Threshold corrections to the MSSM finite-temperature Higgs potential

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Online

- **Introduction**
- **Finite temperature corrections of squarks**
- **Thermal evolution and the critical temperature**
- **Summary**

CALC'2009
Dubna, Russia, July 10-20, 2009

Some brief *T*=0 history from CALC'2003

THDM: Fields

$$\Phi_1 = \begin{pmatrix} \phi_1^+(x) \\ \phi_1^0(x) \end{pmatrix} = \begin{pmatrix} -i\omega_1^+ \\ \frac{1}{\sqrt{2}}(v_1 + \eta_1 + i\chi_1) \end{pmatrix},$$

$$\Phi_2 = e^{i\xi} \begin{pmatrix} \phi_2^+(x) \\ \phi_2^0(x) \end{pmatrix} = e^{i\xi} \begin{pmatrix} -i\omega_2^+ \\ \frac{1}{\sqrt{2}}(v_2 e^{i\zeta} + \eta_2 + i\chi_2) \end{pmatrix}$$

$$\langle\Phi_1\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \qquad \langle\Phi_2\rangle = \frac{e^{i\xi}}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 e^{i\zeta} \end{pmatrix} \equiv \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 e^{i\theta} \end{pmatrix}.$$

$$\mathrm{tg}\,\beta = \frac{v_2}{v_1}, \qquad v^2 \equiv v_1^2 + v_2^2 = (246\ \mathrm{GeV})^2.$$

[Akhmetzyanova E.N., *D M.V.*, Dubinin M.N. Higgs Bosons in the Two-Doublet Model with CP Violation Phys.Rev.D. V.71. N7. 2005. P.075008
Violation of CP invariance in the two-doublet Higgs sector of the MSSM.
E.N. Akhmetzyanova, *M.V. D* , M.N. Dubinin 2006. 58pp. Phys.Part.Nucl.37:677-734,2006.]

Effective THDM potential with explicit CP violation

General hermitian renormalized SU(2)xU(1) invariant potential

$$U(\Phi_1,\Phi_2) = -\mu_1^2(\Phi_1^\dagger\Phi_1) - \mu_2^2(\Phi_2^\dagger\Phi_2) - \mu_{12}^2(\Phi_1^\dagger\Phi_2) - \overset{*}{\mu}{}_{12}^2(\Phi_2^\dagger\Phi_1) +$$

$$+\frac{\lambda_1}{2}(\Phi_1^\dagger\Phi_1)^2 + \frac{\lambda_2}{2}(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) +$$

$$+\frac{\lambda_5}{2}(\Phi_1^\dagger\Phi_2)(\Phi_1^\dagger\Phi_2) + \frac{\overset{*}{\lambda}_5}{2}(\Phi_2^\dagger\Phi_1)(\Phi_2^\dagger\Phi_1) +$$

$$+\lambda_6(\Phi_1^\dagger\Phi_1)(\Phi_1^\dagger\Phi_2) + \overset{*}{\lambda}_6(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_1) +$$

$$+\lambda_7(\Phi_2^\dagger\Phi_2)(\Phi_1^\dagger\Phi_2) + \overset{*}{\lambda}_7(\Phi_2^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1)$$

$$\Phi_1^\dagger\Phi_2 \xrightarrow{CP} \Phi_2^\dagger\Phi_1$$

$$\lambda_{5,6,7} \xrightarrow{CP} \overset{*}{\lambda}_{5,6,7}$$

μ_{12}^2,

$\lambda_5, \lambda_6, \lambda_7$

complex

U ***is CP*-invariant at the M_{SUSY} scale,**

because $\lambda_{5,6,7}=0$

CP ***U_{eff}***

m_{top} M_{SUSY} *μ - mass-energy scale*

Eff. potential method or Feynman diags

$\varphi=\arg(\lambda_{6,7})=\arg(\lambda_5)/2$

Scalar sector for MSSM

The main contribution to self-couplings due to Yukawa 3rd generation couplings.

The corresponding potential with CPV sources

$$\mathcal{V}^0 = \mathcal{V}_M + \mathcal{V}_\Gamma + \mathcal{V}_\Lambda + \mathcal{V}_{\widetilde{Q}} \, ,$$

$$\mathcal{V}_M = (-1)^{i+j} m_{ij}^2 \Phi_i^\dagger \Phi_j + M_{\widetilde{Q}}^2 \left(\widetilde{Q}^\dagger \widetilde{Q}\right) + M_{\widetilde{U}}^2 \widetilde{U}^* \widetilde{U} + M_{\widetilde{D}}^2 \widetilde{D}^* \widetilde{D} \, ,$$

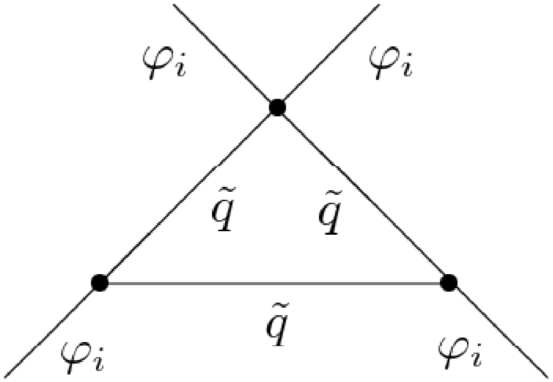
$$\mathcal{V}_\Gamma = \Gamma_i^D \left(\Phi_i^\dagger \widetilde{Q}\right) \widetilde{D} + \Gamma_i^U \left(i \Phi_i^T \sigma_2 \widetilde{Q}\right) \widetilde{U} + \Gamma_i^{*D} \left(\widetilde{Q}^\dagger \Phi_i\right) \widetilde{D}^* - \Gamma_i^{*U} \left(i \widetilde{Q}^\dagger \sigma_2 \Phi_i^*\right) \widetilde{U}^*$$

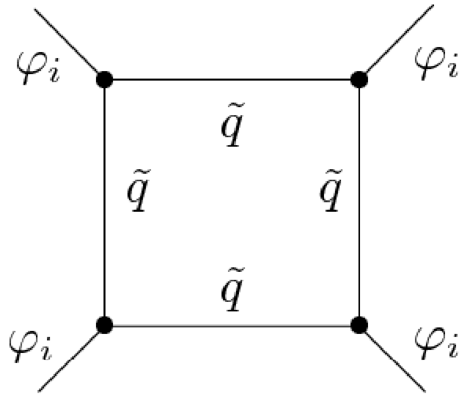
$$\mathcal{V}_\Lambda = \Lambda_{ik}^{jl} \left(\Phi_i^\dagger \Phi_j\right) \left(\Phi_k^\dagger \Phi_l\right) + \left(\Phi_i^\dagger \Phi_j\right) \left[\Lambda_{ij}^Q \left(\widetilde{Q}^\dagger \widetilde{Q}\right) + \Lambda_{ij}^U \widetilde{U}^* \widetilde{U} + \Lambda_{ij}^D \widetilde{D}^* \widetilde{D}\right] +$$

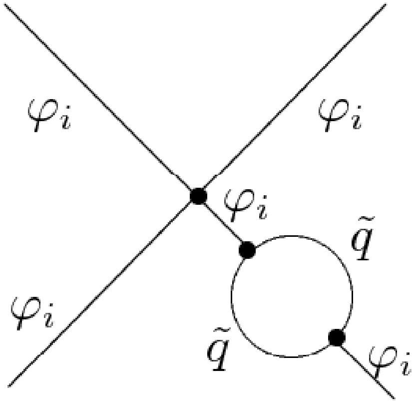
$$+ \overline{\Lambda}_{ij}^Q \left(\Phi_i^\dagger \widetilde{Q}\right) \left(\widetilde{Q}^\dagger \Phi_j\right) + \frac{1}{2} \left[\Lambda \epsilon_{ij} \left(i \Phi_i^T \sigma_2 \Phi_j\right) \widetilde{D}^* \widetilde{U} + \text{э.с.}\right] \, , \quad i, j, k, l = 1, 2$$

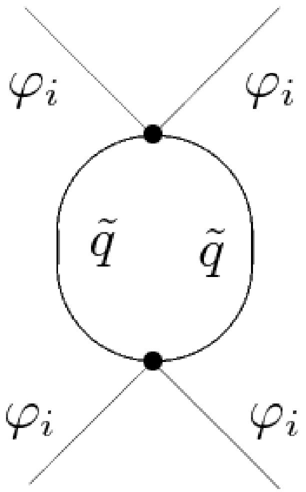
$$\Gamma_{\{1;\,2\}}^U = h_U \{-\mu^*; A_U\}, \qquad \Gamma_{\{1;\,2\}}^D = h_D \{A_D; -\mu^*\}$$

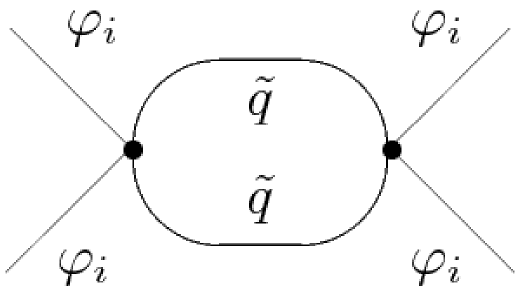
Threshold corrections (left and central diagram) and diagram contributing to the wave-function renormalization (right)

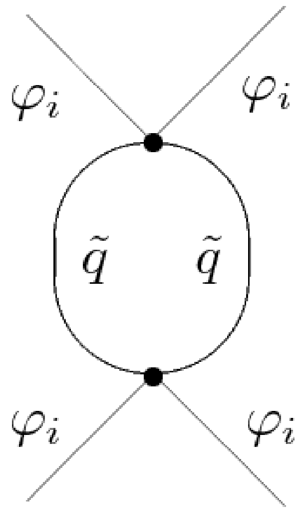








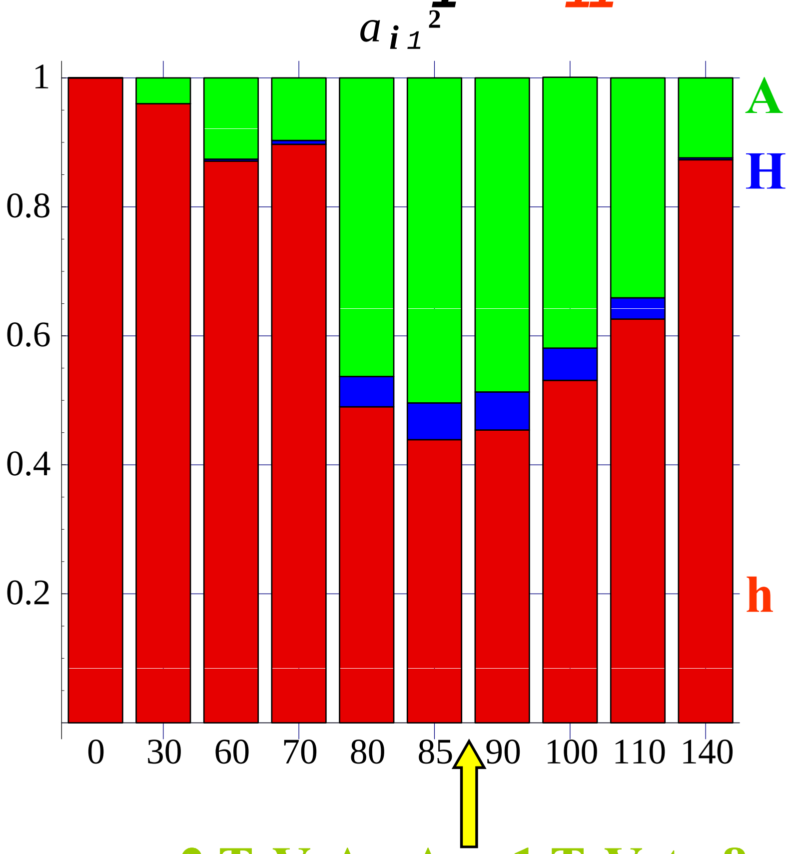


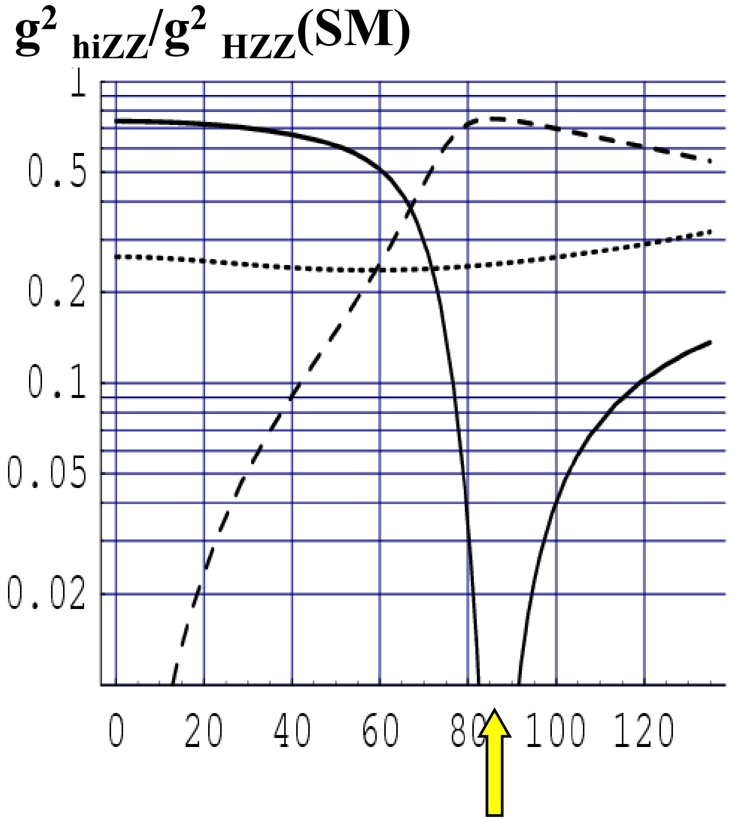


"Fish" diagrams

Matrix elements a_{i1} and coupling with Z-boson

$$h_1 = a_{11}\,h + a_{21}\,H + a_{31}\,A$$





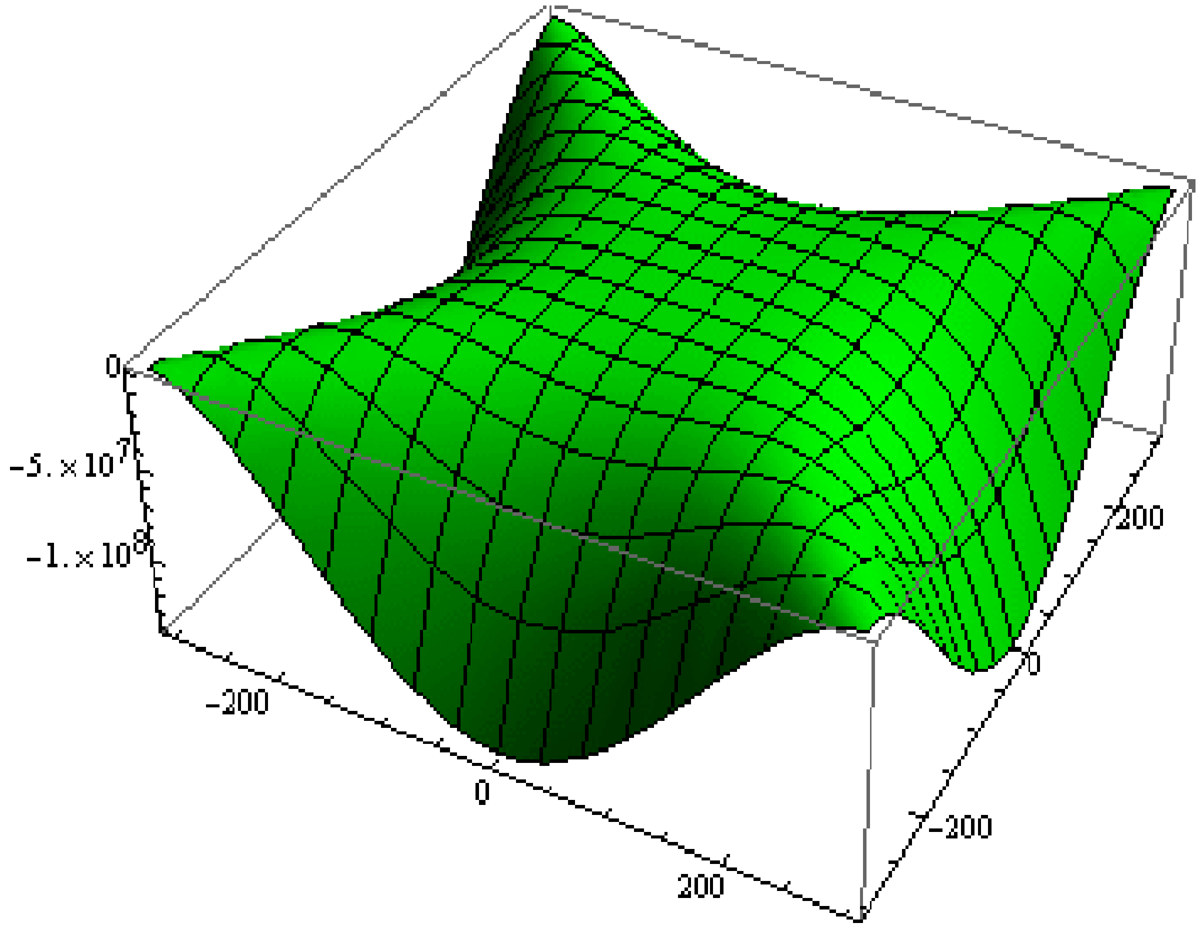
μ = 2 TeV, A_t=A_b= 1 TeV, tg β =5, M_{SUSY} =0.5 TeV, m_H±=150 GeV

Electroweak Baryogenesis

- **Two problems in the Standard Model**
  - **First order phase transition requires m_h< 50 GeV**
  - **Need new sources of CP violation**
- **Minimal Supersymmetric Standard Model**
  - **1st order phase transition is possible if** $m_{\tilde{t}_R} < 160\text{GeV}$
  - **New CP violating phases**

[M. Dolgopolov, M. Dubinin, E. Rykova Threshold corrections to the MSSM finite-temperature Higgs potential. Jan 2009. 26pp. e-Print: arXiv:0901.0524v1]

The zero-temperature two-doublet Higgs potential at the scale M_{SUSY}



Integration and summation method

In the finite temperature field theory Feynman diagrams with boson propagators, containing Matsubara frequencies, lead to structures of the form

$$I[m_1, m_2, ..., m_b] = T \sum_{n=-\infty}^{\infty} \int \frac{d\mathbf{k}}{(2\pi)^3} \prod_{j=1}^{b} \frac{(-1)^b}{(\mathbf{k}^2 + \omega_n^2 + m_j^2)},$$

$$\omega_n = 2\pi n T \ (n = 0, \pm 1, \pm 2, ...),$$

T - temperature

Integration and summation method

$$I[m_1, m_2, ..., m_b] = 2T\,(2\pi T)^{3-2b}\,\frac{(-1)^b \pi^{3/2}}{(2\pi)^3}\,\frac{\Gamma(b-3/2)}{\Gamma(b)}\,S(M, b-3/2),$$

$$S(M, b-3/2) = \int \{\mathrm{d}x\} \sum_{n=1}^{\infty} \frac{1}{(n^2+M^2)^{b-3/2}}, \qquad M^2 \equiv \left(\frac{m}{2\pi T}\right)^2.$$

Integration and summation method

A number of integrals can be easily calculated

$$J \equiv \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{1}{(\mathbf{k}^2 + a_1^2)(\mathbf{k}^2 + a_2^2)},$$

taking a residue in the spherical coordinate system:

$$J = \frac{1}{4\pi(a_1 + a_2)}$$

$a_{1;2}^2$ - the sums of squared frequency and squared mass.

Integration and summation method

Derivatives of first integral with respect to a_1 and a_2 can be used for calculation of integrals

$$J_1[a_1, a_2] \equiv \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{1}{(\mathbf{k}^2 + a_1^2)^2 (\mathbf{k}^2 + a_2^2)} =$$

$$= -\frac{1}{2a_1} \frac{\partial I}{\partial a_1} = \frac{1}{8\pi a_1 (a_1 + a_2)^2},$$

$$J_2[a_1, a_2] \equiv \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{1}{(\mathbf{k}^2 + a_1^2)^2 (\mathbf{k}^2 + a_2^2)^2} =$$

$$= \frac{1}{4a_1 a_2} \frac{\partial^2 I}{\partial a_1 \partial a_2} = \frac{1}{8\pi a_1 a_2 (a_1 + a_2)^3}.$$

Integration and summation method

and

$$J_3[a_1, a_2, a_3] \equiv \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{1}{(\mathbf{k}^2 + a_1^2)(\mathbf{k}^2 + a_2^2)(\mathbf{k}^2 + a_3^2)} =$$

$$= \frac{1}{4\pi(a_1 + a_2)(a_1 + a_3)(a_2 + a_3)},$$

$$J_4[a_1, a_2, a_3] \equiv \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{1}{(\mathbf{k}^2 + a_1^2)^2(\mathbf{k}^2 + a_2^2)(\mathbf{k}^2 + a_3^2)} =$$

$$= \frac{2a_1 + a_2 + a_3}{8\pi a_1(a_1 + a_2)^2(a_1 + a_3)^2(a_2 + a_3)}.$$

Integration and summation method

Substituting

$$a_1 \to \sqrt{4\pi^2 n^2 T^2 + m_1^2} \text{ и } a_2 \to \sqrt{4\pi^2 n^2 T^2 + m_2^2},$$

for

$$J^n = \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{1}{(\mathbf{k}^2 + \omega_n^2 + m_1^2)(\mathbf{k}^2 + \omega_n^2 + m_2^2)},$$

taking the sum over Matsubara frequencies after the integration we get:

$$\sum_{n=-\infty, n\neq 0}^{\infty} J^n = \sum_{n=-\infty, n\neq 0}^{\infty} \frac{1}{4\pi(\sqrt{4\pi^2 n^2 T^2 + m_1^2} + \sqrt{4\pi^2 n^2 T^2 + m_2^2})}.$$

Thus the temperature corrections to effective potential are expressed by summed integrals,

Integration and summation method

after redefinition of mass parameters

$$m_{1;2} \longrightarrow m'_{1;2} = 2\pi T\sqrt{M_{1;2}^2 + n^2}, \quad \text{где } M_{1;2} = \frac{m_{1;2}}{2\pi T},$$

$$I_1 = \frac{-T}{8\pi}\frac{1}{(2\pi T)^3}\sum_{n=-\infty, n\neq 0}^{\infty}\frac{1}{\sqrt{M_1^2+n^2}(\sqrt{M_1^2+n^2}+\sqrt{M_2^2+n^2})^2},$$

$$I_2 = \frac{T}{8\pi}\frac{1}{(2\pi T)^5}\sum_{n=-\infty, n\neq 0}^{\infty}\frac{1}{\sqrt{M_1^2+n^2}\sqrt{M_2^2+n^2}(\sqrt{M_1^2+n^2}+\sqrt{M_2^2+n^2})^3}.$$

The sum of integrals can be expressed by means of the generalized zeta-function

Such forms can be derived if weintroduce Feynman parameters in the integrand of

$$\frac{1}{[\mathbf{k}^2+m_a^2][\mathbf{k}^2+m_b^2]} = \int_0^1 \frac{dx}{([\mathbf{k}^2+m_a^2]x+[\mathbf{k}^2+m_b^2](1-x))^2},$$

and redefine

$$\mathbf{k} \longrightarrow \mathbf{p} = \frac{\mathbf{k}}{2\pi T} \quad \text{и} \quad M^2 = (M_a^2 - M_b^2)x + M_b^2,$$

then we get

$$\frac{1}{[\mathbf{k}^2+m_a^2][\mathbf{k}^2+m_b^2]} = \frac{1}{(2\pi T)^4}\int_0^1 \frac{dx}{[\mathbf{p}^2+n^2+M^2]^2}.$$

The sum of integrals can be expressed by means of the generalized Hurwitz zeta-function

$$d\mathbf{k} = (2\pi T)^3 d\mathbf{p},$$

$$I' = \sum_{n=-\infty, n\neq 0}^{\infty} J = \frac{1}{2\pi T}\int_0^1 dx \sum_{n=-\infty, n\neq 0}^{\infty} \int \frac{d\mathbf{p}}{(2\pi)^3} \frac{1}{[\mathbf{p}^2 + n^2 + M^2]^2},$$

With the help of dimensional regularization or differentiating the integral

$$I' = \frac{1}{16\pi^2 T}\int_0^1 dx\, \zeta(2, \frac{1}{2}, M^2),$$

Where the generalized Hurwitz zeta-function

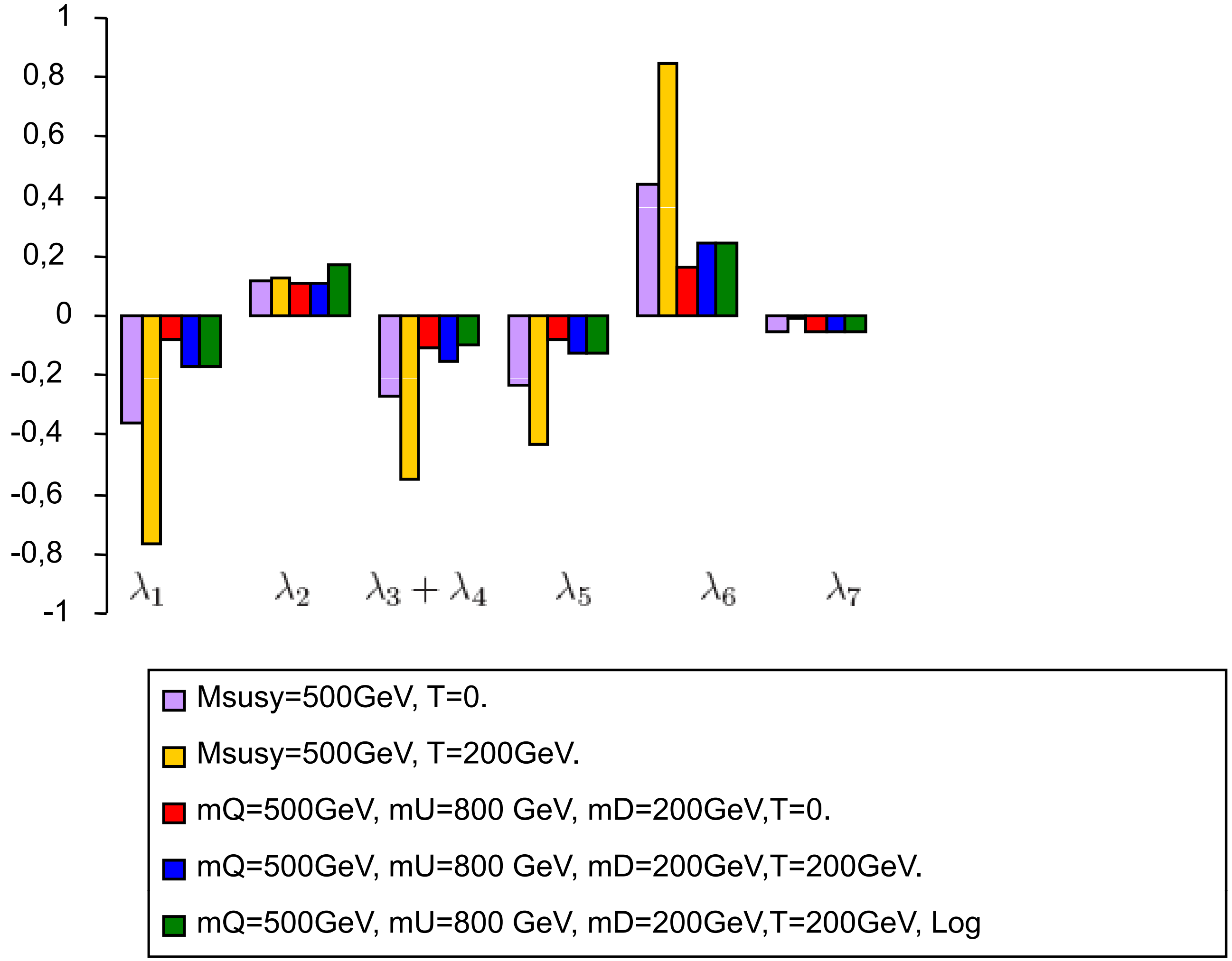
$$\zeta(u, s, t) = \sum_{n=1}^{\infty} \frac{1}{[n^u + t]^s}.$$

The sum of integrals can be expressed by means of the generalized Hurwitz zeta-function

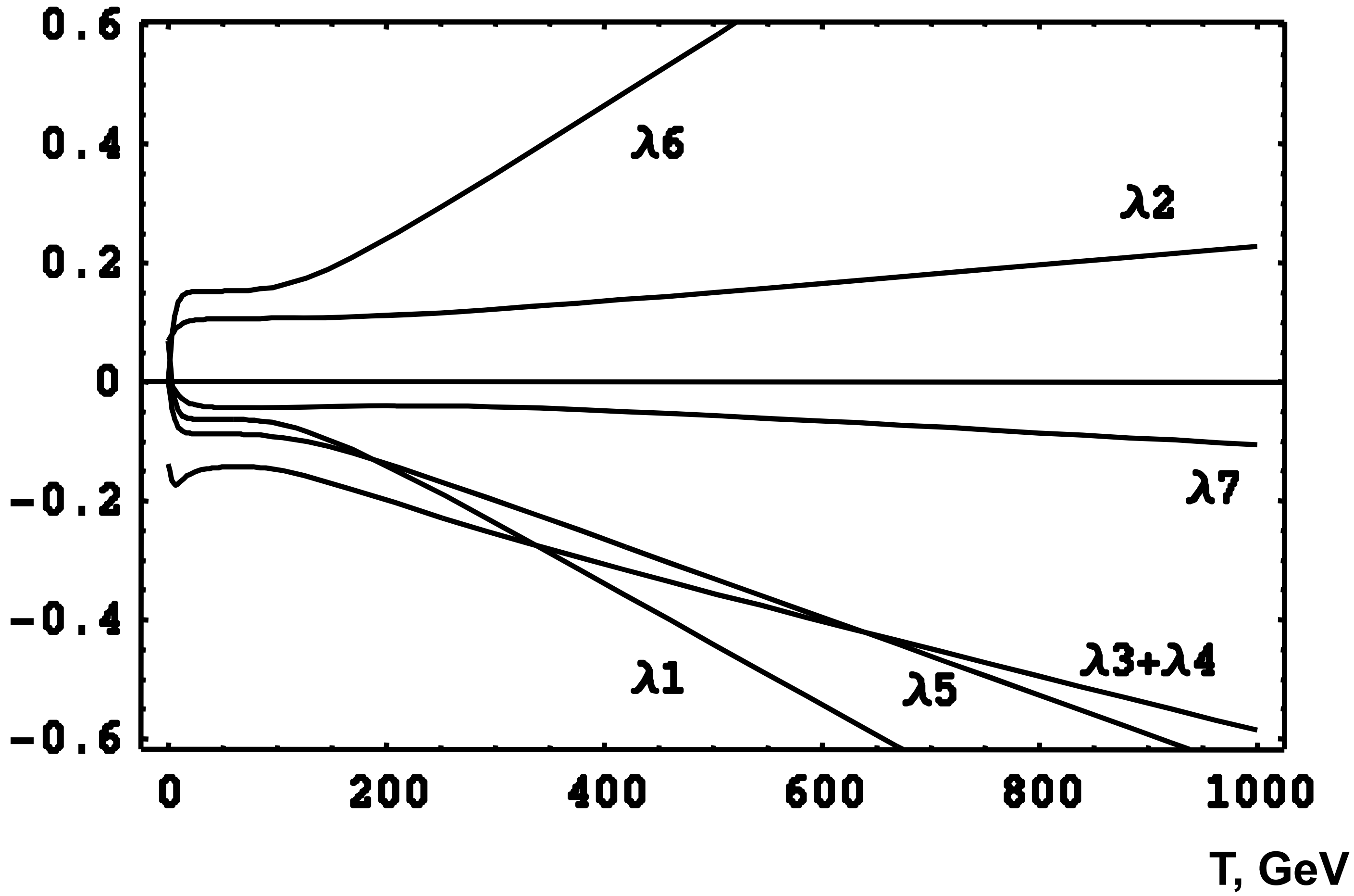
$$I_1 = \frac{T}{2m_a}\frac{\partial}{\partial m_a}I' = -\frac{T}{8\pi}\frac{1}{(2\pi T)^3}\int_0^1 dxx\zeta(2,\frac{3}{2},M^2),$$

$$I_2 = \frac{1}{2m_b}\frac{\partial}{\partial m_b}(I_1) = \frac{3T}{8\pi}\frac{1}{(2\pi T)^5}\int_0^1 dxx(1-x)\zeta(2,\frac{5}{2},M^2).$$

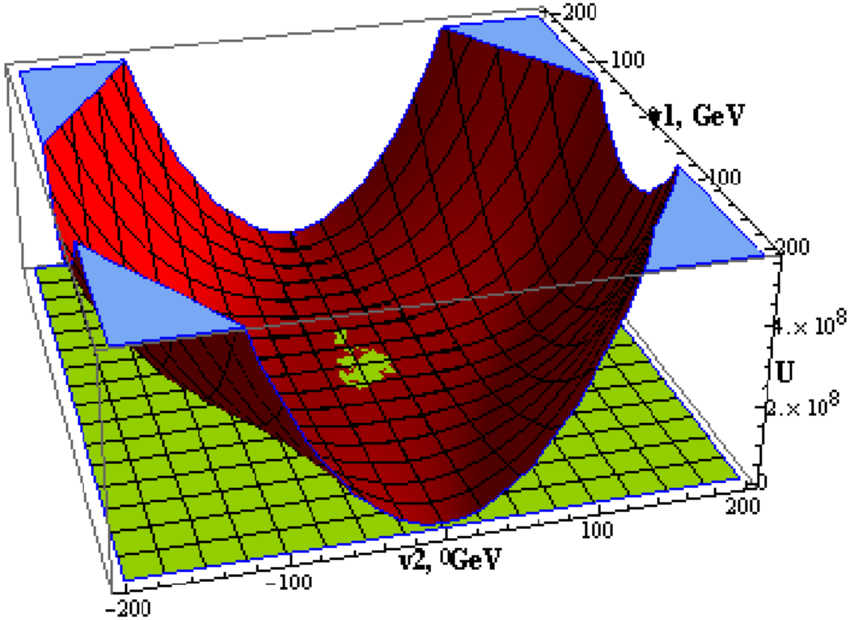
Parameters of the finite temperature effective potential

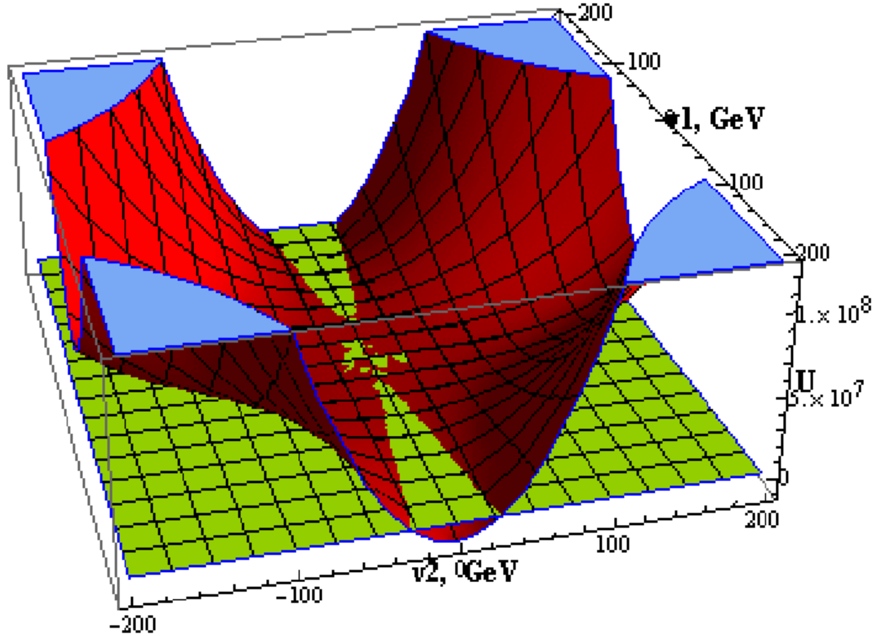


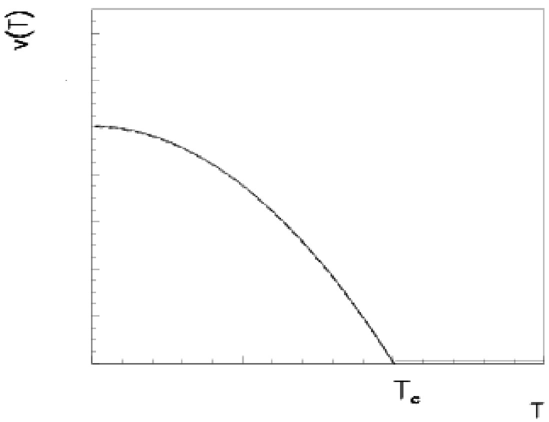
Parameters of the effective potential

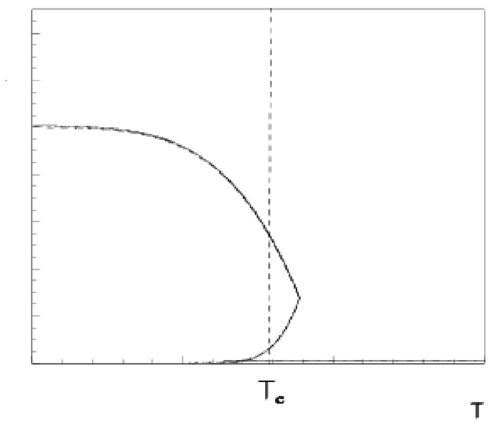


Effective potential U(v1,v2) at the critical temperature T=120 GeV, λ_6 λ_7 =0

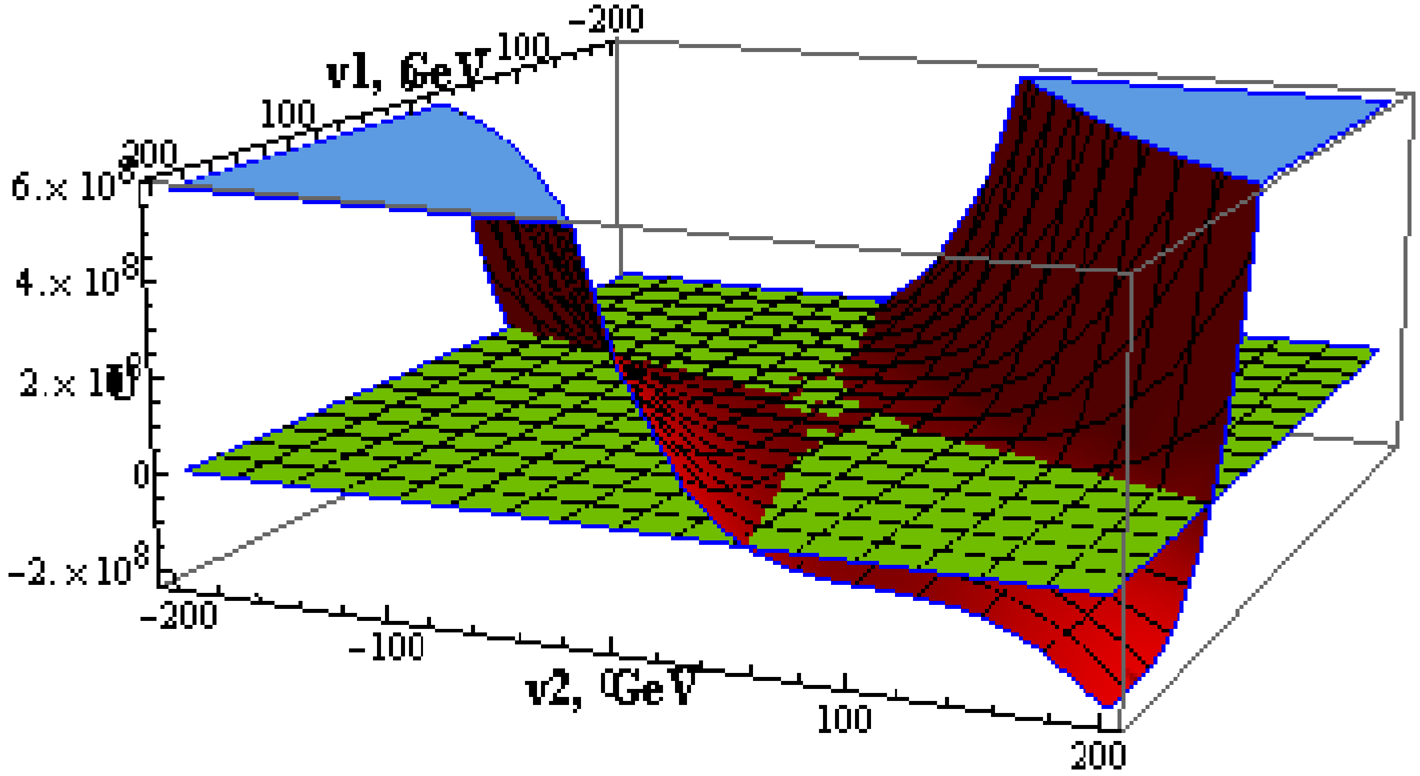








Effective potential U(v1,v2) at the critical temperature T=120 GeV and nonzero λ_6, λ_7



Effective potential at finite temperature

$$v_1(T) = v(T)\cos\bar{\beta}(T), \quad v_2(T) = v(T)\sin\bar{\beta}(T)$$

Mass term

$$U_{mass}(v,\bar{\beta}) = -\frac{v^2}{2}(\mu_1^2\cos^2\bar{\beta} + \mu_2^2\sin^2\bar{\beta}) - \frac{v^2}{2}\mu_{12}^2\sin 2\bar{\beta}$$

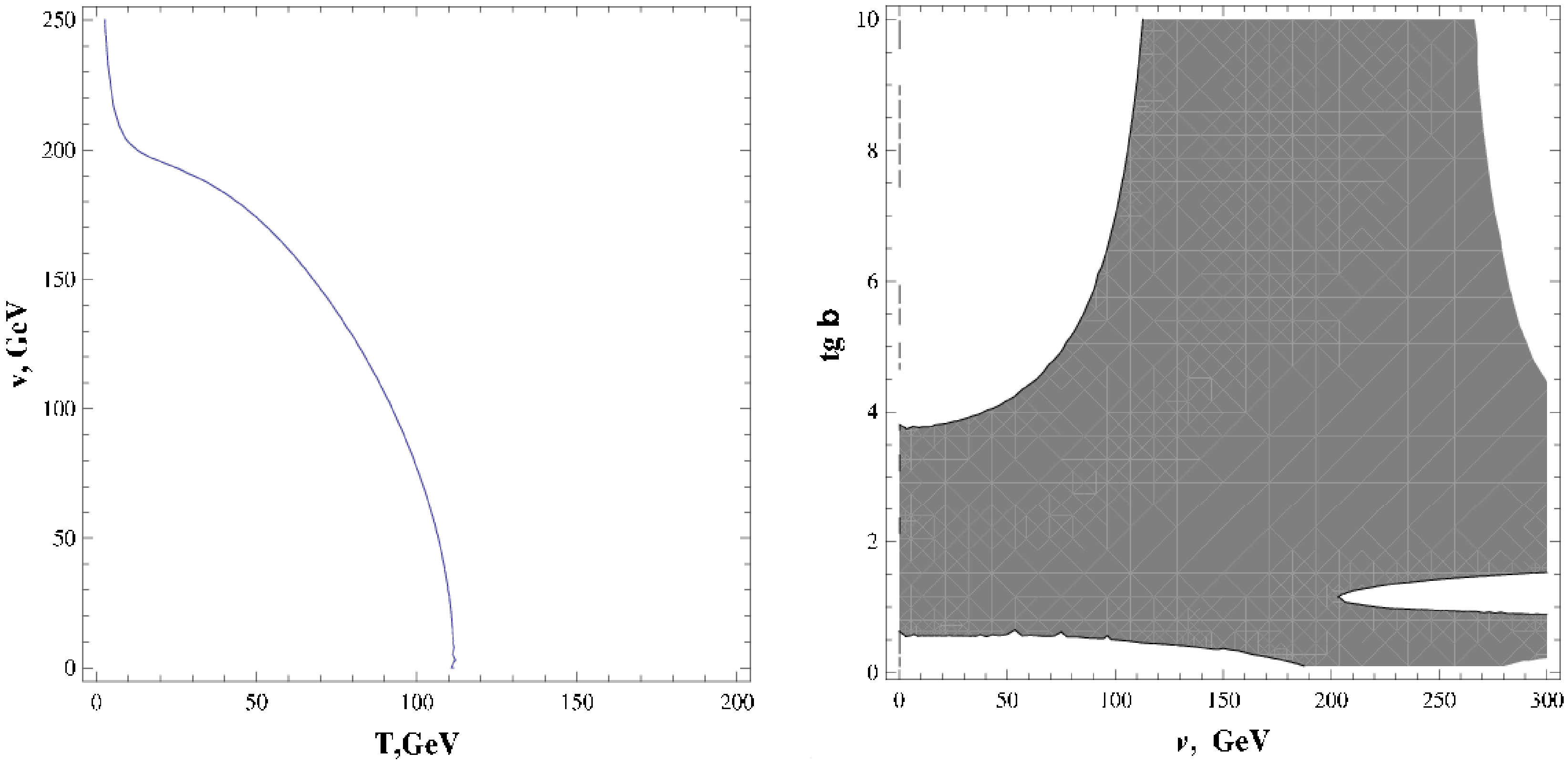
Critical temperature determination

$$\partial U_{mass}/\partial v = 0 \qquad 1/v\ \partial U_{mass}/\partial\bar{\beta} = 0$$

$$\mathtt{tg}2\bar{\beta} = \frac{2\mu_{12}^2}{\mu_1^2 - \mu_2^2}, \qquad (\mu_1^2\mu_2^2 - \mu_{12}^4)[(\mu_1^2 - \mu_2^2)^2 + 4\mu_{12}^4] = 0$$

$$\mu_1^2\mu_2^2 = \mu_{12}^4$$

Evolution of the critical parameters



Effective potential U(v1,v2) at the critical temperature and nonzero λ_6 λ_7

$$\mathrm{tg}2\bar{\beta} = \mathrm{tg}2\beta \frac{1}{\left(\frac{v^2}{2m_A^2} - \alpha_1\right)} \frac{1}{\frac{2\lambda_1 \cos^2\beta - 2\lambda_2 \sin^2\beta}{\cos 2\beta} - \lambda_{345} + \frac{2m_A^2}{v^2} + \alpha_2}$$

$$\alpha_1 = \frac{\lambda_5}{2} + \frac{1}{4}(\lambda_6 \mathrm{ctg}\beta + \lambda_7 \mathrm{tg}\beta),$$

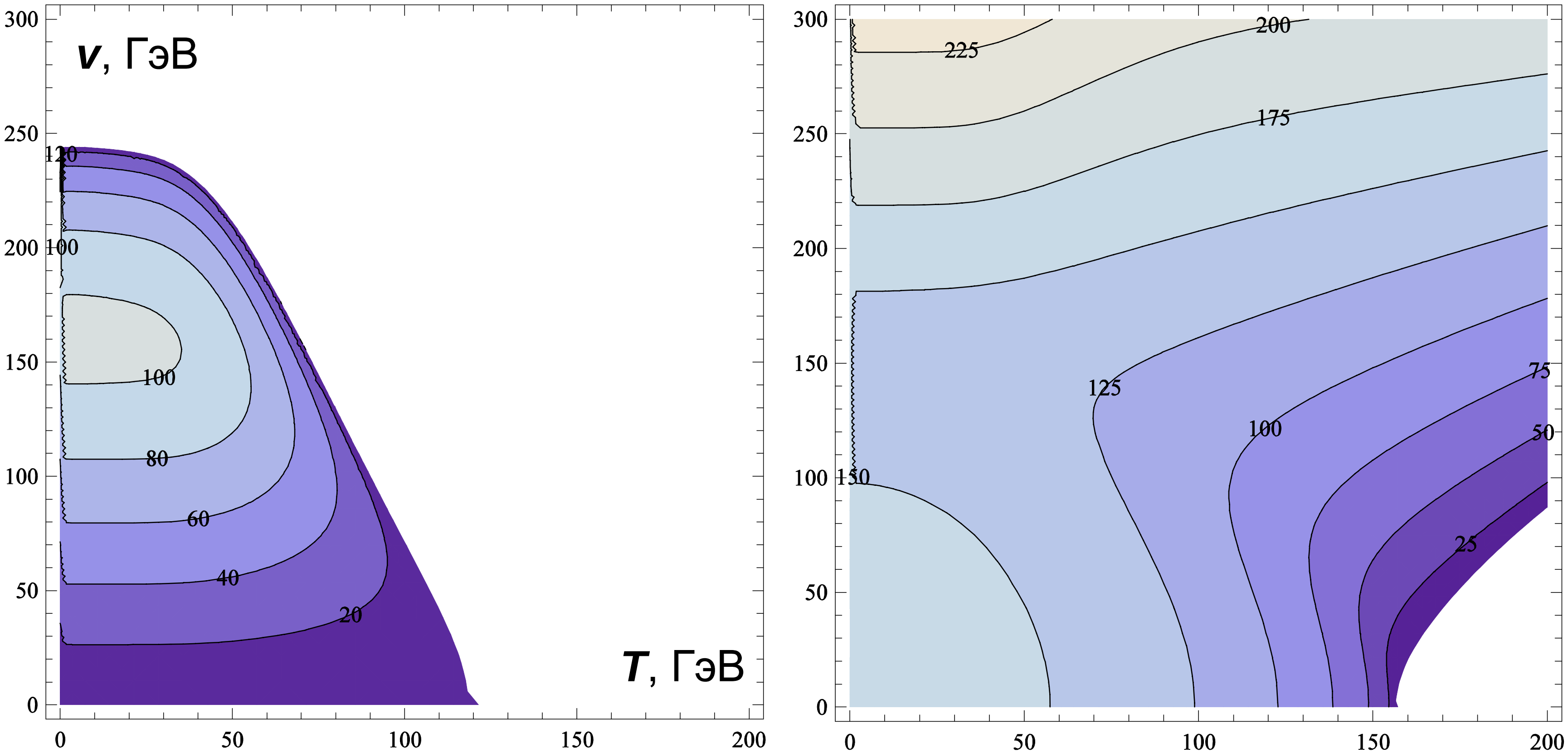
$$\alpha_2 = \lambda_6(\mathrm{tg}2\beta - \mathrm{ctg}\beta) - \lambda_7(\mathrm{tg}\beta + \mathrm{tg}2\beta).$$

$$-\frac{m_A^2}{v^2}(2\lambda_5 + \lambda_6\, \mathrm{ctg}\beta + \lambda_7\, \mathrm{tg}\beta) +$$

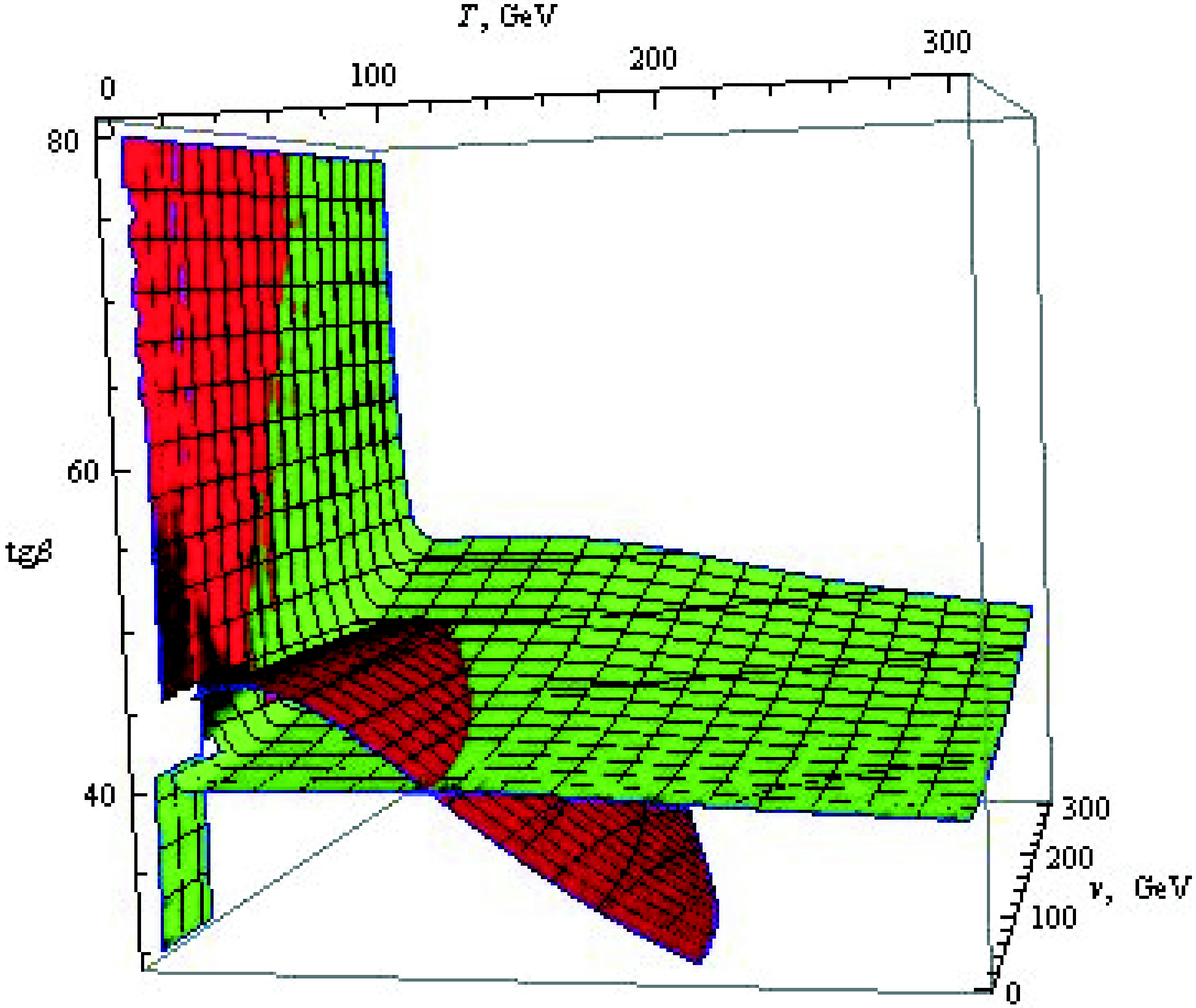
$$+ \frac{v^2}{m_A^2} \left[\frac{2\lambda_1 - 2\lambda_2 \mathrm{tg}^2\beta + \lambda_6(3\mathrm{tg}\beta - \mathrm{ctg}\beta) + \lambda_7(\mathrm{tg}^3\beta - 3\mathrm{tg}\beta)}{1 - \mathrm{tg}^2\beta} - \lambda_{345}\right] = 0.$$

$$\lambda_1 (2\lambda_2 - \lambda_{345})^2 + \lambda_2 (2\lambda_1 - \lambda_{345})^2 + \lambda_{345}(2\lambda_1 - \lambda_{345})(2\lambda_2 - \lambda_{345}) = 0$$

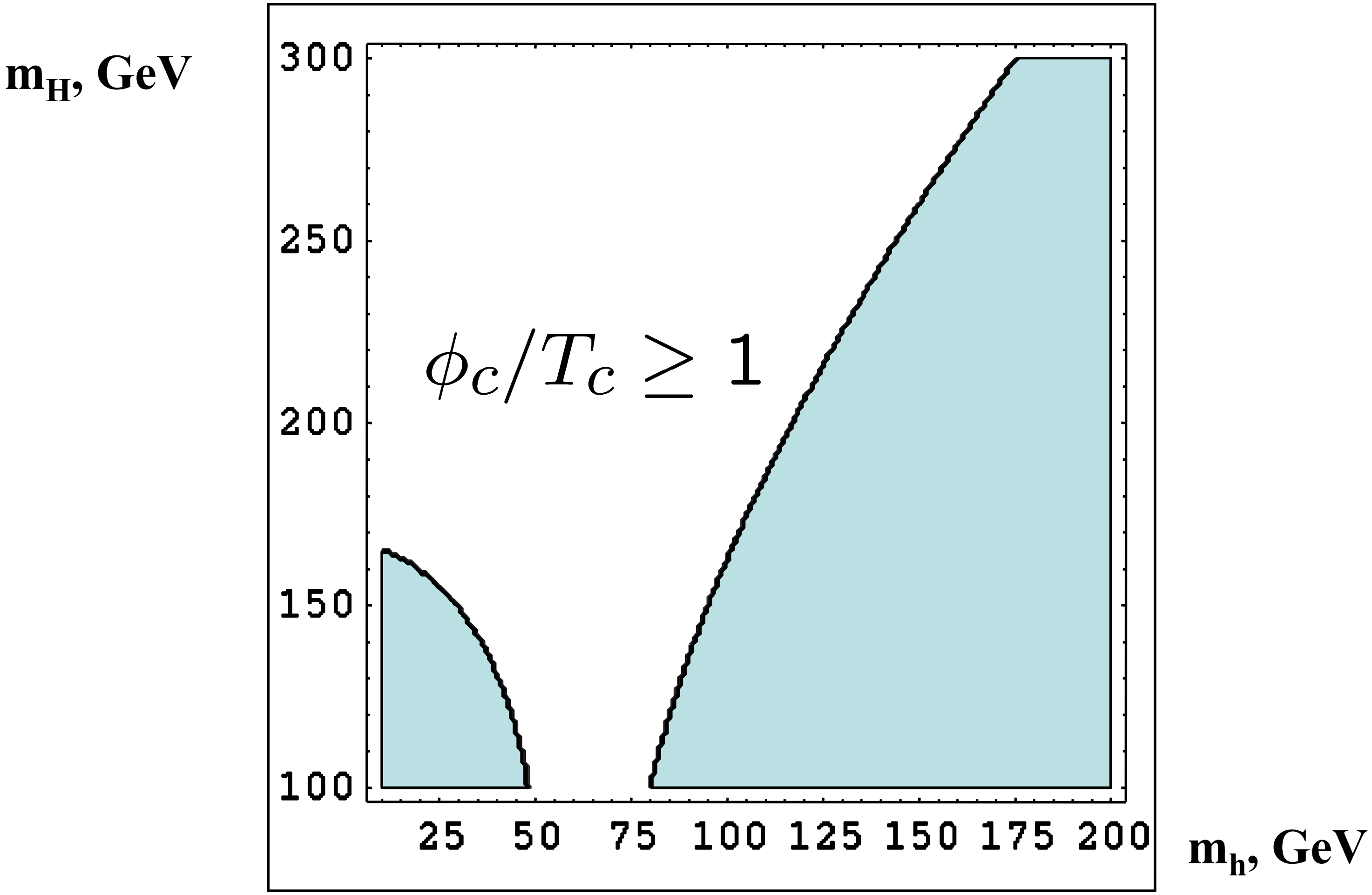
Higgs bosons masses



$\text{tg}\beta = 5,\ m_{H^{\pm}}$ 180 GeV, $A_{t,b} = 1200$ GeV, $\mu = 500$ GeV.


T, GeV
0
100
200
300
80
60
40
tgβ
300
200
100
0
v, GeV

m_h and m_H in the THDM



Conclusions

1. In the MSSM we calculate the 1-loop finite-temperature corrections from the squarks-Higgs bosons sector, reconstruct the effective two-Higgs-doublet potential and study possibilities of the electroweak phase transition in the full MSSM.

2. At large values of A and μ of around 1 TeV, favored indirectly by LEP2 and Tevatron data, the threshold finite-temperature corrections from triangle and box diagrams with intermediate third generation squarks are very substantial.

3. High sensitivity of the low-temperature evolution to the effective two-doublet and the MSSM squark sector parameters is observed, but rather extensive regions of the full MSSM parameter space allow the first-order electroweak phase transition respecting the phenomenological constraints at zero temperature.