Two-loop corrections to the pole masses of heavy quarks in the MSSM

A. Bednyakov\textsuperscript{1}, D.I. Kazakov\textsuperscript{1,2}, A. Sheplyakov\textsuperscript{1}

\textsuperscript{1}Joint Institute for Nuclear Research, Dubna, Russia

\textsuperscript{2}Institute for Theoretical and Experimental Physics, Moscow, Russia
\[
\frac{\Delta m_{t,b}}{m_{t,b}} \equiv \frac{M_{t,b}^{pole} - m_{t,b}^{DR}(\bar{\mu})}{m_{t,b}^{DR}(\bar{\mu})}
\]
Estimates of SUSY particles masses are necessary.

One needs to know values of (running) parameters of MSSM Lagrangian at the “low-energy” (EW $\approx$ 100 GeV, or SUSY breaking $\approx$ 1000 GeV) scale.

For large $\tan \beta$ and/or $m_0$ predicted values of masses are very sensitive to Yukawa couplings of heavy quarks. This dependence is particularly strong for Higgs boson masses.

Thus, it is desirable to have determination of the heavy quarks Yukawa couplings as precise as possible.
In order to evaluate Yukawa couplings from

- $M_{Z}^{pole}, M_{W}^{pole}$
- $M_{t}^{pole}$
- $\alpha_{s}(5)(M_{Z}), \alpha_{em}$
- $m_{b}(m_{b}),$ etc.

we need to calculate relation between the pole and running masses:

$$\frac{\Delta m_{t,b}}{m_{t,b}} \equiv \frac{M_{t,b}^{pole} - m_{t,b}^{DR}(\bar{\mu})}{m_{t,b}^{DR}(\bar{\mu})}$$
Pierce, Matchev, Zhang, hep-ph/9606211:

\[
\frac{\Delta m_{t,b}}{m_{t,b}} = \mathcal{O}(\alpha_s) + \mathcal{O}(y_t^2) + \mathcal{O}(y_b^2) + \mathcal{O}(g^2) + \mathcal{O}(g'^2)
\]

For the \( b \) quark \( \mathcal{O}(\alpha_s) \) terms are reduced significantly by \( \mathcal{O}(y_t^2 + y_b^2) \) ones.

Bednyakov, Onishchenko, Velizhanin, Veretin, hep-ph/0210258:
\( \mathcal{O}(\alpha_s^2) \), obtained by asymptotic expansion in \( m_{t,b}/M_{\text{SUSY}} \), leading term only. For the \( b \) quark this correction is comparable with one-loop one.
- $O(\alpha_s y_t^2 + \alpha_s y_b^2 + y_t^4 + y_b^4)$ corrections for $b$-quark, since these may be of the same order of magnitude as $O(\alpha_s^2)$ ones.
- second-order terms of asymptotic expansion for $t$-quark, since $m_t/M_{SUSY} \approx 0.2$. 

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full connected propagator of a quark:

\[ i (\hat{p} - m - \Sigma(\hat{p}, m_i))^{-1} \]

self-energy of the quark:

\[ \Sigma(\hat{p}, m_i) = \hat{p} \Sigma_V(p^2, m_i^2) + \hat{p} \gamma_5 \Sigma_A(p^2, m_i^2) + m \Sigma_S(p^2, m_i) \]

pole mass \( M_p \):

\[
\left( (1 + \Sigma_V(M_p^2, m_i^2))^2 - \Sigma_A^2(M_p^2, m_i^2) \right) M_p^2 = 0
\]

\[
-m^2 \left( 1 - \Sigma_S(M_p^2, m_i) \right)^2 = 0
\]
perturbative expansion of the pole mass:

\[
\frac{M_p - m}{m} = \alpha M^{(1)} + \alpha^2 M^{(2)},
\]

\[
M^{(1)} = \Sigma^{(1)}(m^2, m_i^2) + \Sigma^{(1)}(m^2, m_i),
\]

\[
M^{(2)} = \Sigma^{(2)}(m^2, m_i^2) + \Sigma^{(2)}(m^2, m_i) + \frac{1}{2} \Sigma_A^{(1)}(m^2, m_i^2)
\]

\[
+ M^{(1)} \left( \Sigma^{(1)}(m^2, m_i^2) \right)
\]

\[
+ 2m^2 \frac{\partial}{\partial p^2} \left( \Sigma^{(1)}(p^2, m_i) + \Sigma^{(1)}(p^2, m_i) \right) \bigg|_{p^2=m^2}
\]
Asymptotic expansion (F.V. Tkachov):

\[ F_\Gamma(p, m, M) = \sum_\gamma F_{\Gamma/\gamma}(p, m) T_\gamma(p_\gamma, m) F_\gamma(p_\gamma, m, M) \]

- \( \gamma \): asymptotically irreducible subgraphs of \( \Gamma \)
- \( T_\gamma(p_\gamma, m) \): Taylor expansion in small masses and external (with respect to \( \gamma \)) momenta
about 160 two-loop propagator-type diagrams

\[ \frac{\Delta m_t}{m_t} \approx 1 + \alpha_s \sum_{n=-1}^{2} m_t^n \sigma_1^{(n)} + \alpha_s^2 \sum_{n=-1}^{2} m_t^n \sigma_2^{(n)} \]
Motivation

Method of calculation

Results

Conclusion

Definition of the pole mass

Asymptotic expansion

$t$ quark self-energy

$b$ quark self-energy

Tools of the trade

\begin{align*}
\gamma &= \Gamma \\
\gamma_0 &= \Gamma \\
\gamma_0 &= \Gamma \\
\end{align*}
about 1200 two-loop propagator-type diagrams

\[ m_b \ll m_t, m_{h0}, m_{H0}, m_{H^+}, m_{\tilde{q}}, m_{\tilde{\chi}^0}, m_{\tilde{\chi}^+}, m_{G_0}, m_{G^+} \]

\[ \Delta m_b \approx 1 + \alpha \sum_{n=-1}^{0} m_b^n \sigma_1^{(n)} + \alpha^2 \sum_{n=-1}^{0} m_b^n \sigma_2^{(n)} \]
other subgraphs give $\mathcal{O}(m_b^2)$ contribution.
Diagram generation: (slightly modified version of) *FeynArts*.

Analytical and numeric calculations: *GiNaC C++ library*.

Asymptotic expansion: *prop2exp C++ library* (based on *GiNaC*).

Recursive reduction of two-loop vacuum integrals to a by integration by parts method, analytic and numeric evaluation of the master-integral: *bubblesii C++ library* (based on *GiNaC* too).

Evaluation of running MSSM parameters and SUSY partner masses: *ffmssmsc C++ library* (forked from *SOFTSUSY* written from scratch).
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SQCD contributions to $\Delta m_t/m_t$
relative value of second-order term
change of predicted values of particle masses
Yukawa and SQCD corrections to $\Delta m_b/m_b$

$m_0 = 1000$ GeV, $A_0 = 0$, $\tan \beta = 50$

$\mathcal{O}(\alpha_s^2)$ corrections are $\approx 30\%$ of one-loop ones
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SQCD contributions to $\Delta m_t/m_t$

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change of predicted values of particle masses

Yukawa and SQCD corrections to $\Delta m_b/m_b$

$A_0 = 0$, $\tan \beta = 50$

$\sigma_{2\text{rel}}^{\text{rel}} \equiv \frac{\Delta^{(2)} m_t/m_t}{\Delta^{(0)} m_t/m_t}$

second-order terms of asymptotic expansion in $m_t/M_{SUSY}$ give negligible contribution

$A. Bednyakov, D.I. Kazakov, A. Sheplyakov$

Two-loop corrections to the pole masses of heavy quarks in the MSSM
Two-loop corrections to \( \Delta m_t/m_t \) yields sizable change (\( \approx 15\% \)) of predicted masses of heavy Higgs bosons and chargino.

This change exceeds discrepancies (indicated by gray region on the plot) between different software for MSSM spectrum calculation.
Two-loop corrections to $\Delta m_t/m_t$ yields sizable change (≈ 15%) of predicted masses of heavy Higgs bosons and chargino.

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Masses of squarks, gluino, and relatively light particles (lightest neutralino, lightest Higgs boson) do not obtain any significant changes.
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Two-loop corrections to the pole masses of heavy quarks in the MSSM
\( \mathcal{O}(\alpha_s^2) \) MSSM corrections to the relation between pole and running masses of the \( t \) quark were evaluated.

- these corrections are \( \approx 30\% \) of one-loop ones
- second-order terms of asymptotic expansion in \( m_t/M_{SUSY} \) give negligible contribution
- this correction yields \( \gtrsim 15\% \) change of predicted masses of heavy Higgs bosons and chargino
- $O(\alpha_s y^2 + y^4)$ MSSM corrections to the relation between pole and running masses of the $b$ quark were evaluated.

- $O(\alpha_s^2)$ correction is positive and of the same order of magnitude as the one-loop MSSM correction.

- $O(\alpha_s y^2 + y^4)$ contribution has the opposite sign in most "interesting" regions of MSSM parameter space and partially compensate $O(\alpha_s^2)$ corrections.

- $O(\alpha_s y^2 + y^4 + \alpha_s^2)$ corrections $\propto 30 - 40\%$ of the one-loop MSSM ones.
Our results can be used for

- calculation of the MSSM mass spectrum
- renormalization group analysis of the Yukawa coupling unification
- dark matter searches, relic density is sensitive to the masses of heavy quarks for large $\tan \beta$