Estimates of statistical uncertainty of new experiments

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Different vacua in 2HDM

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Stages of discussion of future and modern experiments

1. Analytical calculation of amplitudes or cross sections,
2. MC simulation of final states,
3. Extraction of physical quantities

$\Diamond$ Averaged momenta of final particles or their combinations,

Example: W mass.

$\Diamond$ Other quantities obtained by comparison with a model e.g. by least-squares method.

Example – value of anomalous magnetic moment via process $e\gamma \to \mu\nu\nu$

Different uncertainties

Statistical uncertainty δ_{stat}

Systematical uncertainty

We discuss only first.

Standard estimate

Let us have N events $\Rightarrow \delta_{stat} \approx 1/\sqrt{N}$

In some cases this estimate is too optimistic.

How to find?

MC calculation *IMITATE* statistical distribution of real experiment. Therefore, one can determine statistical uncertainty of experiment by REPETITION of MC experiment (what cannot be done in real experiment) – numerical experiment .

Conclusion

One should repeat MC calculation several times with anticipated number of events, varying seed numbers for random number generator. It will give us the statistical parameter spread. This spread must be prepared with standard procedure like physical data spread.

EXAMPLE

$\gamma\gamma \to \mu^+\mu^-\nu\bar{\nu}$ with polarized photons in SM

(Kanishev report)

We consider normalized mean values of longitudinal $p_{\|}^{\mp}$ and transverse $p_{\perp}^{\mp}$ momenta of negative and positive muons in the forward hemisphere ($p_{\|} > 0$, subscript $+$), and charge asymmetry values Δ_L and Δ_T at different photon polarizations (left – γ_-, right – γ_+):

$$P_{L,T+}^{\pm} = \frac{\int p_{\|,\perp}^{\pm} d\sigma}{E_{\gamma max} \int d\sigma}, \quad \Delta_{L,T} = \frac{P_{L,T+}^{-} - P_{L,T+}^{+}}{P_{L,T+}^{-} + P_{L,T+}^{+}}$$

We evaluate these quantities and their statistical uncertainties at $\sqrt{s} = 0.5$ TeV at a given expected number of generated events (about 10^6) by repeating the calculation 5 times with different seed number inputs for MC. We also consider, as an independent set of observations, events obtained by simultaneous change $\lambda_1, \lambda_2 \to -\lambda_1, -\lambda_2, \mu^- \leftrightarrow \mu^+$ (this change should not change distributions due to CP conservation in SM) (equivalently 10 repetition). The table presents averaged values of these quantities together with their relative statistical uncertainties.

Note: High energy photons in Photon Collider will be circularly polarized.

$\gamma_{\lambda_1}\gamma_{\lambda_2}$	N	P_N^- δP_N^-	P_N^+ δP_N^+	Δ_N $\delta\Delta_N$
$\gamma_-\gamma_-$	L	0.599 0.35%	0.170 0.37%	0.557 0.37%
	T	0.338 0.96%	0.150 0.42%	0.386 0.99%
$\gamma_+\gamma_-$	L	0.209 0.82%	0.556 0.34%	−0.454 0.52%
	T	0.159 0.72%	0.249 0.82%	−0.220 2.52%

Statistical uncertainty is 3-10 times larger than $1/\sqrt{N} = 10^{-3}$. It is naturally to expect that these uncertainties will be increased when we take into account non-monocromaticity of photons (consider energy spectra of photons).

This result mean, in particular, that radiative corrections to this process are beyond the accuracy of measurement *(except FSR)*.

We try to use in our work CompHEP. However we could not do it for two reasons.

- Modern version of CompHEP don't allow to change seed number for input of MC without authors.

- Modern version of CompHEP don't allow to consider polarized photon beams.

We used **CalcHEP** which modern version allows both to change seed number for input of MC and to take into account photon polarization.

Different vacua in 2HDM

Motivation

In first moments after Big Bang the temperature of Universe T was very high, in this stage vacuum expectation values of Higgs fields are given by minimum of Gibbs potential Φ. The latter is a sum of Higgs potential $V(\phi)$ and term $aT^2\phi^2$ (in the SM). – We obtain Higgs model with varying in time parameters. At large T potential has EW symmetric minimum at $\langle\phi\rangle = 0$. This stage describes widely discussed phenomenon of *inflation*. During inflatory expansion the Universe become colder, and at some temperature the Gibbs potential transforms into well known form of Higgs model with $\langle\phi\rangle \neq 0$ – we obtain our world with massive particles etc (EWSB). This phase transition determine fate of Universe after inflation.

1. $2\mathcal{HDM}$ – simplest extension of SM Higgs sector, containing 2 scalar doublets (weak isospinors) ϕ_1 and ϕ_2.

2. Higss sector of MSSM – particular case of $2\mathcal{HDM}$

We have in mind Higgs model with parameters varying in time with Lagrangian

$$\mathcal{L} = \mathcal{L}_{gf}^{SM} + T - V + \mathcal{L}_Y\,;$$

$\mathcal{L}_{gf}^{SM}$ – $\mathcal{SM}$ interaction, gauge bosons + fermions,

$\mathcal{L}_Y$ – Yukawa interaction of fermions to scalars,

T – Higgs kinetic term, V – Higgs potential.

With isoscalars

$$x_1 = \phi_1^\dagger \phi_1, \quad x_2 = \phi_2^\dagger \phi_2, \quad x_3 = \phi_1^\dagger \phi_2$$

we have

$$V = -\frac{1}{2}\left[m_{11}^2 x_1 + m_{22}^2 x_2 + \left(m_{12}^2 x_3 + h.c.\right)\right]$$
$$+\frac{\lambda_1}{2} x_1^2 + \frac{\lambda_2}{2} x_2^2 + \lambda_3 x_1 x_3 + \lambda_4 x_3 x_3^\dagger + \left[\frac{\lambda_5}{2} x_3^2 + (\lambda_6 x_1 + \lambda_7 x_2)\, x_3 + h.c.\right].$$

λ_{5-7} and m_{12} are generally complex.

In the extremum of potential $x_i \to \langle x_i \rangle = y_i$ – common numbers.

Vacuum energy V_{vac} is given by the same equation with $x_i \to y_i$.

Vacuum corresponds lowest value of this form.

The extremes of the potential, EWSB

define the v.e.v.'s $\langle\phi_i\rangle$ via

$$\boxed{\frac{\partial V}{\partial \phi_i}(\phi_1 = \langle\phi_1\rangle,\ \ \phi_2 = \langle\phi_2\rangle) = 0.}$$

This equation has trivial electroweak symmetry conserving solution $\langle\phi_1\rangle = 0$, $\langle\phi_2\rangle = 0$ and electroweak symmetry violating solutions.

With accuracy to the choice of z axis in the weak isospin space, most general solution has form

$$\langle\phi_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \ \ \langle\phi_2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} u \\ v_2 e^{i\xi} \end{pmatrix};$$

It is easy to check that $\partial x_1/\partial\phi_2 = \partial x_2/\partial\phi_1 = 0$ and

$$x_3\left(\frac{\partial x_1}{\partial\phi_1}\phi_1\right) - x_1\left(\frac{\partial x_1}{\partial\phi_1}\phi_2\right) = x_3^*\left(\frac{\partial x_3^*}{\partial\phi_1}\phi_2\right) - x_2\left(\frac{\partial x_3^*}{\partial\phi_1}\phi_1\right) = 0,$$
$$x_3\left(\frac{\partial x_3^*}{\partial\phi_1}\phi_1\right)-x_1\left(\frac{\partial x_3^*}{\partial\phi_1}\phi_2\right)=x_3^*\left(\frac{\partial x_1}{\partial\phi_1}\phi_2\right)-x_2\left(\frac{\partial x_1}{\partial\phi_1}\phi_1\right)=x_3x_3^*-x_1x_2.$$

Now, denoting $Z = y_3^* y_3 - y_1 y_2$, the extremum condition can be rewritten as

$$\left\langle x_3\left(\frac{\partial V}{\partial\phi_1}\phi_1\right)-x_1\left(\frac{\partial V}{\partial\phi_1}\phi_2\right)\right\rangle = Z\left(\lambda_4 y_3+\lambda_5^* y_3^*+\lambda_6^* y_1+\lambda_7^* y_2-\frac{m_{12}^{*2}}{2}\right)=0,$$
$$\left\langle x_3^*\left(\frac{\partial V}{\partial\phi_1}\phi_2\right)-x_2\left(\frac{\partial V}{\partial\phi_1}\phi_1\right)\right\rangle = Z\left(\lambda_1 y_1+\lambda_3 y_2+\lambda_6^* y_3^*+\lambda_6 y_3-\frac{m_{11}^{2}}{2}\right)=0,$$
$$\left\langle x_3\left(\frac{\partial V}{\partial\phi_2}\phi_1\right)-x_1\left(\frac{\partial V}{\partial\phi_2}\phi_2\right)\right\rangle = Z\left(\lambda_2 y_2+\lambda_3 y_1+\lambda_7^* y_3^*+\lambda_7 y_3-\frac{m_{22}^{2}}{2}\right)=0.$$

Therefore, two opportunities can be realized for extremum,

$\boxed{Z = 0 \text{ and } Z \neq 0.}$

$$\boxed{Z = y_3^* y_3 - y_1 y_2 \neq 0} \Rightarrow \boxed{u \neq 0}$$

The v.e.v.'s are given by the system of linear algebraic equations with unique solution

$$\begin{aligned}
\lambda_1 y_1 + \lambda_3 y_2 + \lambda_6^* y_3^* + \lambda_6 y_3 &= m_{11}^2/2, \\
\lambda_2 y_2 + \lambda_3 y_1 + \lambda_7^* y_3^* + \lambda_7 y_3 &= m_{22}^2/2, \\
\lambda_4 y_3^* + \lambda_5 y_3 + \lambda_6 y_1 + \lambda_7 y_2 &= m_{12}^2/2.
\end{aligned}$$

In this case it is not possible to split the gauge boson mass matrix into a neutral and charged sector, the interaction of gauge bosons with fermions will not preserve electric charge, photon become massive, etc. – we have a charged vacuum, with a heavy photon! and other nonphysical properties (J. L. Diaz-Cruz et al. (1992,1993).

We are in another word, BUT ...

$$\boxed{Z = y_3^* y_3 - y_1 y_2 = 0} \Rightarrow \boxed{u = 0}$$

Standard vacuum

Another solution of extremum condition

2) with $\boxed{Z = y_3^* y_3 = y_1 y_2} \Rightarrow \boxed{u = 0}$.

$$\Rightarrow \langle\phi_1\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \quad \langle\phi_2\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 e^{i\xi} \end{pmatrix}.$$

It satisfies a condition for $U(1)$ symmetry of electromagnetism. Standard $v_1 = v\cos\beta$, $v_2 = v\sin\beta$ with the $\mathcal{SM}$ constraint $v = \left(G_F\sqrt{2}\right)^{-1/2} = 246$ GeV. It can describe reality.

In this case quantities y_i cannot be considered as independent, the v.e.v.'s cannot be obtained directly by minimization of form V_{vac} in y_i – equations for v.e.v.'s form system of nonlinear equations with many specific features.

Simple model

We construct toy model with $\lambda_1 = \lambda_2 = \lambda_3 = \lambda$, $\lambda_4 = \lambda_6 = \lambda_7 = 0$, $m_{11}^2 = m_{22}^2 = 2m^2$, $m_{12}^2 = 2\mu$ and all real coefficients

$$V = \frac{\lambda}{2}(x_1 + x_2)^2 + \frac{\lambda_5}{2}\left(x_3^2 + x_3^{\dagger 2}\right) - m^2(x_1 + x_2) - \mu^2(x_3 + x_3^\dagger)\,.$$

Stability (positivity) condition reads in this case as $\lambda > |\lambda_5|$.

This model contains extra symmetry $\phi_1 \leftrightarrow \phi_2$, it results in some degenerations which are absent in more realistic case.

$Z \neq 0$, charged vacuum

The vacuum state is given by 2 equations

$$\lambda_5 y_3 - \mu^2 = 0 \;\Rightarrow y_3 = \frac{\mu^2}{\lambda_5}, \quad \lambda(y_1 + y_2) - m^2 = 0 \;\Rightarrow y_1 + y_2 = \frac{m^2}{\lambda}.$$

In this case our degeneration don't allow describe y_1 and y_2 separately. The vacuum energy in this case is independent on relation between v_1 and v_2:

$$V_{vac}^{ch} = -\frac{m^4}{2\lambda} - \frac{\mu^4}{\lambda_5}$$

$Z = 0$, neutral vacua

Let us present set of eq-s for v.e.v.'s in our case:

$$\partial V/\partial\phi_1^\dagger\Big|_{\phi_i=v_i} = \lambda(y_1+y_2)v_1 + \lambda_5 y_3 v_2 - m^2 v_1 - \mu^2 v_2 = 0\,,\ (A)$$
$$\partial V/\partial\phi_2^\dagger\Big|_{\phi_i=v_i} = \lambda(y_1+y_2)v_2 + \lambda_5 y_3^* v_1 - m^2 v_2 - \mu^2 v_1 = 0\,.\ (B)$$

Let us organize:

$$\langle (A)\phi_2^\dagger - (B)^\dagger\phi_1\rangle = (y_2-y_1)(\lambda_5 y_3 - \mu^2) = 0\,. \qquad (1)$$

So that we have two solutions

$$(I):\ y_1 = y_2 \equiv y = v^2/2\ (tan\beta = 1),\ \ y_3 = ye^{i\xi}\,;$$
$$(II):\ y_3 = \mu^2/\lambda_5\,.$$

Solution II

Solution (II) violates our accidental degeneracy. Having in mind (A) we obtain

$$y_1 + y_2 = m^2/\lambda.$$

This sum is the same as in the case of charged vacuum.

$$\Rightarrow \; V_{vac}^{II} = V_{vac}^{ch}\,.$$

In this case condition $Z = 0$ ($y_3^2 = y_1 y_2$) allows specify y_1, y_2 in contrast with the case of $Z \neq 0$ as

$$y_{1,2} \equiv v_{1,2}^2 = \frac{m^2}{2\lambda} \pm \sqrt{\frac{m^4}{4\lambda^2} - \frac{\mu^4}{\lambda_5^2}}\ .$$

Solutions I

For this solution we have with above equations for v.e.v.'

$$V^I_{vac} = \frac{\lambda}{2} v^4 + \frac{\lambda_5}{2} v^4 cos 2\xi - m^2 v^2 - \mu^2 v^2 cos\xi \,.$$

To find extremum, first find minimum in ξ at fixed v. It gives

$$\lambda_5 v^4 sin 2\xi - 2\mu^2 sin\xi \;\Rightarrow \begin{cases} (a) \quad sin\xi = 0\,, \\ (b) \quad cos\xi = \dfrac{\mu^2}{2\lambda_5 v^2}\,. \end{cases}$$

After that we have for solution Ia

$$V^{Ia}_{vac} = \frac{(\lambda + \lambda_5)}{2} v^4 - (m^2 + \mu^2) v^2 \;\Rightarrow\; v^2 = \frac{m^2 + \mu^2}{\lambda + \lambda_5} \Rightarrow$$
$$\Rightarrow V^{Ia}_{vac} = -\frac{(m^2 + \mu^2)^2}{2(\lambda + \lambda_5)}\,,$$

To find vacuum energy for solution Ib, we insert obtained value $cos\xi$ and with $cos2\xi = 2cos^2\xi - 1$. Finally we have

$$V_{vac}^{Ib} = \frac{\lambda}{2}v^4 + \frac{\lambda_5}{2}v^4 \left(2\frac{\mu^4}{4\lambda_5^2 v^4} - 1\right) - m^2 v^2 - \frac{\mu^4}{2\lambda_5} =$$

$$= \frac{(\lambda - \lambda_5)}{2}v^4 - m^2 v^2 - \frac{\mu^4}{4\lambda_5} \Rightarrow$$

$$\Rightarrow v^2 = \frac{m^2}{(\lambda - \lambda_5)}, \quad cos\xi = \frac{\mu^2(\lambda - \lambda_5)}{2m^2\lambda_5} \Rightarrow$$

$$\Rightarrow V_{vac}^{Ib} = -\frac{m^4}{2(\lambda - \lambda_5)} - \frac{\mu^4}{4\lambda_5}.$$

CP violated state based on potential with all real coefficients.

Let us compare energies for different vacua

$$V^{Ia}_{vac} - V^{ch}_{vac} = \frac{(m^2\lambda_5 - \mu^2\lambda)^2}{2\lambda\lambda_5(\lambda+\lambda_5)}$$

$$V^{Ib}_{vac} - V^{ch}_{vac} = \frac{-m^4\lambda_5^2 + \mu^4\lambda(\lambda-\lambda_5)}{4\lambda\lambda_5(\lambda-\lambda_5)}$$

$$V^{Ib}_{vac} - V^{Ia}_{vac} = -\frac{[2m^2\lambda_5 - \mu^2(\lambda-\lambda_5)]^2}{4\lambda_5(\lambda^2-\lambda_5^2)}$$

What extremum is realized as vacuum state, depends on parameters.

In particular at $\mu^2 = m^2/2$, $\lambda_5 = 0.7\lambda$ and denoting $m^4/\lambda = \varepsilon$, we have CP violating vacuum state

$$V^{Ib}_{vac} = -1.75\varepsilon \;\; (cos\xi = 3/28 \approx 0.1)\,, \;\; V^{Ia}_{vac} = -0.66\varepsilon\,, \;\; V^{II}_{vac} \equiv V^{ch}_{vac} = -0.86\varepsilon\,.$$

Variation of parameters of Gibbs potential with temperature and density can change vacua. For each of them neutral direction can different, some of realized during evolution vacuums can be charged, CP violation can appear and disappear. Huge fluctuations at these transitions can be related to a modern structure of matter in Universe, domains of intermediate phases can exist long time influencing evolution of Universe.
This sequence of phase transitions demands detail study.