New Applications of Resummation in Quantum Field Theory

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Outline:

- Introduction
- Review of YFS Theory and Its Extension to QCD
- Extension to QED⊗QCD and Quantum Gravity
- QED⊗QCD Threshold Corrections, Shower/ME Matching and IR-Improved DGLAP Theory at the LHC
- Final State of Hawking Radiation
- Conclusions

Papers byB.F.L.W., S. Jadach and B.F.L. Ward, S. Jadach, *et al.*,B.F.L.W. and S. Yost, M. Phys. Lett. A **14** (1999) 491, hep-ph/0205062; *ibid.* **12** (1997) 2425; *ibid.***19** (2004) 2113; hep-ph/0503189,0508140,0509003,0605054

Motivation

- FNAL/RHIC $t\bar{t}$ PRODUCTION; POLARIZED pp PROCESSES; $b\bar{b}$ PRODUCTION; J/Ψ PRODUCTION: SOFT n(G) EFFECTS ALREADY NEEDED
  $\Delta m_t = 5.1$ GeV with SOFT n(G) UNCERTAINTY $\sim$ 2-3 GeV, ..., ETC.

- FOR THE LHC/TESLA/LC, THE REQUIREMENTS WILL BE EVEN MORE DEMANDING AND OUR QCD SOFT n(G) MC EXPONENTIATION RESULTS WILL BE AN IMPORTANT PART OF THE NECESSARY THEORY – YFS EXPONENTIATED $\mathcal{O}(\alpha_s^2)L$, IN THE PRESENCE OF SHOWERS, ON AN EVENT-BY-EVENT BASIS, WITHOUT DOUBLE COUNTING AND WITH EXACT PHASE SPACE.

- HOW RELEVANT ARE QED HIGHER ORDER CORRECTIONS WHEN QCD IS CONTROLLED AT $\sim 1\%$ PRECISION?

- CROSS CHECK OF QCD LITERATURE:
  1. PHASE SPACE – CATANI, CATANI-SEYMOUR, ALL INITIAL PARTONS MASSLESS
  2. RESUMMATION – STERMAN, CATANI ET AL., BERGER ET AL., ...
  3. NO-GO THEOREMS– TO BE ADDRESSED ELSEWHERE
  4. IR QCD EFFECTS IN DGLAP THEORY

- CROSS CHECK OF QED LITERATURE:

  1. ESTIMATES BY SPIESBERGER, STIRLING, ROTH and WEINZIERL – FEW PER MILLE EFFECTS FROM QED CORRECTIONS TO STR. FN. EVOLUTION.

  2. WELL-KNOWN POSSIBLE ENHANCEMENT OF QED CORRECTIONS AT THRESHOLD, ESPECIALLY IN RESONANCE PRODUCTION

  $\Rightarrow$ HOW BIG ARE THESE EFFECTS AT THE LHC?

- TREAT QED AND QCD SIMULTANEOUSLY IN THE YFS EXPONENTIATION TO ESTIMATE THE ROLE OF THE QED AND TO ILLUSTRATE AN APPROACH TO SHOWER/ME MATCHING.

- QUANTUM GENERAL RELATIVITY:STILL NO PHENOMENOLOGICALLY TESTED THEORY

- OUTSTANDING ISSUES: FINAL STATE OF HAWKING RADIATION, ... – FERTILE GROUND FOR RESUMMATION; SEE ALSO WORK BY REUTER ET AL., LITIM, DONOGHUE ET AL., CAVAGLIA, SOLA ET AL., ETC.

PRELIMINARIES

- WE USE THE GPS CONVENTIONS OF JWW FOR SPINORS; PHOTON-GLUON POLARIZATION VECTORS FOLLOW THEREFROM:

$$(\epsilon^{\mu}_{\sigma}(\beta))^{*} = \frac{\bar{u}_{\sigma}(k)\gamma^{\mu}u_{\sigma}(\beta)}{\sqrt{2}\ \bar{u}_{-\sigma}(k)u_{\sigma}(\beta)}, \quad (\epsilon^{\mu}_{\sigma}(\zeta))^{*} = \frac{\bar{u}_{\sigma}(k)\gamma^{\mu}\mathfrak{u}_{\sigma}(\zeta)}{\sqrt{2}\ \bar{u}_{-\sigma}(k)\mathfrak{u}_{\sigma}(\zeta)}, \tag{1}$$

- REPRESENTATIVE PROCESSES

$pp \rightarrow V + n(\gamma) + m(g) + X \rightarrow \bar{\ell}\ell' + n'(\gamma) + m(g) + X$,

where $V = W^{\pm}, Z$,and $\ell = e, \mu,\ \ell' = \nu_e, \nu_\mu(e, \mu)$

respectively for $V = W^{+}(Z)$, and $\ell = \nu_e, \nu_\mu,\ \ell' = e, \mu$

respectively for $V = W^{-}$.

Quantum Gravity Loop Corrections to Elementary Particle

Progpagators

Review of YFS Theory and Its Extension to QCD

QED CASE – S. Jadach et al., YFS2, YFS3, BHLUMI, BHWIDE, KORALZ, KKMC, YFSWW3, YFSZZ, KoralW

For $e^+(p_1)e^-(q_1) \to \bar{f}(p_2)f(q_2) + n(\gamma)(k_1, \cdot, k_n)$**, renormalization group improved YFS theory (PRD36(1987)939) gives**

$$d\sigma_{exp} = e^{2\alpha\, Re\, B + 2\alpha\, \tilde{B}} \sum_{n=0}^{\infty} \frac{1}{n!} \int \prod_{j=1}^{n} \frac{d^3k_j}{k_j^0} \int \frac{d^4y}{(2\pi)^4} e^{iy(p_1+q_1-p_2-q_2-\sum_j k_j)+D}$$

$$\bar{\beta}_n(k_1, \ldots, k_n) \frac{d^3p_2 d^3q_2}{p_2^0 q_2^0}$$

where the YFS real infrared function $\tilde{B}$ **and the virtual infrared function** B **are known and where we note the usual connections**

$$2\alpha\,\tilde{B} = \int^{k \le K_{max}} \frac{d^3k}{k_0} \tilde{S}(k)$$

$$D = \int d^3k \frac{\tilde{S}(k)}{k^0} \left(e^{-iy\cdot k} - \theta(K_{max} - k)\right) \qquad (2)$$

for the standard YFS infrared emission factor

$$\tilde{S}(k) = \frac{\alpha}{4\pi^2}\left[Q_f Q_{(\bar{f})'}\left(\frac{p_1}{p_1\cdot k} - \frac{q_1}{q_1\cdot k}\right)^2 + (\ldots)\right] \quad (3)$$

if Q_f is the electric charge of f in units of the positron charge. For example, the YFS hard photon residuals $\bar{\beta}_i$ in (1), $i = 0, 1, 2$, are given in S. Jadach *et al.*,CPC102(1997)229 for BHLUMI 4.04 $\Rightarrow$ YFS exponentiated exact $\mathcal{O}(\alpha)$ and LL $\mathcal{O}(\alpha^2)$ cross section for Bhabha scattering via a corresponding Monte Carlo realization of (1).

In hep-ph/0210357(ICHEP02), Acta Phys.Polon.B33,1543-1558,2002, we have extended the YFS theory to QCD:

$$d\hat\sigma_{\rm exp} = \sum_n d\hat\sigma^n$$
$$= e^{\rm SUM_{IR}(QCD)} \sum_{n=0}^{\infty} \int \prod_{j=1}^{n} \frac{d^3k_j}{k_j} \int \frac{d^4y}{(2\pi)^4} e^{iy\cdot(P_1+P_2-Q_1-Q_2-\sum k_j)+D_{\rm QCD}}$$
$$* \tilde{\bar\beta}_n(k_1,\ldots,k_n)\frac{d^3P_2}{P_2^{\,0}}\frac{d^3Q_2}{Q_2^{\,0}} \tag{4}$$

where now the hard gluon residuals $\tilde{\bar\beta}_n(k_1,\ldots,k_n)$ defined by

$$\tilde{\bar\beta}_n(k_1,\ldots,k_n) = \sum_{\ell=0}^{\infty} \tilde{\bar\beta}_n^{(\ell)}(k_1,\ldots,k_n)$$

are free of all infrared divergences to all orders in $\alpha_s(Q)$.

- We stress that the arguments in the earlier papers (**DeLaney** *et al.* **PRD52(1995)108, PLB342(1995)239**) are not really sufficient to derive the respective analog of eq.(4); for, **they did not really expose the compensation between the left over genuine non-Abelian IR virtual and real singularities between** $\int dPh\bar{\beta}_n$ **and** $\int dPh\bar{\beta}_{n+1}$ **respectively that really allows us to isolate** $\tilde{\bar{\beta}}_j$ **and distinguishes QCD from QED, where no such compensation occurs.**

- **Our exponential factor corresponds to the** $N = 1$ **term in the exponent in Gatheral's formula (Phys. Lett.B133(1983)90) for the general exponentiation of the eikonal cross sections for non-Abelian gauge theory; his result is an approximate one in which everything that does not eikonalize and exponentiate is dropped whereas our result (4) is exact.**

Extension to QED⊗QCD and Quantum Gravity

Simultaneous exponentiation of QED and QCD higher order effects, hep-ph/0404087, gives

$$\begin{aligned} B^{nls}_{QCD} &\rightarrow B^{nls}_{QCD} + B^{nls}_{QED} \equiv B^{nls}_{QCED}, \\ \tilde{B}^{nls}_{QCD} &\rightarrow \tilde{B}^{nls}_{QCD} + \tilde{B}^{nls}_{QED} \equiv \tilde{B}^{nls}_{QCED}, \\ \tilde{S}^{nls}_{QCD} &\rightarrow \tilde{S}^{nls}_{QCD} + \tilde{S}^{nls}_{QED} \equiv \tilde{S}^{nls}_{QCED} \end{aligned} \tag{5}$$

which leads to

$$\begin{aligned} d\hat{\sigma}_{\rm exp} = e^{\rm SUM_{IR}(QCED)} \sum_{n,m=0}^{\infty} \int \prod_{j_1=1}^{n} \frac{d^3 k_{j_1}}{k_{j_1}} \\ \prod_{j_2=1}^{m} \frac{d^3 {k'}_{j_2}}{{k'}_{j_2}} \int \frac{d^4 y}{(2\pi)^4} e^{iy\cdot(p_1+q_1-p_2-q_2-\sum k_{j_1}-\sum {k'}_{j_2})+D_{\rm QCED}} \\ \tilde{\bar{\beta}}_{n,m}(k_1,\ldots,k_n;k'_1,\ldots,k'_m)\frac{d^3p_2}{p_2^{\,0}}\frac{d^3q_2}{q_2^{\,0}}, \end{aligned} \tag{6}$$

where the new YFS residuals $\tilde{\bar{\beta}}_{n,m}(k_1,\ldots,k_n;k'_1,\ldots,k'_m)$, with n hard gluons and m hard photons, represent the successive application of the YFS expansion first for QCD and subsequently for QED.

The infrared functions are now

$$\begin{aligned} \mathrm{SUM_{IR}(QCED)} &= 2\alpha_s \Re B^{nls}_{QCED} + 2\alpha_s \tilde{B}^{nls}_{QCED} \\ D_{\mathrm{QCED}} &= \int \frac{dk}{k^0}\left(e^{-iky} - \theta(K_{max} - k^0)\right) \tilde{S}^{nls}_{QCED} \end{aligned} \tag{7}$$

where K_{max} is a dummy parameter – here the same for QCD and QED.

Infrared Algebra(QCED):

$x_{avg}(QED) \cong \gamma(QED)/(1+\gamma(QED))$

$x_{avg}(QCD) \cong \gamma(QCD)/(1+\gamma(QCD))$

$\gamma(A) = \frac{2\alpha_A \mathcal{C}_A}{\pi}(L_s - 1)$, $A = QED, QCD$

$\mathcal{C}_A = Q_f^2, C_F$, respectively, for $A = QED, QCD$

$\Rightarrow$ QCD dominant corrections happen an order of magnitude earlier than those for QED.

$\Rightarrow$ Leading $\bar{\tilde{\beta}}^{(0,0)}_{0,0}$ -level gives a good estimate of the size of the effects we study.

RESUMMED QUANTUM GRAVITY

APPLY (6) TO QUANTUM GENERAL RELATIVITY:

$\Rightarrow$

$$i\Delta'_F(k)|_{\text{resummed}} = \frac{ie^{B''_g(k)}}{(k^2 - m^2 - \Sigma'_s + i\epsilon)} \tag{8}$$

FOR

$$B''_g(k) = -2i\kappa^2 k^4 \frac{\int d^4\ell}{16\pi^4} \frac{1}{\ell^2 - \lambda^2 + i\epsilon} \frac{1}{(\ell^2 + 2\ell k + \Delta + i\epsilon)^2} \tag{9}$$

THIS IS THE BASIC RESULT.

NOTE THE FOLLOWING:

- Σ'_s STARTS IN $\mathcal{O}(\kappa^2)$, SO WE MAY DROP IT IN CALCULATING ONE-LOOP EFFECTS.

- EXPLICIT EVALUATION GIVES, FOR THE DEEP UV REGIME,

$$B''_g(k) = \frac{\kappa^2|k^2|}{8\pi^2}\ln\left(\frac{m^2}{m^2+|k^2|}\right), \qquad (10)$$

$\Rightarrow$ THE RESUMMED PROPAGATOR FALLS FASTER THAN ANY POWER OF $|k^2|$!

- IF m VANISHES, USING THE USUAL $-\mu^2$ NORMALIZATION POINT WE GET $B''_g(k) = \frac{\kappa^2|k^2|}{8\pi^2}\ln\left(\frac{\mu^2}{|k^2|}\right)$ WHICH AGAIN VANISHES FASTER THAN ANY POWER OF $|k^2|$!

THIS MEANS THAT ONE-LOOP CORRECTIONS ARE FINITE!

INDEED, ALL QUANTUM GRAVITY LOOPS ARE UV FINITE(MPLA17(2002)2371)!

QED⊗QCD Threshold Corrections, Shower/ME Matching

and IR-Improved DGLAP Theory at LHC

We shall apply the new simultaneous QED⊗QCD exponentiation calculus to the sinlge Z production with leptonic decay at the LHC (and at FNAL) to focus on the ISR alone, for definiteness. See also the work of Baur *et al.*, **Dittmaier and Kramer, Zykunov for exact $\mathcal{O}(\alpha)$ results and Hamberg** *et al.*, **van Neerven and Matsuura and Anastasiou** *et al.* **for exact $\mathcal{O}(\alpha_s^2)$ results.**

For the basic formula

$$d\sigma_{exp}(pp \to V + X \to \bar{\ell}\ell' + X') = \sum_{i,j} \int dx_i dx_j F_i(x_i) F_j(x_j) d\hat{\sigma}_{exp}(x_i x_j s), \tag{11}$$

we use the result in (6) here with semi-analytical methods and structure functions from Martin *et al.*.

A MC realization will appear elsewhere.

SHOWER/ME MATCHING

- Note the following: In (11) WE DO NOT ATTEMPT AT THIS TIME TO REPLACE HERWIG and/or PYTHIA –
  WE INTEND TO COMBINE OUR EXACT YFS CALCULUS, $d\hat{\sigma}_{exp}(x_i x_j s)$, WITH HERWIG and/or PYTHIA BY USING THEM/IT “IN LIEU” OF $\{F_i\}$.
  A. USE HERWIG/PYTHIA SHOWER FOR $p_T \leq \mu$, YFS nG for $p_T > \mu$.
  B. EXPAND HERWIG/PYTHIA SHOWER FORMULA$\otimes d\sigma_{exp}$ AND ADJUST $\tilde{\bar{\beta}}_{n,m}$ TO EXACTNESS FOR DESIRED ORDER WITH NEW $\tilde{\bar{\beta}}'_{n,m}$
  FIRST USE $\{F_i\}$ TO PICK (x_1, x_2); MAKE EVT WITH $d\sigma_{exp}$; THEN SHOWER EVT USING HERWIG/PYTHIA VIA LES HOUCHES RECIPE.
- THIS COMBINATION OF THEORETICAL CONSTRUCTS CAN BE SYSTEMATICALLY IMPROVED WITH EXACT RESULTS ORDER-BY-ORDER IN α_s, α, WITH EXACT PHASE SPACE.
- THE RECENT ALTERNATIVE PARTON SHOWER ALGORITHM BY JADACH and SKRZYPEK,Acta. Phys. Pol.B35, 745 (2004), CAN ALSO BE USED.
- LACK OF COLOR COHERENCE $\Rightarrow$ ISAJET NOT CONSIDERED HERE.

With this said, we compute , with and without QED, the ratio

$r_{exp} = \sigma_{exp}/\sigma_{Born}$

to get the results (We stress that we *do not* use the narrow resonance approximation here.)

$$r_{exp} = \begin{cases} 1.1901 & ,\text{QCED} \equiv \text{QCD+QED, LHC} \\ 1.1872 & ,\text{QCD, LHC} \\ 1.1911 & ,\text{QCED} \equiv \text{QCD+QED, Tevatron} \\ 1.1879 & ,\text{QCD, Tevatron} \end{cases} \quad (12)$$

$\Rightarrow$

$*$***QED IS AT .3% AT BOTH LHC and FNAL.***

$*$***THIS IS STABLE UNDER SCALE VARIATIONS.***

$*$***WE AGREE WITH BAUR ET AL., HAMBERG ET AL., van NEERVEN and ZIJLSTRA.***

$*$***QED EFFECT SIMILAR IN SIZE TO STR. FN. RESULTS.***

$*$***DGLAP SYNTHESIZATION HAS NOT COMPROMISED THE NORMALIZATION.***

IR-Improved DGLAP Theory

APPLY QCD EXPN THEORY TO DGLAP KERNELS:

$$P_{qq}(z) = C_F F_{YFS}(\gamma_q) e^{\frac{1}{2}\delta_q} \left[\frac{1+z^2}{1-z}(1-z)^{\gamma_q} - f_q(\gamma_q)\delta(1-z) \right] \quad (13)$$

where

$$f_q(\gamma_q) = \frac{2}{\gamma_q} - \frac{2}{\gamma_q+1} + \frac{1}{\gamma_q+2} \quad (14)$$

and

$$\gamma_q = C_F \frac{\alpha_s}{\pi} t = \frac{4C_F}{\beta_0} \quad (15)$$

$$\delta_q = \frac{\gamma_q}{2} + \frac{\alpha_s C_F}{\pi}\left(\frac{\pi^2}{3} - \frac{1}{2}\right) \quad (16)$$

and

$$F_{YFS}(\gamma_q) = \frac{e^{-C_E \gamma_q}}{\Gamma(1+\gamma_q)}. \quad (17)$$

SIMILAR RESULTS HOLD FOR $P_{Gq},\ P_{GG},\ P_{qG}$, GIVING:

$$P_{qq}(z) = C_F F_{YFS}(\gamma_q) e^{\frac{1}{2}\delta_q} \left[\frac{1+z^2}{1-z}(1-z)^{\gamma_q} - f_q(\gamma_q)\delta(1-z) \right], \tag{18}$$

$$P_{Gq}(z) = C_F F_{YFS}(\gamma_q) e^{\frac{1}{2}\delta_q} \frac{1+(1-z)^2}{z} z^{\gamma_q}, \tag{19}$$

$$P_{GG}(z) = 2C_G F_{YFS}(\gamma_G) e^{\frac{1}{2}\delta_G} \{ \frac{1-z}{z} z^{\gamma_G} + \frac{z}{1-z}(1-z)^{\gamma_G} + \frac{1}{2}(z^{1+\gamma_G}(1-z) + z(1-z)^{1+\gamma_G}) - f_G(\gamma_G)\delta(1-z) \}, \tag{20}$$

$$P_{qG}(z) = F_{YFS}(\gamma_G) e^{\frac{1}{2}\delta_G} \frac{1}{2} \{ z^2(1-z)^{\gamma_G} + (1-z)^2 z^{\gamma_G} \}, \tag{21}$$

where

$$\gamma_G = C_G\frac{\alpha_s}{\pi}t = \frac{4C_G}{\beta_0} \tag{22}$$

$$\delta_G = \frac{\gamma_G}{2} + \frac{\alpha_s C_G}{\pi}(\frac{\pi^2}{3} - \frac{1}{2}), \tag{23}$$

$$f_G(\gamma_G) = \frac{n_f}{C_G}\frac{1}{(1+\gamma_G)(2+\gamma_G)(3+\gamma_G)} + \frac{2}{\gamma_G(1+\gamma_G)(2+\gamma_G)} \tag{24}$$

$$+ \frac{1}{(1+\gamma_G)(2+\gamma_G)} + \frac{1}{2(3+\gamma_G)(4+\gamma_G)} \tag{25}$$

$$+ \frac{1}{(2+\gamma_G)(3+\gamma_G)(4+\gamma_G)}. \tag{26}$$

Parton Distributions

Moments of kernels ⇔ Logarithmic exponents for evolution

$$\frac{dM_n^{NS}(t)}{dt} = \frac{\alpha_s(t)}{2\pi} A_n^{NS} M_n^{NS}(t) \tag{27}$$

where

$$M_n^{NS}(t) = \int_0^1 dz z^{n-1} q^{NS}(z,t) \tag{28}$$

and the quantity A_n^{NS} is given by

$$\begin{aligned} A_n^{NS} &= \int_0^1 dz z^{n-1} P_{qq}(z), \\ &= C_F F_{YFS}(\gamma_q) e^{\frac{1}{2}\delta_q} [B(n,\gamma_q) + B(n+2,\gamma_q) - f_q(\gamma_q)] \end{aligned} \tag{29}$$

where $B(x,y)$ is the beta function given by

$$B(x,y) = \Gamma(x)\Gamma(y)/\Gamma(x+y)$$

.

Compare the usual result

$$A_n^{NS^o} \equiv C_F\left[-\frac{1}{2}+\frac{1}{n(n+1)}-2\sum_{j=2}^{n}\frac{1}{j}\right]. \tag{30}$$

- ASYMPTOTIC BEHAVIOR: IR-improved goes to a multiple of $-f_q$, consistent with $\lim_{n\to\infty} z^{n-1} = 0$ for $0 \le z < 1$;
  usual result diverges as $-2C_F \ln n$.
- Different for finite n as well: for $n = 2$ we get, for example, for $\alpha_s \cong .118$,

$$A_2^{NS} = \begin{cases} C_F(-1.33) & , \text{ un-IR-improved} \\ C_F(-0.966) & , \text{ IR-improved} \end{cases} \tag{31}$$

- **For completeness we note**

$$M_n^{NS}(t) = M_n^{NS}(t_0) e^{\int_{t_0}^{t} dt' \frac{\alpha_s(t')}{2\pi} A_n^{NS}(t')}$$
$$= M_n^{NS}(t_0) e^{\bar{a}_n [Ei(\frac{1}{2}\delta_1 \alpha_s(t_0)) - Ei(\frac{1}{2}\delta_1 \alpha_s(t))]} \quad (32)$$
$$\underset{t,t_0 \text{ large with } t>>t_0}{\Longrightarrow} M_n^{NS}(t_0) \left(\frac{\alpha_s(t_0)}{\alpha_s(t)} \right)^{\bar{a'}_n}$$

where $Ei(x) = \int_{-\infty}^{x} dr e^r / r$ **is the exponential integral function,**

$$\bar{a}_n = \frac{2C_F}{\beta_0} F_{YFS}(\gamma_q) e^{\frac{\gamma_q}{4}} [B(n,\gamma_q) + B(n+2,\gamma_q) - f_q(\gamma_q)]$$
$$\bar{a'}_n = \bar{a}_n \left(1 + \frac{\delta_1}{2} \frac{(\alpha_s(t_0) - \alpha_s(t))}{\ln(\alpha_s(t_0)/\alpha_s(t))} \right) \quad (33)$$

with

$\delta_1 = \frac{C_F}{\pi} \left(\frac{\pi^2}{3} - \frac{1}{2} \right)$**.**

Compare with un-IR-improved result where last line in eq.(32) holds exactly with $\bar{a'}_n = 2A_n^{NS^o}/\beta_0$**.**

- Comparison with Moch et al., Vogt et al., Curci et al., etc.,:

  Consider P_{qq} –

$$P^{+}_{ns} = P^{v}_{qq} + P^{v}_{q\bar{q}} \equiv \sum_{n=0}^{\infty} (\frac{\alpha_s}{4\pi})^{n+1} P^{(n)+}_{ns} \quad (34)$$

with

$$P^{(0)+}_{ns}(z) = 2C_F\{\frac{1+z^2}{(1-z)_+} + \frac{3}{2}\delta(1-z)\}, \quad (35)$$

a factor of 2 $\times$ P_{qq}.

Exponentiation $\Rightarrow$

$$P^{+,exp}_{ns}(z) = (\frac{\alpha_s}{4\pi})2P^{exp}_{qq}(z) + F_{YFS}(\gamma_q)e^{\frac{1}{2}\delta_q}\left[(\frac{\alpha_s}{4\pi})^2\{(1-z)^{\gamma_q}\bar{P}^{(1)+}_{ns}(z) + \bar{B}_2\delta\right.$$
$$\left. + (\frac{\alpha_s}{4\pi})^3\{(1-z)^{\gamma_q}\bar{P}^{(2)+}_{ns}(z) + \bar{B}_3\delta(1-z)\}\right] \quad (36)$$

where $P^{exp}_{qq}(z)$ is given above, the resummed residuals $\bar{P}^{(i)+}_{ns}$, $i = 1, 2$ are

related to the exact results $P_{ns}^{(i)+}$, $i = 1, 2$, as follows:

$$\bar{P}_{ns}^{(i)+}(z) = P_{ns}^{(i)+}(z) - B_{1+i}\delta(1-z) + \Delta_{ns}^{(i)+}(z) \tag{37}$$

where

$$\begin{aligned}\Delta_{ns}^{(1)+}(z) &= -4C_F\pi\delta_1\{\frac{1+z^2}{1-z} - f_q\delta(1-z)\}\\ \Delta_{ns}^{(2)+}(z) &= -4C_F(\pi\delta_1)^2\{\frac{1+z^2}{1-z} - f_q\delta(1-z)\}\\ &\quad - 2\pi\delta_1\bar{P}_{ns}^{(1)+}(z)\end{aligned} \tag{38}$$

and

$$\begin{aligned}\bar{B}_2 &= B_2 + 4C_F\pi\delta_1 f_q\\ \bar{B}_3 &= B_3 + 4C_F(\pi\delta_1)^2 f_q - 2\pi\delta_1\bar{B}_2.\end{aligned} \tag{39}$$

Here, the constants $B_i,\ i=2,3$ are given by the results of Moch et al., Vogt et al., Curci et al., etc., as

$$B_2 = 4C_GC_F(\frac{17}{24}+\frac{11}{3}\zeta_2-3\zeta_3)-4C_Fn_f(\frac{1}{12}+\frac{2}{3}\zeta_2)+4C_F^2(\frac{3}{8}-3\zeta_2+6\zeta_3)$$

$$\begin{aligned}B_3 = & 16C_GC_Fn_f(\frac{5}{4}-\frac{167}{54}\zeta_2+\frac{1}{20}\zeta_2^2+\frac{25}{18}\zeta_3)\\ & +16C_GC_F^2(\frac{151}{64}+\zeta_2\zeta_3-\frac{205}{24}\zeta_2-\frac{247}{60}\zeta_2^2+\frac{211}{12}\zeta_3+\frac{15}{2}\zeta_5)\\ & +16C_G^2C_F(-\frac{1657}{576}+\frac{281}{27}\zeta_2-\frac{1}{8}\zeta_2^2-\frac{97}{9}\zeta_3+\frac{5}{2}\zeta_5)\\ & +16C_Fn_F^2(-\frac{17}{144}+\frac{5}{27}\zeta_2-\frac{1}{9}\zeta_3)\\ & +16C_F^2n_F(-\frac{23}{16}+\frac{5}{12}\zeta_2+\frac{29}{30}\zeta_2^2-\frac{17}{6}\zeta_3)\\ & +16C_F^3(\frac{29}{32}-2\zeta_2\zeta_3+\frac{9}{8}\zeta_2+\frac{18}{5}\zeta_2^2+\frac{17}{4}\zeta_3-15\zeta_5),\end{aligned} \quad (40)$$

where ζ_n is the Riemann zeta function evaluated at argument n. The detailed phenomenological consequences of the fully exponentiated 2- and 3-loop DGLAP kernel set will appear elsewhere.

- Wilson's expansion assumes analyticity about $\nu = 2qp = 0$, whereas $\ln(1-z)$ is not so analytic.

FINAL STATE OF HAWKING RADIATION

CONSIDER THE GRAVITON PROPAGATOR IN THE THEORY OF GRAVITY COUPLED TO A MASSIVE SCALAR(HIGGS) FIELD(Feynman). WE HAVE THE GRAPHS

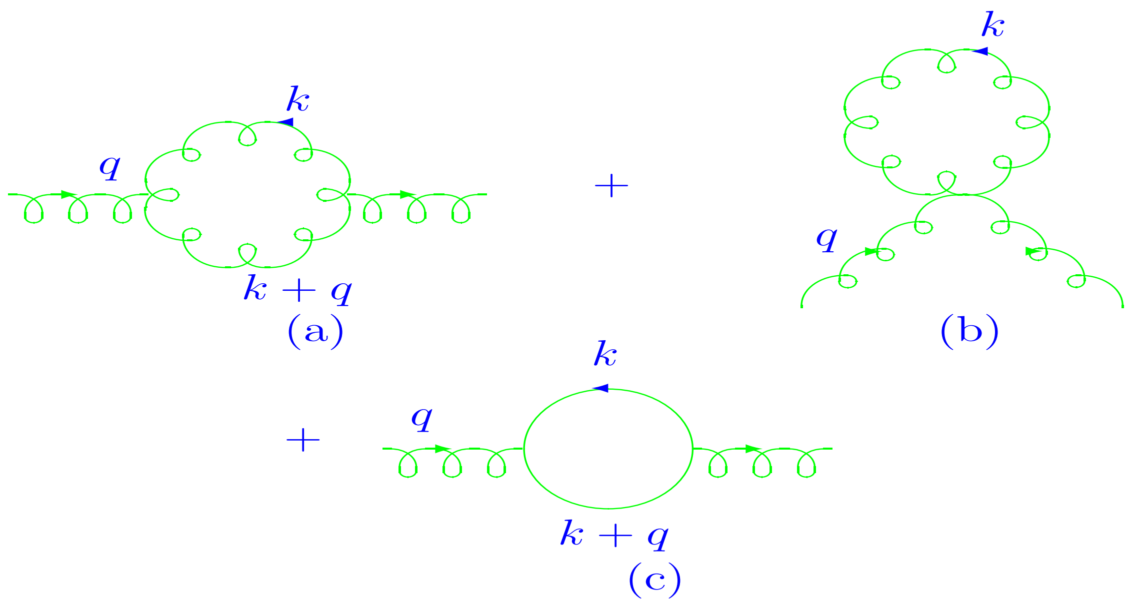

Figure 1: The graviton((a),(b)) and its ghost((c)) one-loop contributions to the graviton propagator. q is the 4-momentum of the graviton.

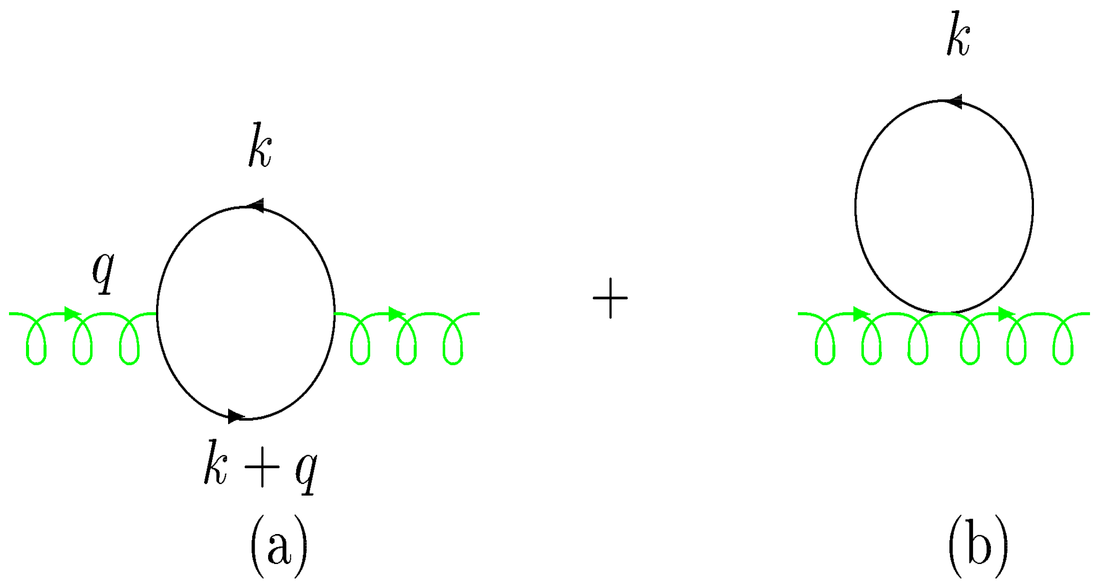

Figure 2: The scalar one-loop contribution to the graviton propagator. q is the 4-momentum of the graviton.

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USING THE RESUMMED THEORY, WE GET THAT THE NEWTON POTENTIAL BECOMES

$$\Phi_N(r) = -\frac{G_N M}{r}(1 - e^{-ar}), \quad (41)$$

FOR

$$a \cong 0.210 M_{Pl}. \quad (42)$$

CONTACT WITH AYMPTOTIC SAFETY APPROACH

- OUR RESULTS IMPLY

$$G(k) = G_N/(1 + \frac{k^2}{a^2})$$

$\Rightarrow$ FIXED POINT BEHAVIOR FOR $k^2 \rightarrow \infty$, IN AGREEMENT WITH THE PHENOMENOLOGICAL ASYMPTOTIC SAFETY APPROACH OF BONNANNO & REUTER IN PRD62(2000) 043008.

- OUR RESULTS IMPLY THAT AN ELEMENTARY PARTICLE HAS

  NO HORIZON WHICH ALSO AGREES WITH BONNANNO'S & REUTER'S

  RESULT THAT A BLACK HOLE WITH A MASS LESS THAN

  $M_{cr} \sim M_{Pl}$

  HAS NO HORIZON.

  BASIC PHYSICS:

  $G(k)$ VANISHES FOR $k^2 \to \infty$.

- A FURTHER “AGREEMENT”: FINAL STATE OF HAWKING RADIATION OF AN ORIGINALLY VERY MASSIVE BLACKHOLE
  BECAUSE OUR VALUE OF THE COEFFICIENT,
  $$\frac{1}{a^2},$$
  OF k^2 IN THE DENOMINATOR OF $G(k)$
  AGREES WITH THAT FOUND BY BONNANNO & REUTER(B-R),
  IF WE USE THEIR PRESCRIPTION FOR THE
  RELATIONSHIP BETWEEN k AND r
  IN THE REGIME WHERE THE LAPSE FUNCTION VANISHES,
  WE GET THE SAME HAWKING RADIATION PHENOMEMNOLOGY AS THEY DO:
  THE BLACK HOLE EVAPORATES IN THE B-R ANALYSIS UNTIL IT REACHES A MASS
  $$M_{cr} \sim M_{Pl}$$
  AT WHICH THE BEKENSTEIN-HAWKING TEMPERATURE VANISHES,
  LEAVING A PLANCK SCALE REMNANT.
- FATE OF REMNANT? IN hep-ph/0503189 $\Rightarrow$ OUR QUANTUM LOOP EFFECTS COMBINED WITH THE $G(r)$ OF B-R IMPLY HORIZON OF THE PLANCK SCALE REMNANT IS OBVIATED – CONSISTENT WITH RECENT RESULTS OF HAWKING.

TO WIT, IN THE METRIC CLASS

$$ds^2 = f(r)dt^2 - f(r)^{-1}dr^2 - r^2 d\Omega^2 \quad (43)$$

THE LAPSE FUNCTION IS, FROM B-R,

$$\begin{aligned} f(r) &= 1 - \frac{2G(r)M}{r} \\ &= \frac{B(x)}{B(x) + 2x^2}\Big|_{x=\frac{r}{G_N M}}, \end{aligned} \quad (44)$$

WHERE

$$B(x) = x^3 - 2x^2 + \Omega x + \gamma\Omega \quad (45)$$

FOR

$$\Omega = \frac{\tilde{\omega}}{G_N M^2} = \frac{\tilde{\omega} M_{Pl}^2}{M^2}. \quad (46)$$

AFTER H-RADIATING TO REGIME NEAR $M_{cr} \sim M_{Pl}$, QUANTUM LOOPS ALLOW US TO REPLACE $G(r)$ WITH $G_N(1-e^{-ar})$ IN THE LAPSE FUNCTION FOR $r < r_>$, THE OUTERMOST SOLUTION OF

$$G(r) = G_N(1-e^{-ar}). \quad (47)$$

IN THIS WAY, WE SEE THAT THE INNER HORIZON MOVES TO NEGATIVE r AND THE OUTER HORIZON MOVES TO $r = 0$ AT THE NEW CRITICAL MASS $\sim 2.38 M_{Pl}$.

NOTE: M. BOJOWALD *et al.*, gr-qc/0503041, – LOOP QG CONCURS WITH GENERAL CONCLUSION.

PREDICTION: THERE SHOULD ENERGETIC COSMIC RAYS AT $E \sim M_{Pl}$ DUE THE DECAY OF SUCH A REMNANT.

Conclusions

YFS THEORY (EEX AND CEEX) EXTENDS TO NON-ABELIAN GAUGE THEORY AND ALLOWS SIMULTANEOUS EXPN OF QED AND QCD WITH PROPER SHOWER/ME MATCHING BUILT-IN.

FOR QED$\otimes$QCD

- FULL MC EVENT GENERATOR REALIZATION IS POSSIBLE.
- SEMI-ANALYTICAL RESULTS FOR QED (AND QCD) THRESHOLD EFFECTS AGREE WITH LITERATURE ON Z PRODUCTION
- AS QED IS AT THE .3% LEVEL, IT IS NEEDED FOR 1% LHC THEORY PREDICTIONS.
- A FIRM BASIS FOR THE COMPLETE $\mathcal{O}(\alpha_s^2, \alpha\alpha_s, \alpha^2)$ MC RESULTS NEEDED FOR THE FNAL/LHC/RHIC/TESLA/LC PHYSICS HAS BEEN DEMONSTRATED AND ALL THE LATTER IS IN PROGRESS, WITH M. Kalmykov, S. Majhi, S. Yost and S. Joseph.

THE THEORY ALLOWS A NEW APPROACH

TO QUANTUM GENERAL RELATIVITY:

- **RESUMMED QG UV FINITE**
- **MANY CONSEQUENCES:**

  BLACK HOLES EVAPORATE TO FINAL MASS $\sim M_{Pl}$

  WITH NO HORIZON

  $\Rightarrow E \sim M_{Pl}$ **COSMIC RAYS,** $\cdots$.