Automating Dipole Subtraction

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Collaboration with

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Reference: Nucl.Phys.Proc.Suppl.183:268-273,2008
arXiv:0807.3701 [hep-ph]

AutoDipole 1.0beta :

//https://indico.desy.de/conferenceOtherViews.py?view=standard&confId=1573

Calculations for Modern and Future Colliders, Dubna Russia, 17 July 2009

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1. Introduction

- Large Hadron Collider (LHC) at CERN

  - Energy Frontier : $\sqrt{S} \simeq 14\mathrm{TeV}$

    $\longrightarrow$ Direct production of Higgs and new particles beyond the Standard Model

  - Proton-Proton collision : $pp \longrightarrow \mathrm{X}$

    $\longrightarrow$ Triggered by the QCD interaction

  - The Standard Model predictions to identify New Physics

$$(\text{New Physics}) = (\text{LHC signals}) - (\text{the SM predictions})$$

  - The rate of QCD processes with high momentum transfer can be predicted by the perturbative expansion in the small strong coupling constant

For example, $\alpha_s(m_t) \simeq 0.1$

■ Perturbative QCD

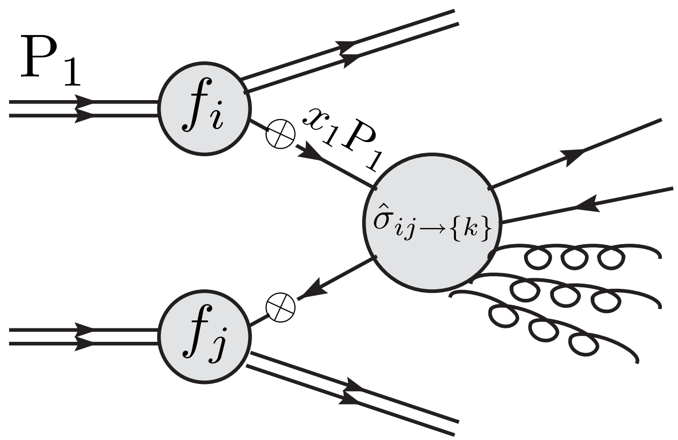
- Master Formula : Factorization of the hard scattering process

$$\sigma_{pp\to X} = \sum_{i,j,\{k\}} \int dx_1 dx_2 f_i(x_1) f_j(x_2) \hat{\sigma}_{ij\to\{k\}}(\alpha_s, Q) \otimes D_{\{k\}\to X}$$

Parton distribution function
(Non-perturbative)

Subprocess partonic cross section
(Perturbative)

Jet algorithm
Parton shower
Hadronization model



- Perturbative expansion of the partonic cross section

$$\hat{\sigma}_{ij\to\{k\}} = \sigma_{\mathrm{LO}}(1 + \alpha_s C_1 + \alpha_s^2 C_2 + \cdots)$$

Leading order (LO)
Next-to-leading order (NLO)
Next-to-next-to-leading order (NNLO)

■ Leading order (LO)

- LO(Tree level) is well automated

Alpgen, CompHep, CalcHEP, FeynArts/FeynCalc, GRACE, HELAC/PHEGAS, MadGraph, $\cdots$

■ Next-to-leading order (NLO)

- LO has a large uncertainty from the renormalization/factorization scale dependences
- NLO is not yet fully automatized
- Process with multi-parton legs are difficult
- LHC priority NLO wish list in Les Houches 2005 (hep-ph/0604120)

process ($V \in \{Z, W, \gamma\}$)	relevant for
1. $pp \to V\,V$ jet	$t\bar{t}H$, new physics
2. $pp \to t\bar{t}\,b\bar{b}$	$t\bar{t}H$
3. $pp \to t\bar{t} + 2\,$jets	$t\bar{t}H$
4. $pp \to V\,V\,b\bar{b}$	VBF$\to H \to VV$, $t\bar{t}H$, new physics
5. $pp \to V\,V + 2\,$jets	VBF$\to H \to VV$
6. $pp \to V + 3\,$jets	various new physics signatures
7. $pp \to V\,V\,V$	SUSY trilepton

-A. Bredenstein, A.Denner, S.Dittmaier, S.Pozzorini
arXiv:0807.1248 0905.0110

- R. Keith Ellis, Kirill Melnikov, Giulia Zanderighi
0901.4101 0906.1445

- C.Berger, Z.Bern, L.Dixon, F.Cordero, D.Forde, T.Gleisberg, H.Ita, D.Kosower, D.Maitre
0902.2760 0907.1984

-These predictions are urgently needed for the successful operation of LHC

-The computation of these radiative corrections is now a very active field

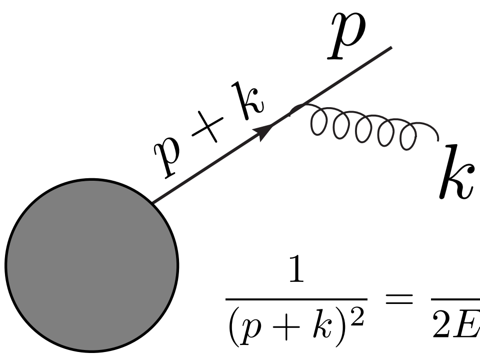
- QCD at NLO : $\sigma_{\mathrm{NLO}} = \sigma_{\mathrm{real}} + \sigma_{\mathrm{virtual}}$

- Real correction

  - One additional gluon to LO

    Because it can not be resolved to LO event in sufficient soft/collinear region

  - Soft and collinear singularities



$$\frac{1}{(p+k)^2} = \frac{1}{2E_k E_p (1 - \beta \cos\theta_{kp})} \longrightarrow \infty$$

Soft limit $\mathrm{E}_k \rightarrow 0$: Soft singularity (IR divergence)

: Detector can not be aware of it

Collinear limit $\theta_{kq} \rightarrow 0$: Collinear singularity $(m_q = 0)$

: Detector can not resolve it to quark

  - Phase space integral of those singularities

	Cutoff		Dimensional regularization
Soft region:	$\int \frac{d^3k}{k}\frac{1}{k^2} \simeq \int_{\mu_k} \frac{dk}{k} \simeq \log \mu_k + \cdots$	$\longleftrightarrow$	$\int \frac{d^{D-1}k}{k}\frac{1}{k^2} \simeq \int_0 \frac{dk}{k^{1+\epsilon}} \simeq -\frac{1}{\epsilon} + \cdots$
Collinear region:	$\int_{-1}^{1} d\cos\theta \frac{1}{1-\cos\theta} \simeq \int_{\mu_\theta} \frac{d\theta}{\theta} \simeq \log \mu_\theta + \cdots$	$\longleftrightarrow$	$\int_0 \frac{d\theta}{\theta^{1+\epsilon}} \simeq -\frac{1}{\epsilon} + \cdots$

■ QCD at NLO : Cancellation of soft/collinear singularities

$$\sigma_{\mathrm{NLO}} = \sigma_{\mathrm{real}} + \sigma_{\mathrm{virtual}}$$

Soft/Collinear

Soft/Collinear

UV

Renormarization

Finite

$+$

Finite in observables

:IR safe (more precisely, soft/collinear safe)

Kinoshita-Lee-Nauenberg theorem

- When an initial parton exists, the collinear subtraction counter term σ_C is added to cancel the collinear singularity from the initial state radiation

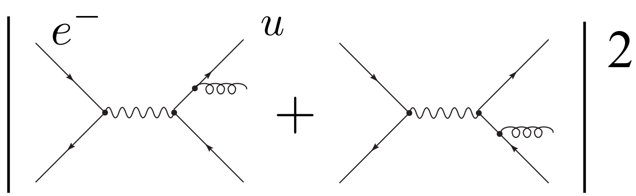
■ The simplest example : $e^- e^+ \to u\bar{u}$

- Typical procedure: Dimensional regularization

Calculate all quantity (phase space and matrix element) in dimension: $\mathrm{D} = 4 - 2\epsilon$

$$\sigma_{\mathrm{NLO}} = \sigma_{\mathrm{real}} + \sigma_{\mathrm{virtual}}$$

$$= \int d\Phi_3 |\mathrm{M}(e^- e^+ \to u\bar{u}g)|^2\Big|_{\text{D-dim}} + \int d\Phi_2 |\mathrm{M}(e^- e^+ \to u\bar{u})|^2_{\text{1-loop}}\Big|_{\text{D-dim}}$$



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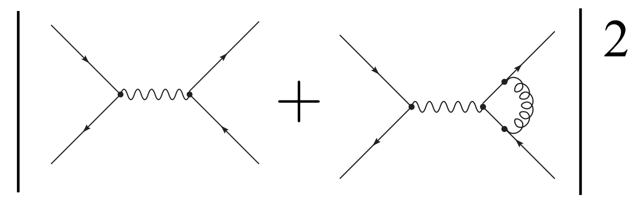


Phase space $\quad \int_0^1 \mathrm{d}x_1 \int_{1-x_1}^1 \mathrm{d}x_2 \frac{1}{[(1-x_1)(1-x_2)(1-x_3)]^\epsilon} \quad \int_0^1 dv \frac{1}{[v(1-v)]^\epsilon}$

$|\mathrm{M}|^2 \quad \frac{x_1^2 + x_2^2 - \epsilon x_3^2}{(1-x_1)(1-x_2)} \quad |\mathrm{M}_{\mathrm{LO}}| \cdot \mathrm{Re}\left[\int dx dy \frac{\Gamma(\epsilon)}{(-xyq^2)^\epsilon}\left((1-\epsilon)^2 - \epsilon\frac{(1-x)(1-y) - \epsilon xy}{xy}\right)\right]$

$$= \sigma_{\mathrm{LO}}^{(\epsilon)} \frac{\alpha_s}{2\pi} C_{\mathrm{F}} \left(\frac{4\pi}{q^2}\right)^\epsilon \frac{4\Gamma(1-\epsilon)^2}{\Gamma(3-3\epsilon)} \left[\left(\underline{\frac{1}{\epsilon^2} - \frac{3}{\epsilon}} + \frac{5}{2} + O(\epsilon)\right) + \left(\underline{-\frac{1}{\epsilon^2} + \frac{3}{\epsilon}} - \frac{7}{4} + O(\epsilon)\right)\right]$$

Cancelation of $1/\epsilon$ poles

$$= \sigma_{\mathrm{LO}} \cdot \frac{\alpha_s}{\pi}$$:Finite results

- This method is not practical for the multi-parton leg processes
  - The complexity and the long expression
  - The phase space integral of n and (n+1) particles in D-dimension

Dipole subtraction

- A general and practical procedure to treat soft/collinear singularities at QCD NLO

S.Catani and M.H.Seymour, Nucl.Phys.B485(1997)291
S.Catani, S.Dittmaier, M.H.Seymour, Z.Trocsanyi, Nucl.Phys.B627(2002)189

1. Construct the counter terms which cancel all soft/collinear singularities
2. Subtract it from $\sigma_{\rm real}$ and add it to $\sigma_{\rm virtual}$

$$\begin{aligned}\sigma_{\rm NLO} &= \sigma_{\rm real} + \sigma_{\rm virtual} \\ &= \left(\sigma_{\rm real} - \sigma_{\rm a}\right) + \left(\sigma_{\rm virtual} + \sigma_{\rm a}\right) \\ &= \int d\Phi_{m+1}\left[|\mathrm{M}_{\rm real}|^2 - \sum_i \mathrm{D}_i\right]\Bigg|_{\rm D=4} + \int d\Phi_m\left[|\mathrm{M}_{\text{1-loop}}|^2 + \int d\Phi_1 \sum_i \mathrm{D}_i\right]\Bigg|_{\rm D=4}\end{aligned}$$

Finite

$$\int_0 dk\left(\frac{1}{k} - \frac{1}{k}\right) < \infty \quad \text{Finite}$$

- Real correction: Instead of reguralizing, subtract
  Calculation (phase space and matrix element) is in 4-dimension

- Dipole term is systematically constructed based on the factorization of soft/collinear singularities
  $\longrightarrow$ reduction to Born level

$$\mathrm{D}_i \simeq \frac{1}{s_i}\mathrm{V}_i \cdot |\mathrm{M}_i|^2_{\rm Born}$$

Singular part

dipole splitting function (Universal)

- Integration of dipole term is analitically done once for all

■ Multi-parton leg processes

- Dipole subtraction makes it possible
- It requires many dipole terms and repeats the same kinds of calculation at huge times (Order 100 dipoles)
- The algorithm is a combinatorial way

⇨ The automatization is required and it is possible

■ Our aim

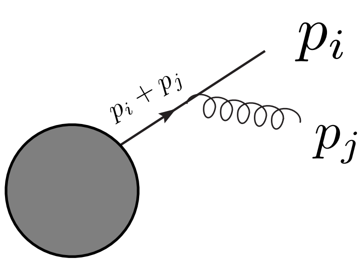
1. Automatize the dipole subtraction
2. Apply it to the QCD backgrounds and the relevant signals in LHC

-There is recent work in the same direction

- T. Gleisberg and F. Krauss, Eur.Phys.J.C53(2008)501, arXiv0709.2881
- M.H. Seymour and C. Tevlin, arXiv0803.2231
- R. Frederix and T. Gehrmann and N. Greiner, JHEP0809:122, arXiv0808.2128
- M. Czakon, C.G. Papadopoulos, M. Worek, arXiv0905.0883

2. Dipole Subtraction

- Soft limit factorization



$$\mathrm{M}_{m+1}(p_i, p_j) = \epsilon_\mu^{a\star} \bar{u}(p_i) i g_s t^a \gamma^\mu \frac{i(\not{p}_i + \not{p}_j)}{(p_i + p_j)^2} \mathrm{M}_m(p_i + p_j)$$

Eikonal approximation: $|\vec{p}_j| \ll |\vec{p}_i|$

$$\simeq \epsilon_\mu^{a\star} g_s \frac{p_i^\mu}{p_i \cdot p_j} \bar{u}(p_i)_\alpha (t^a)_{\alpha\beta} \mathrm{M}_m(p_i)_\beta$$

- No spin correlation
- Color correlation

$$p_j = \lambda q_j \qquad \lambda \to 0$$

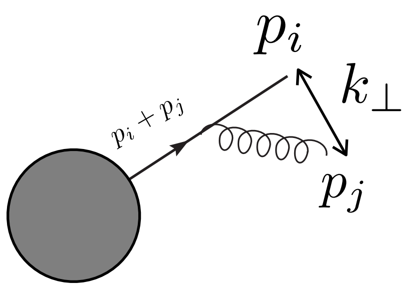
$$< 1, \cdots, i, \cdots, j, \cdots, m+1 || 1, \cdots, i, \cdots, j, \cdots m+1 >_{m+1}$$

$$\longrightarrow \qquad -\frac{1}{\lambda^2} 4\pi\alpha_s < 1, \cdots, i, \cdots, m+1 | [J^\mu]^\dagger J_\mu | 1, \cdots, i, \cdots m+1 >_m$$

Eikonal current: $J^\mu = \sum_i \mathrm{T}_i \frac{p_i^\mu}{p_i \cdot q_j}$

$$\longrightarrow \qquad -\frac{1}{\lambda^2} 8\pi\alpha_s \sum_i \frac{1}{p_i \cdot q_j} \sum_{k(\neq i)} < 1, \cdots, i, \cdots, m+1 | \frac{p_i \cdot p_k}{(p_i + p_k) \cdot q_j} \mathrm{T}_i \cdot \mathrm{T}_k | \underbrace{1, \cdots, i, \cdots m+1 >_m}_{\text{Reduced Born}}$$

■ Collinear limit factorization



$p_i^\mu = zp^\mu + k_\perp^\mu - \frac{k_\perp^2}{z}\frac{n^\mu}{2p\cdot n}$ $\quad$ $p_j^\mu = (1-z)p^\mu - k_\perp^\mu - \frac{k_\perp^2}{1-z}\frac{n^\mu}{2p\cdot n}$ $\quad$ $2p_i\cdot p_j = -\frac{k_\perp^2}{z(1-z)}$

$k_\perp \to 0$

$$< 1,\cdots,i,\cdots,j,\cdots,m+1||1,\cdots,i,\cdots,j,\cdots m+1>_{m+1}$$

$$\longrightarrow \quad \frac{1}{p_i\cdot p_j}4\pi\alpha_s < 1,\cdots,ij,\cdots,m+1|\hat{P}_{(ij),i}(z,k_\perp)|1,\cdots,ij,\cdots m+1>_m$$

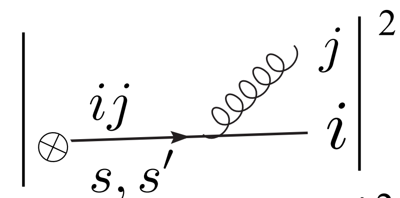
- Self-square of the matrix element has the leading singularity

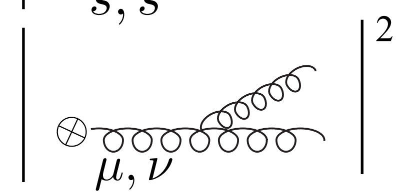
-Altarelli-Parisi splitting function: $\hat{P}_{(ij),i}(z,k_\perp)$

Square of the splitting amplitudes

$$< s|\hat{P}_{qq}(z,k_\perp)|s' >= \delta_{ss'}C_F\left[\frac{1+z^2}{1-z}\right]$$

$$< \mu|\hat{P}_{gg}(z,k_\perp)|\nu >= 2C_A\left[\underbrace{-g_{\mu\nu}}_{\sum_{\lambda=L,R}\epsilon_\mu^{(\lambda)}\epsilon_\nu^{(\lambda)\star}}\left(\frac{z}{1-z}+\frac{1-z}{z}\right) - 2z(1-z)\underbrace{\frac{k_\perp^\mu k_\perp^\nu}{k_\perp^2}}_{\text{Different helicity}}\right]$$

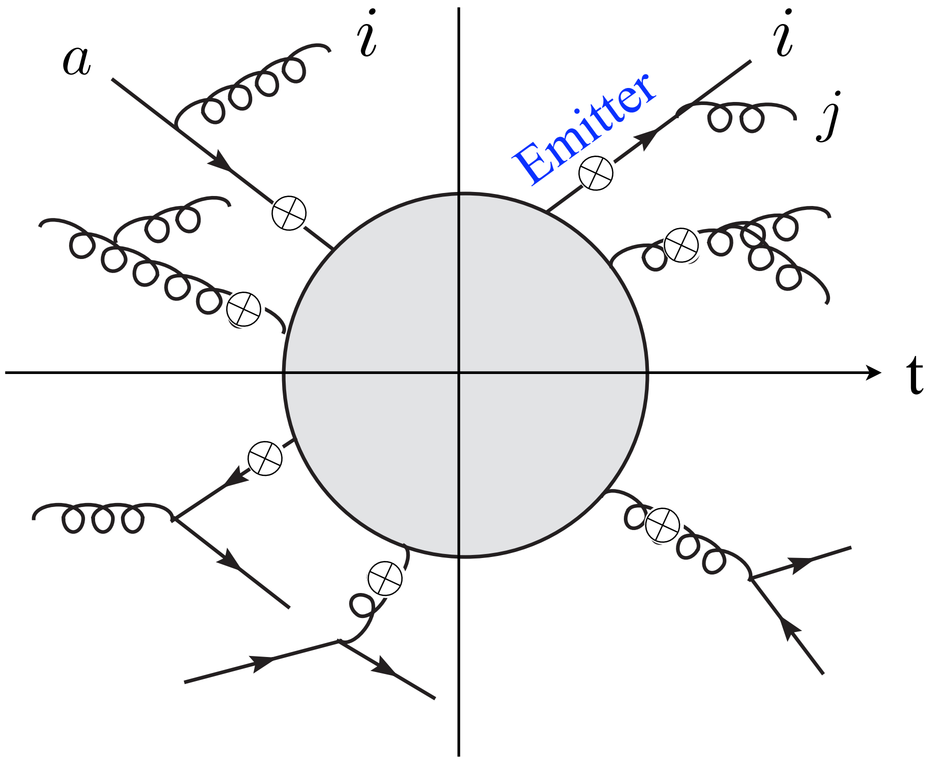




- Gluon spin correlation
- No color correlation

■ Construction of dipole terms

1. Choose emitter pair



Choose all possible leg-pair which matches one of the seven patterns

Initial parton=a,b Final parton=i,j,k

(a, i) or (i, j)

2. Choose spectator

Choose a different leg from emitter pair

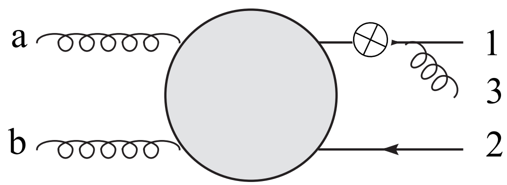
Spectator : $k \neq i, j$ $\quad b \neq a$

emitter pair \ spectator	k	b
(i, j)	$\mathrm{D}_{ij,k}$ $(k \neq i, j)$	$\mathrm{D}_{ij}^{\,b}$
(a, i)	$\mathrm{D}^{ai}_{\;k}$ $(k \neq i)$	$\mathrm{D}^{ai,b}$ $(b \neq a)$

3. Use dipole formulae

$$\mathrm{D}_{ij,k}(p_1,\cdots,p_{m+1}) = -\frac{1}{2p_i\cdot p_j} < 1,\cdots,\tilde{ij},\cdots,\tilde{k},\cdots,m{+}1|\frac{\mathrm{T}_k\cdot\mathrm{T}_{ij}}{\mathrm{T}_{ij}^2}\mathrm{V}_{ij,k}|1,\cdots,\tilde{ij},\cdots,\tilde{k},\cdots,m{+}1>_m$$

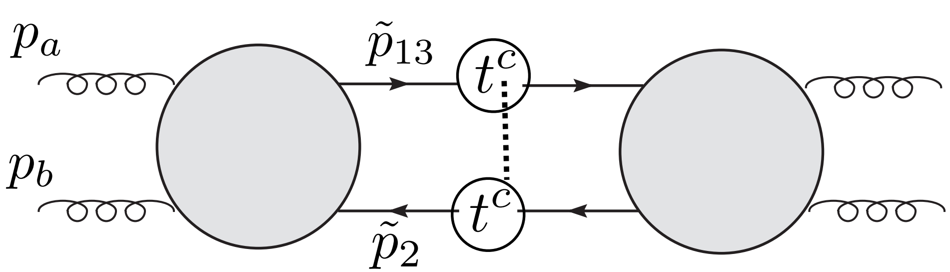
Example : $g(a)g(b)\to u(1)\bar{u}(2)g(3)$



$$\mathrm{D}_{13,2}(p_1,p_2,p_3,p_a,p_b) = -\frac{1}{2p_1\cdot p_3}\langle gg\to\tilde{u}\tilde{\bar{u}}|\frac{\mathrm{T}_{\bar{u}}\cdot\mathrm{T}_{ug}}{\mathrm{T}_{ug}^2}\mathrm{V}_{13,2}|gg\to\tilde{u}\tilde{\bar{u}}\rangle_2$$

- Dipole splitting function : $\mathrm{V}_{13,2}(z,y) = \delta_{ss'}8\pi\alpha C_F\left[\frac{2}{1-z_i(1-y_{ij,k})}-(1+z_i)\right]$

- Color linked Born squared (CLBS): $\langle gg\to\tilde{u}\tilde{\bar{u}}|\mathrm{T}_{\bar{u}}\cdot\mathrm{T}_{ug}|gg\to\tilde{u}\tilde{\bar{u}}\rangle_2$



$$\mathrm{T}^a_{\mathrm{X}} = \begin{cases} t^a & (\mathrm{X} = \text{quark}) \\ f^a & (\mathrm{X} = \text{gluon}) \end{cases}$$

- Reduced momenta satisfy the energy-momentum conservation and on-shell condition

$$p_a^\mu + p_b^\mu = \tilde{p}_{13}^\mu + \tilde{p}_2^\mu \qquad \tilde{p}_{13}^2 = \tilde{p}_2^2 = 0$$

⇨ Make it possible to reduce into the physical born amplitude, which can be calculated by the well automated LO softwares

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$$\tilde{p}_{ij}^\mu = p_i^\mu + p_j^\mu - \frac{y_{ij,k}}{1-y_{ij,k}}p_k^\mu \qquad \tilde{p}_k^\mu = \frac{1}{1-y_{ij,k}}p_k^\mu \qquad z_i = \frac{p_i\cdot p_k}{p_j\cdot p_k + p_i\cdot p_k} \qquad y_{ij,k} = \frac{p_i\cdot p_j}{p_i\cdot p_j + p_j\cdot p_k + p_k\cdot p_i}$$

Soft/collinear limits of dipole terms

$$|\mathrm{M}|^2_{\mathrm{real}} - \sum_i \mathrm{D}_i \quad \text{: Soft and collinear safe}$$

$$\mathrm{D}_{ij,k} = -\frac{1}{s_{ij}} V_{ij,k} < \mathrm{T}_{ij} \cdot \mathrm{T}_k >_m$$

$$-\frac{1}{\lambda^2} 8\pi\alpha_s \frac{1}{p_i \cdot q_j} \frac{p_i \cdot p_k}{(p_i + p_k)\cdot q_j} < \mathrm{T}_i \cdot \mathrm{T}_k >_m \quad \text{(Soft limit)}$$

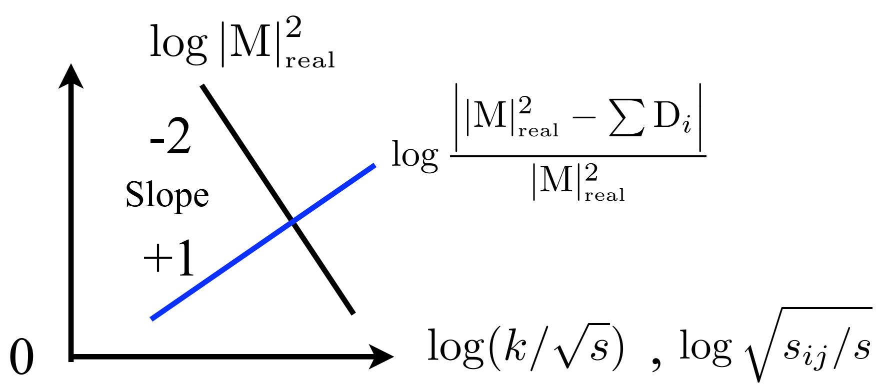
$$\sum_{\substack{k=1 \\ (k \neq i)}}^{m} < \mathrm{T}_i \cdot \mathrm{T}_k >_m = -\mathrm{T}_i^2 < | >_m$$

$$\frac{1}{p_i \cdot p_j} 4\pi\alpha_s \hat{P}_{(ij),i}(z, k_\perp) < | >_m \quad \text{(Collinear limit)}$$

All soft/collinear singularities are cancelled by the dipole terms

Limiting behavior can be predicted

$$|\mathrm{M}|^2_{\mathrm{real}} - \sum_i \mathrm{D}_i = \begin{cases} \dfrac{1}{k^2}(a_0 + a_1 k + a_2 k^2 + \cdots) - \dfrac{1}{k^2} a_0 = \dfrac{1}{k}(a_1 + a_2 k + \cdots) & \text{Soft } (k \to 0) \\ \dfrac{1}{s_{ij}}(b_0 + b_1\sqrt{s_{ij}} + b_2 s_{ij} + \cdots) - \dfrac{1}{s_{ij}} b_0 = \dfrac{1}{\sqrt{s_{ij}}}(b_1 + b_2\sqrt{s_{ij}} + \cdots) & \text{Collinear } (\theta_{ij} \to 0) \end{cases}$$



3. Automatization

- Package: AutoDipole Version 1.0beta (publicly available)

K. Hasegawa, S.Moch, and P.Uwer
Nucl.Phys.Proc.Suppl.183:268-273,2008 (arXiv:0807.3701 [hep-ph])

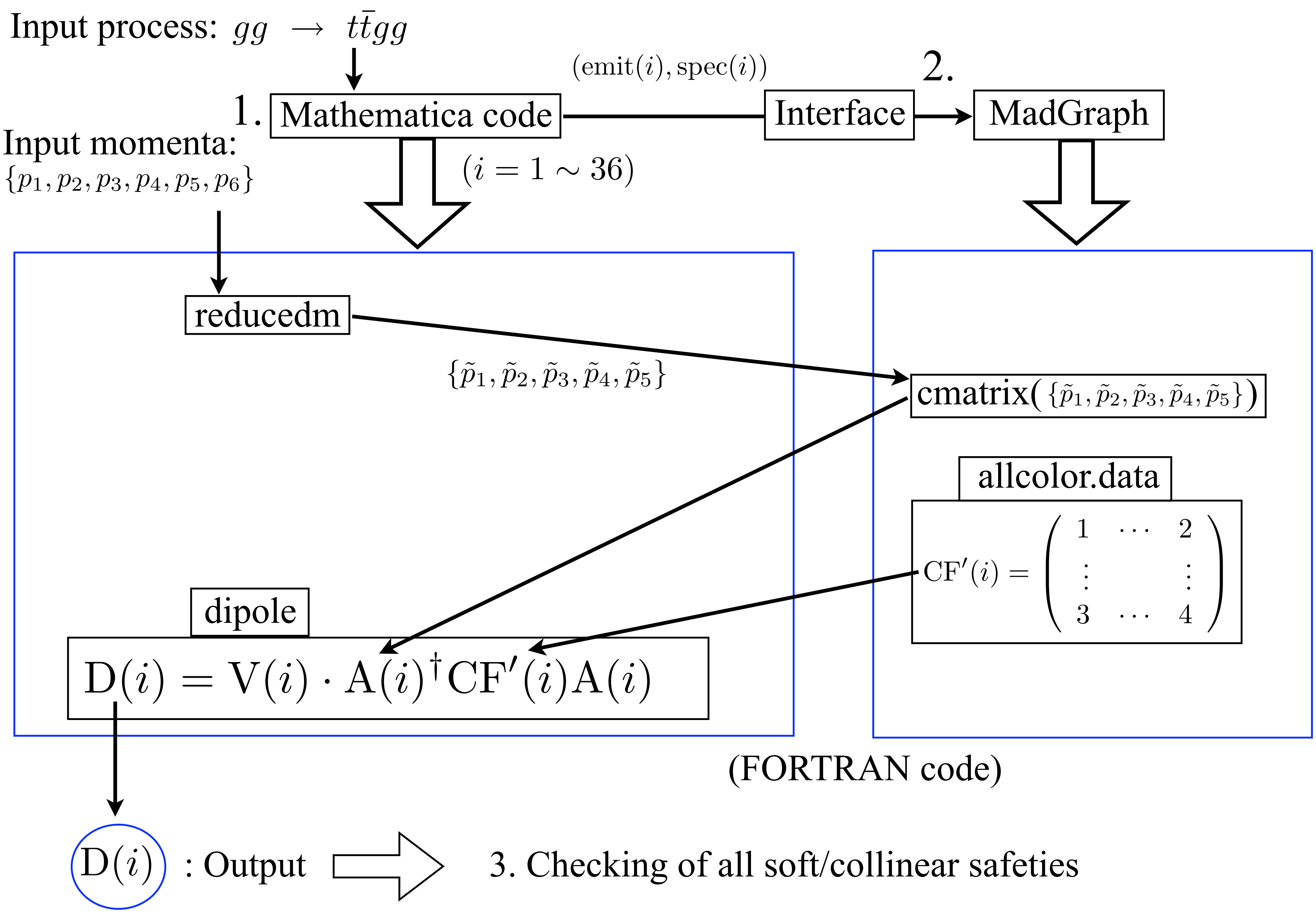
- Includes the subtracted real emission part : $|\mathrm{M}|^2_{\mathrm{real}} - \sum_i \mathrm{D}_i$

- Mathematica code and an interface with MadGraph

- Mathematica code generate the Fortran routines

- Check all soft/collinear safeties

■ Scheme to calculate $|\mathrm{M}|^2 - \sum_i \mathrm{D}_i$ in AutoDipole



■ 1. Mathematica code

Input: $gg \to u\bar{u}g$ (Process at NLO real correction)

↓

Creation of dipole terms (Write down all D_i except for CLBS)

↓

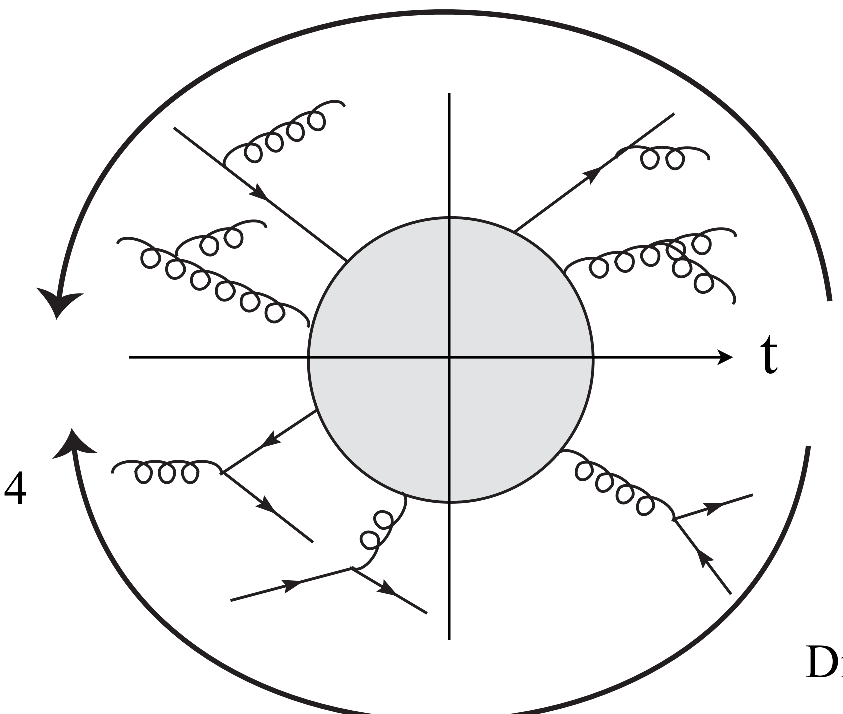
Show the contents of all dipoles and all soft/collinear limits

↓

Write Fortran files: dipole.f reducedm.f and interface for MadGraph

- Creation order

Dipole 1 $\mathrm{B1} = \mathrm{M}_0(gg \to u\bar{u})$



Dipole 4

$\mathrm{B4} = \mathrm{M}_0(ug \to ug)$

Dipole 2 $\mathrm{B2} = \mathrm{M}_0(gg \to gg)$

Dipole 3

■ 2. MadGraph with our interface

- MadGraph

T. Stelzer and W.F. Long, Phys.Commun.81(1994) 357, hep-ph/9401258
Johan Alwall et al, JHEP 0709:028,2007, arXiv:0706.2334

- An automated general LO in the Standard Model, MSSM, and some others models
- Write down the Fortran codes to calculate the matrix element squared
- Numerical evaluation of the helicity amplitude
- In order to calculate the helicity amplitude, HELAS library is used

K. Hagiwara, H. Murayama, I. Watanabe, Nucl.Phys.B367(1991)257

- Color decomposition

$$\mathrm{M} = \sum_a C_a J_a \qquad J_1 = A_1 - A_3 + \cdots$$: Joint amplitude

- Our interface

- Normal Born squared $\longrightarrow$ Color linked Born squared

$< 1, \cdots, m||1, \cdots, m >_m$

$< 1, \cdots, m|\mathrm{T}_i \cdot \mathrm{T}_k|1, \cdots, m >_m$

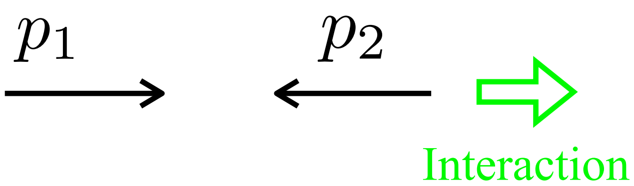
$\searrow$ Different helicity Born squared

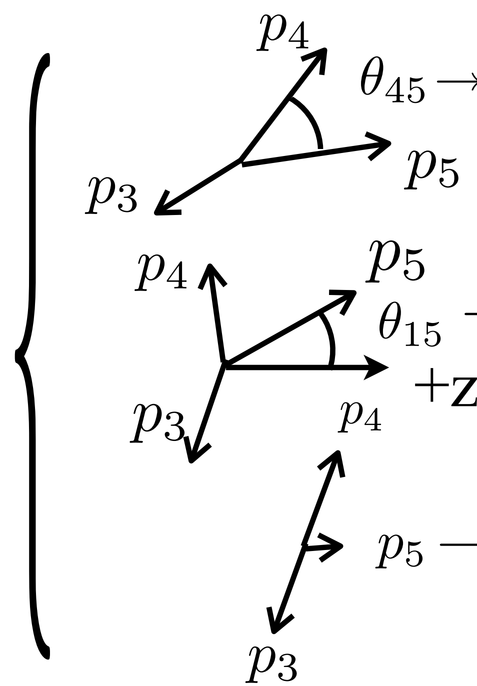
$< 1, \cdots, (i, \lambda), \cdots, m||1, \cdots, (i, \lambda'), \cdots, m >_m$

■ 3. Check the soft/collinear limits

- Configurate all soft/collinear limits

$g(1)g(2) \to u(3)\bar{u}(4)g(5)$





$\theta_{45} \to 0$: Colllinear limit of Final-Final spliting

$\theta_{15} \to 0$: Colllinear limit of Initial-Final spliting

$p_5 \to 0$: Soft limit

- Check the cancellation of the singularities

■ Status of AutoDipole

- We checked the cancellation of all soft/collinear singularities in the following real emission processes

	$2 \to 3$	$2 \to 4$	$2 \to 5$	$2 \to 6$
Massless (Including lepton)	$e^+e^- \to u\bar{u}g$ $e^-u \to e^-ug$ $e^-g \to e^-u\bar{u}$ $u\bar{u} \to e^+e^-g$			
(Parton only)	$gg \to u\bar{u}g$ $gg \to 3g$ $u\bar{u}g \to d\bar{d}g$	$gg \to u\bar{u}gg$ $gg \to 4g$	$u\bar{u} \to d\bar{d}ggg$	
(Including W/Z boson)		$\bar{u}u \to W^+W^-gg$	$gg \to W^+\bar{u}dgg$	
Massive (Including lepton)	$e^+e^- \to t\bar{t}g$			
(Parton only)	$gg \to t\bar{t}g$	$gg \to t\bar{t}gg$ $u\bar{u} \to t\bar{t}gg$ $ug \to t\bar{t}ug$ $\bar{u}g \to t\bar{t}\bar{u}g$ $gg \to t\bar{t}u\bar{u}$ $u\bar{u} \to t\bar{t}u\bar{u}$	$gg \to t\bar{t}ggg$ $gg \to t\bar{t}b\bar{b}g$	$u\bar{u} \to t\bar{t}b\bar{b}gg$

■ Status of AutoDipole - continued

- Agreements with the independent results

- $gg \to t\bar{t}gg$ $u\bar{u} \to t\bar{t}gg$ $ug \to t\bar{t}ug$ $\bar{u}g \to t\bar{t}\bar{u}g$ $gg \to t\bar{t}u\bar{u}$ $u\bar{u} \to t\bar{t}gg$

  - The modes of NLO real emission process to $pp \to t\bar{t} + 1\mathrm{jet}$

  - All dipoles completely agree with the results in

    S. Dittmaier, P. Uwer and S. Weinzierl, arXiv:0810.0452 and
    Phys.Rev.Lett.98(2007)262002,hep-ph/0703120

- $u\bar{u} \to W^+W^-gg$

  - One mode of NLO real emission process to $pp \to W^+W^- + 1\mathrm{jet}$

  - All 10 dipoles completely agree with the results in

    S. Dittmaier, S. Kallweit, P. Uwer, Phys.Rev.Lett.100(2008)062003,
    arXiv:0710.1577 [hep-ph]

4. Use with example

■ Download package and install

- Download the package from the CAPP09 Website

```
//https://indico.desy.de/conferenceOtherViews.py?view=standard&confId=1573
```

- Decompress it

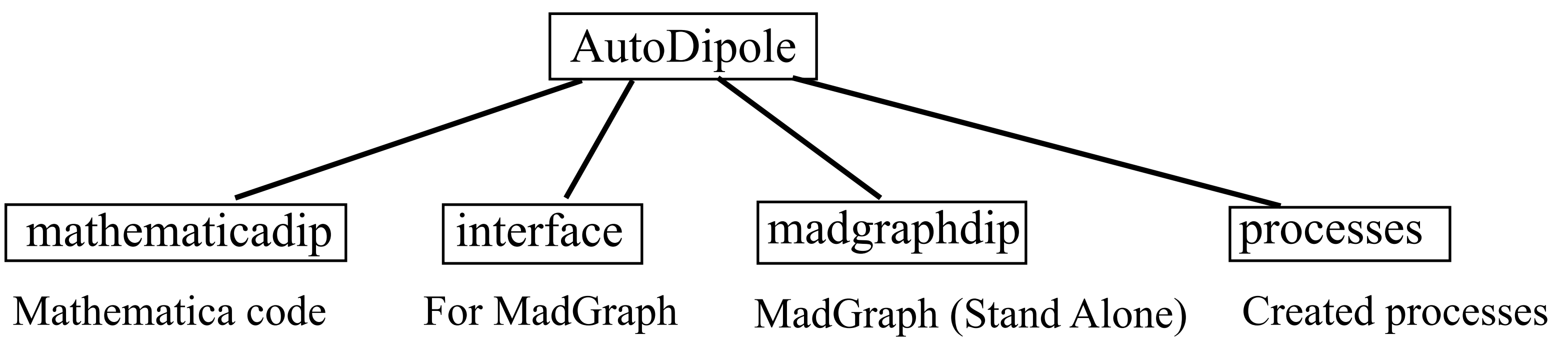
```
tar xvf AutoDipole_V1.0beta.tar
```

- Install MadGraph

```
cd ./AutoDipole_V1.0beta/madgraphdip/MG_ME_SA_V4.2.6/MadGraphII/
```

```
make
```

■ Directory structure



We are in mathematicadip/

- 0. Set up

- Set up file for Mathematica code :

```
cd mathematicadip/
mathematica parameter.m&
```

```
(* AutoDipole - Packege of an automated dipole subtraction by
Kouhei Hasegawa, Sven Moch, and Peter Uwer, 27.03.2009 *)
```

```
ep=0;
```
$D = 4 - 2\epsilon$

```
kap=2/3;
```
A freedom for non-singular part

```
AlphaS=0.1075205492734706;
```
α_s

```
topmass=174.00;
```
m_{top}

```
bottommass=4.7;
```
m_{bottom}

```
skipdipole={2u,2t};
```
Splitting which is skipped

```
acccut=10^(-5);
```
Cut to define soft/collinear safeties

- Available fields: (u,ubar) (d,dbar) (b,bbar) (t,tbar) (e,ebar) gamma (Wp,Wm) Z

- Available interactions are same with ones in MadGraph

■ 1. Input to Mathematica code and run

We are in mathematicadip/


```
mathematica exedip.nb&
```
- Open file exedip.nb

```
<< driver.m
```
- Includes package

```
Realprocess[{e, ebar}, {u, ubar, g}]
```
- Input real emission process and run

```
      Exit

 In[1]:= << driver.m

 In[2]:= Realprocess[{e, ebar}, {u, ubar, g}]

NLO: {{e, pa}, {ebar, pb}} --> {{u, p[1]}, {ubar, p[2]}, {g, p[3]}}

Masses: {0,0} --> {0, 0, 0}

--------------------------------------------------------------------------

Dipole 1

M0=B1:           {e, ebar} --> {u, ubar}

Reduced momenta: {ptil[1], ptil[2]} --> {ptil[3], ptil[4]}

{Splitting (1):(i,j)=(f,g)}

[1.(ij,k)=(fg,k): Dij,k]


--Dip(1)--
```

■ 1. Input to Mathematica code and run - continued

We are in mathematicadip/

- At the end of Output: Contents of dipole are shown

```
****************************************************
Number of dipoles
[Dipole1] : 2
 B1 : 2
{Splitting (1):(i,j)=(f,g)}:  2 (0)
               [1.(ij,k)=(fg,k): Dij,k]  2 (0)
               [2.(ij,a)=(fg,a): Dij^a]  0 (0)
{Splitting (2):(i,j)=(g,g)}:  0 (0)
               [3.(ij,k)=(gg,k): Dij,k]  0 (0)
               [4.(ij,a)=(gg,a): Dij^a]  0 (0)
{Splitting (3):(a,i)=(f,g)}:  0 (0)
               [5.(ai,k)=(fg,k): D^ai,k]  0 (0)
               [6.(ai,b)=(fg,b): D^ai,b]  0 (0)
{Splitting (4):(a,i)=(g,g)}:  0 (0)
               [7.(ai,k)=(gg,k): D^ai,k]  0 (0)
               [8.(ai,b)=(gg,b): D^ai,b]  0 (0)
-------------------
-------------------
[Total] : 2
 (Massive dipoles : 0)
-------------------
END
```

Type of dipole

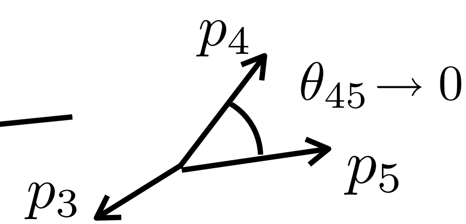
$$\mathrm{D}_{ij,k} = \mathrm{D}_{\text{quark gluon, something}}$$

in final state

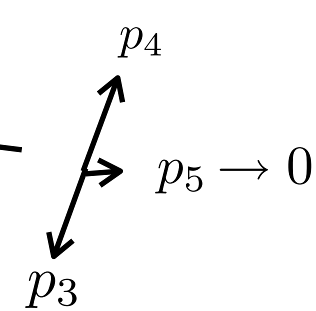
$$\mathrm{D}_{ug,\bar{u}} \quad \mathrm{D}_{\bar{u}g,u}$$

- At the end of Output: All soft/collinear limits and the corresponding dipoles are also shown

```
****************************************************************************

The collinear and soft limits and the corresponding dipoles

NLO: {{e, p[1]}, {ebar, p[2]}} --> {{u, p[3]}, {ubar, p[4]}, {g, p[5]}}

--------------------

Collinear pairs        Corresponding dipoles

1. {3, 5}               {1}

2. {4, 5}               {2}

--------------------

Soft gluon   Collinear assemble    Corresponding dipoles

1. {5}        {1, 2}               {1, 2}

--------------------

END
```



: Colllinear limit of Final-Final spliting



: Soft limit

2. Run of MadGraph with interface

We are in processes/

```
cd ../processes/
```

```
./createdir.csh
```

- Directory : Proc_e-e+_uuxg is produced

This includes the closed Fortran routines to calculate $|\mathrm{M}|^2_{\mathrm{real}} - \sum_i \mathrm{D}_i$

3. Checkings

- Go to directory: Proc_e-e+_uuxg

```
cd Proc_e-e+_uuxg
```

- Check the values of the sum of all dipolles on the 10 phase space points

```
make
```

```
./check
```

	$\vert\mathrm{M}\vert^2_{\mathrm{real}}$	$\sum \mathrm{D}_i$	$\sum \mathrm{D}_i / \vert\mathrm{M}\vert^2_{\mathrm{real}}$	$\left(\vert\mathrm{M}\vert^2_{\mathrm{real}} - \sum \mathrm{D}_i\right) / \vert\mathrm{M}\vert^2_{\mathrm{real}}$
	|M|^2	SumDipole	Ratio	Accuracy
1	0.300494427478150E-05	0.306483766095212E-05	0.101993161293315E+01	-0.199316129331459E-01
2	0.824571736656559E-05	0.831794218053564E-05	0.100875906980063E+01	-0.875906980063354E-02
3	0.108020837472608E-05	0.113477335925199E-05	0.105051338778941E+01	-0.505133877894137E-01
4	0.398997680814314E-06	0.456062859950099E-06	0.114302133039801E+01	-0.143021330398015E+00
5	0.781169201607950E-06	0.814699986512918E-06	0.104292384394565E+01	-0.429238439456497E-01
6	0.348167959120634E-04	0.342689909364597E-04	0.984266071553875E+00	0.157339284461249E-01
7	0.148082090268567E-05	0.156101669648346E-05	0.105415630860717E+01	-0.541563086071683E-01
8	0.459595997449995E-06	0.531616229332302E-06	0.115670334877131E+01	-0.156703348771315E+00
9	0.722990538823266E-06	0.804546570364715E-06	0.111280373277663E+01	-0.112803732776628E+00
10	0.326611878319585E-04	0.329691186754487E-04	0.100942803565732E+01	-0.942803565732195E-02

- Check all soft/collinear limits

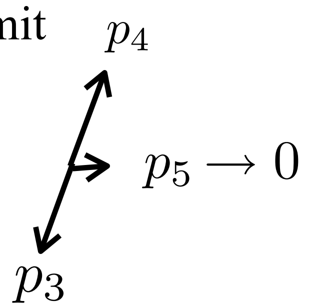
We are in processes/Proc_e-e+_uuxg

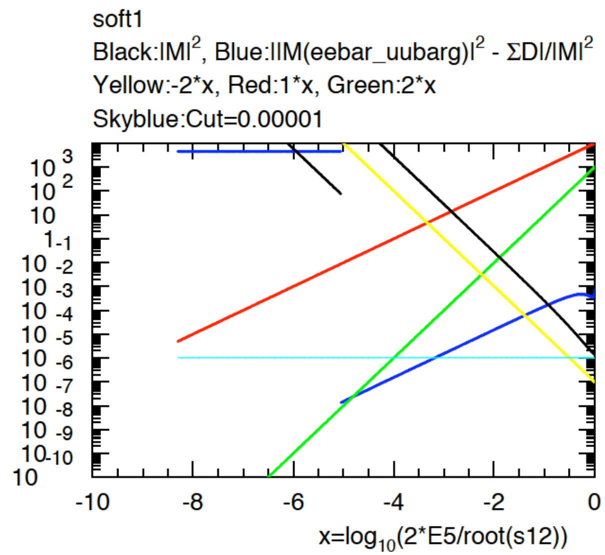
make checkIR

./checkIR

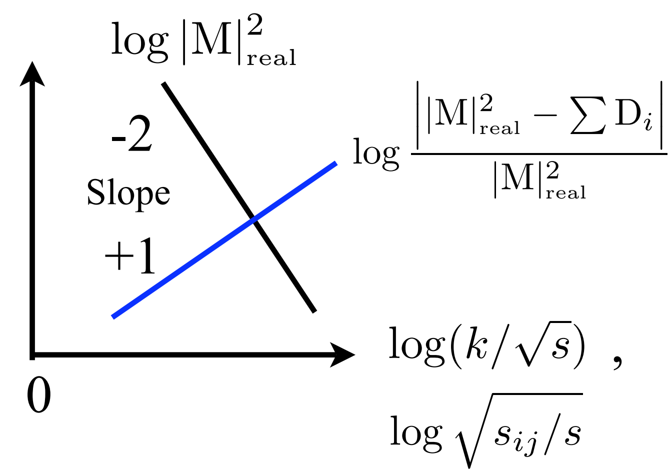
./plotall

- Soft limit

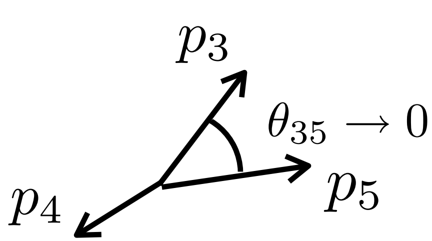


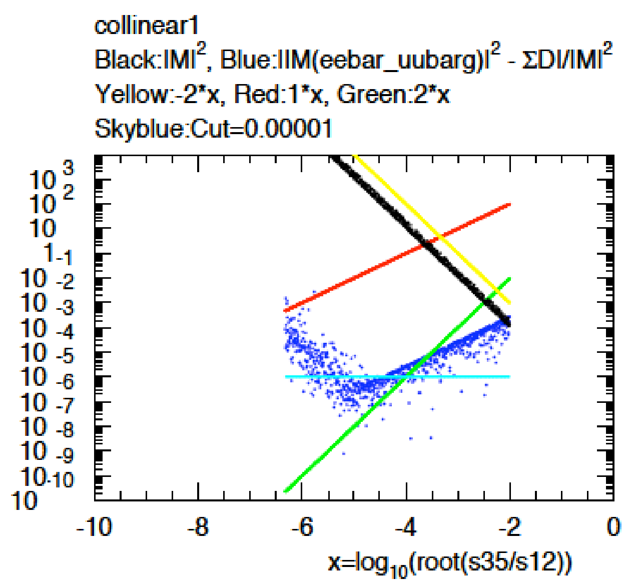


Prediction of the slope



- Collinear limit





$\theta_{45} \to 0$ limit is also similar to this case

We can easily confirm the soft/collinear safeties by seeing these plots, especially the slope

4. Outlook

■ Summary

- Dipole subtraction : General and practical procedure in NLO QCD

- AutoDipole (Version 1.0beta) : Automated dipole subtraction
  - Publicly available
  - Mathematica code and an interface with MadGraph

- Application to some QCD backgrounds in LHC

  Complete agreement at the processes
  - $gg \to t\bar{t}gg \quad u\bar{u} \to t\bar{t}gg \quad \cdots$
  - $\bar{u}d \to W^+W^-gg$

- Use with example $e^-e^+ \to u\bar{u}g$

■ Plan

- The complete packege is publicly available soon

- Compute new NLO QCD predictions for important background at LHC

- Automate the creation of the integrated dipole

Extra Slide

Exercise 2: $gg \rightarrow t\bar{t}g$

We are in mathematicadip/

- 0. Set up

  - Same with Exercise 1   skipdipole={2u,2t}; $\longrightarrow$ t-tbar splitting is skipped

- 1. Input to Mathematica code and run

  Realprocess[{g, g}, {t, tbar, g}]   - Input real emission process

```
In[3]:= Exit

In[1]:= << driver.m

In[2]:= Realprocess[{g, g}, {t, tbar, g}]

I am Dipole

NLO: {{g, pa}, {g, pb}} --> {{t, p[1]}, {tbar, p[2]}, {g, p[3]}}

Masses: {0,0} --> {mt, mt, 0}

----------------------------------------------------------------------

Dipole 1

M0=B1:            {g, g} --> {t, tbar}

Reduced momenta: {ptil[1], ptil[2]} --> {ptil[3], ptil[4]}

{Splitting (1):(i,j)=(f,g)}

[1.(ij,k)=(fg,k): Dij,k]


--Dip(1)--
```

■ 1. Input to Mathematica code and run - continued

We are in mathematicadip/

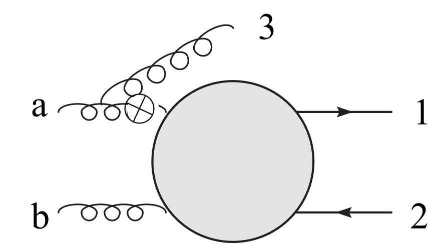
- At the end of Output: Contents of dipole are shown

```
**************************************************

Number of dipoles

[Dipole1] : 12

 B1 : 12

{Splitting (1):(i,j)=(f,g)}:  6 (6)

               [1.(ij,k)=(fg,k): Dij,k]  2 (2)

               [2.(ij,a)=(fg,a): Dij^a]  4 (4)

{Splitting (2):(i,j)=(g,g)}:  0 (0)

               [3.(ij,k)=(gg,k): Dij,k]  0 (0)

               [4.(ij,a)=(gg,a): Dij^a]  0 (0)

{Splitting (3):(a,i)=(f,g)}:  0 (0)

               [5.(ai,k)=(fg,k): D^ai,k]  0 (0)

               [6.(ai,b)=(fg,b): D^ai,b]  0 (0)

{Splitting (4):(a,i)=(g,g)}:  6 (4)

               [7.(ai,k)=(gg,k): D^ai,k]  4 (4)

               [8.(ai,b)=(gg,b): D^ai,b]  2 (0)

-------------------

-------------------

[Total] : 12

 (Massive dipoles : 10)

-------------------

END
```

(ij, k)=(quark gluon, something in final state)

(ij, a)=(quark gluon, something in initial state)

- Gluon radiation from the initial gluon



(ai, k)=(gluon gluon, something in final state)

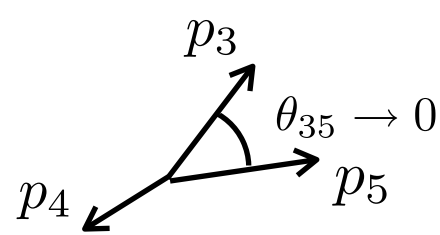
(ai, b)=(gluon gluon, something in initial state)

■ 1. Input to Mathematica code and run - continued

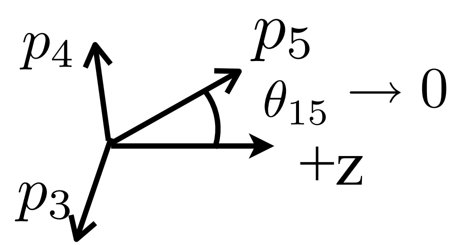
We are in mathematicadip/

-All soft/collinear limits and the corresponding dipoles are also shown

```
******************************************************************************

The collinear and soft limits and the corresponding dipoles

NLO: {{g, p[1]}, {g, p[2]}} --> {{t, p[3]}, {tbar, p[4]}, {g, p[5]}}

-------------------

Collinear pairs          Corresponding dipoles

1. {3, 5}                {1, 3, 4}

2. {4, 5}                {2, 5, 6}

3. {1, 5}                {7, 8, 11}

4. {2, 5}                {9, 10, 12}

Collinear pairs

1.

2.

include a massive quark

-------------------

Soft gluon   Collinear assemble   Corresponding dipoles

1. {5}       {1, 2, 3, 4}         {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}

-------------------

END
```

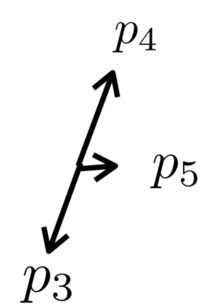


: Colllinear limit of Final-Final spliting



: Colllinear limit of Initial-Final spliting

Collinear limits 1 and 2 do not include the collinear divegences



$p_5 \to 0$: Soft limit

We are in processes/

2. Run of MadGraph with interface

```
cd ../processes/
./createdir.csh
```

- Directory : Proc_gg_ttxg is produced

3. Checkings

```
cd Proc_gg_ttxg
```

- Check the values of the sum of all dipolles on the 10 phase space points

```
make
./check
```

		$\sum \mathrm{D}_i$	$\sum \mathrm{D}_i / \lvert\mathrm{M}\rvert^2_{\mathrm{real}}$	$\left(\lvert\mathrm{M}\rvert^2_{\mathrm{real}} - \sum \mathrm{D}_i\right) / \lvert\mathrm{M}\rvert^2_{\mathrm{real}}$
	IMI^2	SumDipole	Ratio	Accuracy
1	0.400893569363976E-03	0.405385268735139E-03	0.101120421906066E+01	-0.112042190606586E-01
2	0.554612468603335E-03	0.687236683409599E-03	0.123912952252993E+01	-0.239129522529935E+00
3	0.231759860037041E-03	0.308531272164432E-03	0.133125413570374E+01	-0.331254135703743E+00
4	0.262017095449925E-03	0.457611638297119E-03	0.174649534798990E+01	-0.746495347989895E+00
5	0.117434085443178E-03	0.161142425242084E-03	0.137219466251181E+01	-0.372194662511809E+00
6	0.267551495703035E-02	0.267541549586891E-02	0.999962825413785E+00	0.371745862150187E-04
7	0.927338018137340E-03	0.113228428952350E-02	0.122100492741344E+01	-0.221004927413437E+00
8	0.277838316144724E-03	0.509438726318734E-03	0.183357980780941E+01	-0.833579807809412E+00
9	0.353722424746050E-03	0.706243188852582E-03	0.199660281464943E+01	-0.996602814649425E+00
10	0.875738991423606E-03	0.882235650188843E-03	0.100741848750468E+01	-0.741848750467973E-02

3. Checkings - continued

We are in processes/Proc_gg_ttxg

- Check all soft/collinear limits

make checkIR

./checkIR

⟶ Output: resIRcheck   more resIRcheck

```
Cut condition: (IMI^2-SumD)/IMI^2 <  0.100000000000000E-04

 ----------Collinear limits----------
Log10(Root(S_ij/S)) < Cut(i) (i=1,4)
    1   -0.100000000000000E+03
    2   -0.100000000000000E+03
    3   -0.195941167550387E+01
    4   -0.261212258826533E+01
 Maximum value of Cut(i)
    -0.195941167550387E+01
 Corresponding S_ij/S
     0.120552618763782E-03
 ------------------------------------

 -----------Soft limits--------------
Log10(2*E_soft/root(s)) < Softcut(i) (i=1,1)
    1   -0.130917931411337E+01
 Maximum value of Softcut(i)
    -0.130917931411337E+01
 Corresponding (2*E_soft/root(s))^2
     0.240791621767016E-02
 ------------------------------------

 Infrared safeties of all collinear and soft limits are not confirmed
```

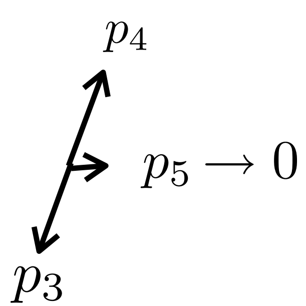
← Confirmation of all soft/collinear safeties

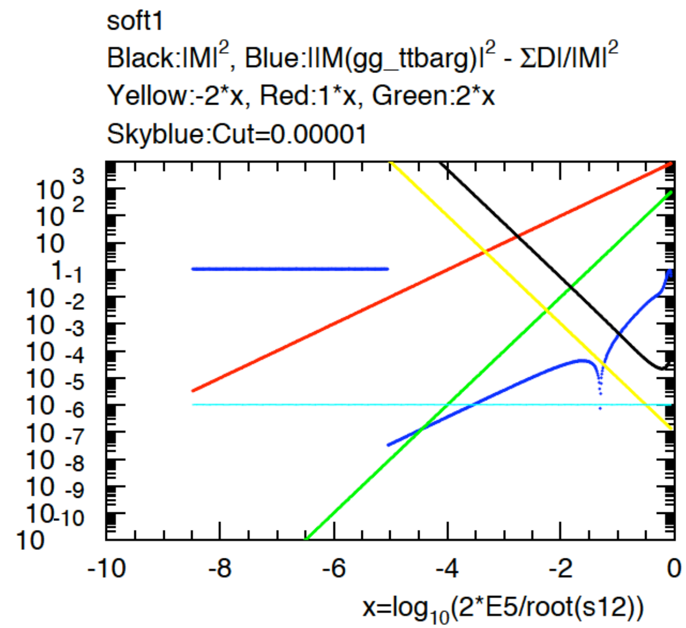
We are in processes/Proc_gg_ttxg

- Plots on all soft/collinear limits

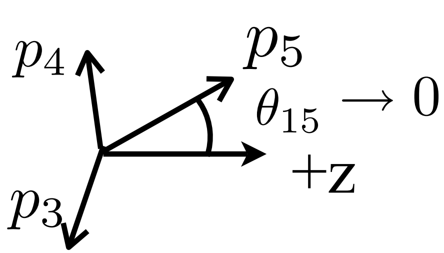
./plotall

- Soft limit

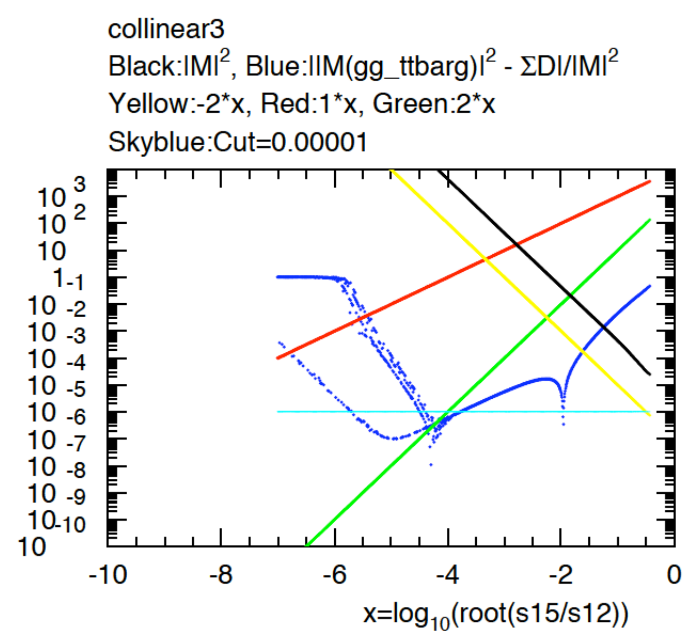




- Collinear limit



-Initial-Final spliting



We can confirm the soft/collinear safeties