SUSY: From the Basics to Phenomenology

Sven Heinemeyer, IFCA (CSIC, Santander)

Dubna, 07/2009

1. SUSY Lagrangian and algebra
2. The MSSM and simplified versions
3. The Higgs sector of the MSSM
4. SUSY at the LHC and the ILC

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SUSY lectures (II):
The MSSM and simplified versions

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1. Soft SUSY-breaking

2. The Minimal Supersymmetric Standard Model (MSSM)

3. Properties of SUSY Theories

4. Simplified versions

1. Soft SUSY-breaking

Exact SUSY: $m_f = m_{\tilde{f}}, \ldots$

$\Rightarrow$ in a realistic model: SUSY must be broken

Only satisfactory way for model of SUSY breaking:

spontaneous SUSY breaking

Specific SUSY-breaking schemes (see below) in general yield effective Lagrangian at low energies, which is supersymmetric except for explicit soft SUSY-breaking terms

Soft SUSY-breaking terms: do not alter dimensionless couplings

(i.e. dimension of coupling constants of soft SUSY-breaking terms > 0)
otherwise: re-introduction of the hierarchy problem

$\Rightarrow$ no quadratic divergences (in all orders of perturbation theory)

scale of SUSY-breaking terms: $M_{\text{SUSY}} \lesssim 1$ TeV

Classification of possible soft breaking terms:

[*L. Girardello, M. Grisaru '82*]

- scalar mass terms: $m^2_{\phi_i} |\phi_i|^2$

- trilinear scalar interactions: $A_{ijk}\phi_i\phi_j\phi_k + \text{h.c.}$

- gaugino mass terms: $\frac{1}{2} m \bar{\lambda}\lambda$

- bilinear terms: $B_{ij}\phi_i\phi_j + \text{h.c.}$

- linear terms: $C_i\phi_i$

$\Rightarrow$ relations between dimensionless couplings unchanged

no additional mass terms for chiral fermions

A. Unconstrained models (MSSM):

agnostic about how SUSY breaking is achieved

no particular SUSY breaking mechanism assumed, parameterization of possible soft SUSY-breaking terms

$\Rightarrow$ relations between dimensionless couplings unchanged
no quadratic divergences

most general case:

$\Rightarrow$ 105 new parameters: masses, mixing angles, phases

Good phenomenological description for universal breaking terms

B. Constrained models (mSUGRA, . . .):

assumption on the scenario that achieves spontaneous SUSY breaking
$\Rightarrow$ prediction for soft SUSY-breaking terms
in terms of small set of parameters

Experimental determination of SUSY parameters

$\Rightarrow$ Patterns of SUSY breaking

Problems can be overcome if SUSY breaking happens in a 'hidden sector', i.e. by fields which have only very small couplings to ordinary matter

SUSY breaking in the hidden sector:

– tree-level (like F- and D-term breaking)

– dynamical breaking (similar to chiral symmetry breaking in QCD), . . .

> SUSY-breaking terms in the MSSM arise radiatively via interaction that communicates SUSY breaking rather than through tree-level couplings to SUSY breaking v.e.v.s

$\Rightarrow$ phenomenology depends mainly on mechanism for communicating SUSY breaking rather than on SUSY-breaking mechanism itself

If mediating interactions are $\approx$ flavor-diagonal

$\Rightarrow$ universal soft-breaking terms

“Hidden sector” :	$\longrightarrow$	Visible sector:
SUSY breaking		MSSM

“Gravity-mediated” : mSUGRA
“Gauge-mediated” : GMSB
“Anomaly-mediated” : AMSB
“Gaugino-mediated”
...

SUGRA: mediating interactions are gravitational

GMSB: mediating interactions are ordinary electroweak and QCD gauge interactions

AMSB, Gaugino-mediation: SUSY breaking happens on a different brane in a higher-dimensional theory

(more details later)

2. The Minimal Supersymmetric Standard Model (MSSM)

MSSM: superpartners for SM fields

SM matter fermions have different quantum numbers than
SM gauge bosons

$\Rightarrow$ need to be placed in different superfields

$\Rightarrow$ no SM fermion is a gaugino

no Higgs is a sfermion (e.g. scalar neutrino)

agnostic about how SUSY breaking is achieved

no particular SUSY breaking mechanism assumed
parameterization of possible soft SUSY-breaking terms

$\Rightarrow$ most general case: 105 new parameters: masses, mixing angles, phases

1. Fermions, sfermions:

left-handed chiral superfields give SM fermions/sfermions
(⇒ the conjugates of right-handed ones appear)

LHχSF Q: quark, squark SU(2) doublets

LHχSF U: up-type quark, squark singlets

LHχSF D: down-type quark, squark singlets

LHχSF L: lepton, slepton SU(2) doublets

LHχSF E: lepton, slepton singlets

⇒ one generation of SM fermions and their superpartners described by five LHχSFs

2. Gauge bosons, gauginos:

Vector superfields:

- gluons g and gluinos $\tilde{g}$
- W bosons $W^{\pm}$, W^0 and winos $\tilde{W}^{\pm}$, $\tilde{W}^0$
- B boson B^0 and bino $\tilde{B}^0$

3. Higgs bosons, higgsinos:

LHχSF

In MSSM: two Higgs doublets needed $\Rightarrow$ two LHχSFs

Comparison with SM case:

$$\mathcal{L}_{\text{SM}} = \underbrace{m_d \bar{Q}_L H d_R}_{\text{d-quark mass}} + \underbrace{m_u \bar{Q}_L \tilde{H} u_R}_{\text{u-quark mass}}$$

$$Q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L, \quad \tilde{H} = i\sigma_2 H^\dagger, \quad H \to \begin{pmatrix} 0 \\ v \end{pmatrix}, \quad \tilde{H} \to \begin{pmatrix} v \\ 0 \end{pmatrix}$$

In SUSY: term $\bar{Q}_L H^\dagger$ not allowed

Superpotential is holomorphic function of chiral superfields, i.e. depends only on φ_i, not on φ_i^*

No soft SUSY-breaking terms allowed for chiral fermions

$\Rightarrow$ H_d and H_u needed to give masses to down- and up-type fermions

Furthermore: two doublets also needed for cancellation of anomalies, quadratic divergences

Chiral supermultiplets of the MSSM:

		spin 0	spin $\frac{1}{2}$	(SU(3)$_c$, SU(2), U(1)$_Y$)
squarks and quarks	Q	$(\tilde{u}_L, \tilde{d}_L)$	(u_L, d_L)	$(3, 2, \frac{1}{6})$
	U	$\tilde{u}_R^*$	$u_R^\dagger$	$(\bar{3}, 1, -\frac{2}{3})$
	D	$\tilde{d}_R^*$	$d_R^\dagger$	$(\bar{3}, 1, \frac{1}{3})$
sleptons and leptons	L	$(\tilde{\nu}, \tilde{e}_L)$	(ν, e_L)	$(1, 2, -\frac{1}{2})$
	E	$\tilde{e}_R^*$	$e_R^\dagger$	$(1, 1, 1)$
higgs and higgsinos	H_u	(h_u^+, h_u^0)	$(\tilde{h}_u^+, \tilde{h}_u^0)$	$(1, 2, \frac{1}{2})$
	H_d	(h_d^0, h_d^-)	$(\tilde{h}_d^0, \tilde{h}_d^-)$	$(1, 2, -\frac{1}{2})$

Vector supermultiplets:

	spin $\frac{1}{2}$	spin 1	(SU(3)$_c$, SU(2), U(1)$_Y$)
gluinos and gluons	$\tilde{g}$	g	$(8, 1, 0)$
winos and W-bosons	$\widetilde{W}^{\pm}, \widetilde{W}^0$	$W^{\pm}, W^0$	$(1, 3, 0)$
bino and B-boson	$\tilde{B}$	B	$(1, 1, 0)$

⇒ MSSM has further symmetry: "R-parity"

all SM-particles and Higgs bosons: even R-parity, $P_R = +1$

all superpartners: odd R-parity, $P_R = -1$

⇒ SUSY particles appear only in pairs

⇒ lightest SUSY particle (LSP) is stable

Soft breaking terms:

$$
\begin{aligned}
\mathcal{L}_{\text{soft}} \;=\; & -\frac{1}{2}\Big(M_1 \tilde{B}\tilde{B} + M_2 \widetilde{W}\widetilde{W} + M_3 \tilde{g}\tilde{g}\Big) + \text{h.c.} \\
& - (m_{H_u}^2 + |\mu|^2) H_u^+ H_u - (m_{H_d}^2 + |\mu|^2) H_d^+ H_d - (b H_u H_d + \text{h.c.}) \\
& - \Big(\tilde{u}_R \mathbf{a_u} \tilde{Q} H_u - \tilde{d}_R \mathbf{a_d} \tilde{Q} H_d - \tilde{e}_R \mathbf{a_e} \tilde{L} H_d\Big) + \text{h.c.} \\
& - \tilde{Q}^+ \mathbf{m_Q^2} \tilde{Q} - \tilde{L}^+ \mathbf{m_L^2} \tilde{L} - \tilde{u}_R \mathbf{m_u^2} \tilde{u}_R^* - \tilde{d}_R \mathbf{m_d^2} \tilde{d}_R^* - \tilde{e}_R \mathbf{m_e^2} \tilde{e}_R^*
\end{aligned} \tag{1}
$$

Most general parameterization of SUSY-breaking terms that keep relations between dimensionless couplings unchanged
$\Rightarrow$ no quadratic divergences

$\mathbf{m_i^2}, \mathbf{a_j}$: 3×3 matrices in family space

$\Rightarrow$ many new parameters

$\rightarrow$ Superpotential of the MSSM

Particle content of the MSSM:

Superpartners for Standard Model particles:

$$\left[u, d, c, s, t, b\right]_{L,R} \quad \left[e, \mu, \tau\right]_{L,R} \quad \left[\nu_{e,\mu,\tau}\right]_L \qquad \text{Spin } \tfrac{1}{2}$$

$$\left[\tilde{u}, \tilde{d}, \tilde{c}, \tilde{s}, \tilde{t}, \tilde{b}\right]_{L,R} \quad \left[\tilde{e}, \tilde{\mu}, \tilde{\tau}\right]_{L,R} \quad \left[\tilde{\nu}_{e,\mu,\tau}\right]_L \qquad \text{Spin } 0$$

$$g \quad \underbrace{W^{\pm}, H^{\pm}} \quad \underbrace{\gamma, Z, H_1^0, H_2^0} \qquad \text{Spin 1 / Spin 0}$$

$$\tilde{g} \quad \tilde{\chi}^{\pm}_{1,2} \quad \tilde{\chi}^0_{1,2,3,4} \qquad \text{Spin } \frac{1}{2}$$

Enlarged Higgs sector:
Two Higgs doublets, physical states: $h^0, H^0, A^0, H^{\pm}$

as usual: Breaking of SU(2) $\times$ U(1)$_Y$ (electroweak symmetry breaking)

$\Rightarrow$ fields with different SU(2) $\times$ U(1)$_Y$ quantum numbers can mix if they have the same SU(3)$_c$, U(1)$_{\text{em}}$ quantum numbers

Squark mixing:

Stop, sbottom mass matrices ($X_t = A_t - \mu/\tan\beta$, $X_b = A_b - \mu\tan\beta$):

$$\mathcal{M}^2_{\tilde{t}} = \begin{pmatrix} M^2_{\tilde{t}_L} + m_t^2 + DT_{t_1} & m_t X_t \\ m_t X_t & M^2_{\tilde{t}_R} + m_t^2 + DT_{t_2} \end{pmatrix} \xrightarrow{\theta_{\tilde{t}}} \begin{pmatrix} m^2_{\tilde{t}_1} & 0 \\ 0 & m^2_{\tilde{t}_2} \end{pmatrix}$$

$$\mathcal{M}^2_{\tilde{b}} = \begin{pmatrix} M^2_{\tilde{b}_L} + m_b^2 + DT_{b_1} & m_b X_b \\ m_b X_b & M^2_{\tilde{b}_R} + m_b^2 + DT_{b_2} \end{pmatrix} \xrightarrow{\theta_{\tilde{b}}} \begin{pmatrix} m^2_{\tilde{b}_1} & 0 \\ 0 & m^2_{\tilde{b}_2} \end{pmatrix}$$

off-diagonal element prop. to mass of partner quark ($\tan\beta \equiv v_u/v_d$)

$\Rightarrow$ mixing important in stop sector (also in sbottom sector for large $\tan\beta$)

$$\text{gauge invariance} \Rightarrow M_{\tilde{t}_L} = M_{\tilde{b}_L} \tag{2}$$

$\Rightarrow$ relation between $m_{\tilde{t}_1}, m_{\tilde{t}_2}, \theta_{\tilde{t}}, m_{\tilde{b}_1}, m_{\tilde{b}_2}, \theta_{\tilde{b}}$

Neutralinos and charginos:

Higgsinos and electroweak gauginos mix

charged:

$$\tilde{W}^+, \tilde{h}_u^+ \to \tilde{\chi}_1^+, \tilde{\chi}_2^+, \qquad \tilde{W}^-, \tilde{h}_d^- \to \tilde{\chi}_1^-, \tilde{\chi}_2^-$$

$\Rightarrow$ charginos: mass eigenstates

mass matrix given in terms of M_2, μ, $\tan\beta$

neutral:

$$\underbrace{\tilde{\gamma}, \tilde{Z},}_{\tilde{W}^0, \tilde{B}^0} \tilde{h}_u^0, \tilde{h}_d^0 \to \tilde{\chi}_1^0, \tilde{\chi}_2^0, \tilde{\chi}_3^0, \tilde{\chi}_4^0$$

$\Rightarrow$ neutralinos: mass eigenstates

mass matrix given in terms of M_1, M_2, μ, $\tan\beta$

$\Rightarrow$ only one new parameter

$\Rightarrow$ MSSM predicts mass relations between neutralinos and charginos

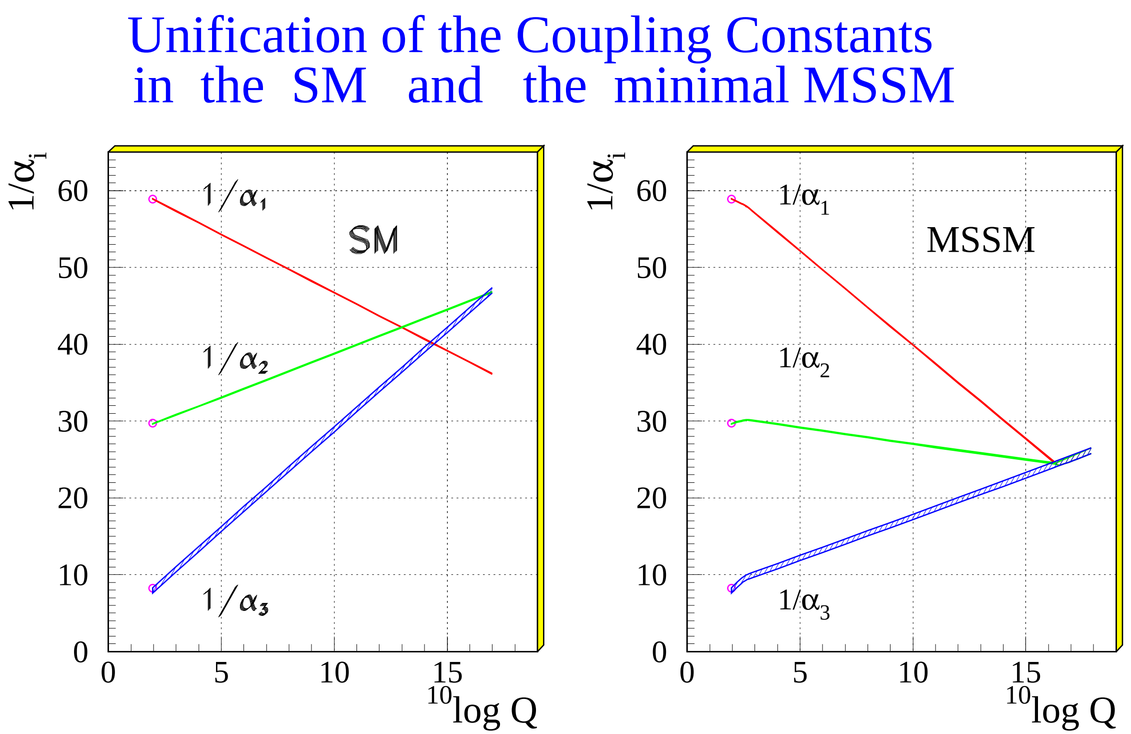
3. Properties of SUSY theories:

Property I: coupling constant unification

[RGE: equations that connect parameters at different energy scales]

→ use RGE's to evolve gauge coupling constants from electroweak scale to the GUT scale

$$\alpha_i(Q_{\text{electroweak}}) \to \alpha_i(Q_{\text{GUT}})$$



gauge couplings do not meet in the SM

they unify in the MSSM
although it was not designed for it!

$\Rightarrow M_{\text{SUSY}} \approx 1$ TeV

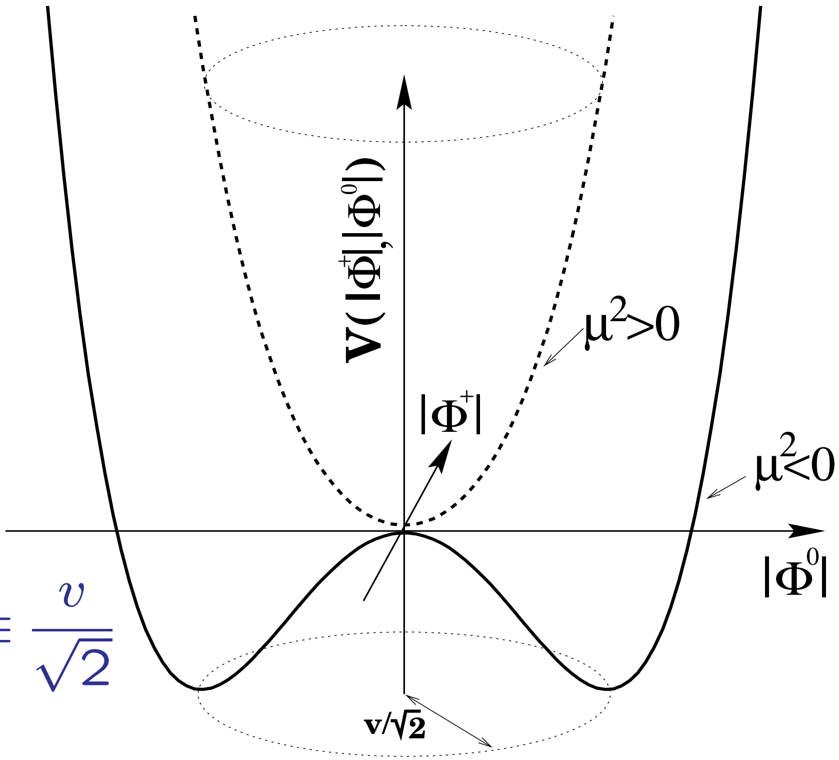
Property II: the Higgs mechanism comes for free

Higgs mechanism needed to give masses to W and Z bosons:

SM: Scalar SU(2) doublet: $\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$

Higgs potential:

$$V(\phi) = \mu^2 \left|\Phi^\dagger \Phi\right| + \lambda \left|\Phi^\dagger \Phi\right|^2, \quad \lambda > 0$$



$\mu^2 < 0$: Spontaneous symmetry breaking

minimum of potential at $|\langle \Phi_0 \rangle| = \sqrt{\frac{-\mu^2}{2\lambda}} \equiv \frac{v}{\sqrt{2}}$

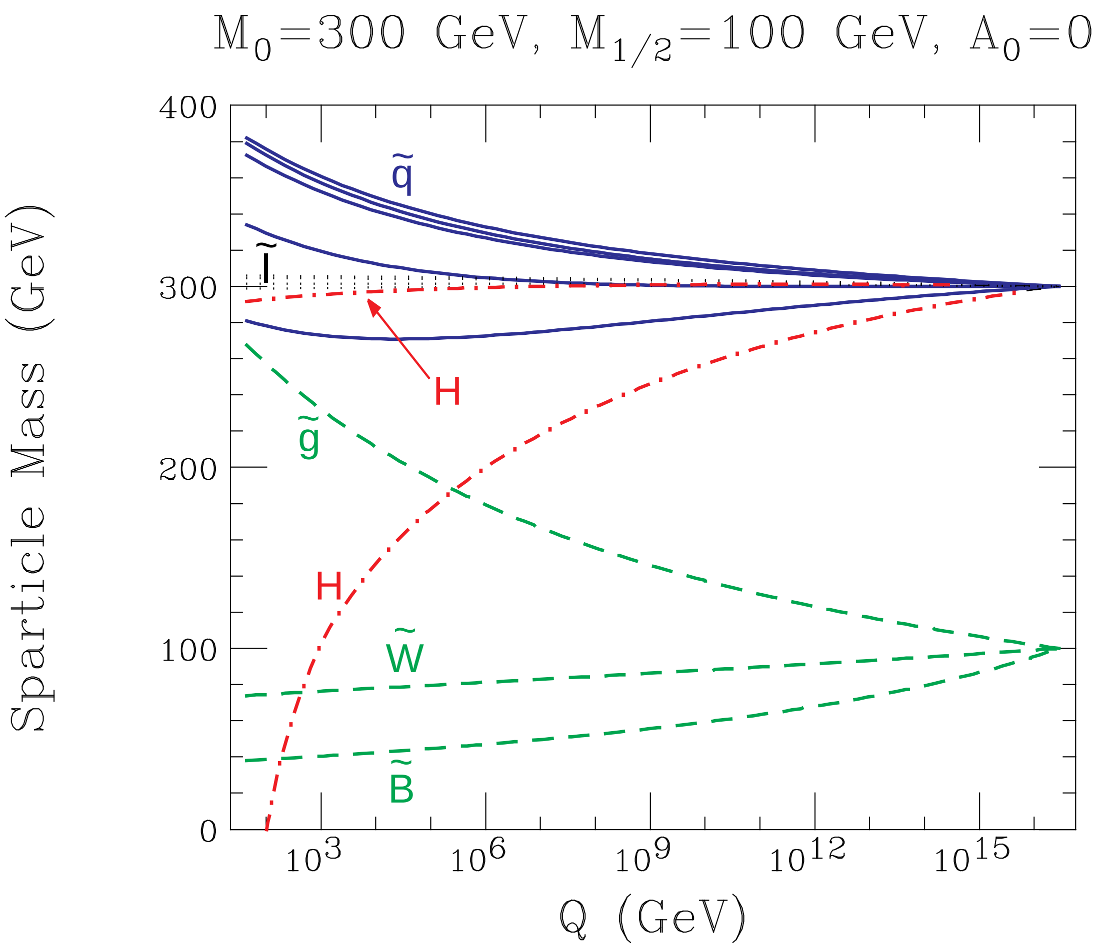
SM: sign of μ has to be set by hand

MSSM: negative sign of μ comes for free provided that . . .

– assume GUT scale (as motivated by coupling constant unification)

– take universal input parameters at the GUT scale (see mSUGRA below)

– run down to the electroweak scale with RGEs



Exactly one parameter turns negative: the "μ" in the Higgs potential

But this only works if

$m_t = 150 \ldots 200$ GeV

and $M_{\text{SUSY}} \approx 1$ TeV

Property III: R parity

Most general gauge-invariant and renormalizable superpotential with chiral superfields of the MSSM:

$$\mathcal{V} = \mathcal{V}_{\text{MSSM}} + \underbrace{\frac{1}{2}\lambda^{ijk} L_i L_j E_k + \lambda'^{ijk} L_i Q_j D_k + \mu'^{i} L_i H_u}_{\text{violate lepton number}} + \underbrace{\frac{1}{2}\lambda''^{ijk} U_i D_j D_k}_{\text{violates baryon number}}$$

If both lepton and baryon number are violated

⇒ rapid proton decay

Minimal choice (MSSM) contains only terms in the Lagrangian with even number of SUSY particles

⇒ additional symmetry: "R parity"

⇒ all SM particles have even R parity, all SUSY particles have odd R parity

Property IV: the LSP

MSSM has further symmetry: "R-parity"

all SM-particles and Higgs bosons: even R-parity, $P_R = +1$

all superpartners: odd R-parity, $P_R = -1$

$\Rightarrow$ SUSY particles appear only in pairs, e.g. $e^+e^- \to \tilde{\chi}_1^+ \tilde{\chi}_1^-$

$\Rightarrow$ lightest SUSY particle (LSP) is stable

(usually the lightest neutralino)

good candidate for Cold Dark Matter

$\Rightarrow M_{\text{SUSY}} \lesssim 1$ TeV

LSP neutral, uncolored $\Rightarrow$ leaves no traces in collider detectors

$\Rightarrow$ Typical SUSY signatures: "missing energy"

Property V: Relations between SUSY parameters

Symmetry properties of MSSM Lagrangian (SUSY, gauge invariance) give rise to coupling and mass relations

Soft SUSY breaking does not affect SUSY relations between dimensionless couplings

E.g.:

gauge boson–fermion coupling

=

gaugino–fermion–sfermion coupling

for U(1), SU(2), SU(3) gauge groups

In SM: all masses are free input parameters
(except M_W–M_Z interdependence)

MSSM:

- Upper bound on mass of lightest $\mathcal{CP}$-even Higgs boson

- Relations between neutralino and chargino masses

- Sfermion mass relations, e.g.

$$m^2_{\tilde{e}_L} = m^2_{\tilde{\nu}_L} - M_W^2 \cos(2\beta)$$

All relations receive corrections from loop effects

$\Leftrightarrow$ effects of soft SUSY breaking, electroweak symmetry breaking

$\Rightarrow$ Experimental verification of parameter relations is a crucial test of SUSY!

Property VI: the mass of the lightest MSSM Higgs boson

Upper bound on m_h in the MSSM:

"Unconstrained MSSM":

M_A, $\tan\beta$, 5 parameters in $\tilde{t}$–$\tilde{b}$ sector, μ, $m_{\tilde{g}}$, M_2

$$m_h \lesssim 135 \text{ GeV}$$

for $m_t = 173.1 \pm 1.3\,\text{GeV}$

(including theoretical uncertainties from unknown higher orders)
$\Rightarrow$ observable at the LHC (possibly at the Tevatron?)

Obtained with:

FeynHiggs

[*S.H., W. Hollik, G. Weiglein '98, '00, '02*]
[*T. Hahn, S.H., W. Hollik, H. Rzehak, G. Weiglein '03 – '09*]

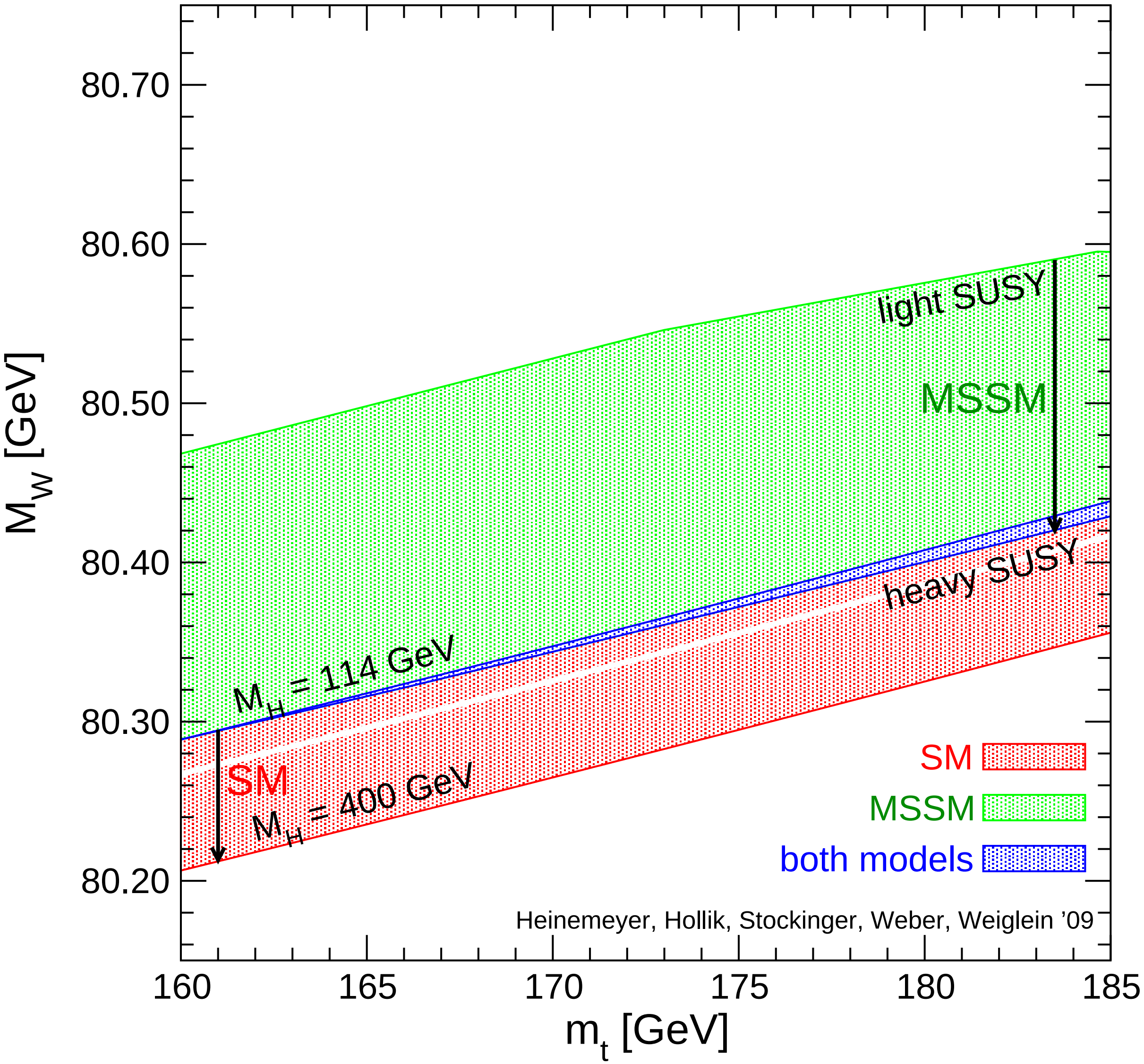
`www.feynhiggs.de`

$\rightarrow$ all Higgs masses, couplings, BRs (easy to link, easy to use :-)

Property VII: the mass of the W boson

Prediction for M_W in the SM and the MSSM :

[S.H., W. Hollik, D. Stockinger, A.M. Weber, G. Weigleim '09]



MSSM band:
scan over
SUSY masses

overlap:
SM is MSSM-like
MSSM is SM-like

SM band:
variation of M_H^{SM}

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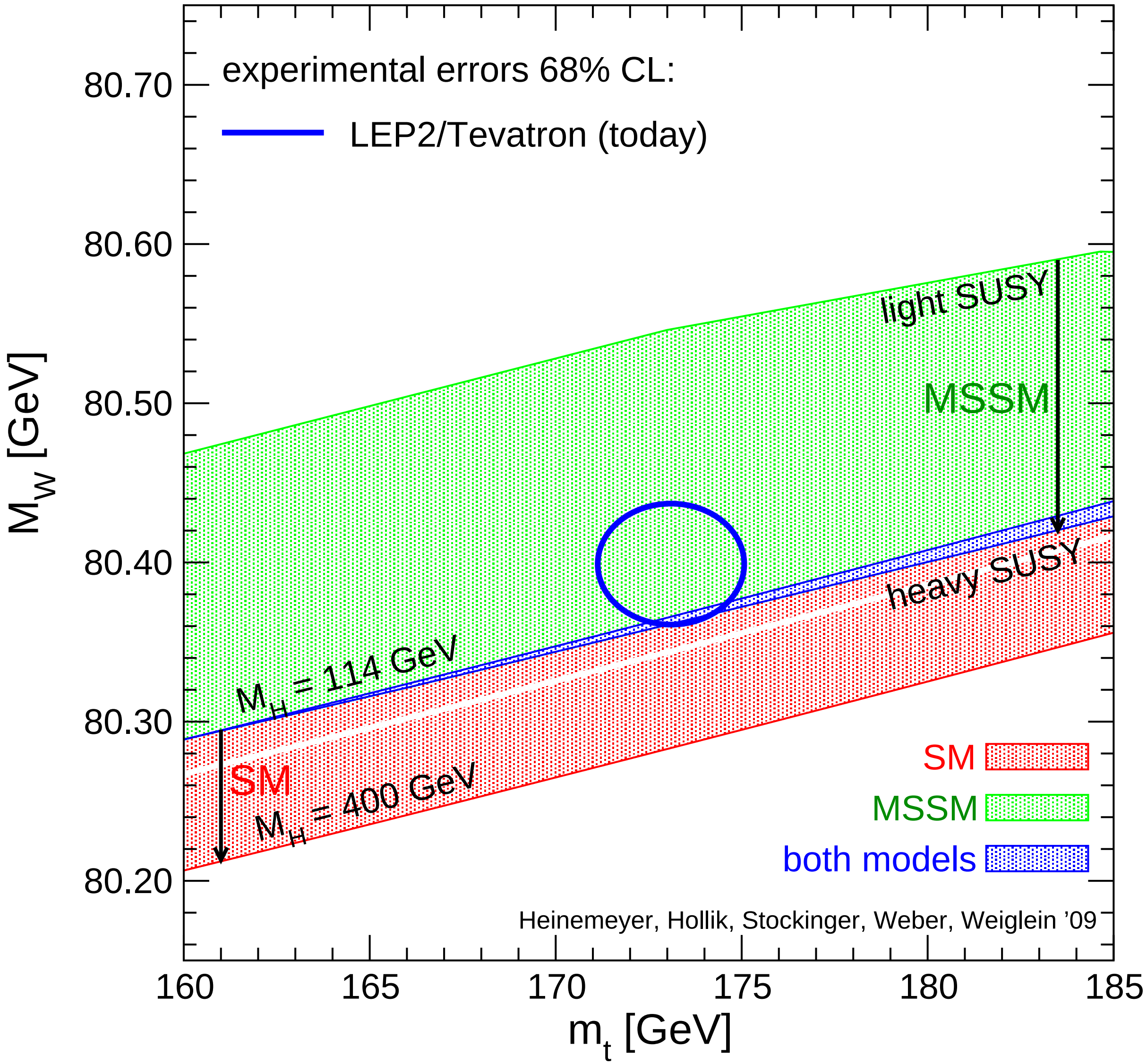

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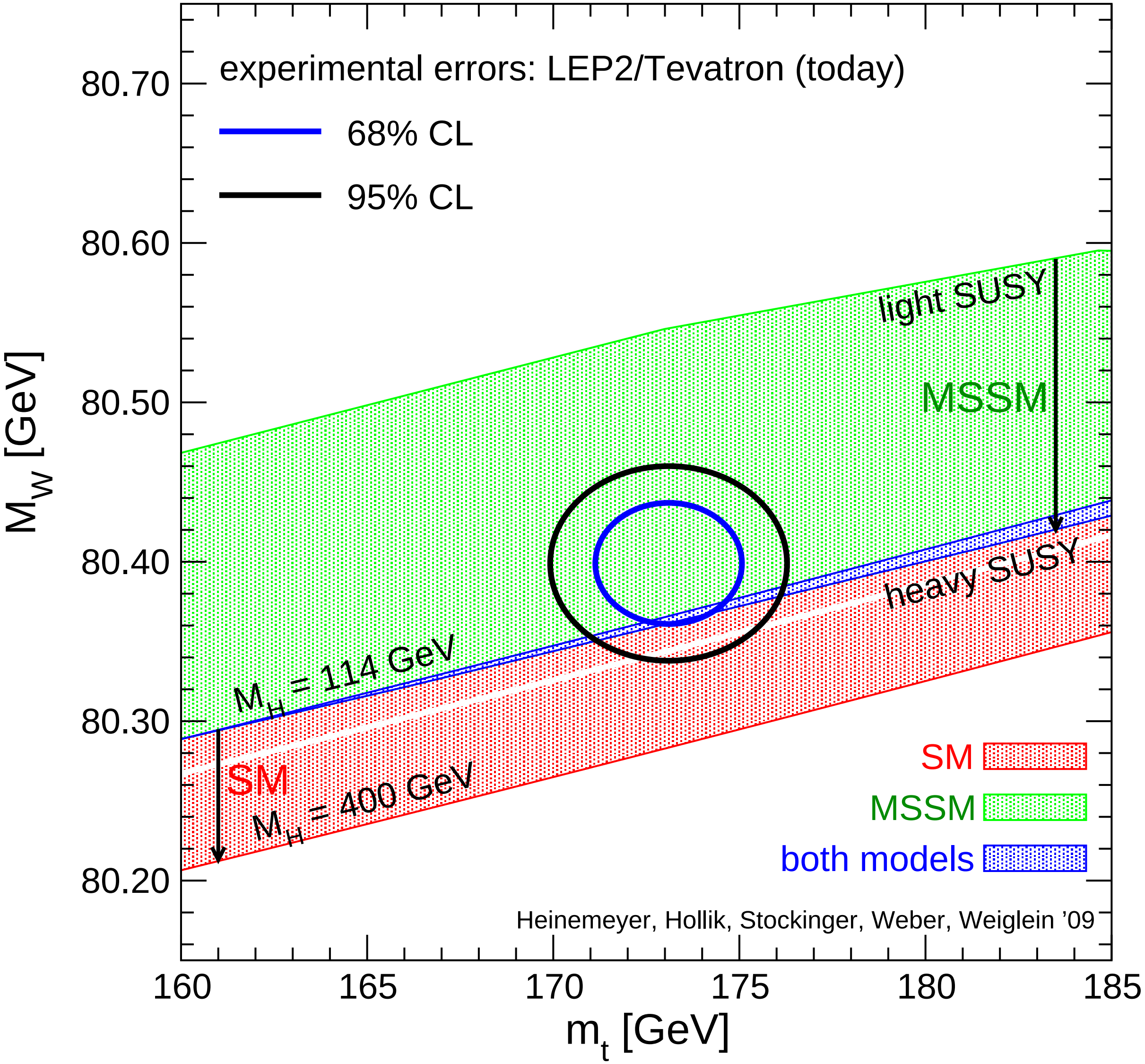

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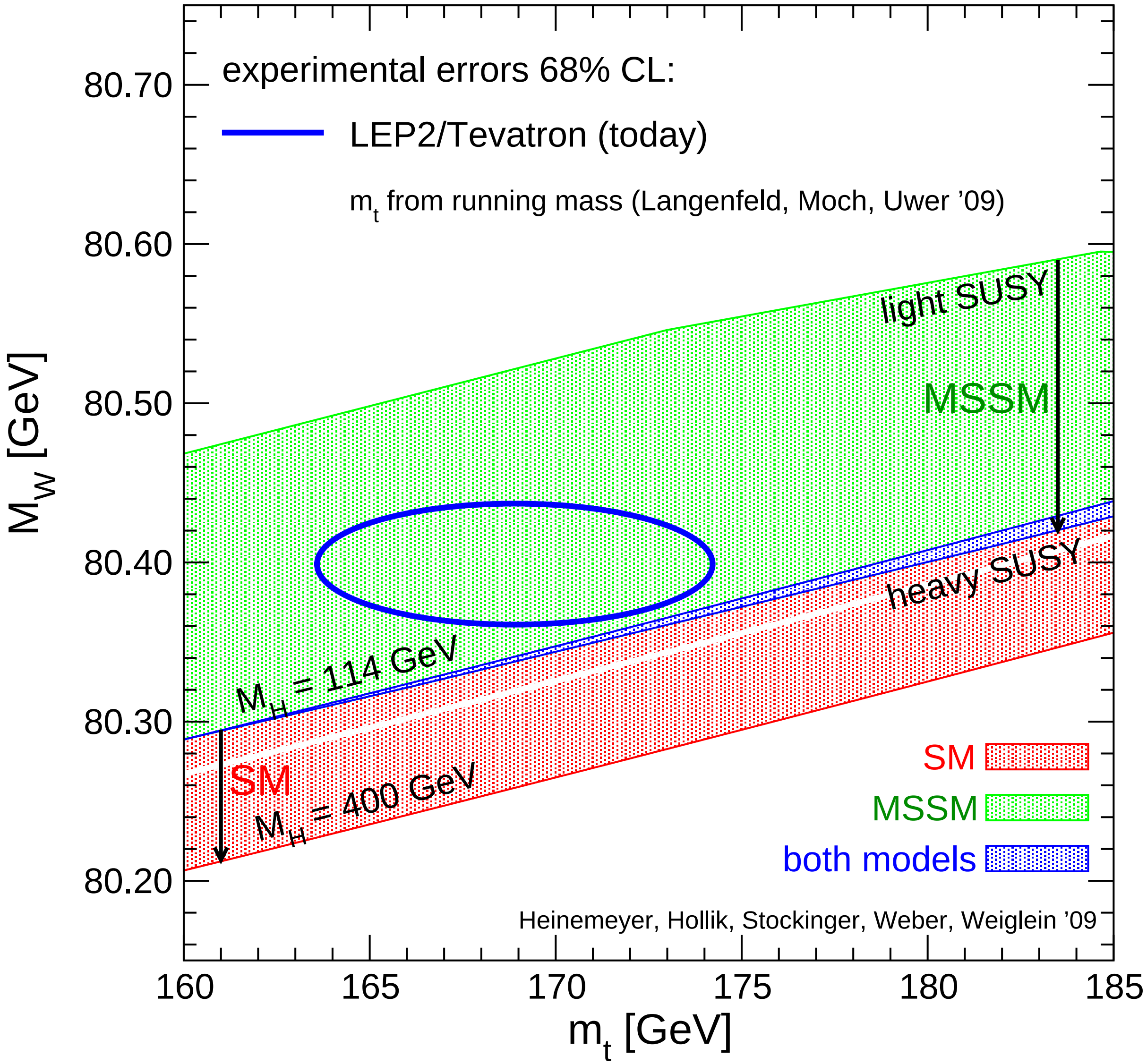


MSSM band:
scan over
SUSY masses

overlap:
SM is MSSM-like
MSSM is SM-like

SM band:
variation of $M_H^{\rm SM}$

4. Constrained Models: CMSSM/mSUGRA, mGMSB, mAMSB

Remember:

⇒ phenomenology depends mainly on mechanism for communicating SUSY breaking rather than on SUSY-breaking mechanism itself

"Hidden sector":	$\longrightarrow$	Visible sector:
SUSY breaking		MSSM

"Gravity-mediated": CMSSM, mSUGRA
"Gauge-mediated": GMSB
"Anomaly-mediated": AMSB
...

SUGRA: mediating interactions are gravitational

GMSB: mediating interactions are ordinary electroweak and QCD gauge interactions

AMSB: SUSY breaking on a different brane in a higher-dimensional theory

All constrained models are special versions of the MSSM !!!

4A. Gravity-mediated SUSY breaking

⇒ Quantum field theory of supergravity: graviton and gravitino

QFT with spin 2 and spin $\frac{3}{2}$ field is not renormalizable

⇒ cannot be extended to arbitrarily high energies

⇒ QFT of supergravity has to be interpreted as effective theory

contains non-renormalizable terms prop. to inverse powers of M_{Pl}

Best candidate for fundamental theory: string theory

SUSY breaking in hidden sector:

⇒ supergravity Lagrangian contains non-renormalizable terms that communicate between hidden and visible sector $\sim 1/M_{\mathrm{Pl}}^n$

Dimensional analysis:

SUSY breaking in hidden sector by v.e.v. $\langle F \rangle$ (dim $\langle F \rangle = \text{mass}^2$)
coupling $\sim 1/M_{\text{Pl}}$

require $m_{\text{soft}} \to 0$ for $\langle F \rangle \to 0$ (no SUSY breaking) and for $M_{\text{Pl}} \to \infty$ (vanishing gravitational interaction)

$$\Rightarrow \quad m_{\text{soft}} \approx \frac{\langle F \rangle}{M_{\text{Pl}}}$$

Wanted: $m_{\text{soft}} \lesssim 1$ TeV (hierarchy problem)

$\Rightarrow \sqrt{\langle F \rangle} \approx 10^{11}$ GeV : scale of SUSY breaking in hidden sector

In general: $m_{\text{gravitino}} = m_{\frac{3}{2}} \approx \frac{\langle F \rangle}{M_{\text{Pl}}}$

$\Rightarrow$ $m_{\frac{3}{2}} \approx m_{\text{soft}}$, gravitational interactions

$\Rightarrow$ gravitino not important for collider phenomenology

Non-renormalizable terms in supergravity Lagrangian:

$$\mathcal{L}_{\rm NR} = -\frac{1}{M_{\rm Pl}} F_X \sum_a \frac{1}{2} f_a \lambda^a \lambda^a + \text{h.c.} - \frac{1}{M_{\rm Pl}^2} F_X F_X^* k^i_j \varphi_i \varphi^{*j} - \frac{1}{M_{\rm Pl}} F_X (\frac{1}{6} y'^{ijk} \varphi_i \varphi_j \varphi_k + \frac{1}{2} \mu'^{ij} \varphi_i \varphi_j) + \text{h.c.}$$

F_X : (auxiliary) field for a chiral supermultiplet X in the hidden sector

φ_i, λ^a: scalar and gaugino fields in the MSSM

If $\sqrt{\langle F_X \rangle} \sim 10^{10}$–$10^{11}$ GeV

$\Rightarrow$ soft SUSY-breaking terms of MSSM with $m_{\rm soft} \approx 10^2$–$10^3$ GeV

Assumption of a "minimal" form of the supergravity Lagrangian

$\Rightarrow$ soft-breaking terms which obey "universality" and "proportionality"

Results in exactly the known MSSM Lagrangian (1)

$$
\begin{aligned}
\mathcal{L}_{\text{soft}} \;=\; & -\frac{1}{2}\left(M_1 \tilde{B}\tilde{B} + M_2 \widetilde{W}\widetilde{W} + M_3 \tilde{g}\tilde{g}\right) + \text{h.c.} \\
& - (m_{H_u}^2 + |\mu|^2) H_u^+ H_u - (m_{H_d}^2 + |\mu|^2) H_d^+ H_d - (b H_u H_d + \text{h.c.}) \\
& - \left(\tilde{u}_R \mathbf{a_u} \tilde{Q} H_u - \tilde{d}_R \mathbf{a_d} \tilde{Q} H_d - \tilde{e}_R \mathbf{a_e} \tilde{L} H_d\right) + \text{h.c.} \\
& - \tilde{Q}^+ \mathbf{m_Q^2} \tilde{Q} - \tilde{L}^+ \mathbf{m_L^2} \tilde{L} - \tilde{u}_R \mathbf{m_u^2} \tilde{u}_R^* - \tilde{d}_R \mathbf{m_d^2} \tilde{d}_R^* - \tilde{e}_R \mathbf{m_e^2} \tilde{e}_R^*
\end{aligned}
$$

with 5 independent parameters at the GUT scale:

$$
\begin{aligned}
M_1 = M_2 = M_3 \;&=\; m_{1/2} \\
m_{H_u}^2 = m_{H_d}^2 = \mathbf{m_Q^2} = \mathbf{m_L^2} = \mathbf{m_u^2} = \mathbf{m_d^2} = \mathbf{m_e^2} \;&=\; m_0 \\
\mathbf{a_u} = \mathbf{a_d} = \mathbf{a_e} \;&=\; A_0 \\
&\; b \\
&\; |\mu|^2
\end{aligned}
$$

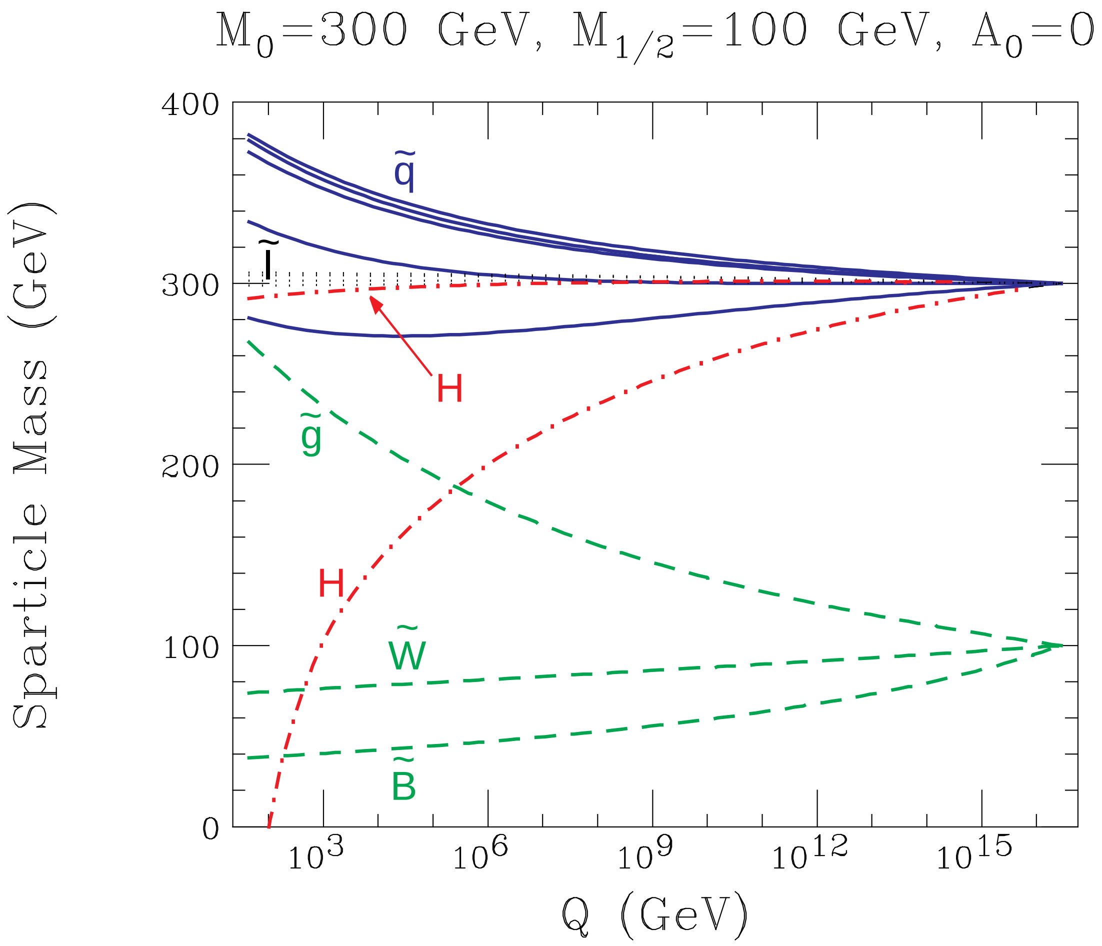
Still to do: parameter(Q_{GUT}) $\to$ parameter($Q_{\text{electroweak}}$)

Low-energy parameters (at the electroweak (EW) scale) via
"Renormalization group equations" (RGEs)

[RGE: equations that connect parameters at different energy scales]

$\Rightarrow$ $M_1, M_2, M_3, m^2_{H_u}, m^2_{H_d}, \mathbf{m^2_Q}, \mathbf{m^2_L}, \mathbf{m^2_u}, \mathbf{m^2_d}, \mathbf{m^2_e}, \mathbf{a_u}, \mathbf{a_d}, \mathbf{a_e}, b, |\mu|^2$ at the EW scale

Example:



Five new parameters, if possible phases are ignored:

$$m_0^2,\ m_{1/2},\ A_0,\ b,\ \mu$$

Final "trick": require radiative electroweak symmetry breaking:
Require correct value of M_Z at the EW scale:

$$\begin{aligned} |\mu|^2 + m_{H_d}^2 &= b \tan\beta - M_Z^2/2 \cos 2\beta \\ |\mu|^2 + m_{H_u}^2 &= b \cot\beta + M_Z^2/2 \cos 2\beta \end{aligned}$$

$\Rightarrow$ $|\mu|$, b given in terms of $\tan\beta$, sign μ

$\Rightarrow$ Scenario characterized by

$$m_0^2,\ m_{1/2},\ A_0,\ \tan\beta,\ \text{sign}\,\mu$$

Usually called 'CMSSM' (constrained MSSM) or 'mSUGRA'

In agreement with all phenomenological constraints (see below)

Summary: "supergravity inspired scenario", "mSUGRA"
characterized by five parameters:

$$m_0^2,\ m_{1/2},\ A_0,\ \tan\beta,\ \text{sign}\,\mu$$

m_0: universal scalar mass parameter

$m_{1/2}$: universal gaugino mass parameter

A_0: universal trilinear coupling

$\tan\beta$: ratio of Higgs vacuum expectation values

sign(μ): sign of supersymmetric Higgs parameter

m_0, $m_{1/2}$, A_0: GUT scale parameters

$\Rightarrow$ particle spectra from renormalization group running to weak scale

Lightest SUSY particle (LSP) is usually lightest neutralino

gaugino masses run in same way as gauge couplings

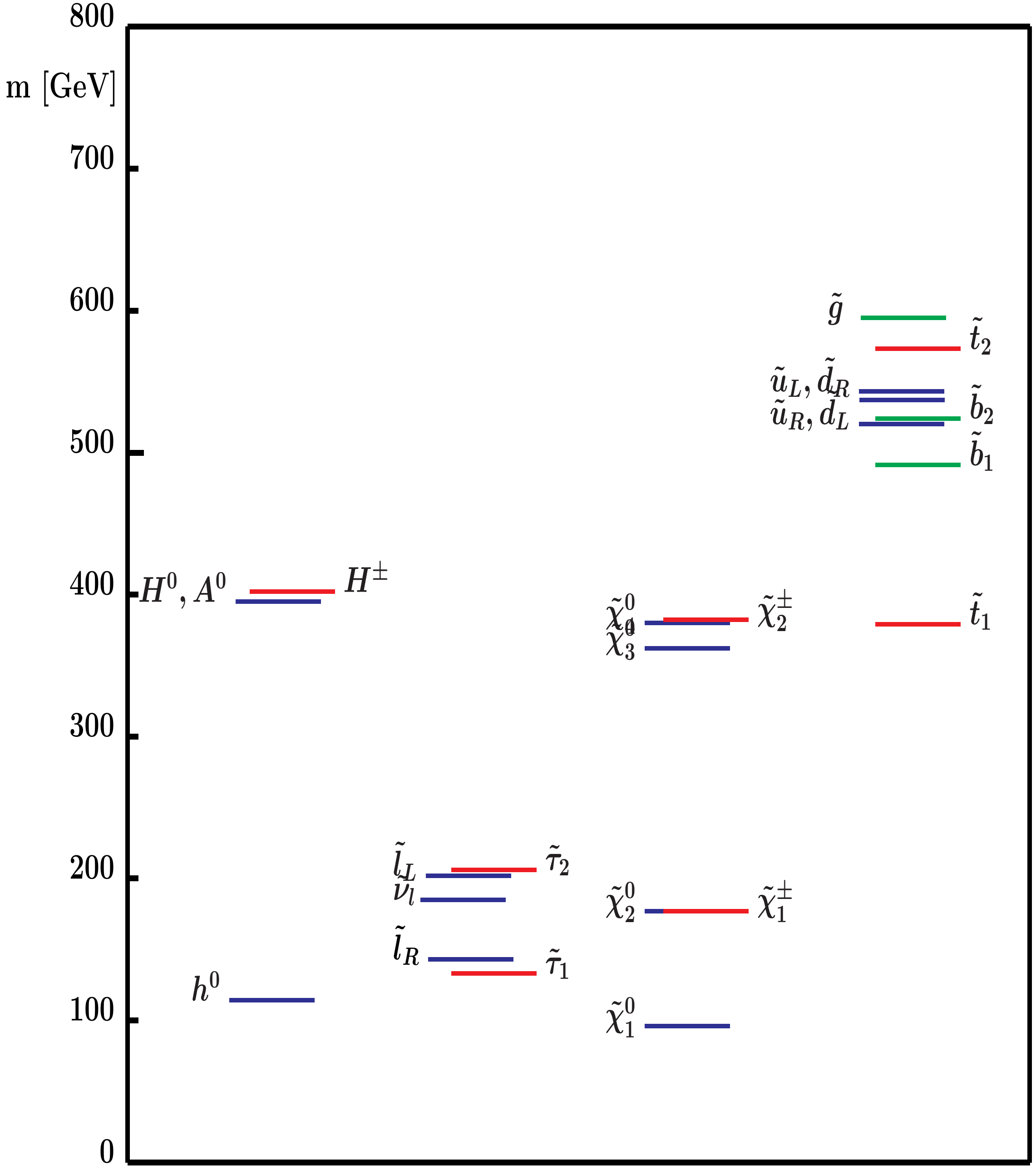
$\Rightarrow$ gluino heavier than charginos, neutralinos

“Typical” mSUGRA scenario

(SPS 1a benchmark scenario):

SPS home page:

`www.ippp.dur.ac.uk/~georg/sps`



4B. (minimal) gauge mediated SUSY breaking: mGMSB

New chiral supermultiplets, “messengers” , couple to SUSY breaking in hidden sector

Couple indirectly to MSSM fields via gauge interactions

$\Rightarrow$ mediation of SUSY breaking via electroweak and QCD gauge interactions

$\Rightarrow$ $\approx$ flavor-diagonal

SUSY breaking already in messenger spectrum

$\Rightarrow$ masses of SUSY particles from loop diagrams with messenger particles, gauge-interaction strength

$$\Rightarrow \quad m_{\text{soft}} \approx \frac{\alpha_i}{4\pi}\frac{\langle F \rangle}{M_{\text{mess}}}, \quad M_{\text{mess}} \sim \sqrt{\langle F \rangle}$$

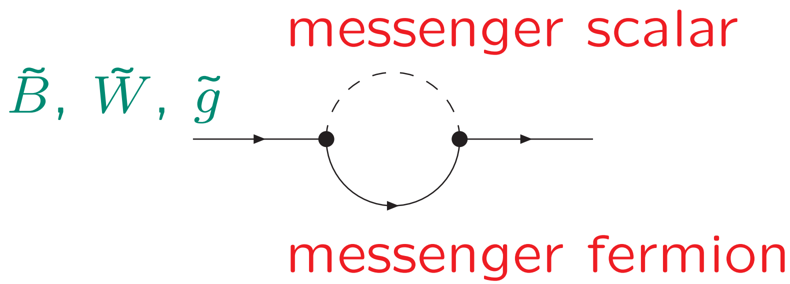
For $m_{\text{soft}} \lesssim 1$ TeV $\Rightarrow \sqrt{\langle F \rangle} \approx 10^4$–$10^5$ GeV

$\Rightarrow$ scale of SUSY breaking in hidden sector much lower than in SUGRA

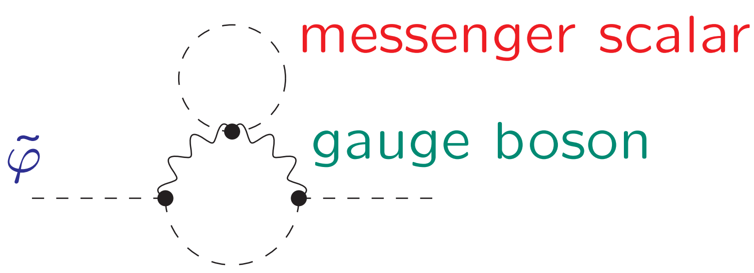
Gravitino mass: $m_{\frac{3}{2}} \approx \frac{\langle F \rangle}{M_{\mathsf{Pl}}} \approx 10^{-9}$ GeV

$\Rightarrow$ Gravitino is always the lightest SUSY particle (LSP)

Gaugino masses generated at one-loop order, $m_\lambda \approx \frac{\alpha_i}{4\pi}$



Scalar masses generated at two-loop order, $m_\varphi^2 \approx \left(\frac{\alpha_i}{4\pi}\right)^2$



⇒ Typical mass hierarchy in GMSB scenario between strongly interacting and weakly interacting particles $\sim \alpha_3/\alpha_2/\alpha_1$

GMSB scenario characterized by

$$M_{\text{mess}}, N_{\text{mess}}, \Lambda, \tan\beta, \text{sign}(\mu)$$

M_{mess}: messenger mass scale

N_{mess}: messenger index (number of messenger multiplets)

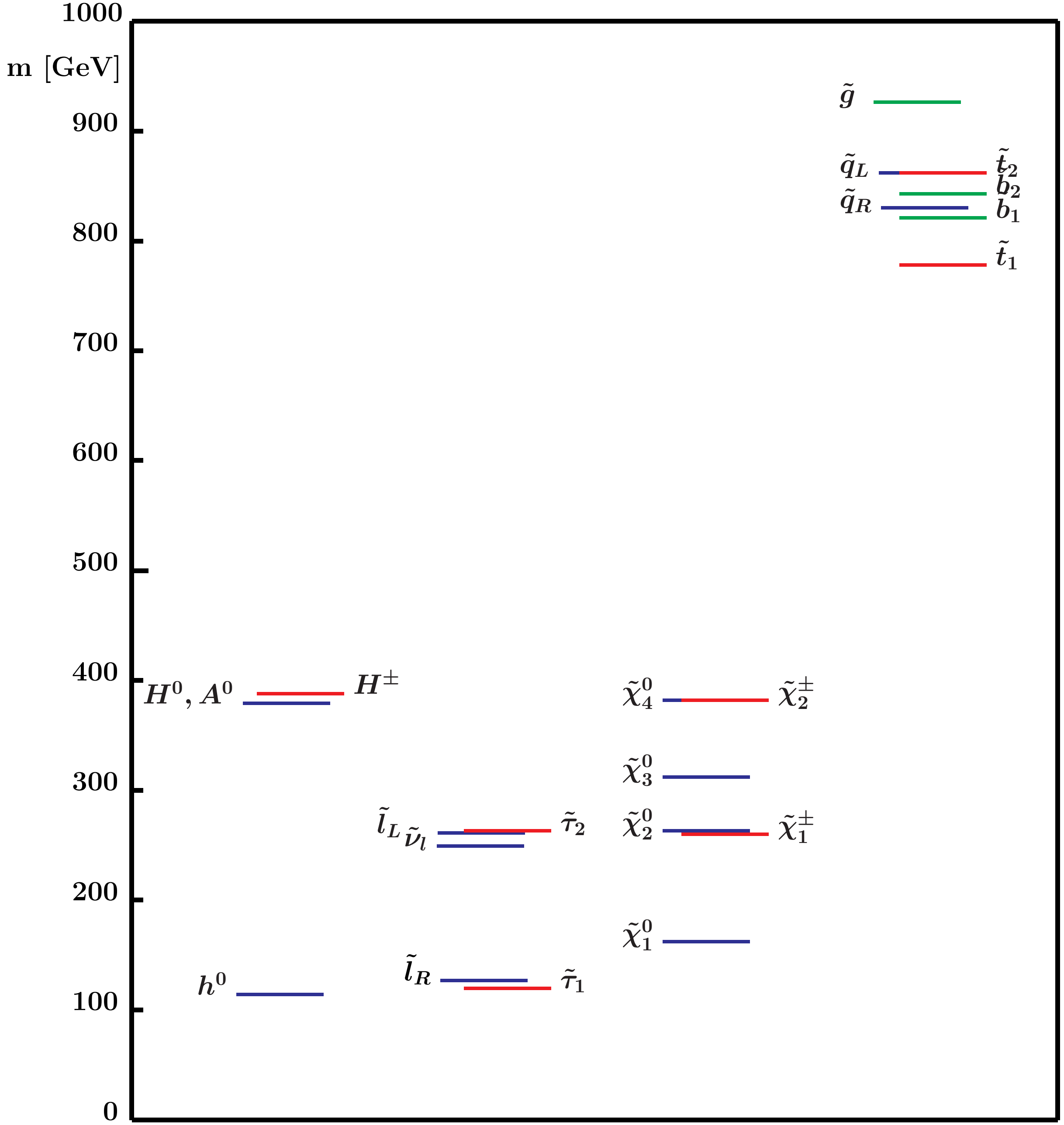
$\Lambda = \langle F \rangle / M_{\text{mess}}$: universal soft SUSY breaking mass scale felt by low-energy sector

LSP is always the gravitino
next-to-lightest SUSY particle (NLSP): $\tilde{\chi}_1^0$ or $\tilde{\tau}_1$

can decay into LSP inside or outside the detector

GMSB scenario with $\tilde{\tau}$ NLSP

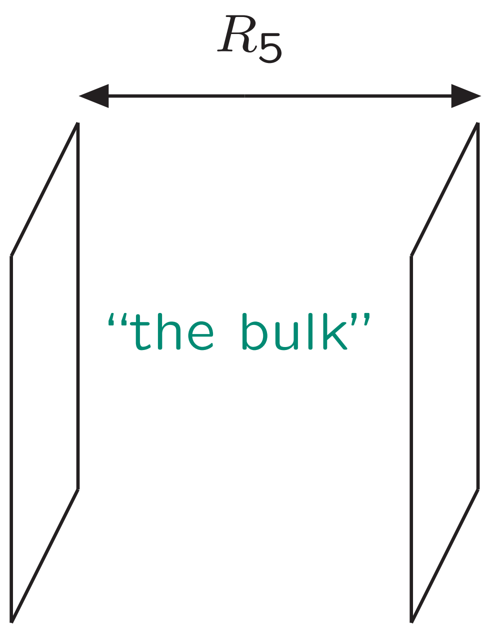
(SPS 7 benchmark scenario):



4C. (minimal) anomaly mediated SUSY breaking: mAMSB

No MSSM in the “bulk”

Anomaly: $\langle F \rangle$ enters RGEs



$$m^2_{\tilde{f}_k} \sim \frac{|\langle F \rangle|^2}{(16\pi^2)^2} g_k^4 + m_0^2 ,$$

$$\sim \frac{m^2_{3/2}}{(16\pi^2)^2} g_k^4 + m_0^2 ,$$

$$M_i \sim \frac{\langle F \rangle}{16\pi^2} g_i^2 \sim \frac{m_{3/2}}{16\pi^2} g_i^2$$

mAMSB scenario characterized by

$$m_{3/2}, m_0, \tan\beta, \mathrm{sign}(\mu)$$

$m_{3/2} = \langle F \rangle / M_{\mathrm{Planck}}$: overall scale of SUSY particle masses

m_0: phenomenological parameter: universal scalar mass term introduced in order to keep squares of slepton masses positive

AMSB scenario (SPS 9):

typical feature: very small neutralino–chargino mass difference

$\Rightarrow \tilde{\chi}_1^\pm \to \tilde{\chi}_1^0 + \pi^\pm$

with very soft pions