Bogoliubov Laboratory of Theoretical Physics

JOINT INSTITUTE FOR NUCLEAR RESEARCH

Form factors of the B-S transitions in the covariant quark model

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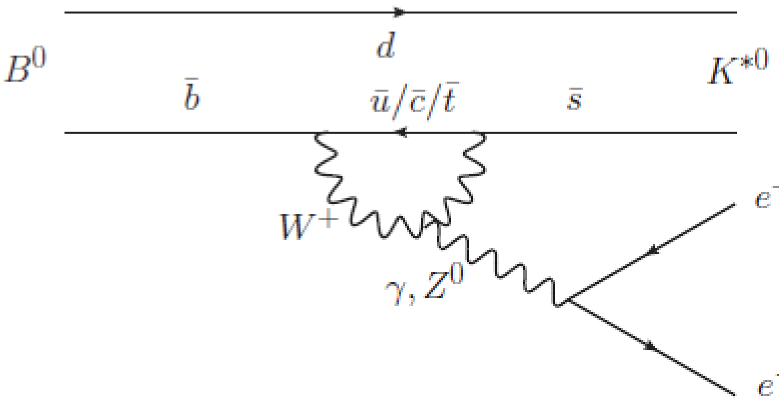
OUTLINE:

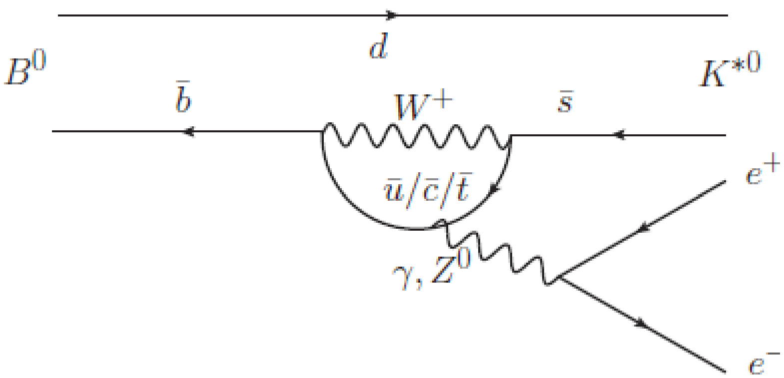
1. B meson
2. $B \to K^*_0(800)$ and $B \to K^*(892)$
3. Scalar mesons: $f_0(500)$, $K^*_0(800)$, $a_0(980)$, $f_0(980)$
4. Covariant quark model
5. Form factors of B-S transition
6. Branching fractions

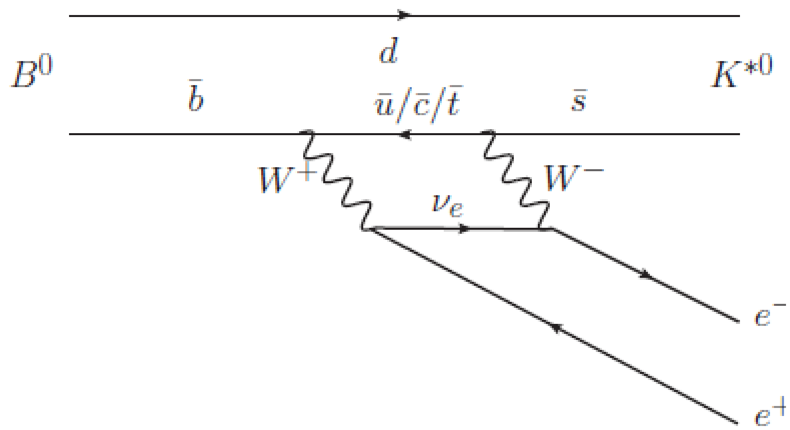
B MESON

Particle	Symbol	Quark content	Charge	Rest mass (Mev/c^2)	Mean lifetime (ps)
Strange B meson	B_s	sb	0	5,366.3±0.6	1.470+0.027 −0.026
Charmed B meson	B_c	cb	+1	6,276±4	0.46±0.07
B meson	B_u	ub	+1	5,279.15±0.31	1.638±0.011
B meson	B_d	db	0	5,279.53±0.33	1.530±0.009

$B \to K^*_0 \ell^+ \ell^-$ and $B \to K^* \ell^+ \ell^-$







*K. Abe et al., BELLE Coll.: Phys. Rev. Lett. **88** (2002) 021801.

B→ K, K* + ℓ⁺ℓ⁻

Decay modes	Branching fractions
$B^+ \to K^*(892)^+ e^+ e^-$	$1.55 \pm 0.31 \times 10^{-6}$
$B^+ \to K^*(892)^+ \mu^+ \mu^-$	$1.12 \pm 0.15 \times 10^{-6}$
$B^+ \to K^+ e^+ e^-$	$5.5 \pm 0.7 \times 10^{-7}$
$B^+ \to K^+ \mu^+ \mu^-$	$4.49 \pm 0.23 \times 10^{-7}$
$B^0 \to K^*(892)^0 e^+ e^-$	$1.03 \pm 0.19 \times 10^{-6}$
$B^0 \to K^*(892)^0 \mu^+ \mu^-$	$1.05 \pm 0.10 \times 10^{-6}$
$B^0 \to K^0 e^+ e^-$	$1.6 \pm 0.9 \times 10^{-7}$
$B^0 \to K^0 \mu^+ \mu^-$	$3.4 \pm 0.5 \times 10^{-7}$

*PDG14’

$B \to K^*(\to K\pi) + \ell^+\ell^-$

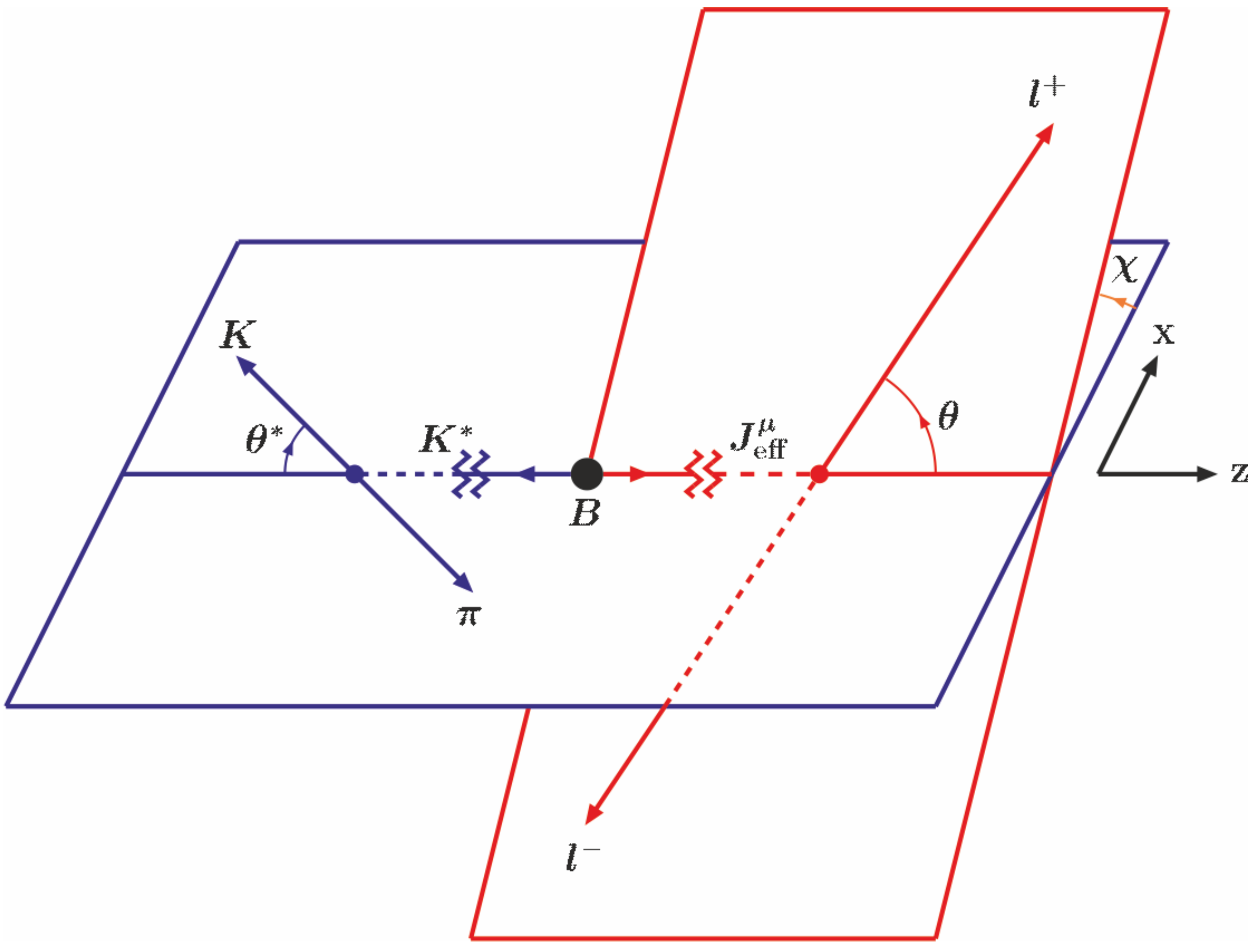
$$\frac{d^4\Gamma}{dq^2\, dcos\theta_K\, dcos\theta_l\, d\chi} = \sum_i P_i(q^2)\, F_i(\theta_K, \theta_l, \chi)$$

$$F_1 = \frac{3}{8}(1 + cos^2\theta) \cdot \frac{3}{4} sin^2\theta_K\ ,$$

$$F_2 = \frac{3}{4} sin^2\theta \cdot \frac{3}{2} cos^2\theta_K\ ,$$

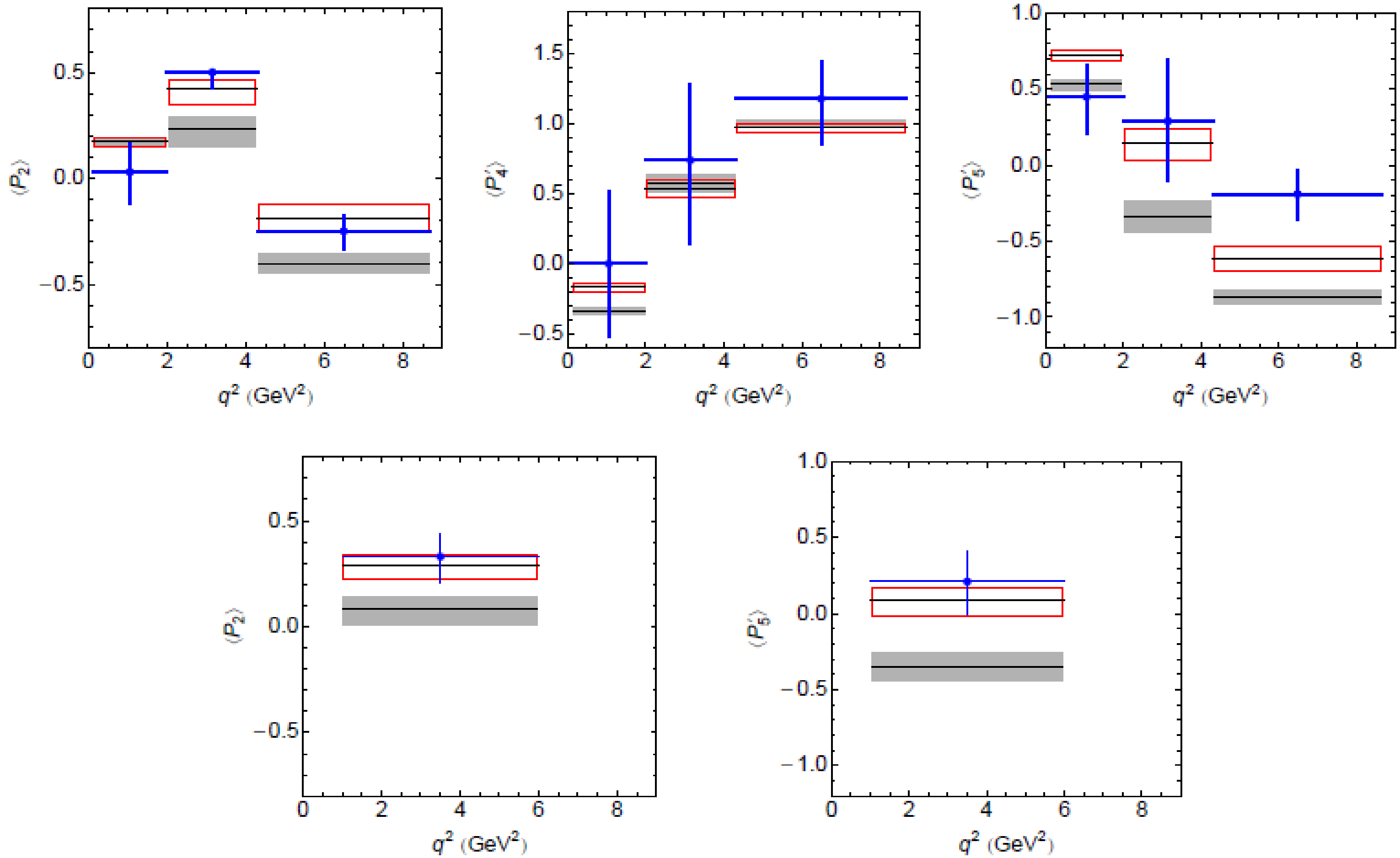
$$F_3 = -\frac{3}{4} sin^2\theta \cdot cos2\chi \cdot \frac{3}{4} sin^2\theta_K\ , \ldots.$$

$B \to K^*(\to K\pi) + \ell^+\ell^-$



* A. Faessler, T. Gutsche, M. A. Ivanov, J. G. Körner, and V. E. Lyubovitskij, Eur. Phys. J. direct C 4, 1 (2002).

$B \to K^*_0(800)$ and $B \to K^*(892)$

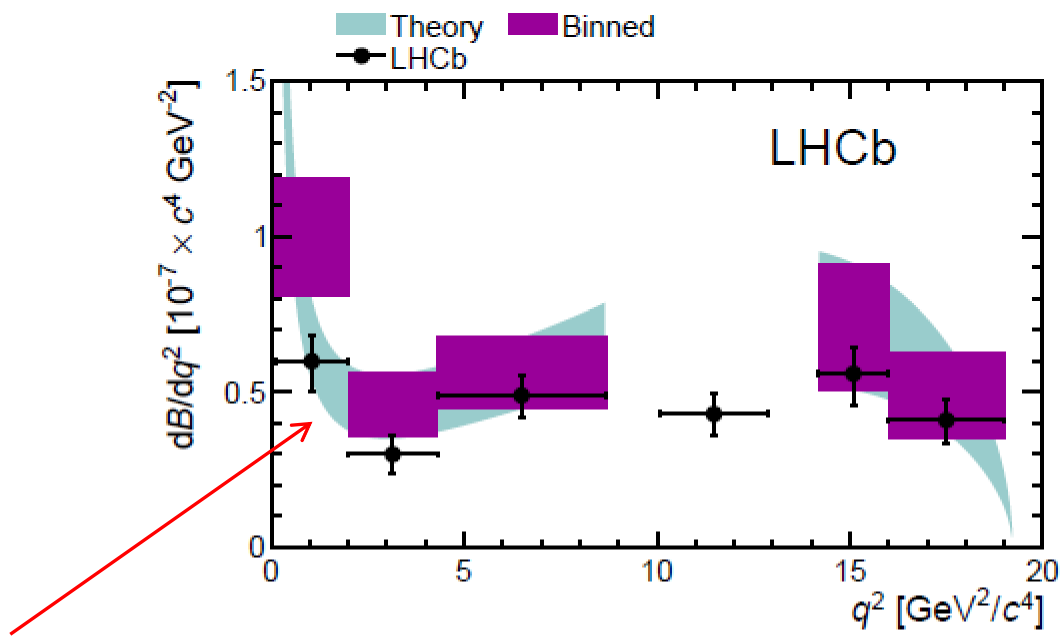


Comparison between the SM predictions (gray boxes), the experimental measurements (blue data points) and the predictions for the scenario with $\mathcal{C}_9^{\rm NP} = -1.5$ and other $\mathcal{C}_i^{\rm NP} = 0$ (red squares).

* J. Matias, Phys. Rev. D **86**, 094024 (2012).

Br(B→K*(Kπ) $\mu^+\mu^-$)

B→K*(Kπ)$\mu^+\mu^-$

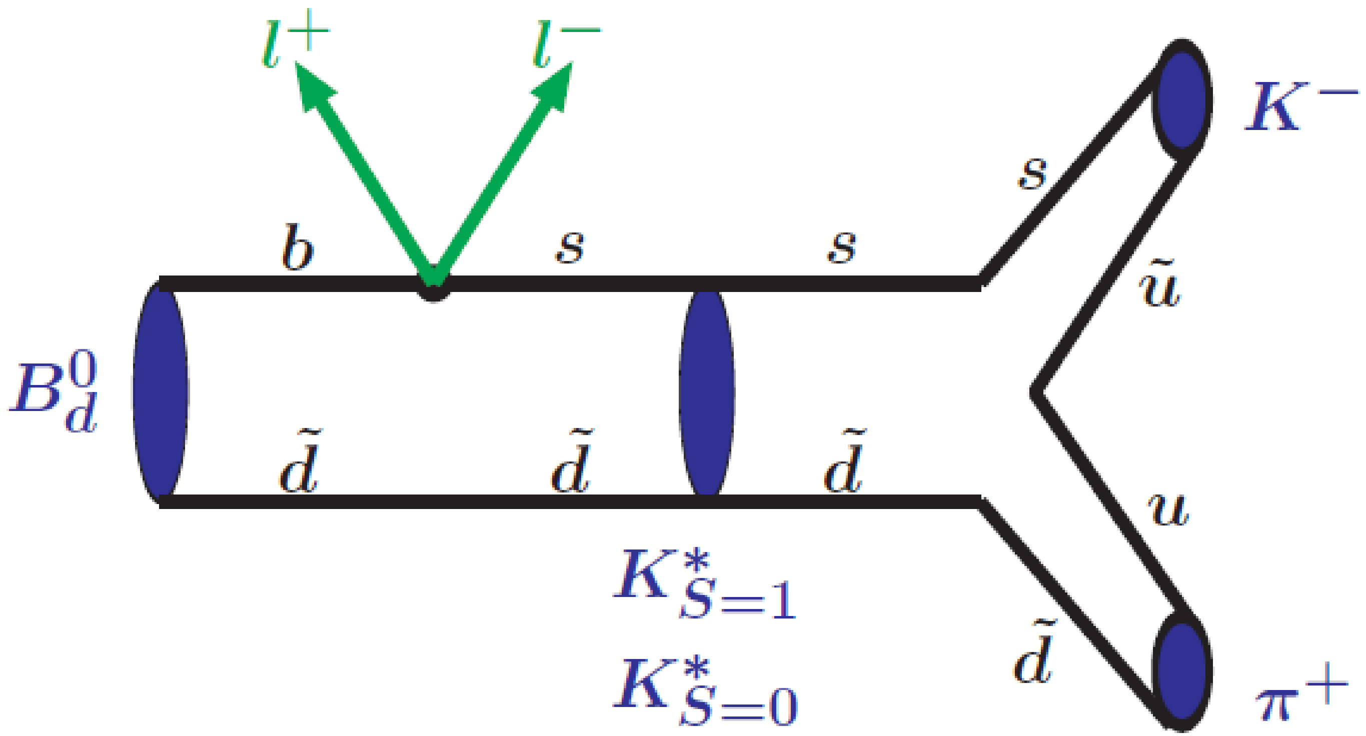


Exp/SM = 3.7σ

New Physics ??

* R. Aaij *et al.* (LHCb Collaboration), Phys. Rev. Lett. **111**, 191801 (2013).

$B \to K^*(\to K\pi) + \ell^+\ell^-$



Scalar mesons

- $S_s \equiv K_0^*(800),\ I\,(J^P) = \frac{1}{2}(0^+),\ m_{K_0^*(800)} = 682 \pm 29$ MeV;
- $S' \equiv f_0(500),\ I^G\,(J^{PC}) = 0^+(0^{++}),\ \ m_{f_0(500)} = 400 - 550$ MeV;
- $S \equiv f_0(980),\ I^G(J^{PC}) = 0^+(0^{++}),\ \ m_{f_0(980)} = 990 \pm 20$ MeV;
- $S^{\pm,0} \equiv a_0^{\pm,0}(980),\ I^G(J^{PC}) = 1^-(0^{++}),\ \ m_{a_0(980)} = 980 \pm 20$ MeV.

$$
\begin{aligned}
\mathcal{L}_{S\bar{q}q} &= \bar{q}\hat{S}q \\
&= S^+\,\bar{u}d + S^-\,\bar{d}u + S^0\,\frac{1}{\sqrt{2}}(\bar{u}u - \bar{d}d) + S_s^+\,\bar{u}s + S_s^0\,\bar{d}s + S_s^-\,\bar{s}u + \bar{S}_s^0\,\bar{s}d \\
&+ S'\left(\cos\delta_S\frac{1}{\sqrt{2}}(\bar{u}u + \bar{d}d) - \sin\delta_S\,\bar{s}s\right) - S\left(\sin\delta_S\frac{1}{\sqrt{2}}(\bar{u}u + \bar{d}d) + \cos\delta_S\,\bar{s}s\right)
\end{aligned}
$$

b → s ℓ⁺ ℓ⁻

$$H_{\rm eff} = -\frac{G_F}{\sqrt{2}}\lambda_t \sum_{i=1}^{10} C_i(\mu) Q_i(\mu)$$

$$\lambda_t \equiv V^{\dagger}_{ts(d)} V_{tb}$$

$$Q_1 = (\bar{s}_i c_j)_{V-A}, (\bar{c}_j b_i)_{V-A}, \qquad Q_2 = (\bar{s}c)_{V-A}(\bar{c}b)_{V-A},$$
$$Q_3 = (\bar{s}b)_{V-A}\sum_q(\bar{q}q)_{V-A}, \qquad Q_4 = (\bar{s}_i b_j)_{V-A}\sum_q(\bar{q}_j q_i)_{V-A},$$
$$Q_5 = (\bar{s}b)_{V-A}\sum_q(\bar{q}q)_{V+A}, \qquad Q_6 = (\bar{s}_i b_j)_{V-A}\sum_q(\bar{q}_j q_i)_{V+A},$$
$$Q_7 = \frac{e}{8\pi^2} m_b (\bar{s}\sigma^{\mu\nu}(1+\gamma_5)b)F_{\mu\nu}, \qquad Q_8 = \frac{g}{8\pi^2} m_b (\bar{s}_i\sigma^{\mu\nu}(1+\gamma_5)\mathbf{T}_{ij} b_j)\mathbf{G}_{\mu\nu},$$
$$Q_9 = \frac{e}{8\pi^2}(\bar{s}b)_{V-A}(\bar{l}l)_V, \qquad Q_{10} = \frac{e}{8\pi^2}(\bar{s}b)_{V-A}(\bar{l}l)_A$$

Effective Hamiltonian

$$M(b \to s\ell^+\ell^-) = \frac{G_F}{\sqrt{2}}\frac{\alpha\lambda_t}{2\pi}\left\{C_9^{\rm eff}\,(\bar{s}O^\mu b)\,\left(\bar{\ell}\gamma_\mu\ell\right) + C_{10}\,(\bar{s}O^\mu b)\,\left(\bar{\ell}\gamma_\mu\gamma_5\ell\right)\right.$$
$$\left. - \frac{2\hat{m}_b}{q^2}\,C_7^{\rm eff}\,\left(\bar{s}\,i\sigma^{\mu\nu}\,(1+\gamma^5)\,q^\nu\,b\right)\,\left(\bar{\ell}\gamma_\mu\ell\right)\right\}$$

$$C_9^{\rm eff} = C_9 + C_0\left\{h(\tilde{m}_c, s) + \frac{3\pi}{\alpha^2}\,\kappa \sum_{V_i=\psi(1s),\psi(2s)} \frac{\Gamma(V_i \to l^+l^-)\,m_{V_i}}{{m_{V_i}}^2 - q^2 - i m_{V_i}\Gamma_{V_i}}\right\}$$
$$- \frac{1}{2}h(1,s)\,(4C_3 + 4C_4 + 3C_5 + C_6)$$
$$- \frac{1}{2}h(0,s)\,(C_3 + 3C_4) + \frac{2}{9}\,(3C_3 + C_4 + 3C_5 + C_6)$$

B-S transition form factors

$$M(H_1 \to H_2\bar{\ell}\ell) = \frac{G_F}{\sqrt{2}} \cdot \frac{\alpha\lambda_t}{2\pi} \cdot \Big\{ C_9^{\rm eff} \langle H_2|\bar{s}O^\mu b|H_1\rangle \bar{\ell}\gamma_\mu \ell + C_{10} \langle H_2|\bar{s}O^\mu b|H_1\rangle \bar{\ell}\gamma_\mu\gamma_5 \ell - \frac{2\hat{m}_b}{q^2} C_7^{\rm eff} \langle H_2|\bar{s}i\sigma^{\mu\nu}(1+\gamma^5)q^\nu b|H_1\rangle \bar{\ell}\gamma_\mu \ell \Big\},$$

where $H_1=B$, $H_2=K^*(s=1)$ *or* $K^*_0(s=0)$

$$< H_2(p_2)\,|\,\bar{s}\,O^\mu\,b\,|H_1(p_1)> \;=\; F_+(q^2)\,P^\mu + F_-(q^2)\,q^\mu\,,$$

$$< H_2(p_2)\,|\,\bar{s}\,i\sigma^{\mu\nu}q_\nu(1+\gamma^5)\,b\,|\,H_1(p_1)> \;=\; -\frac{1}{m_1+m_2}\,\left(P_\mu\,q^2 - q_\mu\,Pq\right)\,F_T(q^2)\,,$$

B-S transition form factors in covariant quark model

$$\mathcal{L}_{\rm int}^{\rm str}(x) = g_M M(x) \int dx_1 \int dx_2 F_M(x, x_1, x_2)\, \bar{q}_1(x_1)\, \Gamma_M\, q_2(x_2) + \text{H.c.}$$

$$F_M(x, x_1, x_2) = \delta^{(4)}(x - \sum_{i=1}^{2} w_i x_i)\, \Phi_M\Big((x_1 - x_2)^2\Big)$$

$$\widetilde{\Phi}_M(-k^2) = \exp(k^2/\Lambda_M^2)$$

$$S_q(k) = \frac{1}{m_q - \not{k} - i\epsilon}$$

Compositeness condition in covariant quark model

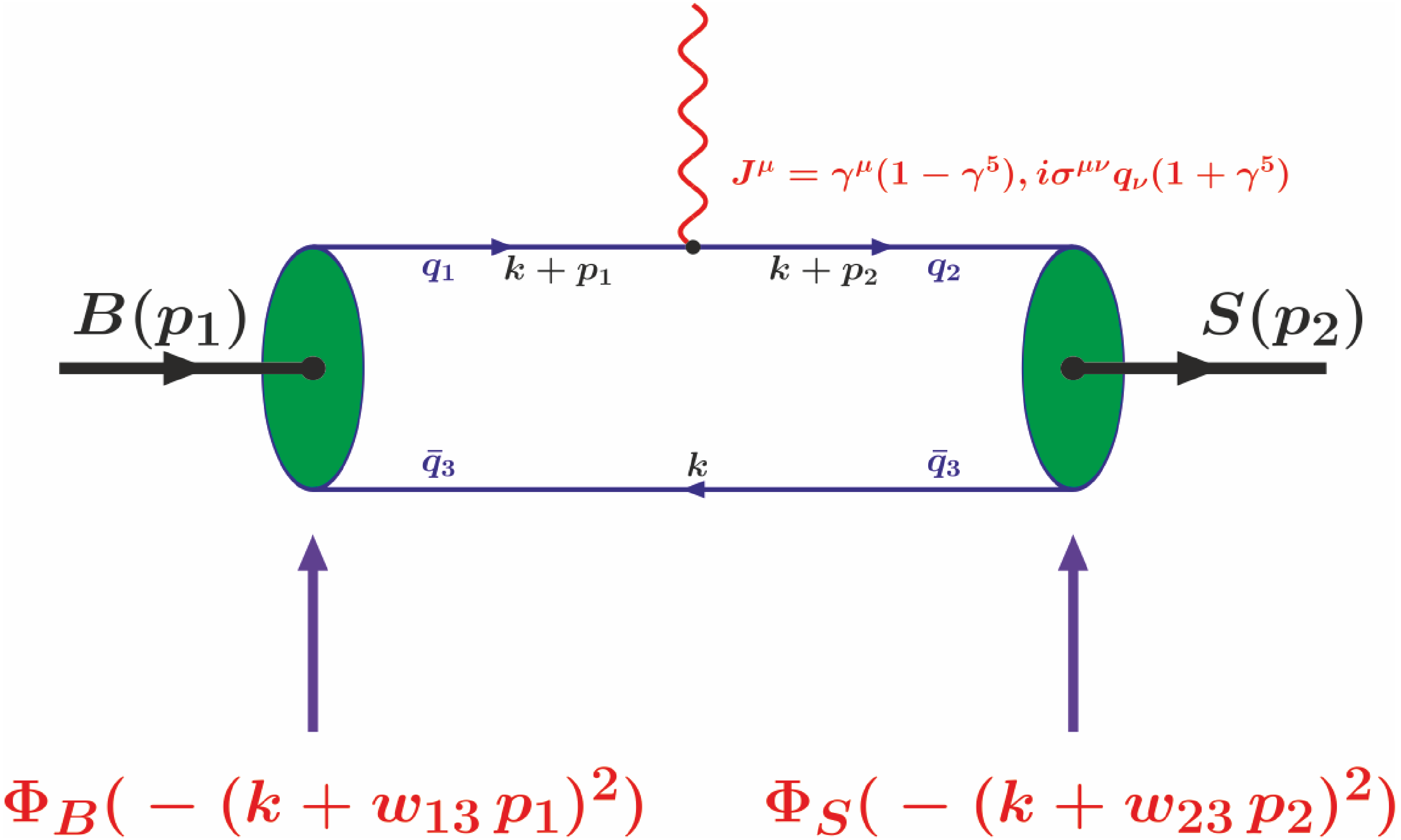
$$Z_M = 1 - g_M^2 \Pi'_M(m_M^2) = 0$$

$$\begin{aligned}
\widetilde{\Pi}'_S(p^2) &= -\frac{1}{2p^2}\, p^\alpha \frac{d}{dp^\alpha} \int \frac{d^4k}{4\pi^2 i}\, \widetilde{\Phi}_S^2(-k^2)\, \mathrm{tr}\Big[S_1(k + w_1 p)\, S_2(k - w_2 p)\Big] \\
&= -\frac{1}{2p^2} \int \frac{d^4k}{4\pi^2 i}\, \widetilde{\Phi}_S^2(-k^2) \Big\{ w_1\, \mathrm{tr}\Big[S_1(k + w_1 p)\, \not{p}\, S_1(k + w_1 p)\, S_2(k - w_2 p)\Big] \\
&\qquad - w_2\, \mathrm{tr}\Big[S_1(k + w_1 p)\, S_2(k - w_2 p)\, \not{p}\, S_2(k - w_2 p)\Big]. \Big\}
\end{aligned}$$

Compositeness condition in covariant quark model

$$
\begin{aligned}
\tilde{\Pi}'_S(p^2) &= \int\limits_0^{1/\lambda^2} \frac{dt\, t}{a_S^2} \int\limits_0^1 d\alpha\, e^{-t\, z_0 + z_1} \\
&\times \frac{t}{32}\Big\{p^2 - 4m_q^2 + \frac{1}{a_S}\left[20 + t\,(1-2\alpha)^2(12m_q^2 - p^2)\right] - \frac{t}{a_S^2}(1-2\alpha)^2(12+p^2 t) + \frac{t^3}{a_S^3}(1-2\alpha)^4 p^2\Big\} \\
z_0 &= \alpha m_q^2 - \alpha(1-\alpha)p^2, \qquad z_1 = \frac{st}{2a_S}(1-2\alpha)^2 p^2, \\
a_S &= 2s + t\,, \qquad s = \frac{1}{\Lambda_S^2}\,.
\end{aligned}
$$

B-S transition diagram



B-S transition matrix elements

$$\langle S_{[\bar{q}_3 q_2]}(p_2)|\bar{q}_2 O^\mu q_1|B_{[\bar{q}_1 q_3]}(p_1)\rangle = F_+^{BS}(q^2)P^\mu + F_-^{BS}(q^2)q^\mu,$$

$$\langle S_{[\bar{q}_3 q_2]}(p_2)|\bar{q}_2(i\sigma^{\mu\nu}q_\nu(1+\gamma^5))q_1|B_{[\bar{q}_1 q_3]}(p_1)\rangle = -\frac{1}{m_1+m_2}(q^2P^\mu - q\cdot Pq^\mu)F_T^{BS}(q^2).$$

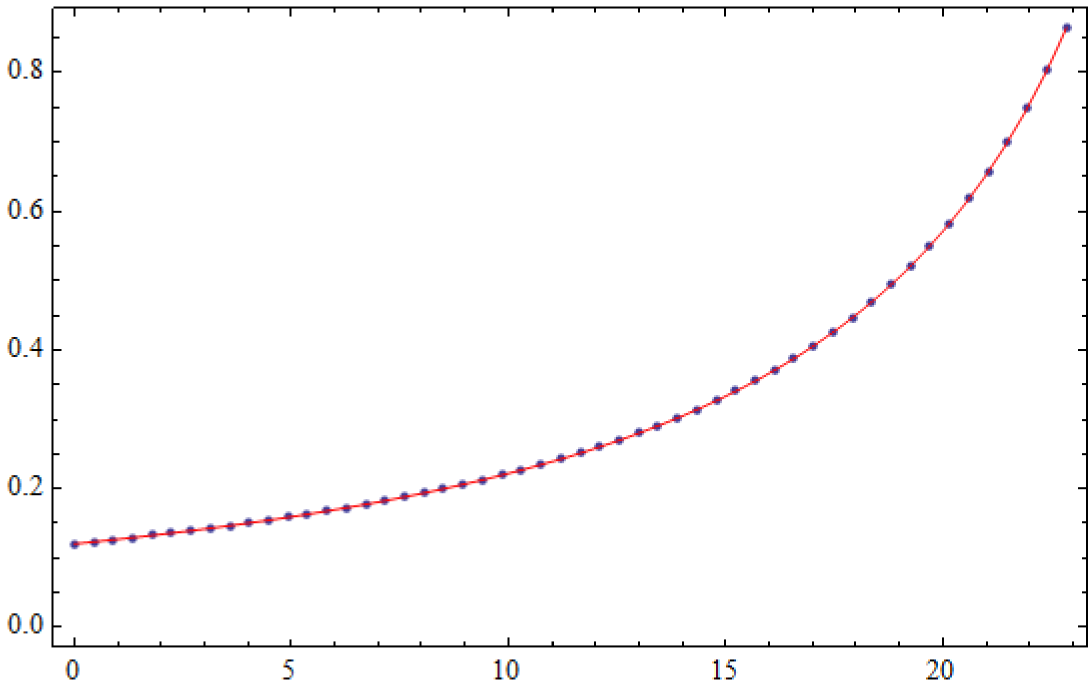
Model parameters

$m_{u/d}$	m_s	m_c	m_b	λ	
0.241	0.428	1.67	5.05	0.181	GeV

The fitted values of the size parameter Λ in GeV

π	K	D	D_s	B	B_s	B_c	η_c	η_b	
0.87	1.02	1.71	1.81	1.96	2.05	2.50	2.06	2.95	
ρ	ω	ϕ	J/ψ	K^*	D^*	D_s^*	B^*	B_s^*	Υ
0.61	0.50	0.91	1.93	0.75	1.51	1.71	1.76	1.71	2.96

Form factors of B-S transition



$$F(q^2) = \frac{F(0)}{1 - as + bs^2}, \qquad s = \frac{q^2}{m_1^2}.$$

Form factors of B-S transition

$q_1 - q_2$	$B - S$	$\Lambda_S = 0.8$			$\Lambda_S = 1.5$		
		$F_+(0)$	a_+	b_+	$F_+(0)$	a_+	b_+
$b - u$	$B_d^0 - a_0^+(980)$	0.144	1.624	0.585	0.192	1.433	0.381
$b - u$	$B_s^0 - K_0^{*+}(800)$	0.138	1.667	0.674	0.274	1.258	0.292
$b - s$	$B_s^0 - f_0(980)$	0.141	1.663	0.651	0.254	1.269	0.262
$b - s$	$B_d^0 - K_0^{*0}(800)$	0.191	1.348	0.407	0.306	0.988	0.108
$b - d$	$B_d^0 - f_0(500)$	0.120	1.448	0.485	0.210	1.067	0.155

Form factors of B-S transition

$q_1 - q_2$	$B - S$	$\Lambda_S = 0.8$			$\Lambda_S = 1.5$		
		$-F_-(0)$	a_-	b_-	$-F_-(0)$	a_-	b_-
$b - u$	$B_d^0 - a_0^+(980)$	0.049	2.144	1.196	0.089	1.723	0.688
$b - u$	$B_s^0 - K_0^{*+}(800)$	0.138	1.727	0.734	0.268	1.291	0.310
$b - s$	$B_s^0 - f_0(980)$	0.140	1.761	0.755	0.253	1.320	0.295
$b - s$	$B_d^0 - K_0^{*0}(800)$	0.199	1.406	0.457	0.296	1.032	0.129
$b - d$	$B_d^0 - f_0(500)$	0.116	1.504	0.536	0.191	1.110	0.180

Form factors of B-S transition

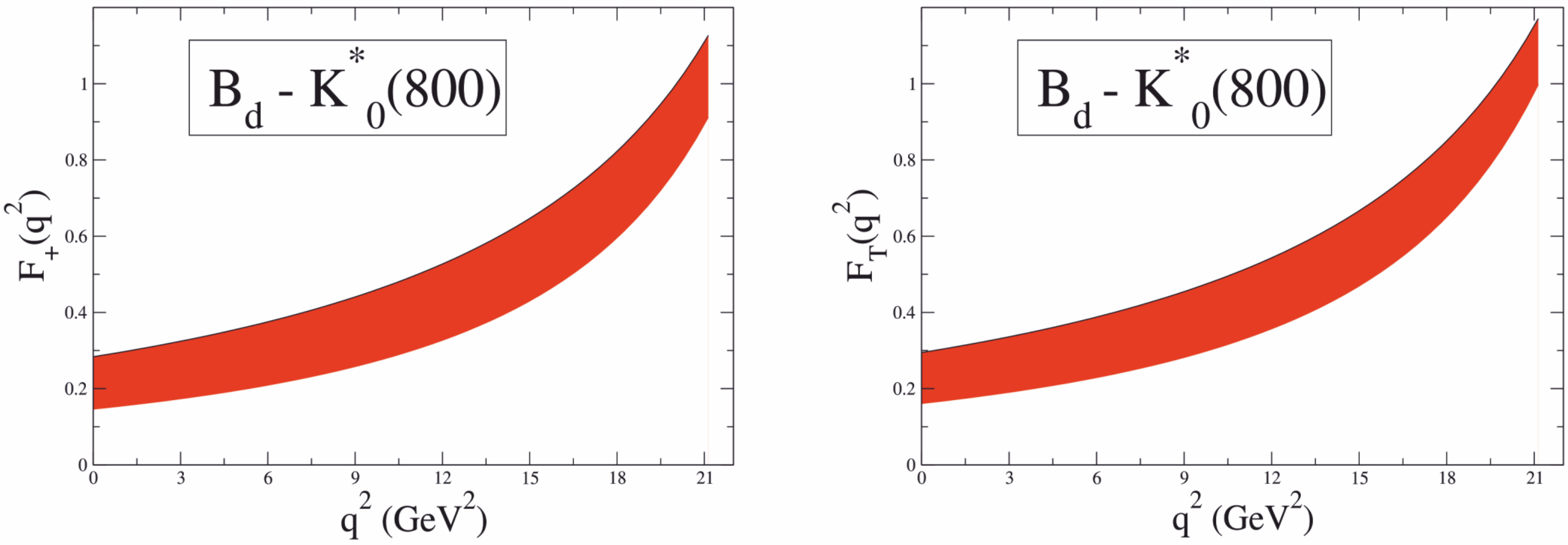
$q_1 - q_2$	$B - S$	$\Lambda_S = 0.8$			$\Lambda_S = 1.5$		
		$F_T(0)$	a_T	b_T	$F_T(0)$	a_T	b_T
$b - s$	$B_s^0 - f_0(980)$	0.165	1.680	0.667	0.285	1.276	0.257
$b - s$	$B_d^0 - K_0^{*\,0}(800)$	0.206	1.367	0.423	0.306	1.005	0.113
$b - d$	$B_d^0 - f_0(500)$	0.124	1.460	0.496	0.203	1.080	0.159

Form factors of B-S transition

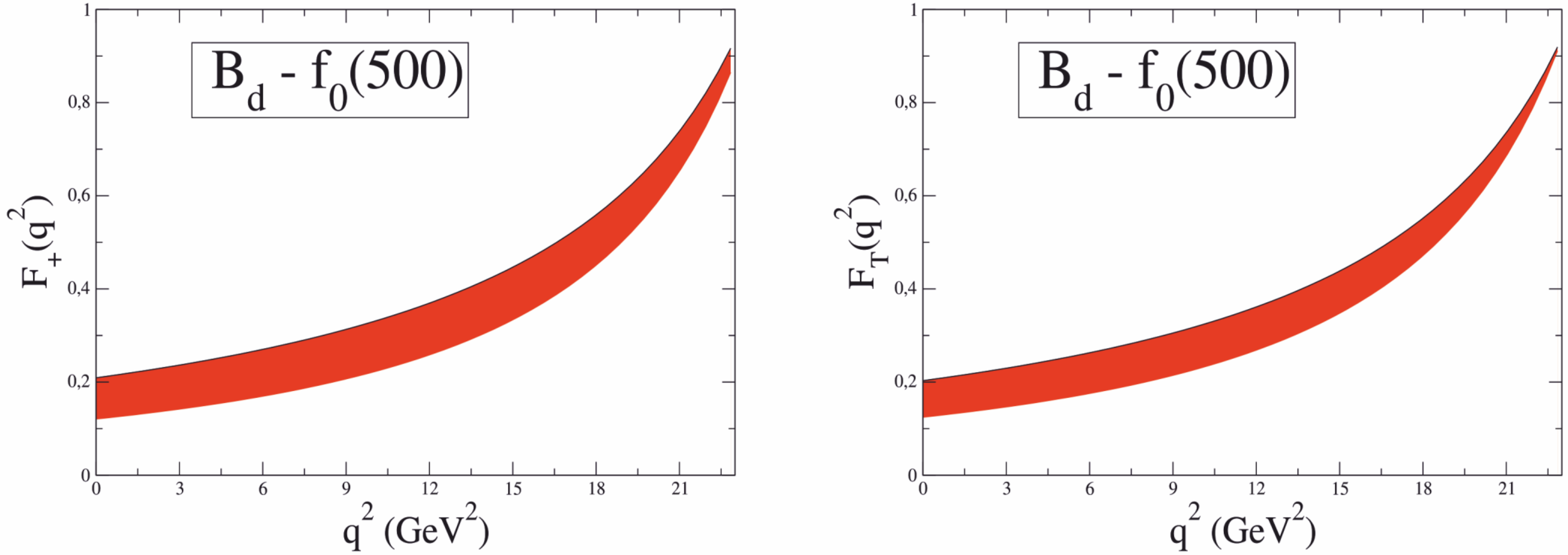
$B-S$	$F(0)$	This work	[36]	[34]	[18]	[31]	[32]	[55]
$B_d^0 - a_0^+(980)$	$F_+(0)$	0.192	0.58	0.56				
$B_s^0 - K_0^{*+}(800)$	$F_+(0)$	0.274	0.44	0.53				
$B_s^0 - f_0(980)$	$F_+(0)$	0.254	0.45	0.44	0.19	0.35	0.12	0.40
	$F_T(0)$	0.285	0.60	0.58	0.23	0.40	-0.08	
$B_d^0 - K_0^{*0}(800)$	$F_+(0)$	0.306	0.50	0.46				
	$F_T(0)$	0.306	0.67	0.58				
$B_d^0 - f_0(500)$	$F_+(0)$	0.210						
	$F_T(0)$	0.203						

[18] P. Colangelo, F. De Fazio, P. Santorelli and E. Scrimieri, Phys. Rev. D **53**, 3672 (1996); Phys. Rev. D **57**, 3186(E) (1998
[31] R. H. Li, C. D. Lu, W. Wang and X. X. Wang, Phys. Rev. D **79**, 014013 (2009) [arXiv:0811.2648 [hep-ph]].
[32] N. Ghahramany and R. Khosravi, Phys. Rev. D **80**, 016009 (2009).
[34] Y. J. Sun, Z. H. Li and T. Huang, Phys. Rev. D **83**, 025024 (2011) [arXiv:1011.3901 [hep-ph]].
[36] Z. G. Wang, Eur. Phys. J. C **75**, 50 (2015) [arXiv:1409.6449 [hep-ph]]
[55] B. El-Bennich, O. Leitner, J.-P. Dedonder and B. Loiseau, Phys. Rev. D **79**, 076004 (2009) [arXiv:0810.5771 [hep-ph]].

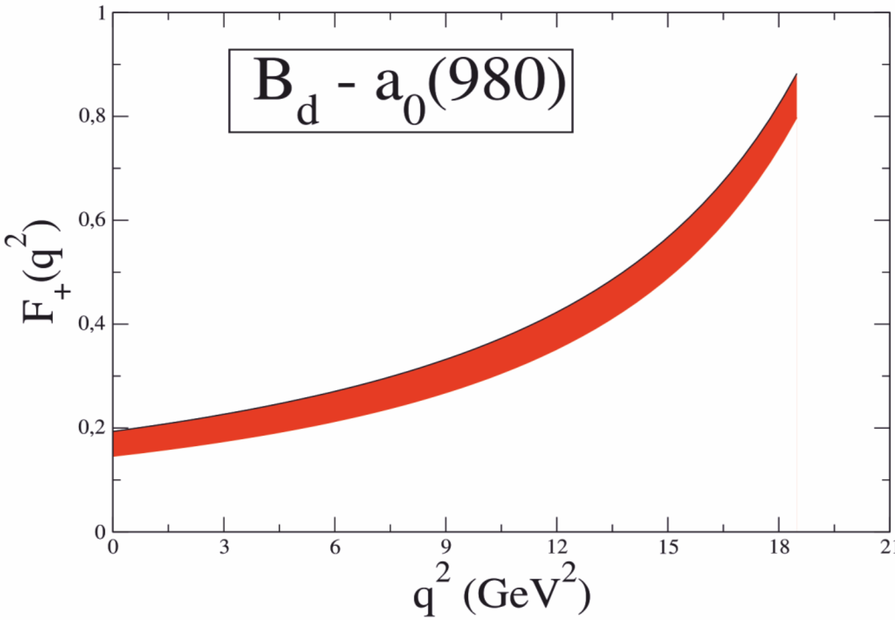
$B_d \rightarrow K_0(800)$

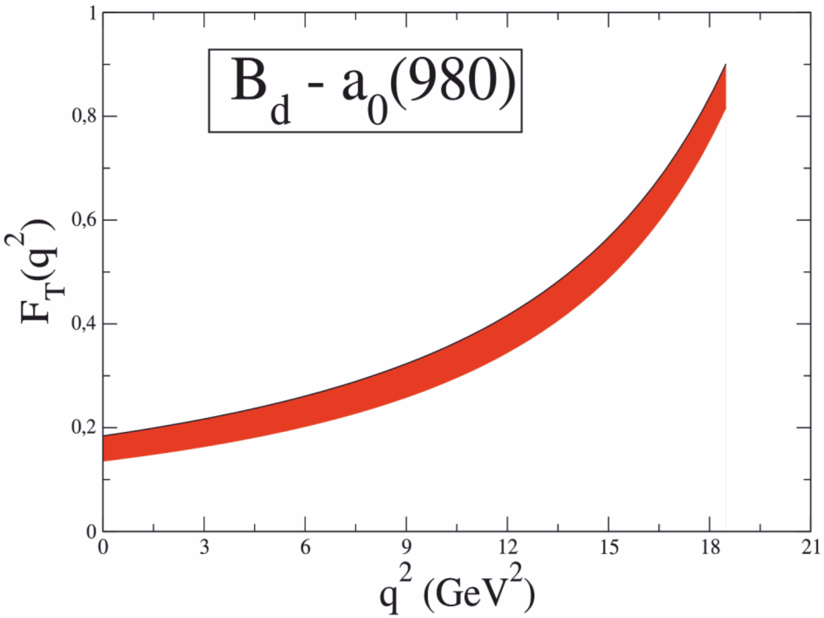


$B_d \to f_0(500)$

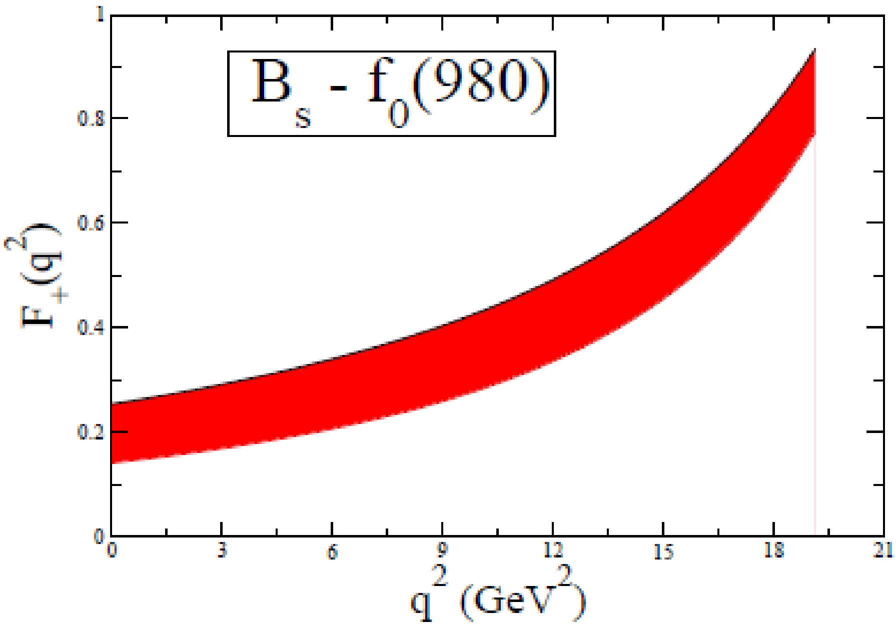


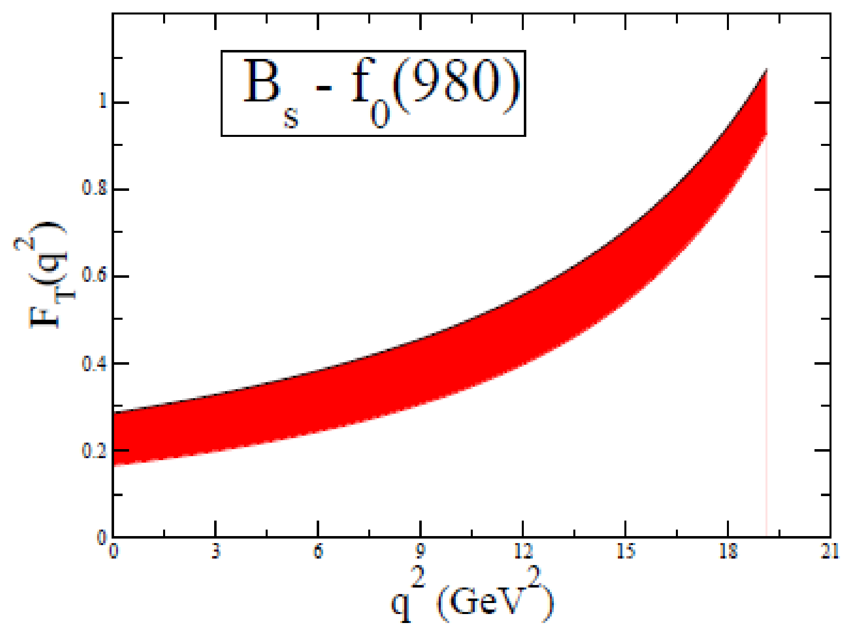
$B_d \rightarrow a_0(980)$





$B_s \to f_0(980)$





Branching fractions

Decay modes	Branching fractions			
	This work ($\Lambda_S = 1.5$ GeV)	[37]	[18]	[31]
$B_d^0 \to a_0^+(980)\mu^-\bar{\nu}_\mu$	0.52×10^{-4}	$(2.74 \pm 0.40) \times 10^{-4}$		1.84×10^{-4}
$B_d^0 \to a_0^+(980)\tau^-\bar{\nu}_\tau$	0.11×10^{-4}	$(1.31 \pm 0.23) \times 10^{-4}$		1.01×10^{-4}
$B_s^0 \to K_0^{*+}(800)\mu^-\bar{\nu}_\mu$	1.23×10^{-4}	$(2.06 \pm 0.31) \times 10^{-4}$		1.42×10^{-4}
$B_s^0 \to K_0^{*+}(800)\tau^-\bar{\nu}_\tau$	0.25×10^{-4}	$(1.07 \pm 0.19) \times 10^{-4}$		0.88×10^{-4}
$B_d^0 \to K_0^{*0}(800)\mu^+\mu^-$	3.47×10^{-7}	$(7.31 \pm 1.21) \times 10^{-7}$		
$B_d^0 \to K_0^{*0}(800)\tau^+\tau^-$	0.61×10^{-7}	$(1.33 \pm 0.36) \times 10^{-7}$		
$B_s^0 \to f_0(980)\mu^+\mu^-$	2.45×10^{-7}	$(5.14 \pm 0.78) \times 10^{-7}$	0.95×10^{-7}	5.21×10^{-7}
$B_s^0 \to f_0(980)\tau^+\tau^-$	0.42×10^{-7}	$(0.74 \pm 0.17) \times 10^{-7}$	1.1×10^{-7}	0.38×10^{-7}
$B_d^0 \to K_0^{*0}(800)\bar{\nu}\nu$	2.53×10^{-6}	$(6.30 \pm 0.97) \times 10^{-6}$		
$B_s^0 \to f_0(980)\bar{\nu}\nu$	1.79×10^{-6}	$(4.39 \pm 0.63) \times 10^{-6}$	0.87×10^{-6}	

[18] P. Colangelo, F. De Fazio, P. Santorelli and E. Scrimieri, Phys. Rev. D **53**, 3672 (1996); Phys. Rev. D **57**, 3186(E) (1998
[31] R. H. Li, C. D. Lu, W. Wang and X. X. Wang, Phys. Rev. D **79**, 014013 (2009) [arXiv:0811.2648 [hep-ph]].
[37] Z. G. Wang, Semi-leptonic $B \to S$ decays in the standard model and in the universal extra dimension mode

S-wave and P-wave contributions in the narrow width-limit

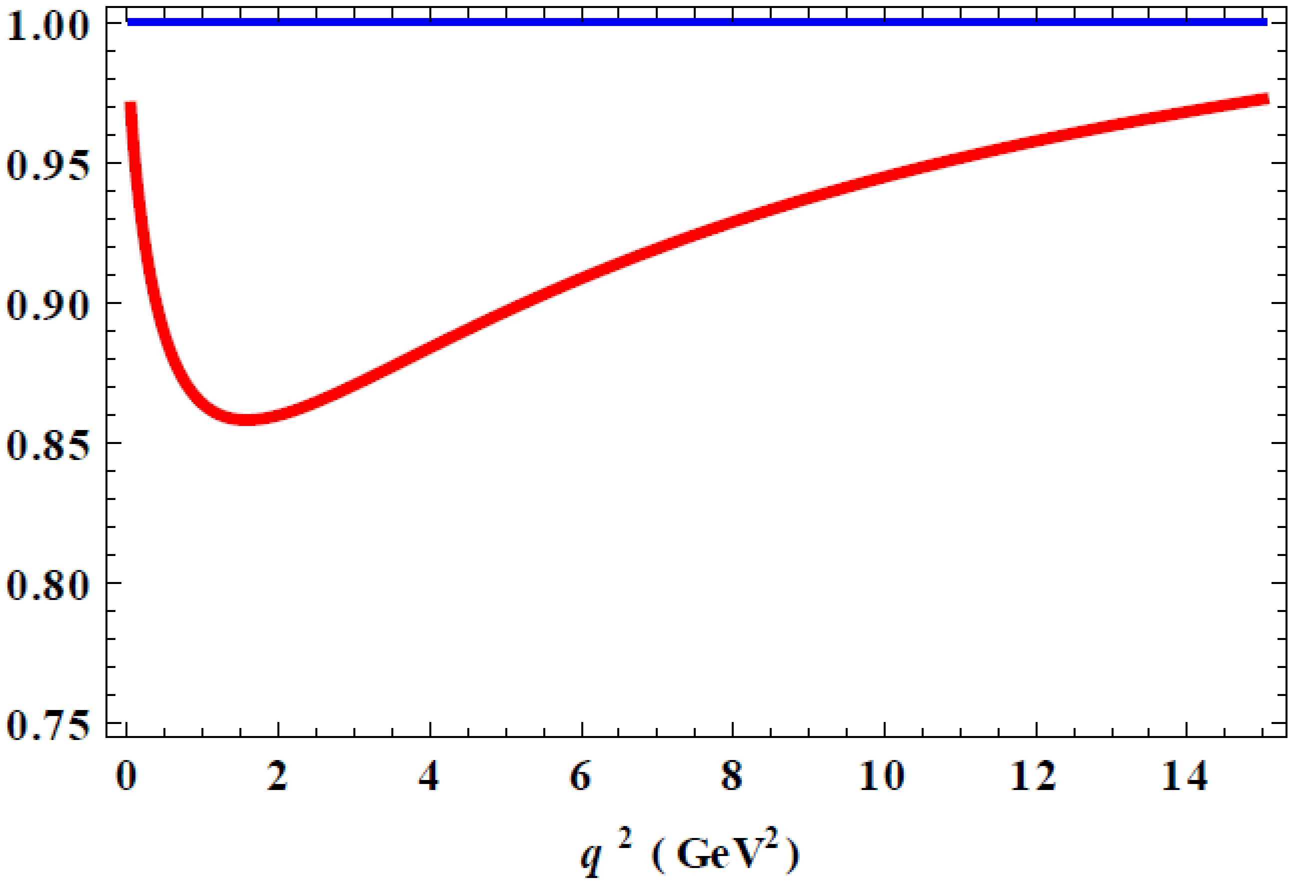
$$\int_{(m_{K^*}-\delta_m)^2}^{(m_{K^*}+\delta_m)^2} dm_{K\pi}^2 |L_S(m_{K\pi}^2)|^2 = 0.17$$

$$\int dm_{K\pi}^2 |L_{K^*}(m_{K\pi}^2)|^2 = \mathcal{B}(K^{*+} \to K^0\pi^+) = \frac{2}{3} \, .$$

* U. G. Meissner and W. Wang, J. High Energy Phys. 01 (2014) 107.

The ratio of the differential decay rate

$$R(q^2) = \frac{2/3\, d\Gamma(B \to K^*(892)\mu^+\mu^-)}{2/3\, d\Gamma(B \to K^*(892)\mu^+\mu^-) + 0.17 d\Gamma(B \to K_0^*(800)\mu^+\mu^-)}$$



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Form factors of the $B - S$ transitions in the covariant quark model

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In the wake of exploring uncertainty in the full angular distribution of the $B \to K\pi + \mu^+\mu^-$ decay caused by the presence of the intermediate scalar K_0^* meson, we perform the straightforward calculation of the $B(B_s) \to S$ (S is a scalar meson) transition form factors in the full kinematical region within the covariant quark model. We restrict ourselves to the scalar mesons below 1 GeV: $a_0(980)$, $f_0(500)$, $f_0(980)$, and $K_0^*(800)$. As an application of the obtained results we calculate the widths of the semileptonic and rare decays $B(B_s) \to S\ell\bar{\nu}$, $B(B_s) \to S\ell\bar{\ell}$, and $B(B_s) \to S\nu\bar{\nu}$. We compare our results with those obtained in other approaches.

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Thank you!

I. INTRODUCTION

Recently, much attention has been paid to the rare flavor-changing neutral current decay $B \to K^*(\to K\pi)\mu^+\mu^-$. One of the reasons for this was the first measurement of form-factor-independent angular observables performed by the LHCb Collaboration [1,2]. It has been claimed that there is a 3.7σ deviation from the Standard Model (SM) prediction for one of the angular observables. Much effort has been spent to explain this deviation by invoking the effects of new physics (NP) (for example, see Refs. [3–9] and references therein). The main emphasis of the above-mentioned papers was on the search for the physical observables that have low sensitivity to the form factors

recoil region [21], the relativistic quark model [22], and the Dyson-Schwinger equations in QCD [23].

The B_s and D_s to $K_0^*(1430)$ transition form factors were calculated in Ref. [24] within an approach based on QCD sum rules. The form factors for the $B \to K_0^*(1430)$ transition have been evaluated in the light-front quark model [25]. The form factors of rare $B \to K_0^*(1430)\ell^+\ell^-$ decay were calculated in Ref. [26] within three-point QCD sum rules. The $B \to S$ transition form factors have been investigated in the light-cone sum rules approach [27]. The transition form factors of $B(B_s)$-meson decay into a scalar meson were studied in Ref. [28] within the perturbative QCD approach. With these form factors, the decay width

$$
\begin{aligned}
& \langle S_{[\bar{q}_3 q_2]}(p_2)\,|\,\bar{q}_2\, O^{\mu}\, q_1\,|B_{[\bar{q}_1 q_3]}(p_1)\rangle \\
= \;& N_c\, g_B\, g_S \int \frac{d^4k}{(2\pi)^4 i}\, \widetilde{\Phi}_B\Big(-(k+w_{13}p_1)^2\Big)\, \widetilde{\Phi}_S\Big(-(k+w_{23}p_2)^2\Big) \\
\times \;& \mathrm{tr}\Big[S_2(k+p_2) O^{\mu}\, S_1(k+p_1)\, \gamma^5\, S_3(k)\Big] \\
= \;& F_+^{BS}(q^2)\, P^{\mu} + F_-^{BS}(q^2)\, q^{\mu}\,,
\end{aligned}
$$

$$
\begin{aligned}
& \langle S_{[\bar{q}_3 q_2]}(p_2)\,|\,\bar{q}_2\, (i\sigma^{\mu\nu} q_\nu (1+\gamma^5))\, q_1\,|B_{[\bar{q}_1 q_3]}(p_1)\rangle \\
= \;& N_c\, g_B\, g_S \int \frac{d^4k}{(2\pi)^4 i}\, \widetilde{\Phi}_B\Big(-(k+w_{13}p_1)^2\Big)\, \widetilde{\Phi}_S\Big(-(k+w_{23}p_2)^2\Big) \\
\times \;& \mathrm{tr}\Big[S_2(k+p_2) i\sigma^{\mu\nu} q_\nu (1+\gamma^5)\, S_1(k+p_1)\, \gamma^5\, S_3(k)\Big] \\
= \;& -\frac{1}{m_1+m_2}\, \big(q^2\, P^{\mu} - q\cdot P\, q^{\mu}\big)\, F_T^{BS}(q^2).
\end{aligned}
$$

B-S transition form factors

$$
\begin{aligned}
M(H_1 \to H_2 \bar{\ell}\ell) \;=\; & \frac{G_F}{\sqrt{2}} \cdot \frac{\alpha \lambda_t}{2\,\pi} \cdot \{ C_9^{\rm eff} \; < H_2 \,|\, \bar{s}\, O^\mu \, b \,|\, H_1 > \; \bar{\ell} \gamma_\mu \ell \\
& + \; C_{10} \; < H_2 \,|\, \bar{s}\, O^\mu \, b \,|\, H_1 > \; \bar{\ell} \gamma_\mu \gamma_5 \ell \\
& - \; \frac{2 \hat{m}_b}{q^2} \, C_7^{\rm eff} \; < H_2 \,|\, \bar{s}\, i \sigma^{\mu\nu} \, (1 + \gamma^5) \, q^\nu \, b \,|\, H_1 > \; \bar{\ell} \gamma_\mu \ell \Big\}
\end{aligned}
$$

where $H_1 = B$, $H_2 = K\,(K_0^*)$.

$$
< H_2(p_2) \,|\, \bar{s}\, O^\mu \, b \,| H_1(p_1) \,> \; = \; F_+(q^2) \, P^\mu + F_-(q^2) \, q^\mu \, ,
$$

$$
< H_2(p_2) \,|\, \bar{s}\, i \sigma^{\mu\nu} q_\nu (1 + \gamma^5) \, b \,|\, H_1(p_1) \,> \; = \; - \frac{1}{m_1 + m_2} \, \left(P_\mu \, q^2 - q_\mu \, P q \right) \, F_T(q^2) \, ,
$$

B-S transition form factors

$$M\left(H_1 \to H_2 + \bar{\ell}\ell\right) = \frac{G_F}{\sqrt{2}} \cdot \frac{\alpha\lambda_t}{2\pi} \left\{T_1^\mu \left(\bar{\ell}\gamma_\mu \ell\right) + T_2^\mu \left(\bar{\ell}\gamma_\mu\gamma_5 \ell\right)\right\}$$

$$T_i^\mu \;=\; \mathcal{F}_+^{(i)}\, P^\mu + \mathcal{F}_-^{(i)}\, q^\mu\,, \qquad (i = 1, 2)\,,$$

$$\mathcal{F}_+^{(1)} \;=\; C_9^{\rm eff}\, F_+ + C_7^{\rm eff}\, F_T\, \frac{2\hat{m}_b}{m_1 + m_2}\,,$$

$$\mathcal{F}_-^{(1)} \;=\; C_9^{\rm eff}\, F_- - C_7^{\rm eff}\, F_T\, \frac{2\hat{m}_b}{m_1 + m_2}\, \frac{Pq}{q^2}\,,$$

$$\mathcal{F}_\pm^{(2)} \;=\; C_{10}\, F_\pm\,.$$