Top-quark processes in SANC

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on behalf of SANC group

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Calculations for Modern and Future Colliders
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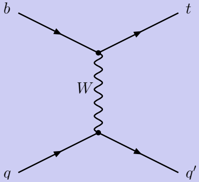
Outline

- introduction to single top physics
- top processes in SANC
- single top production
- top decays
- integrators, cascade approximations
- summary

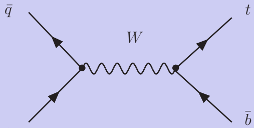
Cross-sections for single top-quark production

σ pb	Tevatron t	LHC t	LHC $\bar{t}$
t-channel	1.15 ± 0.07	150 ± 6	92 ± 4
s-channel	0.54 ± 0.04	7.8 ± 0.7	4.3 ± 0.3
Wt	0.14 ± 0.03	44 ± 5	44 ± 5

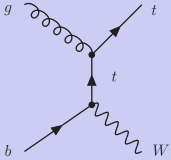
single top-quark production channels



t-channel



s-channel



Wt

t-channel at the LHC

t production

- $ub \to dt \sim 74\%$
- $\bar{d}b \to \bar{u}t \sim 12\%$
- $\bar{s}b \to \bar{c}t \sim 8\%$
- $cb \to st \sim 6\%$

$\bar{t}$ production

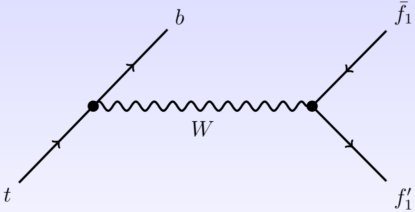
- $d\bar{b} \to u\bar{t} \sim 56\%$
- $\bar{u}\bar{b} \to \bar{d}\bar{t} \sim 20\%$
- $s\bar{b} \to c\bar{t} \sim 13\%$
- $\bar{c}\bar{b} \to \bar{s}\bar{t} \sim 11\%$

Introduction

Top decays

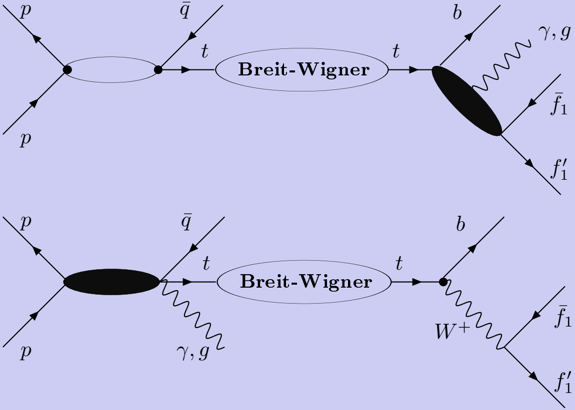
decays ratios

- $t \longrightarrow bl^+\nu_l$ 11% per lepton
- $t \longrightarrow bq\bar{q}'$ 33% for $u\bar{d}$, $c\bar{s}$



Top physics in SANC: cascade approach

top production and decay in cascade



1-loop calculations in SANC

From previous talk:

Precomputation: to precompute as many one-loop diagrams and derived quantities (renormalization constants, etc) as possible (to save CPU time)

Covariant Amplitudes (CA) and scalar Form Factors (FF) — $\mathcal{F}_i$

$\mathcal{A} \propto \gamma_\mu \mathcal{F}_1 + \sigma_{\mu\nu} q_\nu \mathcal{F}_2$

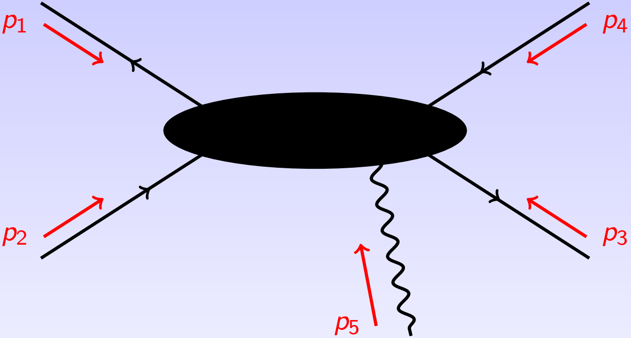
Helicity Amplitudes (HA) — $\mathcal{H}_{\{\lambda_i\}}(\mathcal{F}_i)$

Standard approach: $O \propto |\mathcal{A}|^2$
while in terms of HAs: $O \propto \sum_{\{\lambda_i\}} |\mathcal{H}_{\{\lambda_i\}}|^2$

Accompanying Bremsstrahlung (BR)
Monte-Carlo generator (MC)

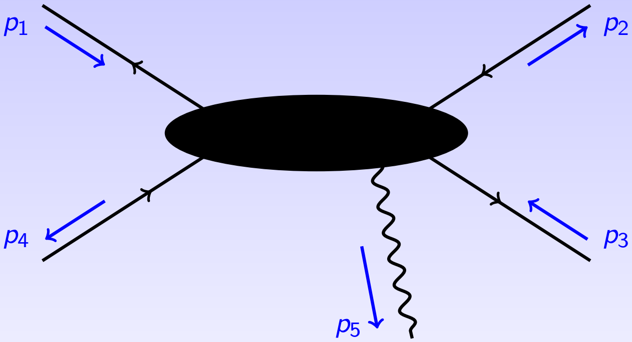
Scalar form factors: multichannel approach

Compute virtual diagrams with all momenta incoming



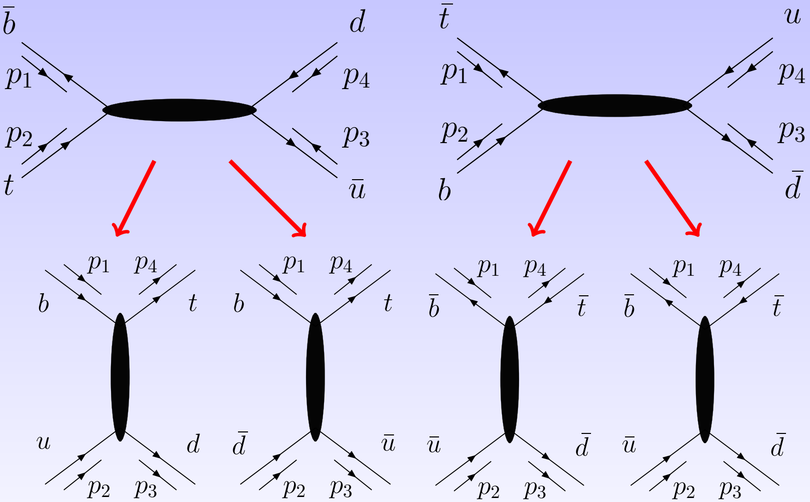
Scalar form factors: multichannel approach

Compute virtual diagrams with all momenta incoming



Get different channels by making permutations of momenta

Convert to t-channel

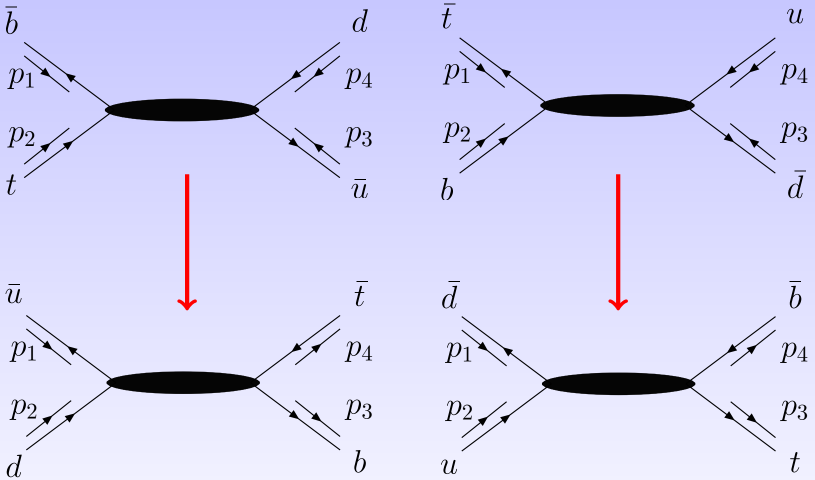


$$p_1 \rightarrow -p_4 \quad p_2 \rightarrow +p_1 \quad p_3 \rightarrow -p_3 \quad p_4 \rightarrow +p_2$$

$$p_1 \rightarrow -p_4 \quad p_2 \rightarrow +p_1 \quad p_3 \rightarrow +p_2 \quad p_4 \rightarrow -p_3$$

$$p_1 \rightarrow +p_1 \quad p_2 \rightarrow -p_4 \quad p_3 \rightarrow +p_2 \quad p_4 \rightarrow -p_3$$

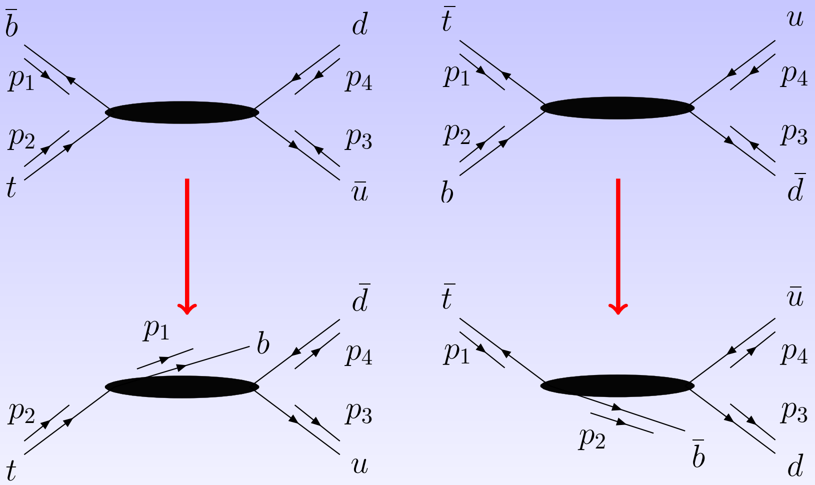
$$p_1 \rightarrow +p_1 \quad p_2 \rightarrow -p_4 \quad p_3 \rightarrow +p_2 \quad p_4 \rightarrow -p_3$$



$$p_1 \longrightarrow -p_3$$
$$p_2 \longrightarrow -p_4$$
$$p_3 \longrightarrow +p_1$$
$$p_4 \longrightarrow +p_2$$

$$p_1 \longrightarrow -p_3$$
$$p_2 \longrightarrow -p_4$$
$$p_3 \longrightarrow +p_1$$
$$p_4 \longrightarrow +p_2$$

Convert to top decay



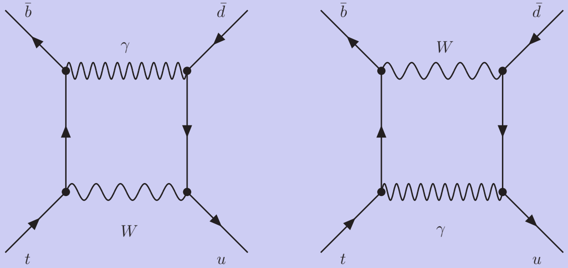
$$
\begin{aligned}
p_1 &\longrightarrow -p_1 \\
p_2 &\longrightarrow +p_2 \\
p_3 &\longrightarrow -p_3 \\
p_4 &\longrightarrow -p_4
\end{aligned}
$$

$$
\begin{aligned}
p_1 &\longrightarrow +p_1 \\
p_2 &\longrightarrow -p_2 \\
p_3 &\longrightarrow -p_3 \\
p_4 &\longrightarrow -p_4
\end{aligned}
$$

EW corrections

Total EW corrections are free from mass and IR singularities.

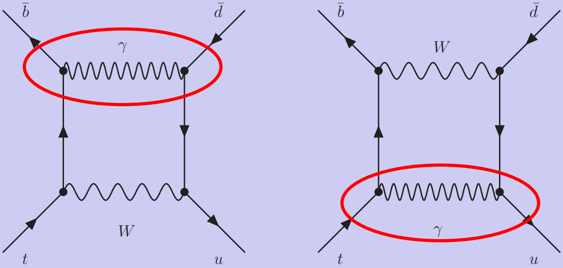
EW boxes



EW corrections

Total EW corrections are free from mass and IR singularities.

EW boxes



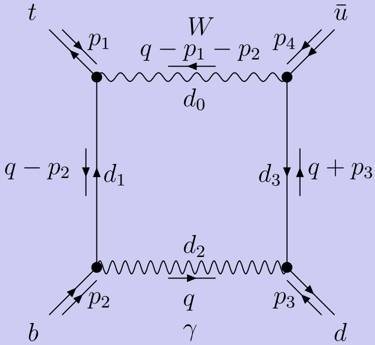
But in straightforward computations we get these singularities in D_0 and C_0 Passarino-Veltman functions, in particular in the EW boxes.

Extraction of singularities

Consider the folowing function

$$i\pi^2 J^d_{WA}(Q^2, T^2; m_b, m_t, m_d, m_u, M_W) = \mu^{4-n}\int d^n q \frac{2q\cdot p_1}{d_0 d_1 d_2 d_3}$$

J^d_{WA} function



Extraction of singularities

$$D_0(-m_b^2, -m_t^2, -m_u^2, -m_d^2, Q^2, T^2; 0, m_b, M_W, m_d) =$$
$$\frac{1}{M_W^2 + Q^2}\Big[$$
$$J(Q^2, T^2; m_b, m_t, m_d, m_u, M_W)$$
$$-C_0(-m_u^2, -m_d^2, Q^2; M_W, m_d, 0)$$
$$+C_0(-m_d^2, -m_b^2, T^2; m_d, 0, m_b)$$
$$\Big]$$

Extraction of singularities

$$D_0(-m_b^2, -m_t^2, -m_u^2, -m_d^2, Q^2, T^2; 0, m_b, M_W, m_d) =$$
$$\frac{1}{M_W^2 + Q^2} \Big[$$
$$J(Q^2, T^2; m_b, m_t, m_d, m_u, M_W)$$
$$-C_0(-m_u^2, -m_d^2, Q^2; M_W, m_d, 0)$$
$$\boxed{+C_0(-m_d^2, -m_b^2, T^2; m_d, 0, m_b)}$$ **IR singular term**
$$\Big]$$

Extraction of singularities

limit $m_u, m_d \to 0$

$$J(Q^2, P^2; m_b, m_t, m_d, m_u, M_W) = \quad J_{sub}(Q^2, P^2; m_b, m_t, M_W) + \left(1 + \frac{Q^2}{m_b^2 + P^2}\right) C_0(-m_u^2, -m_d^2, Q^2; M_W, m_d, 0)$$

limit $m_b \to 0$

$$J_{sub}(Q^2, P^2; m_b, m_t, M_W) = J_{subsub}(Q^2, P^2; 0, m_t, M_W) + \frac{P^2}{Q^2 + m_t^2} C_0(-m_t^2, -m_b^2, Q^2; M_W, m_b, 0).$$

All mass singular C_0 functions cancel analytically in the total EW corrections.

Top production, t-channel: helicity amplitudes

$u + b \to t + d$

$$A_{----} = k\sqrt{s}\Big[2(1 - c_- k_1)\mathcal{F}_{LL} - m_t c_+ (1 - c_- k_1)\mathcal{F}_{LD} - m_t c_+ c_- k_1 \mathcal{F}_{RD}\Big]$$

$$A_{---+} = k \sin\vartheta \left[s(1 - c_- k_1)\mathcal{F}_{LD} - m_t^2 c_+ k_1 \mathcal{F}_{RD}\right]$$

$$A_{+---} = k \sin\vartheta \left[-2 m_t k_1 \mathcal{F}_{LL} + m_t^2 c_+ k_1 \mathcal{F}_{LD} - s(1 - c_- k_1)\mathcal{F}_{RD}\right]$$

$$A_{+--+} = k\sqrt{s} m_t c_+ \left[-k_1 c_- \mathcal{F}_{LD} - (1 - c_- k_1)\mathcal{F}_{RD}\right]$$

$\bar{d} + b \to t + \bar{u}$

$$A_{-++-} = k\sqrt{s} c_+ (\mathcal{F}_{LL} - m_t \mathcal{F}_{LD})$$

$$A_{-+++} = k \sin\vartheta (m_t \mathcal{F}_{LL} - s \mathcal{F}_{LD})$$

$$A_{+++-} = k s \sin\vartheta \mathcal{F}_{RD}$$

$$A_{++++} = -k\sqrt{s} m_t c_+ \mathcal{F}_{RD}$$

$$c_+ = 1 + \cos\vartheta, \quad c_- = 1 - \cos\vartheta, \quad k = \chi(M_W^2, s')\sqrt{s - m_t^2}, \quad k_1 = \frac{2s}{s c_- + m_t^2 c_+}$$

* the limit $m_b \to 0$

Top production, s-channel: helicity amplitudes

$u + \bar{d} \to t + \bar{b}$

$$\begin{aligned}
A_{+---} &= -k\sin\vartheta\left[m_t\mathcal{F}_{LR} + (s - m_t^2)\mathcal{F}_{\mathcal{RD}}\right]\\
A_{+--+} &= -k\sqrt{s}c_+\mathcal{F}_{LL}\\
A_{+-+-} &= -k\sqrt{s}c_-\mathcal{F}_{LR}\\
A_{+-++} &= -k\sin\vartheta\left[m_t\mathcal{F}_{LL} + (s - m_t^2)\mathcal{F}_{LD}\right]
\end{aligned}$$

$\bar{u} + d \to \bar{t} + b$

$$\begin{aligned}
A_{+---} &= -k\sin\vartheta\left[m_t\mathcal{F}_{LL} + (s - m_t^2)\mathcal{F}_{RD}\right]\\
A_{+--+} &= -k\sqrt{s}c_+\mathcal{F}_{LL}\\
A_{+-+-} &= -k\sqrt{s}c_-\mathcal{F}_{LR}\\
A_{+-++} &= -k\sin\vartheta\left[m_t\mathcal{F}_{LR} + (s - m_t^2)\mathcal{F}_{LD}\right]
\end{aligned}$$

$$c_+ = 1 + \cos\vartheta, \qquad c_- = 1 - \cos\vartheta, \qquad k = \chi(M_w^2, s')\sqrt{s - m_t^2}$$

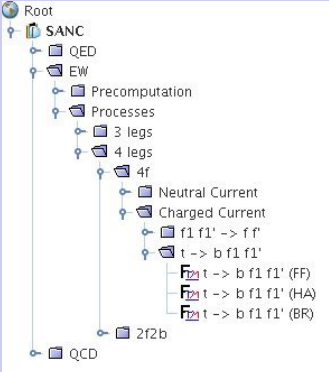
Top production, s-channel: numerical results

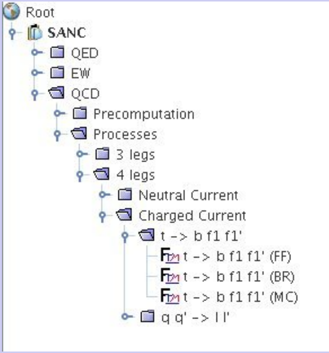
$\sqrt{\hat{s}}/GeV$	300	400	500	1000	2000	5000	14000
$\hat{\sigma}_0$, fb	725.7	534.8	378.3	106.0	27.18	4.378	0.5591
$\hat{\sigma}_1$, fb	750.5	544.5	379.6	100.1	23.43	3.136	0.2796
δ, %, 1)	3.415	1.802	0.3483	-5.562	-13.78	-28.38	-49.99
δ, %, 2)	3.415	1.802	0.3483	-5.562	-13.78	-28.38	-49.99

Table: For the process $u\bar{d} \to t\bar{b}$ the total lowest-order and one-loop corrected cross sections $\hat{\sigma}_0$ and $\hat{\sigma}_1$ in pb in the α EW scheme and corresponding relative one-loop correction δ at the parton level for $\bar{\omega} = 10^{-4}\frac{\sqrt{\hat{s}}}{2}$ GeV, rows 1) and $\bar{\omega} = 10^{-5}\frac{\sqrt{\hat{s}}}{2}$ GeV, rows 2). For the process $\bar{u}d \to \bar{t}b$ — the same.

Top physics in SANC: corrections to top decays

SANC trees for top decays in EW and QCD sectors





Scalar Form Factors (FF), helicity amplitudes (HA), integrated bremsstrahlung (BR), fully differential bremsstrahlung (MC).

Integrators, cascade approximations

To investigate the possibility of cascade approaches in chain of top production and decay we first studied it on $t \longrightarrow bf1f1'$ processes. We considered them as a cascade of $t \longrightarrow bW$ and $W \longrightarrow f1f1'$.

Cascade approximations

- narrow width cascade
- cascade with complex W mass
- pole approximation

Integrators, cascade approximations

narrow width cascade

$$\Gamma_{t\to bl\nu} = \frac{\Gamma^{\rm 1loop}_{t\to Wb}\Gamma^{\rm 1loop}_{W\to l\nu}}{\Gamma_W}\,.$$

or linearized version

$$\Gamma_{t\to bl\nu} = \frac{\Gamma^{\rm Born}_{t\to Wb}\Gamma^{\rm Born}_{W\to l\nu}}{\Gamma_W}\left(1+\delta^{\rm 1loop}_{t\to Wb}+\delta^{\rm 1loop}_{W\to l\nu}\right),$$

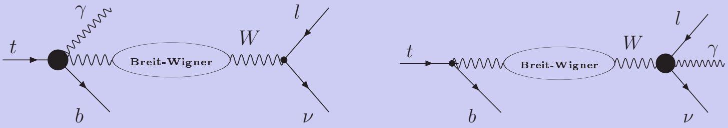
where $\delta^{\rm 1loop} = \Gamma^{\rm 1loop}/\Gamma^{\rm Born} - 1$

Cascade with complex W mass

The same equations but with a complex W mass:
$\widetilde{M}_W^2 = M_W^2 - iM_W\Gamma_W$.

Integrators, cascade approximations

pole approximation



$$
\begin{aligned}
\Gamma^{\rm 1loop}_{t\to bl\nu} = & \frac{1}{k}\int_l^u d\hat{M}^2_W\Big[\Gamma^{\rm 1loop}_{t\to Wb}(\hat{M}^2_W)\Gamma^{\rm Born}_{W\to l\nu}(\hat{M}^2_W) \\
& +\Gamma^{\rm Born}_{t\to Wb}(\hat{M}^2_W)\Gamma^{\rm 1loop}_{W\to l\nu}(\hat{M}^2_W) \\
& -\Gamma^{\rm Born}_{t\to Wb}(\hat{M}^2_W)\Gamma^{\rm Born}_{W\to l\nu}(\hat{M}^2_W)\Big] \\
& \times\frac{M_W}{(\hat{M}^2_W - M^2_W)^2 + M^2_W\Gamma^2_W}
\end{aligned}
$$

The advantage of pole approximation is that it allows fully differential realization, and hence also MC generation.

Numerical results

Results of complete calculation

	$t \to bl^+\bar{\nu}_l$		$t \to bu\bar{d}$	
	Γ^{1l}, MeV	$\delta, \%$	Γ^{1l}, MeV	$\delta, \%$
EW	159.877(3)	6.953(2)	480.341(6)	7.111(1)
QCD	136.73(2)	-8.53(1)	358.72(28)	-20.01(6)

The independence of total widths on b-quark mass.

Cascade approximations for $t \to bl^+\bar{\nu}_l$, EW corrections

	Γ^{1L}, MeV	δ, %
narrow width cascade	162.73	7.033
cascade with complex W mass	162.23	6.83
pole approximation	164.015	7.029

Summary

- We introduced the single top physics sector in SANC system. The building blocks needed for full calculation of single top production with subsequent decay processes were discussed.
- We show present status of this work: the system already includes top decay processes at 1-loop level. The work on the single top production is in progress: s-channel done, t-channel in progress.
- The results of studying different cascade approximations to top decay processes were presented. All considered approximations give good consistent results. Among them pole approximation can be used in MC event generators. This experience gives the ground for using cascade approaches in research of the single top production with subsequent decay processes.
- We get Standard SANC Fortran Modules for these considered processes. They will be used in MC SANC and also could be used in other Monte Carlo programs.

Input parameters in numerical computations

SANC setup

Alpha scheme,

$$
\begin{aligned}
G_F &= 1.16637 \times 10^{-5}\,\mathrm{GeV}^{-2}, & & \\
\alpha(0) &= 1/137.035999, & \alpha_s &= 0.00729735257, \\
M_W &= 80.403\,\mathrm{GeV}, & \Gamma_W &= 2.141\,\mathrm{GeV}, \\
M_Z &= 91.1876\,\mathrm{GeV}, & \Gamma_Z &= 2.4952\,\mathrm{GeV}, \\
M_H &= 120\,\mathrm{GeV}, & m_t &= 174.2\,\mathrm{GeV}, \\
m_u &= 62\,\mathrm{MeV}, & m_d &= 83\,\mathrm{MeV},
\end{aligned}
$$