- Major. neutrino mass $(\Delta L = 2)$ bound from $00\beta\beta$:

$m_{\nu_{ee}} \leq\ 0.2\text{eV}$



Neutrino masses and oscillations: basic physics

- For the simple case of 2×2 mixing, the mass eigenvectors $|\nu_{1,2}\rangle$, do not coincide with the interaction ones $|\nu_{e,\mu}\rangle$

$$\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \end{pmatrix}$$

- Even if at $t = x = 0$ only ν_e

$$\begin{aligned} |\nu_e(t)\rangle &= \cos\theta e^{-iE_1 t}|\nu_1\rangle + \sin\theta e^{-iE_2 t}|\nu_2\rangle \\ E_{1,2} &= \sqrt{p^2 + m_{1,2}^2} \approx p + \frac{m_{1,2}^2}{2p} \end{aligned}$$

at $t > 0$, also ν_μ with probability

$(P_{\nu_a \to \nu_b}(L) = |\langle \nu_a(0)|\nu_b(t)\rangle|^2)$

$$P(\nu_e \rightarrow \nu_\mu) = \sin^2(2\theta)\sin^2\left(\frac{\Delta m^2_{21} t}{4p}\right)$$

- Mixing angle $\leftrightarrow$ Amplitude of oscillations
  $\Delta m^2 \leftrightarrow$ Period of oscillations

Characteristic oscillation length $L_\nu = \frac{4\pi p}{\Delta m^2_{21}}$

Fundamental formulas for all searches

Generate to 3× 3 mixing

- Neutrino Oscillations:

$$|\nu_a> \; = \sum_i U_{ai}|\nu_i>, \quad a = e, \mu, \tau; \; i = 1, 2, 3$$

$$|\nu_j> \; = \sum_b U^\dagger_{jb}|\nu_b>$$

Parameters: 2 independent Δm^2, 3 angles, 1 phase

A straightforward parametrisation is:

$$U = \begin{pmatrix} c_1c_3 & s_1c_3 & s_3e^{i\delta} \\ -s_1c_2 - c_1s_2s_3e^{i\delta} & c_1c_2 - s_1s_2s_3e^{i\delta} & s_2c_3 \\ s_1s_2 - c_1c_2s_3e^{i\delta} & -c_1s_2 - s_1c_2s_3e^{i\delta} & c_2c_3 \end{pmatrix}$$

(A) In SM massless neutrinos:

- No-right handed neutrinos $\Rightarrow$ *no Dirac mass*
- from renormalisability and gauge invariance $\Rightarrow$

  no Majorana mass either

(B) For massive neutrinos, need to extend theory:

1. Extend only lepton sector.
2. Extend only Higgs sector.
3. Extend both Higgs and lepton sector.
4. Add other new particles and interactions.

Signatures of these can be seen at the LHC!

ν_R and See-Saw mechanism

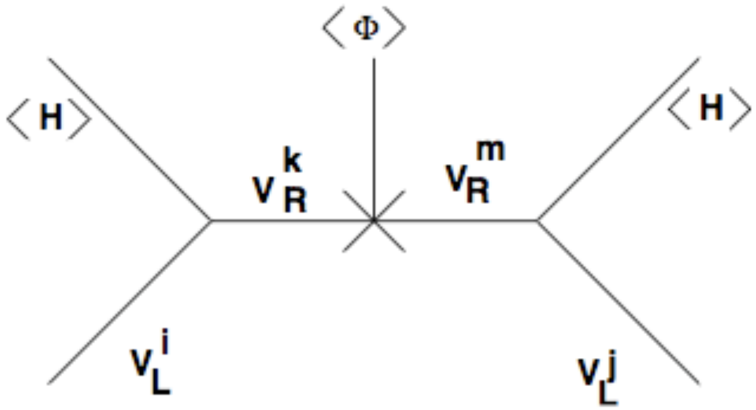
How can we generate naturally light neutrinos?

Combine m_ν^D and M_{ν_R} to write a mass matrix

$$\mathcal{M}_\nu = \begin{pmatrix} 0 & m_\nu^D \\ m_\nu^D & M_{\nu_R} \end{pmatrix}$$

If $M_{\nu_R} \gg m_\nu^D$, a very heavy eigenvalue $M_N \approx M_{\nu_R}$ and a very light

$$m_{eff} \approx \left| \frac{(m_\nu^D)^2}{M_{\nu_R}} \right|$$



For $(m_\nu^D)_{33} \approx (200\,\mathrm{GeV})$ $(\lambda_N \approx \lambda_t)$ and $M_{N_3} \approx \mathcal{O}(10^{13}\,\mathrm{GeV})$,

$m_{eff} \approx 1\ \ \mathrm{eV}$

- **Lepton mixing** analogous to V_{CKM} for quarks

$V_{MNS} = V_\nu^\dagger V_{\ell_L}$: generated both by $\ell^\pm$ and ν

Effects of radiative corrections on neutrino masses and mixing

For i, j, generation indices

$$\frac{1}{m_{eff}^{ij}}\frac{d}{dt}m_{eff}^{ij} = \frac{1}{8\pi^2}\left(-c_i g_i^2 + 3\lambda_t^2 + \frac{1}{2}(\lambda_i^2 + \lambda_j^2)\right)$$

$$16\pi^2\frac{d}{dt}\sin^2 2\theta_{23} = 2\sin^2 2\theta_{23}(1 - 2\sin^2\theta_{23})\lambda_\tau^2\frac{m_{eff}^{33} + m_{eff}^{22}}{m_{eff}^{33} - m_{eff}^{22}}$$

$\sin^2 2\theta_{23}$ affected by quantum corrections if:

(i) λ_τ large (large $\tan\beta$) (ii) $m_{eff}^{33} - m_{eff}^{22}$ small

Semi-analytic and numerical studies $\Rightarrow$

- The mixing can even be amplified/destroyed

Corrections from Massive Neutrinos (L-sector)

$$16\pi^2 \frac{d}{dt}\lambda_t = \left(6\lambda_t^2 + \lambda_N^2 - G_U\right)\lambda_t$$

$$16\pi^2 \frac{d}{dt}\lambda_N = \left(4\lambda_N^2 + 3\lambda_t^2 - G_N\right)\lambda_N$$

$$16\pi^2 \frac{d}{dt}\lambda_b = \left(\lambda_t^2 - G_D\right)\lambda_b$$

$$16\pi^2 \frac{d}{dt}\lambda_\tau = \left(\lambda_N^2 - G_E\right)\lambda_\tau$$

$$t\frac{d}{dt}(m_{\tilde{\ell}}^2)_i^j = \frac{1}{16\pi^2}\left\{(m_{\tilde{\ell}}^2\lambda_\ell^\dagger\lambda_\ell)_i^j + (m_{\tilde{\ell}}^2\lambda_\nu^\dagger\lambda_\nu)_i^j + \ldots\right\}$$

$$\delta m_{\tilde{\ell}} \propto \frac{1}{16\pi}\ln\frac{M_{\rm GUT}}{M_N}\lambda_\nu^\dagger\lambda_\nu m_{\rm SUSY}^2$$

Correlations with quark-charged lepton hierarchies

(i.e. Why the top quark mass so much larger?)

- A family symmetry generates the observed hierarchies

	Q_i	$\bar{U}_i$	$\bar{D}_i$	L_i	$\bar{E}_i$	H_2	H_1
$U(1)$	a_i	a_i	a_i	b_i	b_i	$-2a_3$	wa_3

- The rest of the terms appear once the symmetry is broken

$$Q_i \bar{U}_j H_2 (<\theta> / M)^n$$

Similarly for other fermions, including neutrinos

- Up-mass matrix:

Top coupling $Q_3\bar{U}_3H_2$ 0 charge $\Rightarrow$ allowed

All other couplings forbidden

$$M^{up} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- Suppose singlets θ with non-0 flavor-charges

(singlets expected in realistic models)

Then: invariant terms $Q_i\bar{U}_jH_2(<\theta>/M)^n$

n depending on flavour charges

- Hierarchical mass structures generated for ALL fermions

$SU(5)$

(i) Assume the family symmetry is combined with $SU(5)$

(ii) Use the GUT structure ONLY to constrain $U(1)$ charges

Under this group we have the following relations:

$$
\begin{aligned}
Q_{(q,u^c,e^c)_i} &= Q_i^{10} \\
Q_{(l,d^c)_i} &= Q_i^{\bar{5}} \\
Q_{(\nu_R)_i} &= Q_i^{\nu_R}
\end{aligned}
$$

- M_{up} symmetric
- $M_{\ell^\pm} = M_{down}^T$
- L lepton mixing $\approx$ R down-quark one

Can we obtain acceptable patterns of masses/mixings? i.e.

$$\frac{M_u}{m_t} = \begin{pmatrix} \bar{\epsilon}^6 & \bar{\epsilon}^5 & \bar{\epsilon}^3 \\ \bar{\epsilon}^5 & \bar{\epsilon}^4 & \bar{\epsilon}^2 \\ \bar{\epsilon}^3 & \bar{\epsilon}^2 & 1 \end{pmatrix}, \frac{M_{down}}{m_b} = \begin{pmatrix} \bar{\epsilon}^4 & \bar{\epsilon}^3 & \bar{\epsilon}^3 \\ \bar{\epsilon}^3 & \bar{\epsilon}^2 & \bar{\epsilon}^2 \\ \bar{\epsilon} & 1 & 1 \end{pmatrix}$$

$$\frac{M_\ell}{m_\tau} = \begin{pmatrix} \bar{\epsilon}^4 & \bar{\epsilon}^3 & \bar{\epsilon} \\ \bar{\epsilon}^3 & \bar{\epsilon}^2 & 1 \\ \bar{\epsilon}^3 & \bar{\epsilon}^2 & 1 \end{pmatrix}, m_{eff} \propto \begin{pmatrix} \bar{\epsilon}^2 & \bar{\epsilon} & \bar{\epsilon} \\ \bar{\epsilon} & 1 & 1 \\ \bar{\epsilon} & 1 & 1 \end{pmatrix}$$

$SO(10)$

- All L- and R-handed fermions in the 16 of $SO(10)$
- Both MSSM Higgs fields fit in a single 10 of $SO(10)$ $\Downarrow$

For all fermions, *L-R symmetric textures*, similar structure

(different expansion parameters due to Higgs mixing)

Flipped $SU(5)$

$$Q_{(q,d^c,\nu^c)_i} = Q_i^{10}, \quad Q_{(l,u^c)_i} = Q_i^{\bar{5}}, \quad e^c \text{ singlet of } SU(5)$$

- Symmetric M_{down}
- $m_\nu^D = M_{up}^T$

Minimal Models with Abelian Flavour Symmetries

- Large splitting between fermion masses

  Naturally leads to large neutrino hierarchies

- Unknown phases/order unity coefficients $\Downarrow$

  Difficult to obtain naturally degenerate neutrinos

- In many models lepton hierarchies consistent with mostly SAMSW but LAMSW possible, ie by see-saw conditions

Models with non-Abelian flavour symmetries

- Degenerate ν and $\ell^{\pm}$ textures assuming

  ie that the lepton fields are SO(3) triplets

- Subsequently break SO(3) so as:

  large charged lepton splitting/ small neutrino splitting

- Favour almost-degenerate neutrino textures

- Textures with (almost)-bimaximal mixing predicted

  LAMSW / VO oscillations for solar neutrinos

Flavour symmetries determine soft SUSY terms

$$\mathcal{L}_{m^2} = m_0^2(\phi_1^*\phi_1+\phi_2^*\phi_2+\phi_3^*\phi_3+\left(\frac{\langle\theta\rangle}{M_{fl}}\right)^{q_2-q_1}\phi_1^*\phi_2+\left(\frac{\langle\theta\rangle}{M_{fl}}\right)^{q_3-q_1}\phi_1^*\phi_3+\left(\frac{\langle\theta\rangle}{M_{fl}}\right)^{q_3-q_2}\phi_2^*\phi_3+\text{h.c.}).$$

L-R symmetric

$$\begin{pmatrix} 1 & \tilde{\epsilon}^{|a+2b|} & \tilde{\epsilon}^{|a+b|} \\ \tilde{\epsilon}^{|a+2b|} & 1 & \tilde{\epsilon}^{|b|} \\ \tilde{\epsilon}^{|a+b|} & \tilde{\epsilon}^{|b|} & 1 \end{pmatrix}$$

SU(5)

$$\mathbf{E}_L \sim \begin{pmatrix} 1 & \lambda^2 & \lambda^2 \\ \lambda^2 & 1 & 1 \\ \lambda^2 & 1 & 1 \end{pmatrix} \qquad \mathbf{E}_R \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}$$

$$\begin{aligned} Q_{(q,u^c,e^c)_i} &= Q_i^{10} \\ Q_{(l,d^c)_i} &= Q_i^{\bar{5}} \\ Q_{(\nu_R)_i} &= Q_i^{\nu_R} \end{aligned}$$

$$\frac{M_\ell}{m_\tau} = \begin{pmatrix} \bar{\epsilon}^4 & \bar{\epsilon}^3 & \bar{\epsilon} \\ \bar{\epsilon}^3 & \bar{\epsilon}^2 & 1 \\ \bar{\epsilon}^3 & \bar{\epsilon}^2 & 1 \end{pmatrix}$$

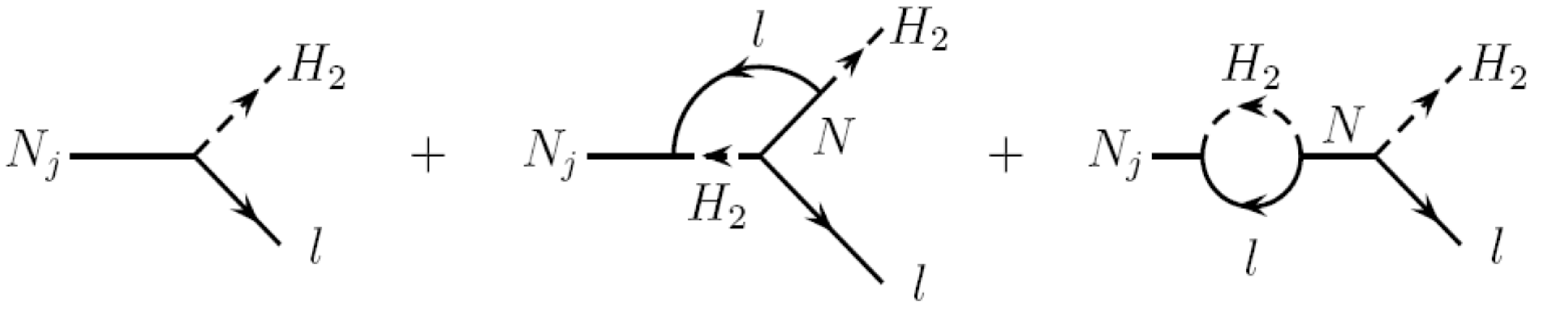
L: (1,0,0)

R: (3,2,0)

Baryonic/Leptonic Asymmetry in Universe

- Violation of B, L $n_B = N_B - N_{\bar{B}}$
- CP violation *(differences in scattering of (anti)-matter)*
- Out-of-Equilibrium Conditions *(no reverse reaction to bring system back to initial state)*

- Neutrino Mass: ΔL & finally ΔB (due to interactions)



Tree level and one-loop diagrams contributing to heavy neutrino decays

Out-of-equilibrium condition:

Decay rates smaller than Hubble parameter H at $T \approx M_{N_1}$

Three-level width of N_1: $\Gamma = \frac{(\lambda^\dagger\lambda)_{11}}{8\pi} M_{N_1}$

Compare with: $H \approx 1.7\, g_*^{1/2}\, \frac{T^2}{M_p}$

$(g_*^{MSSM} \approx 228.75,\ g_*^{SM} = 106.75)$

$$\Rightarrow \frac{(\lambda^\dagger\lambda)_{11}}{14\pi g_*^{1/2}} M_p < M_{N_1}$$

More accurate by looking at Boltzmann equations

CP-violating asymmetry, ϵ

(interference between tree-level and 1-loop amplitudes)

$$\epsilon_j = \frac{1}{(8\pi\lambda^\dagger\lambda)_{11}} \sum_j \mathrm{Im}\left[(\lambda^\dagger\lambda)^2_{1j}\right] f\left(\frac{m^2_{N_j}}{m^2_{N_1}}\right)$$

$$f(y) = \sqrt{y}\left[1 - (1+y)\ln\left(\frac{1+y}{y}\right)\right]$$

Plus self-energy corrections $\tilde{\delta} \propto \frac{M_{N_1}}{(M_{N_2} - M_{N_1})}$

Satisfying the basic conditions in such a framework

gives additional constraints

LFV IN RARE DECAYS AND CONVERSIONS

In SM extensions with $\Delta L_i \neq 0$, non-zero rates for processes such as:

$$\mu \rightarrow e\gamma$$

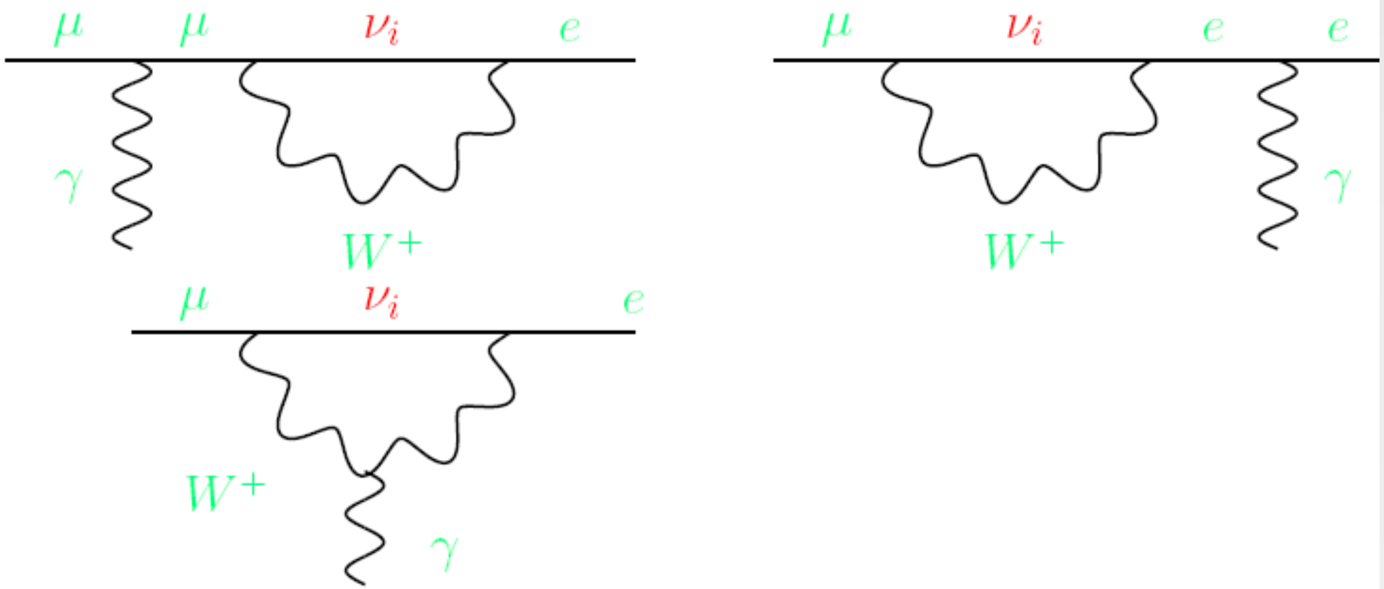
$$\tau \rightarrow \mu\gamma$$

$\mu - e$ conversion on nuclei

Very good expected future BR sensitivities:

$\mu \rightarrow e\gamma$	10^{-14}
$\mu^- Ti \rightarrow e^- Ti$	10^{-18}

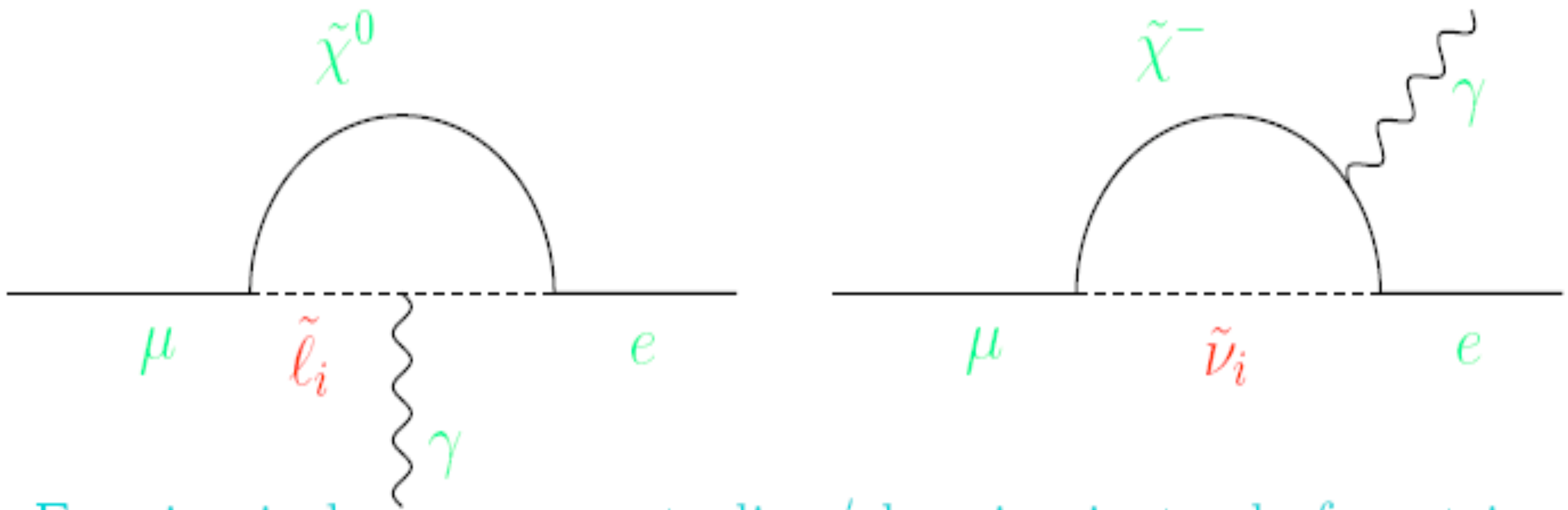
1. $\mu \to e\gamma$ in the SM with $m_{\nu_i} \neq 0$



$\nu_i = \nu_\mu \cos\theta + \nu_e \sin\theta$, $\Gamma = \frac{1}{16}\,\frac{G_F^2\ m_\mu^5\ \alpha}{128\ \pi^4}\left(\frac{m_2^2 - m_1^2}{m_W^2}\right)\sin^2\theta\cos^2\theta$

$BR \leq 10^{-50}$, for Δm_{12}^2 from neutrino data too small!

Very different in SUSY!



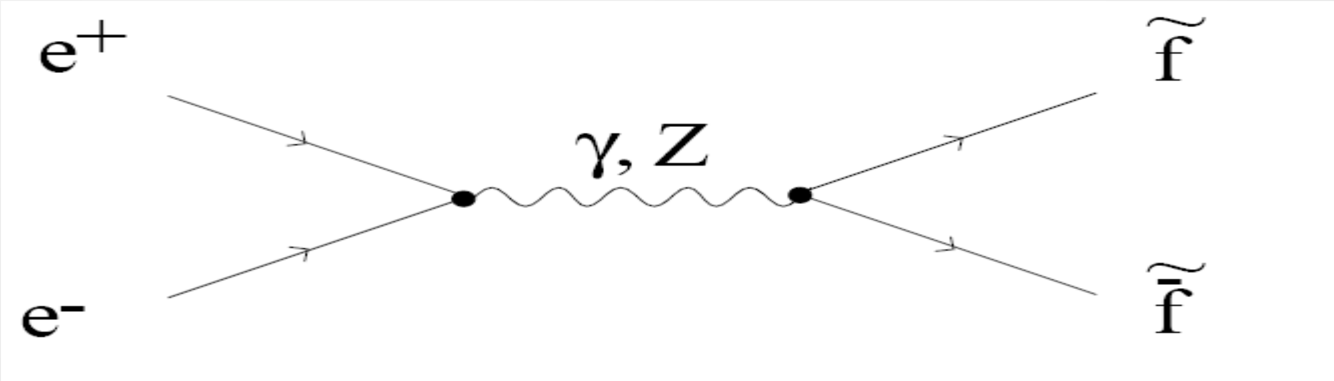
Fermion in loop now neutralino/chargino instead of neutrino

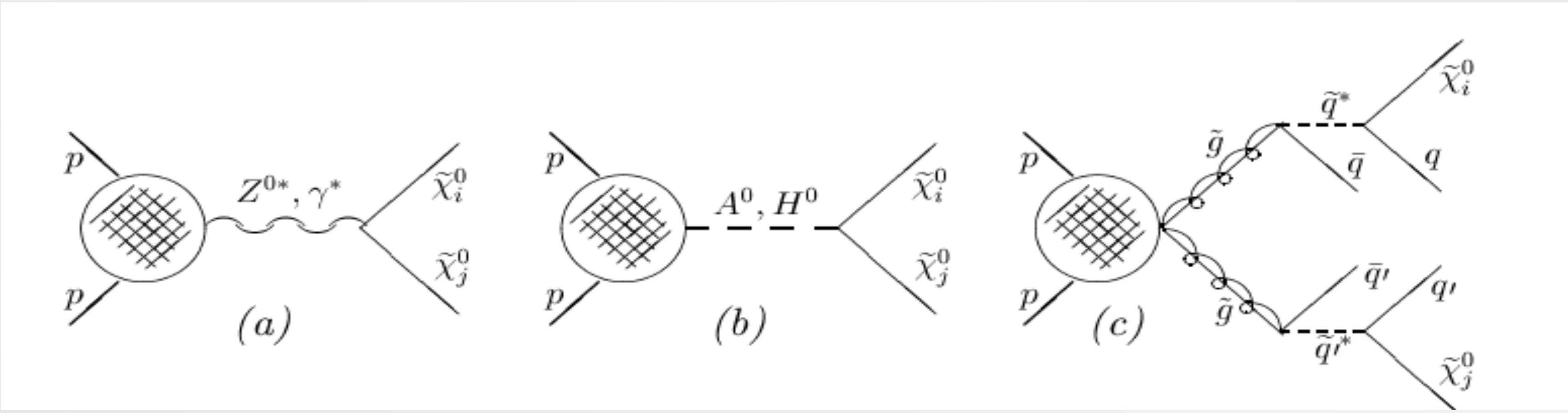
$(m_{\tilde{\chi}^0}, m_{\tilde{\chi}^\pm} \gg m_\nu \Rightarrow \text{large rates})$

Passing to Colliders

SUSY Interactions: All of the SM

+ vertices where 2 particles substituted by s-particles

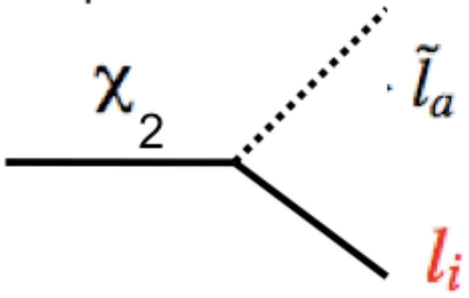




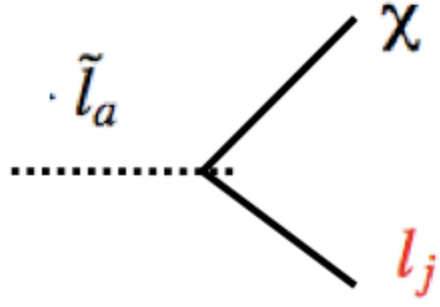
Simplest SUSY Models: “Missing Energy Signatures”

Example of FC versus LFV

Lepton pairs in neutralino decays:



$$\chi_2 \to \tilde{l}_a \; l_i$$
$$\downarrow$$
$$\chi \; l_j$$



LC: Decays involving leptons of same flavour

LFV: $$\chi_2 \to \chi + \tau^{\pm}\mu^{\mp}$$

How large LFV?

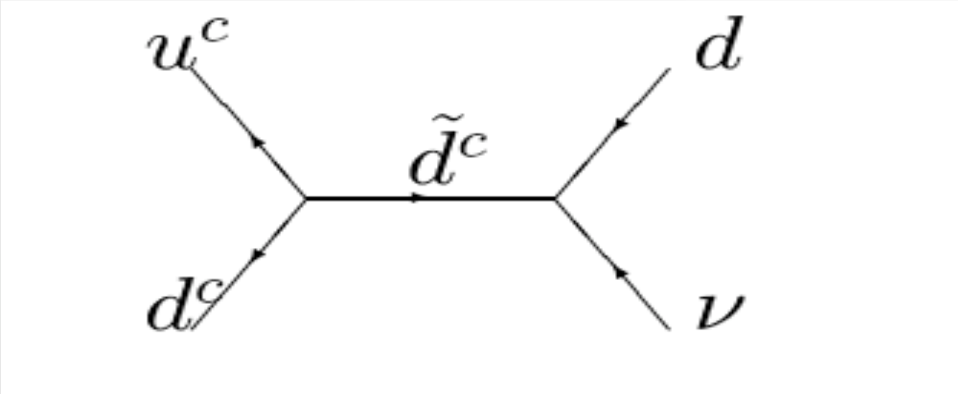
Depends on sfermion mixing (back to MODEL BUILDING)

SUSY & R-violation?

In addition to interactions generating fermion masses

also $\lambda_{ijk} L_i L_j \bar{E}_k \qquad \lambda^{'}_{ijk} L_i Q_j \bar{D}_k \qquad \lambda^{''}_{ijk} \bar{U}_i \bar{D}_j \bar{D}_k$

- These violate *B & L [45 couplings from SU(2),SU(3)]*
- If they co-exist, unacceptably fast *p decay*



Choices:

X R-parity *(SM: +1 , SUSY: -1)*

Kills all terms with ΔL, ΔB

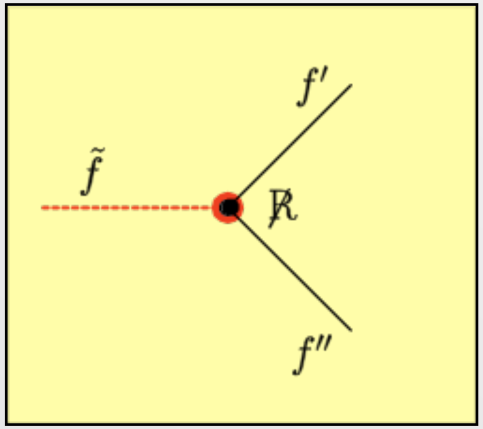
***Stable LSP**, possibly dark matter*

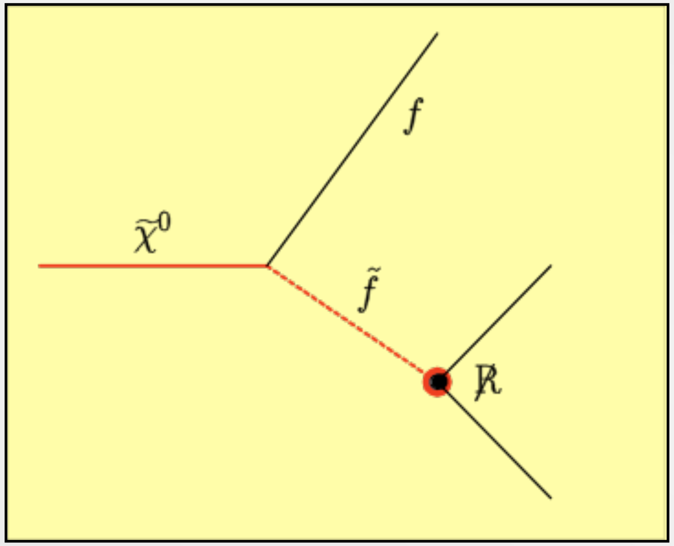
Collider Signal: Missing Energy

✓ Symmetries i.e. Permiting only *ΔL* ***or*** *ΔB*

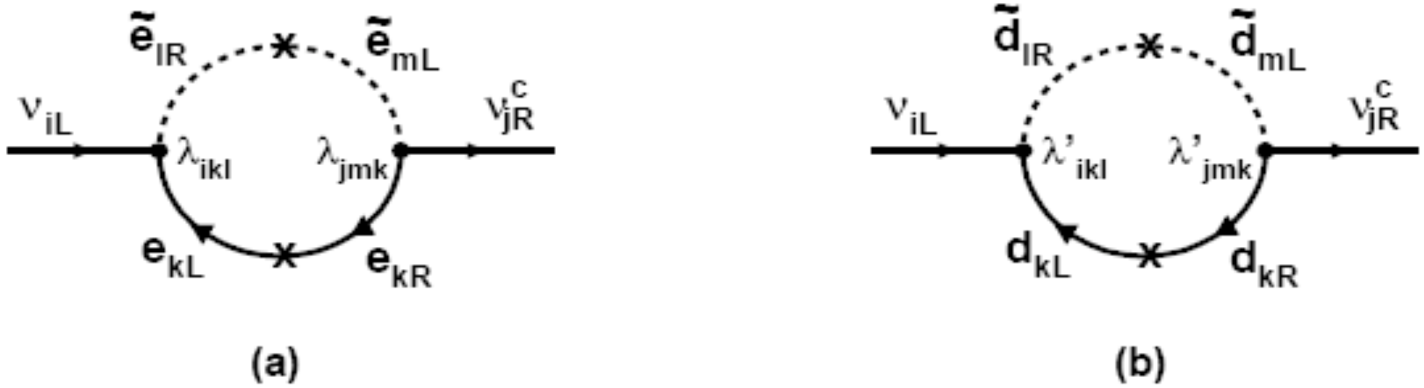
***LSP: Unstable** (do we lose SUSY Dark Matter?)*

Colliders: Multi-lepton and/or Multi-jet events



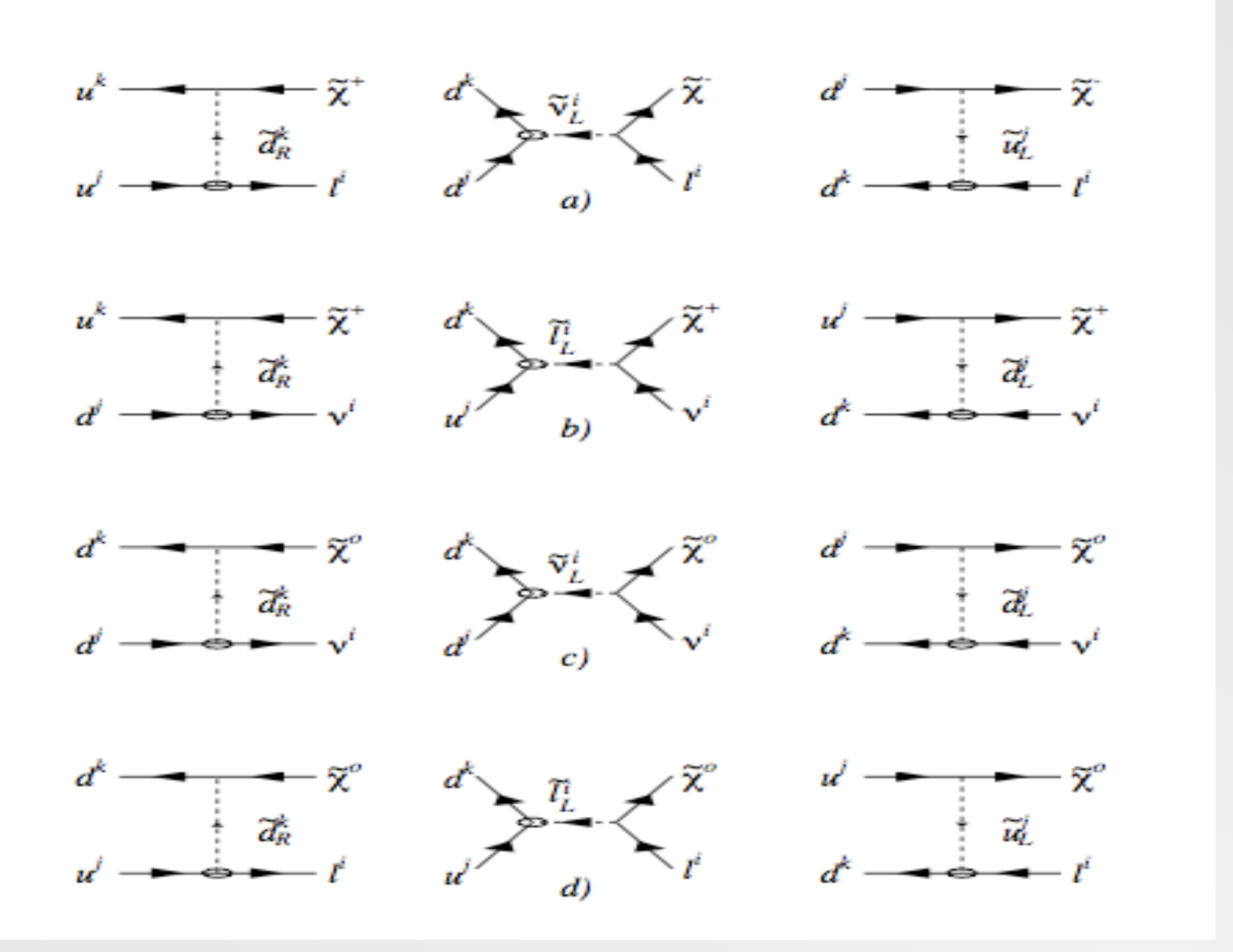


Neutrino Masses in R-violation



$$M^\nu_{ij}|_\lambda = \frac{1}{16\pi^2} \sum_{k,l,m} \lambda_{ikl}\lambda_{jmk}\, m_{e_k} \frac{(\tilde{m}^{e\,2}_{LR})_{ml}}{m^2_{\tilde{e}_{Rl}} - m^2_{\tilde{e}_{Lm}}} \ln\left(\frac{m^2_{\tilde{e}_{Rl}}}{m^2_{\tilde{e}_{Lm}}}\right) + (i \leftrightarrow j)$$

$$M^\nu_{ij}|_{\lambda'} = \frac{3}{16\pi^2} \sum_{k,l,m} \lambda'_{ikl}\lambda'_{jmk}\, m_{d_k} \frac{(\tilde{m}^{d\,2}_{LR})_{ml}}{m^2_{\tilde{d}_{Rl}} - m^2_{\tilde{d}_{Lm}}} \ln\left(\frac{m^2_{\tilde{d}_{Rl}}}{m^2_{\tilde{d}_{Lm}}}\right) + (i \leftrightarrow j)$$



Results strongly depend on flavour-structure of operators

LECTURE 1: SUMMARY

- Neutrino data provides the first experimental indication for physics BSM and for ΔL & LFV

- Link of neutrino textures to other fermion masses has significant implications for flavour structure of underlying theory and for model building

- This flavour structure may be visible through concrete processes in colliders (minimal schemes & beyond, TO BE DISCUSSED…)

- Additional motivation/constraints if one believes in leptogenesis (& SUSY Dark Matter, TO BE DISCUSSED…)