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Charged Current Deep Inelastic Scattering at three loops

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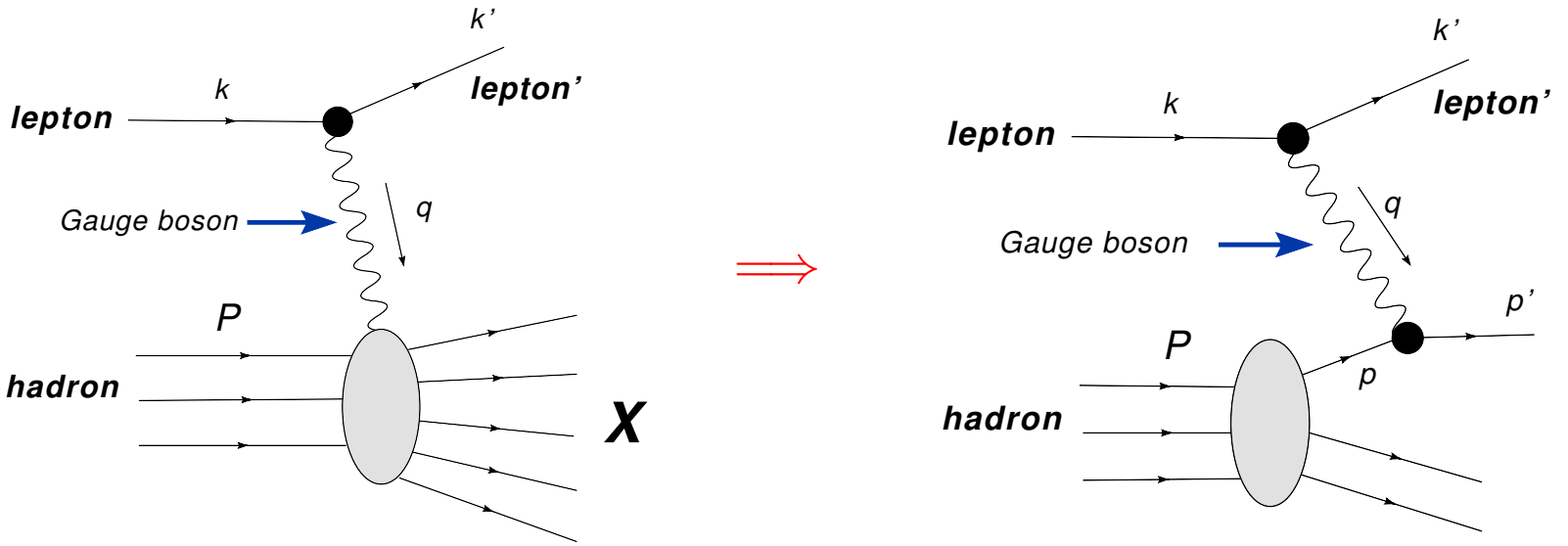
Structure Functions

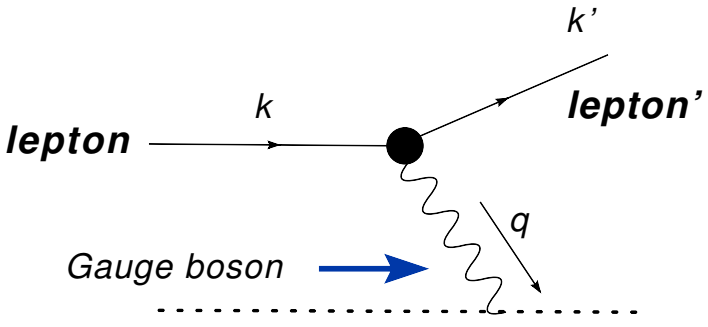
- Deep-inelastic lepton-hadron scattering ($e^{\pm}p$, $e^{\pm}n$, νp, $\bar{\nu}p$, ... - collisions)



- *Gauge boson*:
  γ, Z^0, $W^{\pm}$

- **Kinematic variables**
  - momentum transfer $Q^2 = -q^2 > 0$
  - Bjorken variable $x = Q^2/(2p \cdot q)$

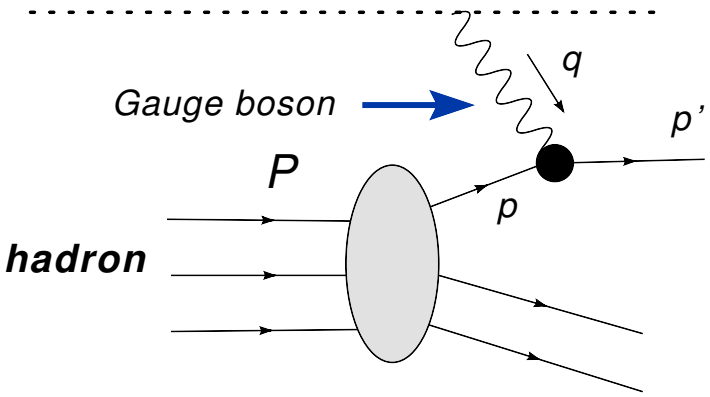
$$d\sigma \sim L^{\mu\nu} W_{\mu\nu}$$



$\Rightarrow$ $L^{\mu\nu}$-leptonic tensor

$$L^{\mu\nu} = A \times (k^\mu k'^\nu + k^\nu k'^\mu - k \cdot k' g^{\mu\nu}) + B \times \left(\mathrm{i}\epsilon^{\mu\nu\alpha\beta} k^\alpha k'^\beta\right)$$

- Coefficients A and B are real and depend on the process.
- γ^5 involved $\Rightarrow$ $B \neq 0$!!!



$\Rightarrow$ $W_{\mu\nu}$-hadronic tensor

$$W_{\mu\nu} = e_{\mu\nu}\frac{1}{2x}F_L(x,Q^2) + d_{\mu\nu}\frac{1}{2x}F_2(x,Q^2) + \mathrm{i}\epsilon_{\mu\nu\alpha\beta}\frac{p^\alpha q^\beta}{p\cdot q}F_3(x,Q^2)$$

$$e_{\mu\nu} = g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}, \qquad d_{\mu\nu} = -g_{\mu\nu} - p_\mu p_\nu\frac{4x^2}{q^2} - (p_\mu q_\nu + p_\nu q_\mu)\frac{2x}{q^2}$$

we are interested in the Mellin moments of the structure functions:

$$F_i^N(Q^2) = \int_0^1 dx x^{N-2}F_i(x,Q^2), \qquad i = 2, L; \qquad F_3^N = \int_0^1 dx x^{N-1}F_3(x,Q^2)$$

Factorization of structure functions

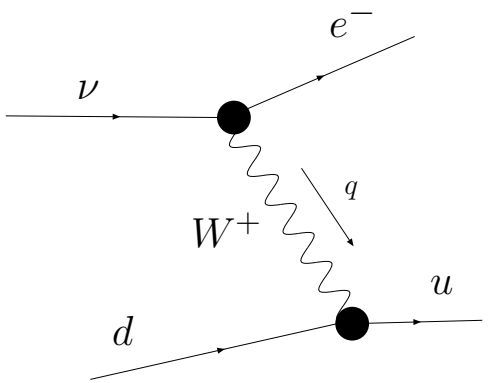
- Structure function $F_i(x, Q^2)$ are given as convolution

$$F_i(x, Q^2) = \sum_{p=partons} \int_x^1 \frac{dz}{z} f_p\left(\frac{x}{z}, \mu^2\right) C_{i,p}\left(z, \frac{Q^2}{\mu^2}, \alpha_s\right), \quad i = 2, L, 3.$$

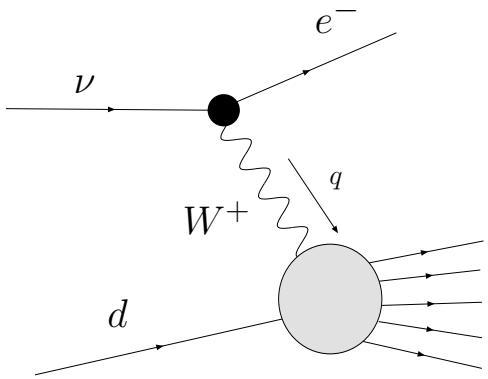
- Parton Distribution Functions $f_p\left(\frac{x}{z}, \mu^2\right)$ are extracted from data [HERA, Tevatron, fixed target exp.]
- Coefficient functions $C_{i,p}\left(z, \frac{Q^2}{\mu^2}, \alpha_s\right)$ are calculable in QCD

Higher order QCD corrections (CC DIS)

- Parton level $\Rightarrow$



- Beyond LO $\Rightarrow$

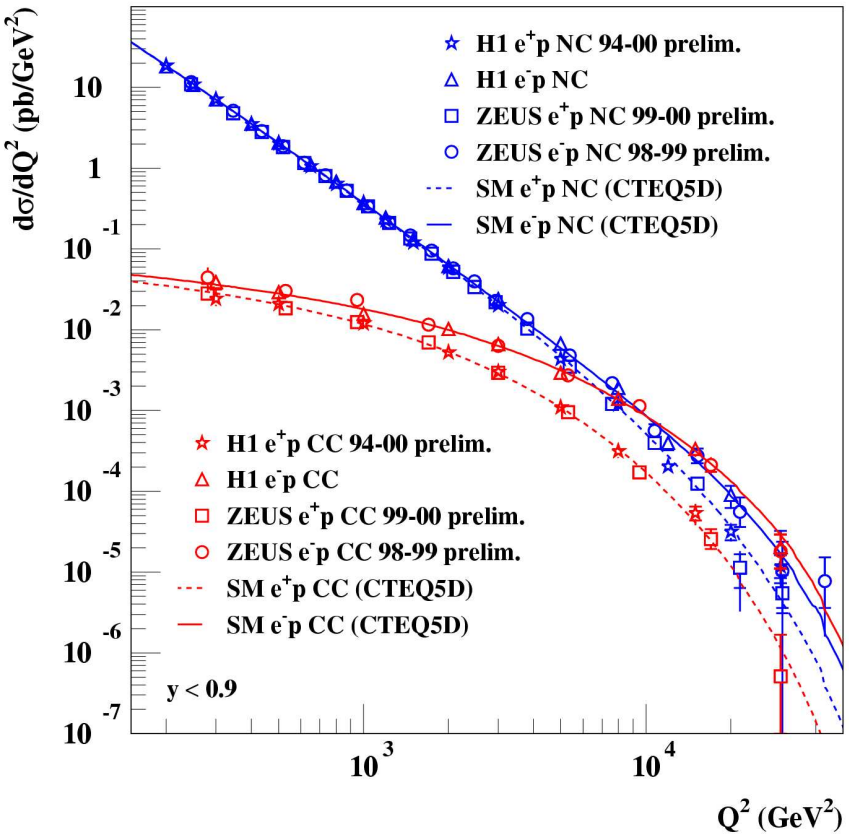


▲ Exchange via $W^{\pm}$ gauge boson
  – Vector and Axial-Vector interaction: $a\gamma^{\mu} + b\gamma^{\mu}\gamma^{5}$

▲ One has to take into account all possible final states up to fixed order in α_s (including virtual corrections)
  – Our calculations: up to α_s^3

- Physical motivation

CC DIS at HERA



$\Longleftarrow$ Electroweak measurements at HERA

CC and NC equal at high energies

$\sin^2\theta_w$ from NuTeV

The Paschos-Wolfenstein relation

Exact relation for massless quarks and isospin zero target

[Paschos,Wolfenstein'73, Llewelin Smith'83]

$$R^- = \frac{\sigma^\nu_{NC} - \sigma^{\bar\nu}_{NC}}{\sigma^\nu_{CC} - \sigma^{\bar\nu}_{CC}} = \frac{1}{2} - \sin^2\theta_W$$

QCD corrections to the Paschos-Wolfenstein relation

Second moments of PDFs $q^- = \int dx\, x(q - \bar q)$, expand in isoscalar combination $u^- + d^-$

[Davidson, Forte, Gambino, Rius, Strumia, hep-ph/0112302]

$$R^- = \frac{1}{2} - \sin^2\theta_W + \left[1 - \frac{7}{3}\sin^2\theta_W + \frac{8\alpha_s}{9\pi}\left(\frac{1}{2} - \sin^2\theta_W\right) + \mathcal{O}({\alpha_s}^2)\right] \times$$

$$\left(\frac{u^- - d^-}{u^- + d^-} - \frac{s^-}{u^- + d^-} + \frac{c^-}{u^- + d^-}\right)$$

main uncertainties in s^-

Current state in the determination of the structure functions

- F_i^{ep} via one photon exchange:
  LO - Gross, Wilczek'73;Altarelli ,Parisi'77 (*)
  NLO - Bardeen, Buras, Duke, Muta'78 (**)
  NNLO - Zijlstra, van Neerven'92 (***)
  N^{3}LO - Moch, Vogt, Vermaseren'05 (****)
  $i = 2, L$-needs even Mellin moments (MM), $i = 3$ needs odd MM

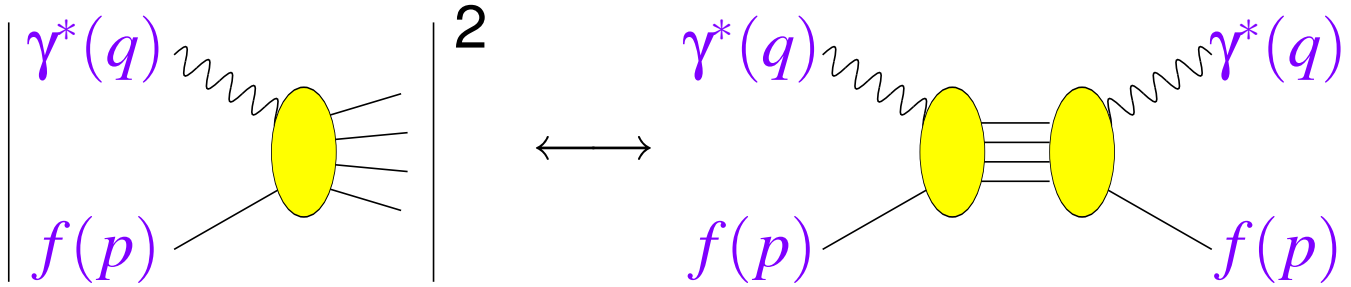
- $F_{2,L}^{\nu p+\bar{\nu}p}$ and $F_3^{\nu p+\bar{\nu}p}$:
  LO, NLO - (**), NNLO - (***), N^{3}LO - (****)
  Again even and odd MM moments correspondingly [Balin, Love, Nanopoulos'74;Politzer'74] (#)

- $F_{2,L}^{\nu p-\bar{\nu}p}$ and $F_3^{\nu p-\bar{\nu}p}$:
  Now (!!!) odd MM for the first case and even for the second case (#). It is the main difference in the determination of these structure functions.
  LO, NLO - (**), NNLO - (***)
  Our calculation - up to N^{3}LO ($\sim {\alpha_s}^3$) for the coefficient functions .

Optical theorem

- The hadronic tensor is related to the imaginary part of the forward Compton scattering amplitude

  – α_s^3 calculation in DIS with help of loop technology



$$W_{\mu\nu}(p,q) = \frac{1}{2\pi} ImT_{\mu\nu}(p,q)$$

Calculatuions up to 3 loops

The calculation

- DIS structure functions
  $F_{2,L}^{\nu p \pm \bar{\nu} p}$, $F_3^{\nu p \pm \bar{\nu} p}$ - 1076 diagrams up to 3 loops
- latest version of FORM [Vermaseren, version 3.1,7-oct-2002.]

The tools

- QGRAF $\mapsto$ generation of diagrams for DIS structure functions [Nogueira'93]

calculation of diagrams $\mapsto$

- MINCER

IBP identity [Chetyrkin, Tkachev'81]

$$0 = \int dk_i \frac{\partial}{\partial k_i^\mu} \left[(k_i - k_j)^\mu \times \text{Integrand}(..., k_i, ...) \right]$$

- MINCER in FORM [Larin, Tkachev, Vermaseren'91]

Renormalization subtle point for F_3 [Larin, Vermaseren,91]

- insertion of γ^5 with the Larin prescription $\mapsto$

$$\gamma_\mu\gamma_5 = \mathrm{i}\frac{1}{3!}\epsilon_{\mu\nu\sigma\tau}\gamma^\rho\gamma^\sigma\gamma^\tau$$

This definition violates the axial Ward identity which is to be restored by an additional renormalization. The necessary Z_A in $\overline{MS}$ is

$$Z_A = 1 + \left(\frac{\alpha_s(\mu^2)}{4\pi}\right)^2 \frac{1}{\varepsilon}\left[\frac{22}{3}C_A C_F - \frac{4}{3}C_F n_f\right] + \mathcal{O}({\alpha_s}^3)$$

The treatment of γ^5 in $D = 4 - 2\varepsilon$ introduces an extra finite renormalization with Z_5 . It is derived in $\overline{MS}$ from

$$(R_{\overline{MS}}V_\mu)\gamma_5 = Z_5(R_{\overline{MS}}A_\mu),$$

$R_{\overline{MS}}$ - denotes renormalization operation in the $\overline{MS}$ scheme to remove UV divergencies.

$$Z_5 = 1 - \frac{\alpha_s(\mu^2)}{\pi}C_F + \left(\frac{\alpha_s(\mu^2)}{4\pi}\right)^2\left[22C_F^2 - \frac{107}{9}C_A C_F + \frac{2}{9}C_F n_f\right] + \mathcal{O}({\alpha_s}^3)$$

The check

- Sum rules calculations in DIS

  ▲ Gross - Llewellyn Smith sum rule and Bjorken sum rule

  [Larin, Vermaseren,91]

$$\int_0^1 dx(F_1^{\bar{\nu}p}(x,Q^2)-F_1^{\nu p}(x,Q^2)) = 1, \quad \int_0^1 dx(F_3^{\bar{\nu}p}(x,Q^2)+F_3^{\nu p}(x,Q^2)) = 6$$

  ▲ Adler sum rule, Gottfried sum rule [Broadhurst, Kataev, Maxwell'04]

  Conjucture of colour coefficients of coefficient functions "even" - "odd"

  Difference $\sim (C_F - C_A/2)$

- Calculations with gauge parameter of fixed low order Mellin moments

$$\mathrm{i}\frac{-g^{\mu\nu} + (1-\xi)q^\mu q^\nu}{q^2 - \mathrm{i}\epsilon}$$

First Results

Results for 6'th Mellin Moment of $F_{2,L}^{\nu p+\bar{\nu}p}$ calculated with gauge parameter ξ

$\Longrightarrow$

```
Result6MellinMoment=
 + ep^-3*cf*nf^2*proj2*a^3 * ( 45376/2835 )

       + ep^-3*cf*nf^2*proj2*fl02*a^3 * (  - 7744/19845 )

       + ep^-3*cf*ca*nf*proj2*a^3 * (  - 499136/2835 )

       + ep^-3*cf*ca*nf*proj2*fl02*a^3 * ( 76472/15435 )

       + ep^-3*cf*ca^2*proj2*a^3 * ( 1372624/2835 )

       + ep^-3*cf^2*nf*proj2*a^3 * (  - 8042896/33075 )

       + ep^-3*cf^2*nf*proj2*fl02*a^3 * ( 2745248/694575 )

       + ep^-3*cf^2*ca*proj2*a^3 * ( 44235928/33075 )

       + ep^-3*cf^3*proj2*a^3 * ( 2851206632/3472875 )

       + ep^-2*cf*nf*proj2*a^2 * (  - 11344/315 )

       + ep^-2*cf*nf*proj2*fl02*a^2 * ( 1936/2205 )

       + ep^-2*cf*nf^2*projL*a^3 * ( 128/63 )

       + ep^-2*cf*nf^2*projL*fl02*a^3 * ( 704/2205 )

       + ep^-2*cf*nf^2*proj2*a^3 * ( 24865088/297675 )

       + ep^-2*cf*nf^2*proj2*fl02*a^3 * (  - 7616288/2083725 )

       + ep^-2*cf*ca*proj2*a^2 * ( 62392/315 )

       + ep^-2*cf*ca*nf*projL*a^3 * (  - 1408/63 )

       + ep^-2*cf*ca*nf*projL*fl02*a^3 * (  - 6952/1715 )

       + ep^-2*cf*ca*nf*proj2*a^3 * (  - 278999824/297675 )

       + ep^-2*cf*ca*nf*proj2*fl02*a^3 * ( 632446364/14586075 )

       + ep^-2*cf*ca^2*projL*a^3 * ( 3872/63 )

       + ep^-2*cf*ca^2*proj2*a^3 * ( 148961968/59535 )

       + ep^-2*cf^2*proj2*a^2 * ( 4021448/11025 )

       + ep^-2*cf^2*nf*projL*a^3 * (  - 22688/735 )

       + ep^-2*cf^2*nf*projL*fl02*a^3 * (  - 105424/77175 )

       + ep^-2*cf^2*nf*proj2*a^3 * (  - 452277148/496125 )

       + ep^-2*cf^2*nf*proj2*fl02*a^3 * ( 706249276/24310125 )

       + ep^-2*cf^2*ca*projL*a^3 * ( 124784/735 )

       + ep^-2*cf^2*ca*proj2*a^3 * ( 318925714/70875 )

       + ep^-2*cf^3*projL*a^3 * ( 8042896/77175 )
```



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       + ep^-2*cf^3*proj2*a^3 * ( 193359459838/121550625 )

       + ep^-1*cf*proj2*a * ( 11344/105 )

       + ep^-1*cf*nf*projL*a^2 * (  - 64/21 )

       + ep^-1*cf*nf*projL*fl02*a^2 * (  - 352/735 )

       + ep^-1*cf*nf*proj2*a^2 * (  - 352924/3675 )

       + ep^-1*cf*nf*proj2*fl02*a^2 * ( 132788/25725 )

       + ep^-1*cf*nf^2*projL*a^3 * ( 63008/2205 )

       + ep^-1*cf*nf^2*projL*fl02*a^3 * ( 805408/231525 )

       + ep^-1*cf*nf^2*proj2*a^3 * ( 11064863104/31255875 )

       + ep^-1*cf*nf^2*proj2*fl02*a^3 * (  - 668677264/31255875 )

       + ep^-1*cf*ca*projL*a^2 * ( 352/21 )

       + ep^-1*cf*ca*proj2*a^2 * ( 1061458/2205 )

       + ep^-1*cf*ca*nf*projL*a^3 * (  - 12232672/33075 + 512/7*z3 )

       + ep^-1*cf*ca*nf*projL*fl02*a^3 * (  - 72036532/1620675 )

       + ep^-1*cf*ca*nf*proj2*a^3 * (  - 47221693441/10418625 + 61568/63*z3 )

       + ep^-1*cf*ca*nf*proj2*fl02*a^3 * ( 50568844114/218791125 + 5632/2205*
         z3 )

       + ep^-1*cf*ca^2*projL*a^3 * ( 38160616/33075 - 2816/7*z3 )

       + ep^-1*cf*ca^2*proj2*a^3 * ( 1742475480083/125023500 - 75611168/11025*
         z3 )

       + ep^-1*cf^2*projL*a^2 * ( 22688/735 )

       + ep^-1*cf^2*proj2*a^2 * ( 461811682/1157625 )

       + ep^-1*cf^2*nf*projL*a^3 * (  - 4246112/33075 - 1024/7*z3 )

       + ep^-1*cf^2*nf*projL*fl02*a^3 * (  - 267890384/24310125 )

       + ep^-1*cf^2*nf*proj2*a^3 * (  - 207007922507/72930375 - 32384/63*z3 )

       + ep^-1*cf^2*nf*proj2*fl02*a^3 * ( 6166968943/43758225 - 26752/2205*z3
          )

       + ep^-1*cf^2*ca*projL*a^3 * ( 150516896/165375 + 106368/245*z3 )

       + ep^-1*cf^2*ca*proj2*a^3 * ( 32362552717471/2187911250 - 2818272/1225*
         z3 )

       + ep^-1*cf^3*projL*a^3 * (  - 526407736/2701125 + 181504/245*z3 )

       + ep^-1*cf^3*proj2*a^3 * ( 17849453368609/10939556250 + 47021696/11025*
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  z3 )

+ proj2 * ( 16 )

+ cf*projL*a * ( 32/7 )

+ cf*proj2*a * ( 3896/315 )

+ cf*nf*projL*a^2 * (  - 19616/735 )

+ cf*nf*projL*fl02*a^2 * (  - 281384/77175 )

+ cf*nf*proj2*a^2 * (  - 958728089/3472875 )

+ cf*nf*proj2*fl02*a^2 * ( 156115511/8103375 )

+ cf*nf^2*projL*a^3 * ( 3284384/15435 )

+ cf*nf^2*projL*fl02*a^3 * ( 75506944/3472875 )

+ cf*nf^2*proj2*a^3 * ( 1285477058744/1093955625 - 1043648/2835*z3 )

+ cf*nf^2*proj2*fl02*a^3 * (  - 180894407804/1531537875 + 297152/19845*
  z3 )

+ cf*ca*projL*a^2 * ( 2014408/11025 - 384/7*z3 )

+ cf*ca*proj2*a^2 * ( 483662519/277830 - 33216/35*z3 )

+ cf*ca*nf*projL*a^3 * (  - 34580648092/10418625 + 768/7*z4 + 12037216/
  11025*z3 )

+ cf*ca*nf*projL*fl02*a^3 * (  - 3593116634/14586075 - 22016/1575*z3 )

+ cf*ca*nf*projL*fl11*a^3 * ( 5869993/7875 - 7680/7*z5 + 7552/21*z3 )

+ cf*ca*nf*proj2*a^3 * (  - 185596210943447/13127467500 + 30784/21*z4
   + 583131488/99225*z3 )

+ cf*ca*nf*proj2*fl02*a^3 * ( 194988584182433/160811476875 + 2816/735*
  z4 - 6887656/46305*z3 )

+ cf*ca*nf*proj2*fl11*a^3 * (  - 7439888/2625 + 28160/7*z5 - 96832/105*
  z3 )

+ cf*ca^2*projL*a^3 * ( 244541735537/20837250 + 8320/7*z5 - 4224/7*z4
   - 78641128/11025*z3 )

+ cf*ca^2*proj2*a^3 * ( 226508049563851/5834430000 + 46016/7*z5 -
  37805584/3675*z4 - 2591997496/99225*z3 )

+ cf^2*projL*a^2 * ( 681032/25725 + 768/7*z3 )

+ cf^2*proj2*a^2 * ( 124821228631/243101250 + 3008/5*z3 )

+ cf^2*nf*projL*a^3 * ( 68865586472/72930375 - 1536/7*z4 - 101504/49*z3
   )

+ cf^2*nf*projL*fl02*a^3 * (  - 15173112008/156279375 + 126208/3675*z3
   )
```



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+ cf^2*nf*projL*fl11*a^3 * (  - 46959944/23625 + 20480/7*z5 - 60416/63*
  z3 )

+ cf^2*nf*proj2*a^3 * (  - 2253512214831097/153153787500 - 16192/21*z4
   + 83738416/11025*z3 )

+ cf^2*nf*proj2*fl02*a^3 * ( 2347676150813041/3216229537500 - 13376/735
  *z4 - 37354496/138915*z3 )

+ cf^2*nf*proj2*fl11*a^3 * ( 59519104/7875 - 225280/21*z5 + 774656/315*
  z3 )

+ cf^2*ca*projL*a^3 * (  - 1227903231868/364651875 - 3840*z5 + 159552/
  245*z4 + 871554016/77175*z3 )

+ cf^2*ca*proj2*a^3 * ( 4229331857573299/61261515000 + 62784/7*z5 -
  4227408/1225*z4 - 57102186536/1157625*z3 )

+ cf^3*projL*a^3 * (  - 7011150686566/1093955625 + 20480/7*z5 + 272256/
  245*z4 + 116194976/25725*z3 )

+ cf^3*proj2*a^3 * ( 170295294522201013/6432459075000 - 18304*z5 +
  23510848/3675*z4 - 20777060296/3472875*z3 )

+ ep*proj2 * (  - 16 )

+ ep*cf*projL*a * ( 552/35 )

+ ep*cf*proj2*a * ( 6332/105 )

+ ep*cf*nf*projL*a^2 * (  - 1963088/15435 )

+ ep*cf*nf*projL*fl02*a^2 * (  - 41615962/2701125 )

+ ep*cf*nf*proj2*a^2 * (  - 1026393555199/1458607500 + 363008/945*z3 )

+ ep*cf*nf*proj2*fl02*a^2 * ( 248522310551/3403417500 - 74048/6615*z3 )

+ ep*cf*ca*projL*a^2 * ( 3755125936/3472875 - 576/7*z4 - 117632/245*z3
   )

+ ep*cf*ca*proj2*a^2 * ( 413577123433/116688600 - 49824/35*z4 -
  62710288/33075*z3 )

+ ep*cf^2*projL*a^2 * (  - 57710903164/72930375 + 1152/7*z4 + 703552/
  735*z3 )

+ ep*cf^2*proj2*a^2 * ( 49105805823569/11344725000 + 4512/5*z4 -
  19899136/4725*z3 )

+ ep^2*cf*projL*a * ( 9656/315 )

+ ep^2*cf*proj2*a * ( 7738/45 - 11344/45*z3 );
```

Conclusion and Outlook

- Charged Current DIS calculations up to $\alpha_s{}^3$ are important.
- We use **QGRAF**, **FORM**, **MINCER** and in addition our **own codes** to perform calculations.
- First results are obtained. **Gauge invariance check** is still not finished.