Modification of dipole picture fo scattering for baryon

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Outline

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- Baryon Wilson loop
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Definitions

Introduce the light cone vectors n_1 and n_2

$$n_1 = (1,0,0,1)\,, \quad n_2 = \frac{1}{2}(1,0,0,-1)\,, \quad n_1^+ = n_2^- = n_1 n_2 = 1$$

For any p define $p^\pm$

$$p^+ = pn_2 = \frac{1}{2}\left(p^0 + p^3\right), \qquad p^- = pn_1 = p^0 - p^3,$$

$$p^2 = 2p^+p^- - \vec{p}^{\,\mathbf{2}};$$

The scalar products:

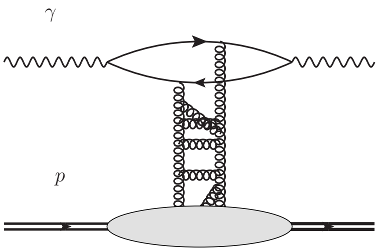
$$p = p^+ n_1 + p^- n_2 + p_\perp, \quad (p\,k) = p^\mu k_\mu = p^+ k^- + p^- k^+ - \vec{p}\vec{k}.$$

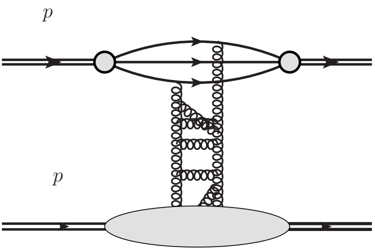
Definitions

Wilson line describing interaction with external field b_η^- made of slow gluons with $p^+ < e^\eta$

$$U_{\vec{z}}^\eta = Pe^{ig\int_{-\infty}^{+\infty} dz^+ b_\eta^-(z^+,\vec{z})}, \quad b_\eta^- = \int \frac{d^4p}{(2\pi)^4} e^{-ipz} b^-(p)\,\theta(e^\eta - p^+).$$

Motivation





Dipole picture
$s \gg Q^2 \gg \Lambda^2_{QCD}$

?

$$\sigma_{\gamma*}(s, Q^2) = \int d^2\mathbf{r}|\Psi_{\gamma*}(\mathbf{r}, Q^2)|^2\, \sigma_{dip}(\mathbf{r}, s), \quad \sigma_{dip}(\mathbf{r}, s) = 2\int d\mathbf{b}(1 - \frac{1}{N_c}F(\mathbf{b}, \mathbf{r}, s))$$

$\mathbf{r} = \mathbf{r_1} - \mathbf{r_2}$ — dipole size, $\mathbf{b} = \frac{1}{2}(\mathbf{r_1} + \mathbf{r_2})$ — impact parameter, $F = tr(U_1 U_2^\dagger)$, — dipole Green function, $U_i = U_i^\eta$ — Wilson lines, describing fast moving quarks interacting with the target.

η — rapidity divide, gluons with $p^+ > e^\eta$ belong to photon wavefunction, gluons with $p^+ < e^\eta$ belong to Wilson lines, describing the field of the target.

Motivation

$tr(U_1 U_2^\dagger)$ obeys the LO Balitsky-Kovchegov evolution equation

$$\frac{\partial tr(U_1 U_2^\dagger)}{\partial \eta} = \frac{\alpha_s}{2\pi^2} \int d\vec{z}_4 \frac{\vec{z}_{12}^{\,2}}{\vec{z}_{14}^{\,2}\vec{z}_{42}^{\,2}} \left[tr(U_1 U_4^\dagger) tr(U_4 U_2^\dagger) - N_c tr(U_1 U_2^\dagger) \right].$$

LO equation was obtained in 1996-99, NLO — in 2007-2010 (Balitsky and Chirilli).

Previous work

- M. Praszalowicz and A. Rostworowski, 1998 - Proton wave function with one and two gluon emissions was studied. Indication that new color structures, not only dipoles and three-quark singlets (like proton) appear.
- Y. Hatta, E. Iancu, K. Itakura and L. McLerran, 2005 - Odderon in the color glass condensate was studied. Linear evolution equation for 3-quark Wilson line (its C-odd part) was obtained in the coordiante representation. It was shown that this equation is equivalent to the BKP equation in the momentum representation.
- J. Bartels and L. Motyka, 2008 - Wave function, impact factor were studied. Gluon radiation was diagonalized into the evolution of 2-quark, 3-quark, and 4-quark states in C-even and C-odd states obeying the BKP equations with nonlinear terms.

Shock wave

For a fast moving particle with the velocity $-\beta$ and the field strength tensor $\mathbb{F}(x^+, x^-, \vec{x})$ in its rest frame, in the observer's frame the field will look like

$$\mathfrak{F}^{-i}\left(y^+, y^-, \vec{y}\right) = \lambda \mathbb{F}^{-i}(\lambda y^+, \frac{1}{\lambda} y^-, \vec{y}) \to \delta\left(y^+\right) \mathfrak{F}^i\left(\vec{y}\right),$$

$$\mathfrak{F}^{-i} \gg \mathfrak{F}^{\cdots}$$

in the Regge limit $\lambda \to +\infty$, $\lambda = \sqrt{\frac{1+\beta}{1-\beta}}$.

Therefore the natural choice for the gauge is $b^{i,+} = 0$,

b^- is the solution of the equations

$$\frac{\partial b^-}{\partial y^i} = \delta(y^+)\mathfrak{F}^i\left(\vec{y}\right), i.e.$$

$$b^\mu\left(y\right) = \delta(y^+) B\left(\vec{y}\right) n_2^\mu$$

It is the shock-wave field.

Propagator in the shock wave background

Choose the gluon field $\mathcal{A}$ in the gauge $\mathcal{A}n_2 = 0$ as a sum of external classical b and quantum A.

$$\mathcal{A} = A + b, \quad b^{\mu}(x) = \delta(x^{+})B(\vec{x})\, n_2^{\mu}.$$

The A-b interaction lagrangian has only one vertex

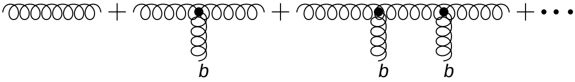
$$\mathcal{L}_i = \frac{g}{2} f^{acb}(b^{-})^{c} g_{\perp}^{\alpha\beta} \left[A_{\alpha}^{a} \frac{\overleftrightarrow{\partial}}{\partial x^{-}} A_{\beta}^{b} \right].$$

The free propagator $G_0^{\mu\nu}(x^{+}, p^{+}, \vec{p}) =$

$$= \frac{-d_0^{\mu\nu}(p^{+}, p_{\perp})}{2p^{+}} e^{-i\frac{\vec{p}^{\,2} x^{+}}{2p^{+}}} \left(\theta(x^{+})\theta(p^{+}) - \theta(-x^{+})\theta(-p^{+})\right) + n_2^{\mu} n_2^{\nu} \ldots,$$

$$d_0^{\mu\nu}(p) = g_{\perp}^{\mu\nu} - \frac{p_{\perp}^{\mu} n_2^{\nu} + p_{\perp}^{\nu} n_2^{\mu}}{p^{+}} - \frac{n_2^{\mu} n_2^{\nu} \vec{p}^{\,2}}{(p^{+})^{2}}.$$

Propagator in the shock-wave background



Sum the diagrams

- b does not depend on x^-, hence the conservation of p^+,
- $b \sim \delta(x^+)$, hence $e^{-i\frac{\vec{p}^{\,2}(x_1^+ - x_2^+)}{2p^+}} \to 1$ in every internal vertex,
- $g_\perp^{\mu\nu} d_{0\nu\rho} g_\perp^{\rho\sigma} = g_\perp^{\mu\sigma}$, hence no dependence on $\vec{p} \Longrightarrow$ conservation of $\vec{x}$ in every internal vertex

Propagator in the shock-wave background:

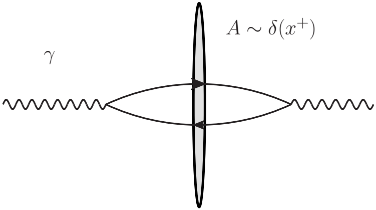
$$G_{\mu\nu}(x,y)|_{x^+>0>y^+} = 2iA^\mu(x) \int d^4z\,\delta(z^+) F^{+i}(z) \frac{U_{\vec{z}}}{\overrightarrow{\Longrightarrow} \atop \frac{\partial}{\partial z^-}} F^{+i}(z) A^\nu(y)\,.$$

where the interaction with b is through Wilson line

$$U_{\vec{z}} = P e^{ig\int_{y^+}^{x^+} dz^+ b^-(z^+,\vec{z})}.$$

Shock wave formalism

LO I. Balitsky 1996, NLO I. Balitsky and G. Chirilli 2006-2013



Color field of a fast moving particle $A^- \sim \delta(z^+)A^\eta(z_\perp)$
$A^\eta(z_\perp)$ contains slow components with rapidities $< \eta$

Quark propagator in such an external field $G(x, y) \sim U^\eta(z_\perp)$

DIS matrix element contains a Wilson loop = color dipole operator $U^\eta_{12} = tr(U^\eta(z_{1\perp})U^{\eta\dagger}(z_{2\perp}))$.

Baryon (3 quark) Wilson loop — 3QWL

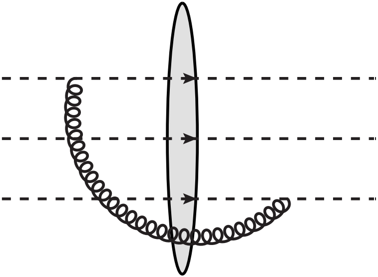
$$B_{123} = \varepsilon^{i'j'h'} \varepsilon_{ijh} U(\vec{z}_1)^i_{i'} U(\vec{z}_2)^j_{j'} U(\vec{z}_3)^h_{h'} = U_1 \cdot U_2 \cdot U_3.$$

- B_{123} is gauge invariant since under a gauge rotation the Wilson lines change

$$U(\vec{z}_1)^i_{i'} \to V(x)^i_k U(\vec{z}_1)^k_{k'} V(y)^{k'}_{i'}, \quad V \in SU(3).$$

- $\varepsilon^{i'j'h'} U^i_{i'} U^j_{j'} U^h_{h'} = \varepsilon^{ijh}$,
- $\varepsilon_{ijh} \varepsilon^{i'j'h'} U^i_{i'} U^j_{j'} = 2(U^\dagger)^{h'}_h, \quad \varepsilon_{ijh} \varepsilon^{i'j'h'} (U^\dagger)^i_{i'} (U^\dagger)^j_{j'} = 2U^{h'}_h,$
- $U_i \cdot U_j \cdot U_k = (U_i U_l^\dagger) \cdot (U_j U_l^\dagger) \cdot (U_k U_l^\dagger).$
- $B^\eta_{iij} = U_i \cdot U_i \cdot U_j = 2tr(U_j U_i^\dagger)$, i.e. quark-diquark and quark-antiquark systems are described by the same operator.

LO Evolution equation for a baryon Wilson loop



$$\Delta B_r = \frac{\Delta\eta\alpha_s}{\pi^2}\int d\vec{z}_4 (U_4^{\eta_2})^{ab}$$

$$\times\left[\frac{1}{\vec{z}_{41}^{\,2}}\left(t^a U_1^{\eta_2} t^b\right)\cdot U_2^{\eta_2}\cdot U_3^{\eta_2} + (1\leftrightarrow 2) + (1\leftrightarrow 3)\right.$$

$$+\frac{\vec{z}_{41}\vec{z}_{42}}{\vec{z}_{41}^{\,2}\vec{z}_{42}^{\,2}}\left((t^a U_1^{\eta_2})\cdot(U_2^{\eta_2} t^b) + (U_1^{\eta_2} t^b)\cdot(t^a U_2^{\eta_2})\right)\cdot U_3^{\eta_2}$$

$$\left. + (1\leftrightarrow 3) + (2\leftrightarrow 3)\right].$$

Evolution equation for Baryon Green function

Using SU(3) identities

$$U_4^{ba} = 2tr(t^b U_4 t^a U_4^\dagger), \quad t_{ij}^a t_{kl}^a = \frac{1}{2}\delta_{il}\delta_{kj} - \frac{1}{2N_c}\delta_{ij}\delta_{kl}$$

the evolution equation reads

$$\frac{\partial B_{123}^\eta}{\partial \eta} = \frac{\alpha_s}{2\pi^2}\int d\vec{z}_4 \left[\left\{\frac{C_1}{\vec{z}_{41}^{\,2}} + (1 \leftrightarrow 2) + (1 \leftrightarrow 3)\right\}\right.$$

$$\left. + \left\{\frac{\vec{z}_{41}\vec{z}_{42}}{\vec{z}_{41}^{\,2}\vec{z}_{42}^{\,2}} C_{12} + (1 \leftrightarrow 3) + (2 \leftrightarrow 3)\right\}\right].$$

where

$$C_1 = tr\left(U_1^\eta U_4^{\eta\dagger}\right) B_{423}^\eta - 3B_{123}^\eta,$$

$$C_{12} = 2B_{123}^\eta - \left(U_2^\eta U_4^{\eta\dagger} U_1^\eta + U_1^\eta U_4^{\eta\dagger} U_2^\eta\right) \cdot U_4^\eta \cdot U_3^\eta.$$

Evolution equation for baryon Green function

Then we can use the SU(3) identity

$$\left(U_2 U_4^\dagger U_1 + U_1 U_4^\dagger U_2\right) \cdot U_4 \cdot U_3 = -B_{123}^\eta +$$

$$+\frac{1}{2}(B_{144}^\eta B_{324}^\eta + B_{244}^\eta B_{314}^\eta - B_{344}^\eta B_{214}^\eta)$$

to rearrange the equation in the closed way

$$\frac{\partial B_{123}^\eta}{\partial \eta} = \frac{\alpha_s 3}{4\pi^2} \int d\vec{z}_4 \left[\frac{\vec{z}_{12}^{\,2}}{\vec{z}_{41}^{\,2} \vec{z}_{42}^{\,2}} (-B_{123}^\eta + \right.$$

$$\left. + \frac{1}{6}(B_{144}^\eta B_{324}^\eta + B_{244}^\eta B_{314}^\eta - B_{344}^\eta B_{214}^\eta)) \quad + (1 \leftrightarrow 3) + (2 \leftrightarrow 3) \right].$$

In the large N_c limit $\langle B_{144}^\eta B_{324}^\eta \rangle \to \langle B_{144}^\eta \rangle \langle B_{324}^\eta \rangle$.

confirmed by JIMWLK results (A. Kovner M. Lublinsky Y. Mulian)

SU(3) identity

Using $U_i \cdot U_j \cdot U_k = (U_i U_l^\dagger) \cdot (U_j U_l^\dagger) \cdot (U_k U_l^\dagger)$, one can rewrite the identity as

$$-\left(U_2 U_4^\dagger U_1 U_4^\dagger + U_1 U_4^\dagger U_2 U_4^\dagger\right) \cdot E \cdot (U_3 U_4^\dagger)$$

$$= (U_1 U_4^\dagger) \cdot (U_2 U_4^\dagger) \cdot (U_3 U_4^\dagger)$$

$$-tr(U_1 U_4^\dagger)(U_3 U_4^\dagger) \cdot (U_2 U_4^\dagger) \cdot E - tr(U_2 U_4^\dagger)(U_3 U_4^\dagger) \cdot (U_1 U_4^\dagger) \cdot E$$

$$+tr(U_3 U_4^\dagger)(U_2 U_4^\dagger) \cdot (U_1 U_4^\dagger) \cdot E.$$

and prove it expanding the Levi-Civita symbols as

$$\varepsilon_{ijh}\varepsilon^{i'j'h'} = \begin{vmatrix} \delta_i^{i'} & \delta_i^{j'} & \delta_i^{h'} \\ \delta_j^{i'} & \delta_j^{j'} & \delta_j^{h'} \\ \delta_h^{i'} & \delta_h^{j'} & \delta_h^{h'} \end{vmatrix}.$$

Quark-diquark limit

$B^{\eta}_{122} = U_1 \cdot U_2 \cdot U_2 = 2tr(U_1 U_2^{\dagger})$, i.e. quark-diquark and quark-antiquark systems are described by the same operator. The evolution equation should go into the dipole Balitsky-Kovchegov evolution equation as $\vec{z}_{23} \to 0$

$$\frac{\partial tr(U_1 U_2^{\dagger})}{\partial \eta} = \frac{\alpha_s}{2\pi^2} \int d\vec{z}_4 \frac{\vec{z}_{12}^{\,2}}{\vec{z}_{14}^{\,2} \vec{z}_{42}^{\,2}} \left[tr(U_1 U_4^{\dagger}) tr(U_4 U_2^{\dagger}) - N_c tr(U_1 U_2^{\dagger}) \right].$$

Indeed this is the case

$$\frac{\partial B^{\eta}_{122}}{\partial \eta} = \frac{\alpha_s 3}{4\pi^2} \int d\vec{z}_4 \left[\frac{\vec{z}_{12}^{\,2}}{\vec{z}_{41}^{\,2} \vec{z}_{42}^{\,2}} (-B^{\eta}_{122} + \right.$$

$$\left. + \frac{1}{6} (B^{\eta}_{144} B^{\eta}_{224} + B^{\eta}_{244} B^{\eta}_{214} - B^{\eta}_{244} B^{\eta}_{214})) \quad + (1 \to 2) + (2 \leftrightarrow 2) \right].$$

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$$\left. + \frac{1}{6} (B^{\eta}_{144} B^{\eta}_{224} + B^{\eta}_{244} B^{\eta}_{214} - B^{\eta}_{244} B^{\eta}_{214})) \;\; + (1 \to 2) + (2 \leftrightarrow 2) \right].$$

C-even case

We build C-even and C-odd operators with
$B^{\eta}_{\bar{1}\bar{2}\bar{3}} = \varepsilon^{i'j'h'}\varepsilon_{ijh}U^{\eta\dagger}(\vec{z}_1)^{i}_{i'}\,U^{\eta\dagger}(\vec{z}_2)^{j}_{j'}\,U^{\eta\dagger}(\vec{z}_3)^{h}_{h'} = U_1^{\dagger}\cdot U_2^{\dagger}\cdot U_3^{\dagger}$

$$B^{+}_{123} = B^{\eta}_{123} + B^{\eta}_{\bar{1}\bar{2}\bar{3}} - 12, \quad B^{-}_{123} = B^{\eta}_{123} - B^{\eta}_{\bar{1}\bar{2}\bar{3}}$$

$$B^{+}_{123} = \frac{1}{2}(B^{+}_{133} + B^{+}_{211} + B^{+}_{322}) + \tilde{B}^{+}_{123},$$

where $\tilde{B}^{+}_{123}$ works from the 4-gluon exchange. In SU(3)

$$B_{iij} = 2tr(U_j U_i^{\dagger})$$

$\rightarrow B^{+}_{123}$ splits into 3 LO C-even BK Green functions and one NLO contribution. cf. Bartels and Motyka 2007 ?

C-odd case

$$\frac{\partial B^-_{123}}{\partial \eta} = \frac{\alpha_s 3}{4\pi^2} \int d\vec{z}_4 \frac{\vec{z}_{12}^{\,2}}{\vec{z}_{14}^{\,2} \vec{z}_{42}^{\,2}} \Big[B^-_{423} + B^-_{143} - B^-_{123}$$

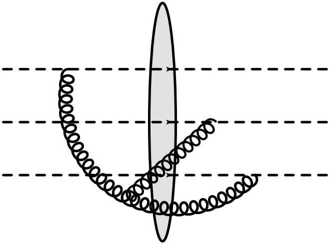
$$- B^-_{124} - B^-_{443} + B^-_{424} + B^-_{144} + \frac{1}{12} \left(B^+_{144} B^-_{324} + B^+_{244} B^-_{314} - B^+_{344} B^-_{214} \right)$$

$$+ \frac{1}{12} \left(B^-_{144} B^+_{324} + B^-_{244} B^+_{314} - B^-_{344} B^+_{214} \right) \Big] + (2 \leftrightarrow 3) + (1 \leftrightarrow 3).$$

The linear part of this result coincides with the linear result of Hatta, Iancu, Itakura, McLerran 2005, which they proved to coincide with the BKP equation.

NLO corrections

NLO evolution of 2 Wilson lines with open indices from Balitsky and Chirilli 2013



$$
\begin{aligned}
\frac{\partial B_{123}}{\partial \eta} = \frac{\alpha_s(\mu^2)}{8\pi^2} \int d\vec{r}_0 \Bigg[& (B_{100}B_{320} + B_{200}B_{310} - B_{300}B_{210} - 6B_{123}) \\
& \times \left\{ \frac{\vec{r}_{12}^{\,2}}{\vec{r}_{01}^{\,2}\vec{r}_{02}^{\,2}} - \frac{3\alpha_s}{4\pi}\beta \left[\ln\left(\frac{\vec{r}_{01}^{\,2}}{\vec{r}_{02}^{\,2}}\right)\left(\frac{1}{\vec{r}_{02}^{\,2}} - \frac{1}{\vec{r}_{01}^{\,2}}\right) - \frac{\vec{r}_{12}^{\,2}}{\vec{r}_{01}^{\,2}\vec{r}_{02}^{\,2}} \ln\left(\frac{\vec{r}_{12}^{\,2}}{\tilde{\mu}^2}\right)\right]\right\} \\
& - \frac{\alpha_s}{\pi} \ln\frac{\vec{r}_{20}^{\,2}}{\vec{r}_{21}^{\,2}} \ln\frac{\vec{r}_{10}^{\,2}}{\vec{r}_{21}^{\,2}} \left\{ \frac{1}{2}\left[\frac{\vec{r}_{13}^{\,2}}{\vec{r}_{10}^{\,2}\vec{r}_{30}^{\,2}} - \frac{\vec{r}_{32}^{\,2}}{\vec{r}_{30}^{\,2}\vec{r}_{20}^{\,2}}\right](B_{100}B_{320} - B_{200}B_{310}) \right. \\
& \left. - \frac{\vec{r}_{12}^{\,2}}{\vec{r}_{10}^{\,2}\vec{r}_{20}^{\,2}} \left(9B_{123} - \frac{1}{2}\left[2\left(B_{100}B_{320} + B_{200}B_{130}\right) - B_{300}B_{120}\right]\right)\right\} + (1 \leftrightarrow 3) + (2 \leftrightarrow 3) \Bigg]
\end{aligned}
$$

NLO corrections

$$
-\frac{\alpha_s^2}{8\pi^4}\int d\vec{r}_0 d\vec{r}_4 \; \Big[\{\tilde{L}_{12}\left(U_0 U_4{}^\dagger U_2\right)\cdot\left(U_1 U_0{}^\dagger U_4\right)\cdot U_3
$$

$$
+L_{12}\Big[\left(U_0 U_4{}^\dagger U_2\right)\cdot\left(U_1 U_0{}^\dagger U_4\right)\cdot U_3 + tr\left(U_0 U_4{}^\dagger\right)\left(U_1 U_0{}^\dagger U_2\right)\cdot U_3\cdot U_4
$$

$$
-\frac{3}{4}[B_{144}B_{234}+B_{244}B_{134}-B_{344}B_{124}]+\frac{1}{2}B_{123}\Big]
$$

$$
+(M_{13}-M_{12}-M_{23}+M_2^{13})\Big[\left(U_0 U_4{}^\dagger U_3\right)\cdot\left(U_2 U_0{}^\dagger U_1\right)\cdot U_4
$$

$$
+\left(U_1 U_0{}^\dagger U_2\right)\cdot\left(U_3 U_4{}^\dagger U_0\right)\cdot U_4\Big] + (\text{all 5 permutations } 1\leftrightarrow 2\leftrightarrow 3)\} + \;(0\leftrightarrow 4)\Big]
$$

$$
-\frac{\alpha_s^2 n_f}{16\pi^4}\int d\vec{r}_0 d\vec{r}_4\Big[\Big\{\Big(\frac{1}{3}(U_1 U_0{}^\dagger U_4 + U_4 U_0{}^\dagger U_1)\cdot U_2\cdot U_3 - \frac{1}{9}B_{123}tr(U_0{}^\dagger U_4)
$$

$$
+(U_1 U_0{}^\dagger U_2)\cdot U_3\cdot U_4 + \frac{1}{6}B_{123} - \frac{1}{4}(B_{013}B_{002}+B_{001}B_{023}-B_{012}B_{003})
$$

$$
+\;\;(1\leftrightarrow 2)\Big) + (0\leftrightarrow 4)\Big\} L_{12}^q + (1\leftrightarrow 3) + (2\leftrightarrow 3)\Big]
$$

$$
\beta\ln\frac{1}{\tilde{\mu}^2} = \left(\frac{11}{3}-\frac{2}{3}\frac{n_f}{3}\right)\ln\left(\frac{\mu^2}{4e^{2\psi(1)}}\right)+\frac{67}{9}-\frac{\pi^2}{3}-\frac{10}{9}\frac{n_f}{3}.
$$

This equation has correct dipole limit.

Remains to compare with JIMWLK results (A. Kovner M. Lublinsky Y. Mulian)

NLO corrections

Pomeron contribution $L_{12}(0 \leftrightarrow 4) = L_{12}$

$$L_{12} = \left[\frac{1}{\vec{r}_{01}{}^2\vec{r}_{24}{}^2 - \vec{r}_{02}{}^2\vec{r}_{14}{}^2}\left(-\frac{\vec{r}_{12}{}^4}{8}\left(\frac{1}{\vec{r}_{01}{}^2\vec{r}_{24}{}^2} + \frac{1}{\vec{r}_{02}{}^2\vec{r}_{14}{}^2}\right) + \frac{\vec{r}_{12}{}^2}{\vec{r}_{04}{}^2} - \frac{\vec{r}_{02}{}^2\vec{r}_{14}{}^2 + \vec{r}_{01}{}^2\vec{r}_{24}{}^2}{4\vec{r}_{04}{}^4}\right)\right.$$
$$\left. + \frac{\vec{r}_{12}{}^2}{8\vec{r}_{04}{}^2}\left(\frac{1}{\vec{r}_{02}{}^2\vec{r}_{14}{}^2} - \frac{1}{\vec{r}_{01}{}^2\vec{r}_{24}{}^2}\right)\right] \ln\left(\frac{\vec{r}_{01}{}^2\vec{r}_{24}{}^2}{\vec{r}_{14}{}^2\vec{r}_{02}{}^2}\right) + \frac{1}{2\vec{r}_{04}{}^4}.$$

$$L^q_{12} = \frac{1}{\vec{r}_{04}{}^4}\left\{\frac{\vec{r}_{02}{}^2\vec{r}_{14}{}^2 + \vec{r}_{01}{}^2\vec{r}_{24}{}^2 - \vec{r}_{04}{}^2\vec{r}_{12}{}^2}{2(\vec{r}_{02}{}^2\vec{r}_{14}{}^2 - \vec{r}_{01}{}^2\vec{r}_{24}{}^2)} \ln\left(\frac{\vec{r}_{02}{}^2\vec{r}_{14}{}^2}{\vec{r}_{01}{}^2\vec{r}_{24}{}^2}\right) - 1\right\}.$$

2-point contribution to odderon $\tilde{L}_{12}(0 \leftrightarrow 4) = -\tilde{L}_{12}$

$$\tilde{L}_{12} = \frac{\vec{r}_{12}{}^2}{8}\left[\frac{\vec{r}_{12}{}^2}{\vec{r}_{01}{}^2\vec{r}_{02}{}^2\vec{r}_{14}{}^2\vec{r}_{24}{}^2} - \frac{1}{\vec{r}_{01}{}^2\vec{r}_{04}{}^2\vec{r}_{24}{}^2} - \frac{1}{\vec{r}_{02}{}^2\vec{r}_{04}{}^2\vec{r}_{14}{}^2}\right] \ln\left(\frac{\vec{r}_{01}{}^2\vec{r}_{24}{}^2}{\vec{r}_{14}{}^2\vec{r}_{02}{}^2}\right).$$

New structures

$$M_{12} = \frac{\vec{r}_{12}{}^2}{16}\left[\frac{\vec{r}_{12}{}^2}{\vec{r}_{01}{}^2\vec{r}_{02}{}^2\vec{r}_{14}{}^2\vec{r}_{24}{}^2} - \frac{1}{\vec{r}_{01}{}^2\vec{r}_{04}{}^2\vec{r}_{24}{}^2} - \frac{1}{\vec{r}_{02}{}^2\vec{r}_{04}{}^2\vec{r}_{14}{}^2}\right] \ln\left(\frac{\vec{r}_{01}{}^2\vec{r}_{02}{}^2}{\vec{r}_{14}{}^2\vec{r}_{24}{}^2}\right).$$

$$M_2^{13} = \frac{1}{4\vec{r}_{01}{}^2\vec{r}_{34}{}^2}\left(\frac{\vec{r}_{12}{}^2\vec{r}_{23}{}^2}{\vec{r}_{02}{}^2\vec{r}_{24}{}^2} - \frac{\vec{r}_{14}{}^2\vec{r}_{23}{}^2}{\vec{r}_{04}{}^2\vec{r}_{24}{}^2} - \frac{\vec{r}_{03}{}^2\vec{r}_{12}{}^2}{\vec{r}_{02}{}^2\vec{r}_{04}{}^2} + \frac{\vec{r}_{13}{}^2}{\vec{r}_{04}{}^2}\right) \ln\left(\frac{\vec{r}_{02}{}^2}{\vec{r}_{24}{}^2}\right).$$

NLO corrections: quasi-conformal kernel

To construct composite conformal operators we use the model (I. Balitsky and G. Chirilli 2009, A. Kovner M. Lublinsky Y. Mulian 2014)

$$O^{conf} = O + \frac{1}{2}\frac{\partial O}{\partial \eta}\Big|_{\frac{\vec r_{mn}^{\,2}}{\vec r_{im}^{\,2}\vec r_{in}^{\,2}} \to \frac{\vec r_{mn}^{\,2}}{\vec r_{im}^{\,2}\vec r_{in}^{\,2}} \ln\left(\frac{\vec r_{mn}^{\,2} a}{\vec r_{im}^{\,2}\vec r_{in}^{\,2}}\right)},$$

where a is an arbitrary constant. For the conformal 3QWL operator we have the following ansatz

$$B_{123}^{conf} = B_{123} + \frac{\alpha_s 3}{8\pi^2}\int d\vec r_4 \left[\frac{\vec r_{12}^{\,2}}{\vec r_{41}^{\,2}\vec r_{42}^{\,2}} \ln\left(\frac{a\vec r_{12}^{\,2}}{\vec r_{41}^{\,2}\vec r_{42}^{\,2}}\right)\right.$$

$$\left.\times(-B_{123} + \frac{1}{6}(B_{144}B_{324} + B_{244}B_{314} - B_{344}B_{214})) + (1 \leftrightarrow 3) + (2 \leftrightarrow 3)\right]$$

If we put $\vec r_2 = \vec r_3$, then

$$B_{122}^{conf} = B_{122} + \frac{\alpha_s 3}{4\pi^2}\int d\vec r_4 \frac{\vec r_{12}^{\,2}}{\vec r_{41}^{\,2}\vec r_{42}^{\,2}} \ln\left(\frac{a\vec r_{12}^{\,2}}{\vec r_{41}^{\,2}\vec r_{42}^{\,2}}\right)(-B_{122} + \frac{1}{6}B_{144}B_{224}).$$

Composite operators

for $(-B_{123}+\frac{1}{6}(B_{144}B_{324}+B_{244}B_{314}-B_{344}B_{214}))$ we have

$$(-3B_{123}+\frac{1}{2}(B_{144}B_{324}+B_{244}B_{314}-B_{344}B_{214}))^{conf}$$

$$=(-3B_{123}+\frac{1}{2}(B_{144}B_{324}+B_{244}B_{314}-B_{344}B_{214}))$$

$$+\frac{\alpha_s}{8\pi^2}\int d\vec{r}_0\left(A_{34}\frac{\vec{r}_{34}^{\,2}}{\vec{r}_{03}^{\,2}\vec{r}_{04}^{\,2}}\ln\left(\frac{\vec{r}_{34}^{\,2}a}{\vec{r}_{03}^{\,2}\vec{r}_{04}^{\,2}}\right)+A_{13}\frac{\vec{r}_{13}^{\,2}}{\vec{r}_{03}^{\,2}\vec{r}_{01}^{\,2}}\ln\left(\frac{\vec{r}_{13}^{\,2}a}{\vec{r}_{03}^{\,2}\vec{r}_{01}^{\,2}}\right)\right.$$

$$+A_{23}\frac{\vec{r}_{23}^{\,2}}{\vec{r}_{03}^{\,2}\vec{r}_{02}^{\,2}}\ln\left(\frac{\vec{r}_{23}^{\,2}a}{\vec{r}_{03}^{\,2}\vec{r}_{02}^{\,2}}\right)+A_{14}\frac{\vec{r}_{14}^{\,2}}{\vec{r}_{01}^{\,2}\vec{r}_{04}^{\,2}}\ln\left(\frac{\vec{r}_{14}^{\,2}a}{\vec{r}_{01}^{\,2}\vec{r}_{04}^{\,2}}\right)$$

$$\left.+A_{24}\frac{\vec{r}_{24}^{\,2}}{\vec{r}_{02}^{\,2}\vec{z}_{04}^{\,2}}\ln\left(\frac{\vec{r}_{24}^{\,2}a}{\vec{r}_{02}^{\,2}\vec{z}_{04}^{\,2}}\right)+A_{12}\frac{\vec{r}_{12}^{\,2}}{\vec{r}_{01}^{\,2}\vec{r}_{02}^{\,2}}\ln\left(\frac{\vec{r}_{12}^{\,2}a}{\vec{r}_{01}^{\,2}\vec{r}_{02}^{\,2}}\right)\right).$$

Quasi-conformal kernel

$\sim n_f$ part does not change

$$\langle K_{NLO} \otimes B_{123}^{conf} \rangle = -\frac{\alpha_s^2}{8\pi^4} \int d\vec{r}_0 d\vec{r}_4 \ \Big(\Big\{ \tilde{L}_{12}^{C} \left(U_0 U_4{}^{\dagger} U_2 \right) \cdot \left(U_1 U_0{}^{\dagger} U_4 \right) \cdot U_3$$

$$+ L_{12}^{C} \Big[\left(U_0 U_4{}^{\dagger} U_2 \right) \cdot \left(U_1 U_0{}^{\dagger} U_4 \right) \cdot U_3 + tr \left(U_0 U_4{}^{\dagger} \right) \left(U_1 U_0{}^{\dagger} U_2 \right) \cdot U_3 \cdot U_4$$

$$- \frac{3}{4} [B_{144} B_{234} + B_{244} B_{134} - B_{344} B_{124}] + \frac{1}{2} B_{123} \Big]$$

$$+ M_{12}^{C} \Big[\left(U_0 U_4{}^{\dagger} U_3 \right) \cdot \left(U_2 U_0{}^{\dagger} U_1 \right) \cdot U_4 + \left(U_1 U_0{}^{\dagger} U_2 \right) \cdot \left(U_3 U_4{}^{\dagger} U_0 \right) \cdot U_4 \Big]$$

$$+ n_f(\ldots) + \ (\text{all 5 permutations } 1 \leftrightarrow 2 \leftrightarrow 3) \Big\} + (0 \leftrightarrow 4) \Big)$$

$$- \frac{\alpha_s^2}{8\pi^3} \int d\vec{r}_0 \left(\frac{\beta}{2} \left[\ln \left(\frac{\vec{r}_{01}^{\,2}}{\vec{r}_{02}^{\,2}} \right) \left(\frac{1}{\vec{r}_{02}^{\,2}} - \frac{1}{\vec{r}_{01}^{\,2}} \right) - \frac{\vec{r}_{12}^{\,2}}{\vec{r}_{01}^{\,2} \vec{r}_{02}^{\,2}} \ln \left(\frac{\vec{r}_{12}^{\,2}}{\tilde{\mu}^2} \right) \right] \right.$$

$$\left. \times \left(\frac{3}{2} (B_{100} B_{230} + B_{200} B_{130} - B_{300} B_{210}) - 9 B_{123} \right) + (1 \leftrightarrow 3) + (2 \leftrightarrow 3) \right)$$

$$- \frac{\alpha_s^2}{32\pi^3} \int d\vec{r}_0 \left(B_{003} B_{012} \left[\frac{\vec{r}_{32}{}^2}{\vec{r}_{03}{}^2 \vec{r}_{02}{}^2} \ln^2 \left(\frac{\vec{r}_{32}{}^2 \vec{r}_{10}{}^2}{\vec{r}_{13}{}^2 \vec{r}_{20}{}^2} \right) - \frac{\vec{r}_{12}{}^2}{\vec{r}_{01}{}^2 \vec{r}_{02}{}^2} \ln^2 \left(\frac{\vec{r}_{12}{}^2 \vec{r}_{30}{}^2}{\vec{r}_{13}{}^2 \vec{r}_{20}{}^2} \right) \right] \right.$$

$$\left. + (\text{all 5 permutations } 1 \leftrightarrow 2 \leftrightarrow 3) \right).$$

Remains to compare with JIMWLK results (A. Kovner M. Lublinsky Y. Mulian)

Quasi-conformal kernel

$$L_{12}^{C} = L_{12} + \frac{\vec{r}_{12}^{\,2}}{4\vec{r}_{01}^{\,2}\vec{r}_{04}^{\,2}\vec{r}_{24}^{\,2}} \ln\left(\frac{\vec{r}_{02}^{\,2}\vec{r}_{14}^{\,2}}{\vec{r}_{04}^{\,2}\vec{r}_{12}^{\,2}}\right) + \frac{\vec{r}_{12}^{\,2}}{4\vec{r}_{02}^{\,2}\vec{r}_{04}^{\,2}\vec{r}_{14}^{\,2}} \ln\left(\frac{\vec{r}_{01}^{\,2}\vec{r}_{24}^{\,2}}{\vec{r}_{04}^{\,2}\vec{r}_{12}^{\,2}}\right),$$

$$\tilde{L}_{12}^{C} = \tilde{L}_{12} + \frac{\vec{r}_{12}^{\,2}}{4\vec{r}_{01}^{\,2}\vec{r}_{04}^{\,2}\vec{r}_{24}^{\,2}} \ln\left(\frac{\vec{r}_{02}^{\,2}\vec{r}_{14}^{\,2}}{\vec{r}_{04}^{\,2}\vec{r}_{12}^{\,2}}\right) - \frac{\vec{r}_{12}^{\,2}}{4\vec{r}_{02}^{\,2}\vec{r}_{04}^{\,2}\vec{r}_{14}^{\,2}} \ln\left(\frac{\vec{r}_{01}^{\,2}\vec{r}_{24}^{\,2}}{\vec{r}_{04}^{\,2}\vec{r}_{12}^{\,2}}\right),$$

$$\begin{aligned} M_{12}^{C} = {} & \frac{\vec{r}_{12}^{\,2}}{16\vec{r}_{02}^{\,2}\vec{r}_{04}^{\,2}\vec{r}_{14}^{\,2}} \ln\left(\frac{\vec{r}_{01}^{\,2}\vec{r}_{02}^{\,2}\vec{r}_{34}^{\,4}}{\vec{r}_{03}^{\,4}\vec{r}_{14}^{\,2}\vec{r}_{24}^{\,2}}\right) + \frac{\vec{r}_{12}^{\,2}}{16\vec{r}_{01}^{\,2}\vec{r}_{04}^{\,2}\vec{r}_{24}^{\,2}} \ln\left(\frac{\vec{r}_{03}^{\,4}\vec{r}_{04}^{\,4}\vec{r}_{12}^{\,4}\vec{r}_{24}^{\,2}}{\vec{r}_{01}^{\,2}\vec{r}_{02}^{\,6}\vec{r}_{14}^{\,2}\vec{r}_{34}^{\,4}}\right) \\ & + \frac{\vec{r}_{23}^{\,2}}{16\vec{r}_{02}^{\,2}\vec{r}_{04}^{\,2}\vec{r}_{34}^{\,2}} \ln\left(\frac{\vec{r}_{01}^{\,4}\vec{r}_{03}^{\,2}\vec{r}_{24}^{\,6}\vec{r}_{34}^{\,2}}{\vec{r}_{02}^{\,2}\vec{r}_{04}^{\,4}\vec{r}_{14}^{\,4}\vec{r}_{23}^{\,4}}\right) + \frac{\vec{r}_{23}^{\,2}}{16\vec{r}_{03}^{\,2}\vec{r}_{04}^{\,2}\vec{r}_{24}^{\,2}} \ln\left(\frac{\vec{r}_{02}^{\,2}\vec{r}_{03}^{\,2}\vec{r}_{14}^{\,4}}{\vec{r}_{01}^{\,4}\vec{r}_{24}^{\,2}\vec{r}_{34}^{\,2}}\right) \\ & + \frac{\vec{r}_{13}^{\,2}}{16\vec{r}_{03}^{\,2}\vec{r}_{04}^{\,2}\vec{r}_{14}^{\,2}} \ln\left(\frac{\vec{r}_{02}^{\,4}\vec{r}_{14}^{\,2}\vec{r}_{34}^{\,2}}{\vec{r}_{01}^{\,2}\vec{r}_{03}^{\,2}\vec{r}_{24}^{\,4}}\right) + \frac{\vec{r}_{13}^{\,2}}{16\vec{r}_{01}^{\,2}\vec{r}_{04}^{\,2}\vec{r}_{34}^{\,2}} \ln\left(\frac{\vec{r}_{02}^{\,4}\vec{r}_{14}^{\,2}\vec{r}_{34}^{\,2}}{\vec{r}_{01}^{\,2}\vec{r}_{03}^{\,2}\vec{r}_{24}^{\,4}}\right) \\ & + \frac{\vec{r}_{03}^{\,2}\vec{r}_{12}^{\,2}}{8\vec{r}_{01}^{\,2}\vec{r}_{02}^{\,2}\vec{r}_{04}^{\,2}\vec{r}_{34}^{\,2}} \ln\left(\frac{\vec{r}_{01}^{\,2}\vec{r}_{03}^{\,2}\vec{r}_{24}^{\,4}}{\vec{r}_{02}^{\,2}\vec{r}_{04}^{\,2}\vec{r}_{12}^{\,2}\vec{r}_{34}^{\,2}}\right) + \frac{\vec{r}_{23}^{\,2}\vec{r}_{12}^{\,2}}{8\vec{r}_{01}^{\,2}\vec{r}_{02}^{\,2}\vec{r}_{24}^{\,2}\vec{r}_{34}^{\,2}} \ln\left(\frac{\vec{r}_{02}^{\,2}\vec{r}_{12}^{\,2}\vec{r}_{34}^{\,2}}{\vec{r}_{01}^{\,2}\vec{r}_{23}^{\,2}\vec{r}_{24}^{\,2}}\right) \\ & + \frac{\vec{r}_{14}^{\,2}\vec{r}_{23}^{\,2}}{8\vec{r}_{01}^{\,2}\vec{r}_{04}^{\,2}\vec{r}_{24}^{\,2}\vec{r}_{34}^{\,2}} \ln\left(\frac{\vec{r}_{01}^{\,2}\vec{r}_{04}^{\,2}\vec{r}_{23}^{\,2}\vec{r}_{24}^{\,2}}{\vec{r}_{02}^{\,4}\vec{r}_{14}^{\,2}\vec{r}_{34}^{\,2}}\right), \end{aligned}$$

All these functions are conformally invariant

In the quark-diquark limit $\vec{r}_3 \to \vec{r}_2$

$$\Big\{ M_{12}^{C} \Big[\left(U_0 U_4{}^{\dagger} U_3 \right) \cdot \left(U_2 U_0{}^{\dagger} U_1 \right) \cdot U_4 + \left(U_1 U_0{}^{\dagger} U_2 \right) \cdot \left(U_3 U_4{}^{\dagger} U_0 \right) \cdot U_4 \Big]$$

$$+ \text{(all 5 permutations } 1 \leftrightarrow 2 \leftrightarrow 3) \Big\} + (0 \leftrightarrow 4)$$

$$\to 2\tilde{L}_{12}^{C} \Big[tr \left(U_0{}^{\dagger} U_4 \right) \left(tr \left(U_2{}^{\dagger} U_0 U_4{}^{\dagger} U_1 \right) + tr \left(U_2{}^{\dagger} U_1 U_4{}^{\dagger} U_0 \right) \right)$$

$$+ 2 tr \left(U_0{}^{\dagger} U_1 \right) tr \left(U_2{}^{\dagger} U_4 \right) tr \left(U_4{}^{\dagger} U_0 \right) - (0 \leftrightarrow 4) \Big],$$

$$\Big\{ \tilde{L}_{12}^{C} \left(U_0 U_4{}^{\dagger} U_2 \right) \cdot \left(U_1 U_0{}^{\dagger} U_4 \right) \cdot U_3 + \text{(all 5 permutations } 1 \leftrightarrow 2 \leftrightarrow 3) \Big\} + (0 \leftrightarrow 4)$$

$$\to 2\tilde{L}_{12}^{C} \Big[tr \left(U_4{}^{\dagger} U_0 \right) \left(tr \left(U_0{}^{\dagger} U_1 U_2{}^{\dagger} U_4 \right) + tr \left(U_0{}^{\dagger} U_4 U_2{}^{\dagger} U_1 \right) \right) - (0 \leftrightarrow 4) \Big],$$

$$L_{12}^{C} \Big[\left(U_0 U_4{}^{\dagger} U_2 \right) \cdot \left(U_1 U_0{}^{\dagger} U_4 \right) \cdot U_3 + tr \left(U_0 U_4{}^{\dagger} \right) \left(U_1 U_0{}^{\dagger} U_2 \right) \cdot U_3 \cdot U_4 + \frac{1}{2} B_{123}$$

$$- \frac{3}{4} [B_{144} B_{234} + B_{244} B_{134} - B_{344} B_{124}] + \text{(all 5 permutations } 1 \leftrightarrow 2 \leftrightarrow 3) \Big] + (0 \leftrightarrow 4)$$

$$\to 4L_{12}^{C} \Big[tr \left(U_2{}^{\dagger} U_1 \right) - 3 tr \left(U_0{}^{\dagger} U_1 \right) tr \left(U_2{}^{\dagger} U_0 \right) + tr \left(U_0{}^{\dagger} U_1 \right) tr \left(U_2{}^{\dagger} U_4 \right) tr \left(U_4{}^{\dagger} U_0 \right)$$

$$- tr \left(U_0{}^{\dagger} U_1 U_4{}^{\dagger} U_0 U_2{}^{\dagger} U_4 \right) + (0 \leftrightarrow 4) \Big].$$

we get the dipole result

$$\langle K_{NLO} \otimes B_{122}^{conf} \rangle = -\frac{\alpha_s^2}{2\pi^4} \int d\vec{r}_0 d\vec{r}_4 \left(\left\{ \left(\tilde{L}_{12}^C + L_{12}^C \right) tr\left(U_0{}^{\dagger} U_1 \right) tr\left(U_2{}^{\dagger} U_4 \right) tr\left(U_4{}^{\dagger} U_0 \right) \right.\right.$$

$$\left.\left. + L_{12}^C \left[tr\left(U_2{}^{\dagger} U_1 \right) - 3 tr\left(U_0{}^{\dagger} U_1 \right) tr\left(U_2^{\dagger} U_0 \right) - tr\left(U_0{}^{\dagger} U_1 U_4^{\dagger} U_0 U_2{}^{\dagger} U_4 \right) \right] \right\} + (0 \leftrightarrow 4) \right)$$

$$-\frac{3\alpha_s^2}{2\pi^3} \int d\vec{r}_0 \frac{11}{6} \left[\ln\left(\frac{\vec{r}_{01}^{\,2}}{\vec{r}_{02}^{\,2}} \right) \left(\frac{1}{\vec{r}_{02}^{\,2}} - \frac{1}{\vec{r}_{01}^{\,2}} \right) - \frac{\vec{r}_{12}^{\,2}}{\vec{r}_{01}^{\,2} \vec{r}_{02}^{\,2}} \ln\left(\frac{\vec{r}_{12}^{\,2}}{\tilde{\mu}^2} \right) \right]$$

$$\times \left(tr\left(U_0{}^{\dagger} U_1 \right) tr\left(U_2{}^{\dagger} U_0 \right) - 3 tr\left(U_2{}^{\dagger} U_1 \right) \right).$$

This is twice the gluon part of the BK kernel.

$$B_{122} = 2tr(U_1 U_2^{\dagger})$$

Linearization

In the 3-gluon approximation

$$\langle K_{NLO}\otimes B_{123}^{conf}\rangle \overset{3g}{=} -\frac{9\alpha_s^2}{8\pi^4}\int d\vec{r}_0 d\vec{r}_4 (L_{12}^C+L_{13}^C+L_{23}^C-\frac{n_f}{54}(L_{12}^q+L_{13}^q+L_{23}^q))(B_{044}+B_{004}-12)$$

$$-\frac{\alpha_s^2 n_f}{24\pi^4}\int d\vec{r}_0 d\vec{r}_4 \ \{(2B_{014}-B_{001}-B_{144})(L_{12}^q+L_{13}^q-2L_{32}^q)+(1\leftrightarrow 3)+(1\leftrightarrow 2)\}$$

$$+\frac{27\alpha_s^2}{4\pi^2}\zeta(3)(3-\delta_{23}-\delta_{13}-\delta_{21})(B_{123}-6)$$

$$-\frac{9\alpha_s^2}{64\pi^4}\int d\vec{r}_0 d\vec{r}_4 \ (F_0(B_{040}-B_{044})+\{F_{140}+(0\leftrightarrow 4)\}\,B_{140}+(\text{all 5 perm.}1\leftrightarrow 2\leftrightarrow 3))$$

$$-\frac{9\alpha_s^2}{64\pi^3}\int d\vec{r}_0 \left(\tilde{F}_{100}B_{100}+\tilde{F}_{230}B_{230}+(1\leftrightarrow 3)+(1\leftrightarrow 2)\right)$$

$$-\frac{9\alpha_s^2}{16\pi^3}\int d\vec{r}_0 \left(\beta\left[\ln\left(\frac{\vec{r}_{01}^{\,2}}{\vec{r}_{02}^{\,2}}\right)\left(\frac{1}{\vec{r}_{02}^{\,2}}-\frac{1}{\vec{r}_{01}^{\,2}}\right)-\frac{\vec{r}_{12}^{\,2}}{\vec{r}_{01}^{\,2}\vec{r}_{02}^{\,2}}\ln\left(\frac{\vec{r}_{12}^{\,2}}{\tilde{\mu}^2}\right)\right]\right.$$

$$\left.\times(B_{100}+B_{230}+B_{200}+B_{130}-B_{300}-B_{210}-B_{123}-6)+(1\leftrightarrow 3)+(2\leftrightarrow 3)\right).$$

Here $\delta_{ij}=1$, if $\vec{r}_i=\vec{r}_j$ and $\delta_{ij}=0$ otherwise.

$$\tilde{F}_{100} = \left(\frac{\vec{r}_{12}{}^2}{\vec{r}_{01}{}^2 \vec{r}_{02}{}^2} - \frac{\vec{r}_{13}{}^2}{\vec{r}_{01}{}^2 \vec{r}_{03}{}^2} - \frac{2\vec{r}_{23}{}^2}{\vec{r}_{02}{}^2 \vec{r}_{03}{}^2} \right) \ln^2 \left(\frac{\vec{r}_{02}{}^2 \vec{r}_{13}{}^2}{\vec{r}_{01}{}^2 \vec{r}_{23}{}^2} \right) + \frac{\vec{r}_{23}{}^2}{2\vec{r}_{02}{}^2 \vec{r}_{03}{}^2} \ln^2 \left(\frac{\vec{r}_{03}{}^2 \vec{r}_{12}{}^2}{\vec{r}_{02}{}^2 \vec{r}_{13}{}^2} \right)$$

$$+ \tilde{S}_{123} I \left(\frac{\vec{r}_{12}{}^2}{\vec{r}_{01}{}^2 \vec{r}_{02}{}^2}, \frac{\vec{r}_{13}{}^2}{\vec{r}_{01}{}^2 \vec{r}_{03}{}^2}, \frac{\vec{r}_{23}{}^2}{\vec{r}_{02}{}^2 \vec{r}_{03}{}^2} \right) + (2 \leftrightarrow 3),$$

$$\tilde{F}_{230} = \left(\frac{2\vec{r}_{12}{}^2}{\vec{r}_{01}{}^2 \vec{r}_{02}{}^2} - \frac{\vec{r}_{23}{}^2}{2\vec{r}_{02}{}^2 \vec{r}_{03}{}^2} \right) \ln^2 \left(\frac{\vec{r}_{03}{}^2 \vec{r}_{12}{}^2}{\vec{r}_{02}{}^2 \vec{r}_{13}{}^2} \right) + \left(\frac{\vec{r}_{13}{}^2}{\vec{r}_{01}{}^2 \vec{r}_{03}{}^2} - \frac{\vec{r}_{12}{}^2}{\vec{r}_{01}{}^2 \vec{r}_{02}{}^2} \right) \ln^2 \left(\frac{\vec{r}_{02}{}^2 \vec{r}_{13}{}^2}{\vec{r}_{01}{}^2 \vec{r}_{23}{}^2} \right)$$

$$- \tilde{S}_{123} I \left(\frac{\vec{r}_{12}{}^2}{\vec{r}_{01}{}^2 \vec{r}_{02}{}^2}, \frac{\vec{r}_{13}{}^2}{\vec{r}_{01}{}^2 \vec{r}_{03}{}^2}, \frac{\vec{r}_{23}{}^2}{\vec{r}_{02}{}^2 \vec{r}_{03}{}^2} \right) + (2 \leftrightarrow 3).$$

$$\tilde{S}_{123} = \left(\frac{\vec{r}_{12}{}^4}{\vec{r}_{01}{}^4 \vec{r}_{02}{}^4} + \frac{\vec{r}_{13}{}^4}{\vec{r}_{01}{}^4 \vec{r}_{03}{}^4} + \frac{\vec{r}_{23}{}^4}{\vec{r}_{02}{}^4 \vec{r}_{03}{}^4} - \frac{2\vec{r}_{13}{}^2 \vec{r}_{12}{}^2}{\vec{r}_{01}{}^4 \vec{r}_{02}{}^2 \vec{r}_{03}{}^2} - \frac{2\vec{r}_{23}{}^2 \vec{r}_{12}{}^2}{\vec{r}_{01}{}^2 \vec{r}_{02}{}^4 \vec{r}_{03}{}^2} - \frac{2\vec{r}_{13}{}^2 \vec{r}_{23}{}^2}{\vec{r}_{01}{}^2 \vec{r}_{02}{}^2 \vec{r}_{03}{}^4} \right)$$

is the square of the area of the triangle with the corners at $r_{1,2,3}$ after the inversion.

$$I(a, b, c) = \int_0^1 \frac{dx}{a(1-x) + bx - cx(1-x)} \ln \left(\frac{a(1-x) + bx}{cx(1-x)} \right)$$

is symmetric w.r.t. interchange of its arguments function.

Linearization

$$F_0 = \frac{\vec{r}_{12}{}^2}{2\vec{r}_{14}{}^2\vec{r}_{24}{}^2}\left(\frac{\vec{r}_{24}{}^2}{\vec{r}_{02}{}^2\vec{r}_{04}{}^2}\ln\left(\frac{\vec{r}_{01}{}^2\vec{r}_{02}{}^2\vec{r}_{34}{}^4}{\vec{r}_{14}{}^2\vec{r}_{24}{}^2\vec{r}_{03}{}^4}\right) - \frac{\vec{r}_{13}{}^2}{\vec{r}_{01}{}^2\vec{r}_{03}{}^2}\ln\left(\frac{\vec{r}_{01}{}^2\vec{r}_{13}{}^2\vec{r}_{24}{}^2}{\vec{r}_{03}{}^2\vec{r}_{12}{}^2\vec{r}_{14}{}^2}\right)\right.$$
$$\left. + \frac{2\vec{r}_{34}{}^2}{\vec{r}_{03}{}^2\vec{r}_{04}{}^2}\ln\left(\frac{\vec{r}_{01}{}^2\vec{r}_{02}{}^2\vec{r}_{34}{}^2}{\vec{r}_{03}{}^2\vec{r}_{04}{}^2\vec{r}_{12}{}^2}\right)\right) - (0 \leftrightarrow 4).$$

$$F_{140} = \frac{\vec{r}_{12}{}^2}{\vec{r}_{02}{}^2\vec{r}_{04}{}^2\vec{r}_{14}{}^2}\ln\left(\frac{\vec{r}_{02}{}^2\vec{r}_{04}{}^2\vec{r}_{12}{}^2\vec{r}_{34}{}^4}{\vec{r}_{03}{}^4\vec{r}_{14}{}^2\vec{r}_{24}{}^4}\right)$$
$$-\frac{\vec{r}_{01}{}^2\vec{r}_{23}{}^2}{\vec{r}_{02}{}^2\vec{r}_{03}{}^2\vec{r}_{04}{}^2\vec{r}_{14}{}^2}\ln\left(\frac{\vec{r}_{01}{}^2\vec{r}_{24}{}^2\vec{r}_{34}{}^2}{\vec{r}_{04}{}^2\vec{r}_{14}{}^2\vec{r}_{23}{}^2}\right) - \frac{\vec{r}_{23}{}^2\vec{r}_{12}{}^2}{\vec{r}_{02}{}^2\vec{r}_{03}{}^2\vec{r}_{14}{}^2\vec{r}_{24}{}^2}\ln\left(\frac{\vec{r}_{02}{}^2\vec{r}_{14}{}^2\vec{r}_{23}{}^2}{\vec{r}_{03}{}^2\vec{r}_{12}{}^2\vec{r}_{24}{}^2}\right)$$
$$+\frac{\vec{r}_{23}{}^2}{\vec{r}_{03}{}^2\vec{r}_{04}{}^2\vec{r}_{24}{}^2}\ln\left(\frac{\vec{r}_{02}{}^2\vec{r}_{34}{}^2}{\vec{r}_{04}{}^2\vec{r}_{23}{}^2}\right) + \frac{\vec{r}_{02}{}^2\vec{r}_{13}{}^2}{\vec{r}_{01}{}^2\vec{r}_{03}{}^2\vec{r}_{04}{}^2\vec{r}_{24}{}^2}\ln\left(\frac{\vec{r}_{01}{}^2\vec{r}_{02}{}^2\vec{r}_{34}{}^4}{\vec{r}_{03}{}^2\vec{r}_{04}{}^2\vec{r}_{13}{}^2\vec{r}_{24}{}^2}\right).$$

All the functions F are conformally invariant.

Linearization

In the 3-gluon approximation

$$B^{+}_{123} \stackrel{3g}{=} \frac{1}{2}(B^{+}_{133} + B^{+}_{211} + B^{+}_{322}).$$

Therefore for model of the composite operator we use

$$B^{+conf}_{123} \stackrel{3g}{=} \frac{1}{2}(B^{+conf}_{133} + B^{+conf}_{211} + B^{+conf}_{322})$$

and

$$\langle K_{NLO} \otimes B^{+conf}_{123}\rangle \stackrel{3g}{=} \frac{1}{2}\langle K_{NLO} \otimes (B^{+conf}_{133} + B^{+conf}_{211} + B^{+conf}_{322})\rangle.$$

This equality imposes the following constraints

$$0 = \{F_{140} + (0 \leftrightarrow 4)\} + (\text{all 5 permutations } 1 \leftrightarrow 2 \leftrightarrow 3),$$

$$0 = \int d\vec{r}_0 \tilde{F}_{230},$$

$$0 = \int \frac{d\vec{r}_4}{\pi} \left(\{F_{140} + (0 \leftrightarrow 4)\} + (2 \leftrightarrow 3)\right) + \tilde{F}_{100} + \frac{1}{2}\tilde{F}_{230}|_{1\leftrightarrow 3} + \frac{1}{2}\tilde{F}_{230}|_{1\leftrightarrow 2}.$$

They are satisfied.

Linearized C-odd exchange

$$\frac{\partial B_{123}^{-conf}}{\partial \eta} \stackrel{3g}{=} \frac{3\alpha_s(\mu^2)}{4\pi^2} \int d\vec{r}_0 \left[\left(B_{100}^{-conf} + B_{320}^{-conf} + B_{200}^{-conf} + B_{310}^{-conf} \right. \right.$$

$$\left. - B_{300}^{-conf} - B_{210}^{-conf} - B_{123}^{-conf} \right)$$

$$\times \left(\frac{\vec{r}_{12}^{\,2}}{\vec{r}_{01}^{\,2}\vec{r}_{02}^{\,2}} - \frac{3\alpha_s}{4\pi}\beta \left[\ln\left(\frac{\vec{r}_{01}^{\,2}}{\vec{r}_{02}^{\,2}}\right)\left(\frac{1}{\vec{r}_{02}^{\,2}} - \frac{1}{\vec{r}_{01}^{\,2}}\right) - \frac{\vec{r}_{12}^{\,2}}{\vec{r}_{01}^{\,2}\vec{r}_{02}^{\,2}} \ln\left(\frac{\vec{r}_{12}^{\,2}}{\tilde{\mu}^2}\right) \right] \right)$$

$$\left. + (1 \leftrightarrow 3) + (2 \leftrightarrow 3) \right] + \frac{27\alpha_s^2}{4\pi^2}\zeta(3)(3 - \delta_{23} - \delta_{13} - \delta_{21})B_{123}^{-}$$

$$-\frac{\alpha_s^2 n_f}{24\pi^4} \int d\vec{r}_0 d\vec{r}_4 \, \left\{ (2B_{014}^{-} - B_{001}^{-} - B_{144}^{-}) \left(L_{12}^{q} + L_{13}^{q} - 2L_{32}^{q}\right) + (1 \leftrightarrow 3) + (1 \leftrightarrow 2) \right\}$$

$$-\frac{9\alpha_s^2}{64\pi^3} \int d\vec{r}_0 \left(\tilde{F}_{100} B_{100}^{-} + \tilde{F}_{230} B_{230}^{-} + (1 \leftrightarrow 3) + (1 \leftrightarrow 2) \right)$$

$$-\frac{9\alpha_s^2}{64\pi^4} \int d\vec{r}_0 d\vec{r}_4 \, \left(2F_0 B_{040}^{-} + \{F_{140} + (0 \leftrightarrow 4)\} B_{140}^{-} + (\text{all 5 perm. } 1 \leftrightarrow 2 \leftrightarrow 3) \right).$$

Linearized C-odd exchange for a dipole

The BK equation for the C-odd part of the color dipole operator $B^{-}_{122} = 2tr(U_1 U_2^{\dagger}) - 2tr(U_1^{\dagger} U_2)$ in the 3-gluon approximation reads

$$\frac{\partial B^{-conf}_{122}}{\partial \eta} \overset{3g}{=} \frac{3\alpha_s(\mu^2)}{2\pi^2} \int d\vec{r}_0 (B^{-conf}_{100} + B^{-conf}_{220} - B^{-conf}_{122})$$
$$\times \left(\frac{\vec{r}_{12}^{\,2}}{\vec{r}_{01}^{\,2} \vec{r}_{02}^{\,2}} - \frac{3\alpha_s}{4\pi} \beta \left[\ln\left(\frac{\vec{r}_{01}^{\,2}}{\vec{r}_{02}^{\,2}} \right) \left(\frac{1}{\vec{r}_{02}^{\,2}} - \frac{1}{\vec{r}_{01}^{\,2}} \right) - \frac{\vec{r}_{12}^{\,2}}{\vec{r}_{01}^{\,2} \vec{r}_{02}^{\,2}} \ln\left(\frac{\vec{r}_{12}^{\,2}}{\tilde{\mu}^2} \right) \right] \right)$$
$$- \frac{9\alpha_s^2}{2\pi^4} \int d\vec{r}_0 d\vec{r}_4 \, \tilde{L}^C_{12} B^{-}_{044} + \frac{27\alpha_s^2}{2\pi^2} \zeta(3) B^{-}_{122}$$
$$- \frac{\alpha_s^2 n_f}{12\pi^4} \int d\vec{r}_0 d\vec{r}_4 \{ (2B^{-}_{014} - B^{-}_{001} - B^{-}_{144}) - (2B^{-}_{024} - B^{-}_{002} - B^{-}_{244}) \} \, L^q_{12}.$$

This equation contains the nondipole 3QWL operators in its quark part.

Discussion

- 3QWL operator is the basic operator describing C-odd exchange in the linear regime (3-gluon approximation).
- Generic C-odd Wilson line operators (without derivatives) can be reduced to baryon Wilson loop operators in the linear regime, e.g. C-odd part of a quadrupole operator $tr(U_1 U_2^\dagger U_3 U_4^\dagger)$ in $SU(3)$ can be decomposed into a sum of 3QWLs

$$2tr(U_1 U_2^\dagger U_3 U_4^\dagger) - 2tr(U_4 U_3^\dagger U_2 U_1^\dagger) \stackrel{3g}{=}$$

$$\stackrel{3g}{=} B_{144}^- + B_{322}^- - B_{433}^- - B_{211}^- + B_{124}^- + B_{234}^- - B_{123}^- - B_{134}^-.$$

- Even C-odd part of a color dipole evolves via baryon Wilson loops in NLO.
- C-odd observables will depend on 3QWL starting from NLO accuracy.

Discussion

- The evolution equation for the C-odd part of the 3QWL operator is the generalization of the BKP equation for odderon exchange to the saturation regime.
- However, it is valid for the colorless object, i.e. for the function $B^{-}_{ijk} = B^{-}\left(\vec{r}_i, \vec{r}_j, \vec{r}_k\right)$, which vanishes as $\vec{r}_i = \vec{r}_j = \vec{r}_k$.
- The linear approximation of the equation for the C-odd part of the 3QWL should be equivalent to the NLO BKP for odderon exchange acting in the space of such functions (cf. Bartels Fadin Lipatov Vacca 2012).
- One may try to restore the full NLO BKP kernel from our result via the technique similar to the one developed for the 2-point operators (Fadin Fiore AG Papa 2011).

Results and outlook

- LO and NLO evolution equation for 3QWL.
- Quasi-conformal equation for composite 3QWL operator.
- Linearized quasi-conformal equation in 3-g approximation.
- Linearized equation for a dipole depending on 3QWLs in 3-g approximation.

- Baryon Wilson loop is a natural SU(3) model for low-x proton Green function $\rightarrow$ phenomenology.
- Solutions.

Thank you for your attention!