Meson mass spectrum and the Fermi coupling in the Covariant Confined Quark Model

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Outline

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- **Model**
- **Fermi and Yukawa interactions**
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Motivation

- **One of the puzzles of hadron physics is the origin of the hadron masses. The Standard Model (particularly, QCD) operates only with fundamental particles (quarks, leptons, neutrinos), gauge bosons and the Higgs.**

- **It is not yet clear how to explain an appearance of the numerous number of the observed hadrons and elucidate the generation of their masses.**

- **The calculation of hadron mass characteristics comparable to the precision of experimental data still remains one of the major problems in QCD and involving of more phenomenological inputs are needed.**

- **CCQM has been successfully applied for:**
  - **calculation of the leptonic decay constants,**
  - **basic electromagnetic decay widths and form factors**

needed for semi-leptonic, non-leptonic and rare decays of *B* mesons and Λ_b baryons.

CCQ Model (Short Review)

Basic Assumptions:

T. Branz et al., Phys. Rev. **D81**, 034010 (2010).

- Hadrons *H(x)* are elementary particles - consist of constituent quarks. Hadrons interact by means of *quark exchanges* with hadron-quark coupling g_H.

$$L_{\rm int} = g_H H(x) J_H(x)$$

- The matrix element between the *physical state* and the corresponding *bare state* is determined by the renormalization constant.

$$Z_H = \left\langle H_{bare} \,|\, H_{phys} \right\rangle^2 , \qquad H_{bare} = Z_H^{1/2} H_{phys}$$

- The renormalized hadron-quark coupling g_{ren} is determined by the **compositeness condition** which eliminates the bare fields from consideration.

$$Z_H = 1 - g_{ren}^2 \tilde{\Pi}'_H (M_H^2) = 0$$

- **Infrared confinement** is introduced to guarantee the absence of all possible *thresholds* corresponding to quark production. It allows to use the same values for the constituent quark masses.

Compositeness Condition

- **Yukawa-type model**

$$L_Y = \bar{q}(i\hat{\partial} - m)q + \phi_0(\Box - M_0^2)\phi_0 + g_0\phi_0(\bar{q}\,\Gamma\, q)$$

$$Z_Y = \int \delta\phi_0 \int \delta\bar{q} \int \delta q \exp\left\{ i \int dx\, L_Y(x) \right\}$$

Integrating out over the quark fields (Gaussian integral)

$$Z_Y = \int \delta\phi_0 \exp\left\{ \frac{i}{2}\left(\phi_0(\Box - M_0^2)\phi_0\right) + \frac{i\, g_0^2}{2}\left(\phi_0 \tilde{\Pi} \phi_0\right) \right\}$$

$$\cdot \exp\left\{ -\sum_{n=3}^{\infty} \frac{i^n g_0^n}{n} \int dx_1 \ldots \int dx_n\, \phi_0(x_1) \ldots \phi_0(x_n) \cdot tr\left\{ \Gamma S(x_1 - x_2) \ldots \Gamma S(x_n - x_1) \right\} \right\}$$

where the mass function of the boson field (zero-spin)

$$\Pi(x-y) \equiv i < T\left\{ (\bar{q}\Gamma q)_x (\bar{q}\Gamma q)_y \right\} > = -i \cdot tr\left\{ \Gamma S(x-y) \Gamma S(y-x) \right\}$$

Expand the mass function at the physical value ***M*** up to the 2nd order

$$\tilde{\Pi}(p^2) = \tilde{\Pi}(M^2) + (p^2 - M^2)\tilde{\Pi}'(M^2) + \tilde{\Pi}^{ren}(p^2),$$

$$\tilde{\Pi}'(p^2) \equiv \frac{d}{dp^2}\tilde{\Pi}(p^2)$$

Bi-linear part of the Lagrangian takes form

$$L_Y^{(2)} = \frac{1}{2}\left(\phi_0\left[\Box - M_0^2 - g_0^2\,\tilde{\Pi}(M^2) + (\Box - M^2)\tilde{\Pi}'(M^2)\right]\phi_0\right) + \frac{g_0^2}{2}\left(\phi_0\tilde{\Pi}^{ren}\phi_0\right)$$

Define:

$$\mathrm{Z} \equiv \frac{1}{1 + g_0^2 \cdot \tilde{\Pi}'(M^2)}$$

Renormalization of boson wave function, Yukawa coupling and mass:

$$\phi_{ren} \equiv Z^{-1/2}\phi_0\,;$$
$$g_{ren} \equiv Z^{1/2} g_0;$$
$$M^2 \equiv M_0^2 - g_0^2\tilde{\Pi}(M^2)$$

Also, Z may be expressed via the renormalized coupling constant as follows:

$$\mathrm{Z} = 1 - g_{ren}^2\tilde{\Pi}'(M^2)$$

Finally, the renormalized generating functional for the Yukawa theory

$$Z_Y^{ren} = \int \delta\phi_{ren} \exp\left\{ \frac{i}{2}\Big(\phi_{ren}(\Box - M^2)\phi_{ren}\Big) + \frac{i\,g_{ren}^2}{2}\Big(\phi_{ren}\tilde{\Pi}^{ren}\phi_{ren}\Big)\right\}$$
$$\cdot \exp\left\{ -\sum_{n=3}^{\infty} \frac{i^n\, g_{ren}^n}{n} \int dx_1 \ldots \int dx_n\, \phi_{ren}(x_1)\ldots\phi_{ren}(x_n)\cdot tr\left\{\Gamma\, S(x_1 - x_2)\ldots\Gamma\, S(x_n - x_1)\right\}\right\}$$

- **Fermi-type model**

$$L_F = \bar{q}(i\hat{\partial} - m)q + \frac{G}{2}(\bar{q}\,\Gamma\,q)^2$$

Gaussian representation:

$$\exp\left\{i\frac{G}{2}(\bar{q}\,\Gamma\,q)^2\right\} \approx \int \delta\phi \exp\left\{-i\frac{1}{2G}(\phi\,\phi) + i\left(\phi(\bar{q}\,\Gamma\,q)\right)\right\}$$

Integrating out over the quark fields (Gaussian integral)

$$Z_F = \int \delta\bar{q} \int \delta q \exp\left\{i\int dx\, L_F(x)\right\}$$
$$= \int \delta\phi \exp\left\{-i\frac{1}{2G}(\phi\,\phi) - \sum_{n=2}^{\infty}\frac{i^n}{n}\int dx_1 ... \int dx_n\, \phi(x_1)...\phi(x_n) \cdot tr\left\{\Gamma\, S(x_1 - x_2)...\Gamma\, S(x_n - x_1)\right\}\right\}$$

Bi-linear terms in boson fields:

$$L_F^{(2)} = \frac{1}{2}\left(\phi\left[-\frac{1}{G} + \Pi(M^2) + (\Box - M^2)\Pi'(M^2)\right]\phi\right) + \frac{1}{2}\left(\phi\,\Pi^{ren}\phi\right)$$

Require condition

$$-\frac{1}{G} + \tilde{\Pi}(M^2) = 0$$

Rescaling of boson field

$$\phi_{ren} = \left[\tilde{\Pi}'(M^2)\right]^{-1/2}\phi$$

$$Z_F^{ren} = \int \delta\phi_{ren} \exp\left\{\frac{i}{2}\left(\phi_{ren}(\Box - M^2)\phi_{ren}\right) + \frac{i}{2}\frac{1}{\tilde{\Pi}'(M^2)}\left(\phi_{ren}\tilde{\Pi}^{ren}\phi_{ren}\right)\right\}$$

$$\cdot \exp\left\{-\sum_{n=3}^{\infty}\frac{i^n}{n}\left[\frac{1}{\tilde{\Pi}'(M^2)}\right]^{n/2}\int dx_1 \ldots \int dx_n\, \phi_{ren}(x_1)\ldots\phi_{ren}(x_n)\cdot tr\left\{\Gamma S(x_1 - x_2)\ldots\Gamma S(x_n - x_1)\right\}\right\}$$

Both renormalized generating functionals are equal if:

$$g_{ren} = \frac{1}{\sqrt{\tilde{\Pi}'(M^2)}} \quad \longrightarrow \quad Z \equiv 1 - g_{ren}^2 \tilde{\Pi}'(M^2) = 0$$

Compositeness Condition

$$\phi_0 = Z^{1/2} \phi_{ren} = 0$$

The vanishing of the wave function renormalization constant (Z=0) in the Yukawa theory may be interpreted as the condition that the bare (unrenormalized) field vanishes for a composite boson.

A. Salam, Nuovo Cim. 25, 224 (1962)
S. Weinberg, Phys. Rev. 130, 776 (1963)

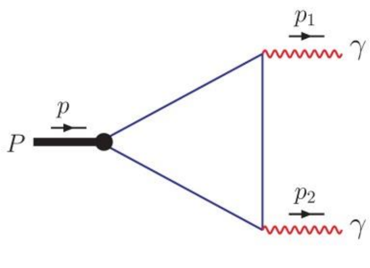
Jouvet condition

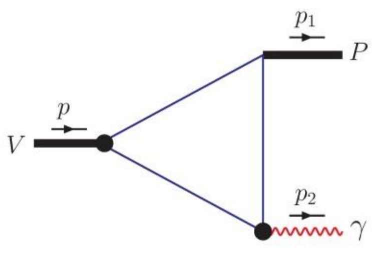
B.Jouvet, Nuovo Cim. 3, 1133 (1956) :

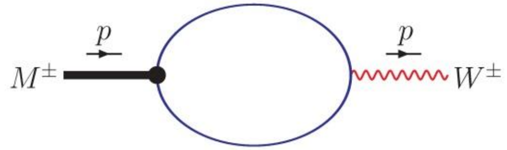
$$as\ Z \to 0: \qquad M_0 \to \infty; \quad g_0 \to \infty \quad by\,keeping \quad \lim_{\substack{g\to 0\\ \mu\to 0}} \frac{g_0^2}{M_0^2 - M^2} = G < \infty$$

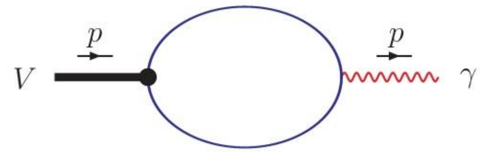
Infrared Confinement

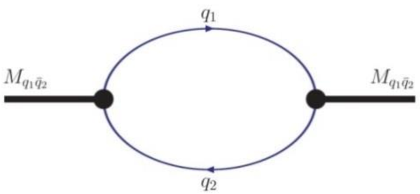
- Matrix elements are combinations of propagators and vertices.











$$-\frac{1}{G}+\tilde{\Pi}(M^2)=0$$

- Schwinger representation of the propagator in a loop:

$$\tilde{S}_m(\hat{k}+\hat{p})=\frac{m+(\hat{k}+\hat{p})}{m^2-(\hat{k}+\hat{p})^2}=\left(m+(\hat{k}+\hat{p})\right)\cdot\int\limits_0^\infty ds\exp\left[-s\left(m^2-(k+p)^2\right)\right]$$

- **Vertex function** in simple Gaussian form:

$$\tilde{\Phi}(-k^2) = \exp\left(\frac{k^2}{\Lambda^2}\right)$$

- Loop integrals over ***k*** (and external momenta ***p***, too) are taken in Euclidean space:

$$k^0 \to ik_4; \qquad p^0 \to ip_4; \qquad k^2 \to -k_E^2 \le 0; \qquad p^2 \to -p_E^2 \le 0$$

- **Loop integrals are absolutely convergent** (*negative quadratic forms in the exponentials*).

- Loop momenta ***k*** (in the numerator) are expressed by exponentials:

$$k_\mu \cdot \exp(2kp) = \frac{1}{2}\frac{\partial}{\partial p_\mu}\exp(2kp)$$

- The kernel of a loop diagram is finally rewritten in the form (***n*** – number of propagators):

$$\Pi(\mathrm{p},\mathrm{k}_1,\ldots,\mathrm{k}_n)=\int_0^\infty ds_1\ldots\int_0^\infty ds_n\,F(s_1,\ldots,s_n)$$

- The Schwinger parameter set is turned into a simplex by using the identity:

$$1=\int_0^\infty dt\cdot\delta\left(t-s_1\,\ldots-s_n\right)$$

- Then,

$$\Pi(\mathrm{p},\mathrm{k}_1,\ldots,\mathrm{k}_n)=\int_0^\infty dt\,t\int_0^1 ds_1\ldots\int_0^1 ds_n\,\delta\left(1-s_1\,\ldots-s_n\right)\,F(ts_1,ts_2,\ldots,ts_n)$$

- Cut off the upper bound over ***t***-integration ($\infty \to 1/\lambda^2$):

$$\Pi^{IRC}(\mathrm{p},\mathrm{k}_1,...,\mathrm{k}_n) = \int_0^{1/\lambda^2} dt\, t \int_0^1 ds_1 ... \int_0^1 ds_n\, \delta\left(1 - s_1 \, ... - s_n\right) F(ts_1, ts_2, ..., ts_n)$$

- The infrared cut-off removes all possible thresholds in the quark loop diagram

.

- The cut-off parameter λ to be the same in all physical processes.

Meson Mass Function within CCQM

- **Interaction Lagrangian**:

$$L_{\rm int} = g_H H(x) J_H(x)$$

- **Quark currents (for mesons):**

$$J_H(x) = \int dx_1 \int dx_2\, F_H(x; x_1, x_2) \cdot \bar{q}(x_2) \Gamma_H q(x_1)$$

$$\Gamma_P = i\gamma^5; \quad \Gamma_V = \gamma^\mu$$

- **Vertex function (translational invariant):**

$$F_H(x; x_1, x_2) = \delta(x - \omega_1 x_1 - \omega_2 x_2)\, \Phi_H(|x_1 - x_2|^2)$$

$$\omega_j = \frac{m_j}{m_1 + m_2}$$

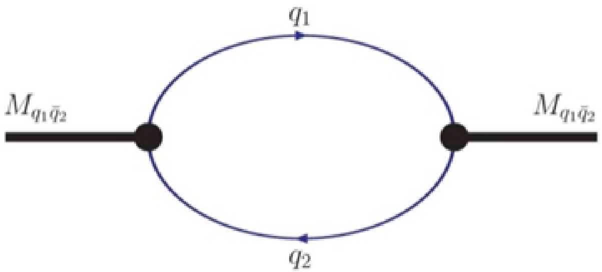
- **Vertex function** in the Gaussian form (its Fourier transformation) :

$$\tilde{\Phi}_H(-p^2) = \exp\left(\frac{p^2}{\Lambda_H^2}\right)$$

- **Quark propagator (**in the Schwinger representation**):**

$$\tilde{S}_{m1}(\hat{p}) = \frac{m_1 + \hat{p}}{m_1^2 - p^2} = \left(m_1 + \hat{p}\right) \cdot \int_0^\infty ds_1 \exp\left[-s\left(m_1^2 - p^2\right)\right]$$

- **Meson mass function (operator)**:



$$\Pi_{PP}(x-y) = +i\left\langle T\left\{J_P(x)J_P(\mathrm{y})\right\}\right\rangle_0$$

$$\Pi_{VV}^{\mu\nu}(x-y) = -i\left\langle T\left\{J_V^{\mu}(x)J_V^{\nu}(\mathrm{y})\right\}\right\rangle_0$$

- **Pseudoscalar and Vector mesons**:

$$\tilde{\Pi}_{PP}(p^2) = \mathrm{N}_c \int \frac{d^4k}{(2\pi)^4 i} \tilde{\Phi}_P^2(-k^2)\,\mathrm{tr}\left\{\gamma^5 \tilde{S}_{m1}(k_1 + p\omega_1)\gamma^5 \tilde{S}_{m2}(k_2 - p\omega_2)\right\}$$

$$\tilde{\Pi}_{VV}^{\mu\nu}(p^2) = \mathrm{N}_c \int \frac{d^4k}{(2\pi)^4 i} \tilde{\Phi}_V^2(-k^2)\,\mathrm{tr}\left\{\gamma^{\mu} \tilde{S}_{m1}(k_1 + p\omega_1)\gamma^{\nu} \tilde{S}_{m2}(k_2 - p\omega_2)\right\}$$

Meson Ground-State Spectrum

- **Meson mass equation**:

$$1 = G \cdot \tilde{\Pi}(M^2)$$

$$\tilde{\Pi}_H(p^2) = \frac{3}{4\pi^2} \int_0^\infty \frac{dt \cdot t}{a_H^2} \int_0^1 ds \exp\left(-t \cdot z_0 + z_H\right) \cdot \left[\frac{n_H}{a_H} + m_1 m_2 + p^2 \left(\omega_1 - \frac{b}{a_H} \right) \left(\omega_2 + \frac{b}{a_H} \right) \right]$$

$$z_0 \equiv s\, m_1^2 + (1-s)\, m_2^2 - s\,(1-s)\, p^2 ; \qquad a_H \equiv t + \frac{2}{\Lambda_H^2} ; \qquad n_P = 2;$$

$$z_H \equiv \frac{2t}{2 + t \cdot \Lambda_H^2} \left(s - \omega_2 \right)^2 p^2 ; \qquad b \equiv t \left(s - \omega_2 \right) ; \qquad n_V = 1$$

- **Branching point**:

$$if: \quad p^2 = (m_1 + m_2)^2 \qquad then: \quad z_0 = p^2 \left[s\,\omega_1^2 + (1-s)\,\omega_2^2 - s\,(1-s) \right] \xrightarrow[s=\omega_2]{} 0$$

$$\text{Integral} \int_0^\infty \frac{dt \cdot t}{a_H^2} \ldots = \text{diverges and a threshold singularity appears!}$$

Removing Singularity by Infrared Cut-off

Cut off the upper bound of t-integral (**infrared cut-off in terms of k-integral**)

$$\text{for } \lambda > 0: \quad \text{no threshold singularity:} \quad \int_0^{1/\lambda^2} \frac{dt\, t}{a_H^2} \ldots = \text{ converges!}$$

T. Branz et al., Phys. Rev. **D81**, 034010 (2010).

$$\tilde{\Pi}_H(p^2) = \frac{3}{4\pi^2} \int_0^{1/\lambda^2} \frac{dt \cdot t}{a_H^2} \int_0^1 ds \exp\left(-t \cdot z_0 + z_H\right) \cdot \left[\frac{n_H}{a_H} + m_1 m_2 + p^2 \left(\omega_1 - \frac{b}{a_H}\right)\left(\omega_2 + \frac{b}{a_H}\right) \right]$$

A meson in the interaction Lagrangian is characterized by parameters

- the coupling constant **g_H**
- the size parameter **Λ_H**
- two constituent quark masses **m_1** & **m_2**
- the infrared confinement parameter **λ** universal for all hadrons.

Hereby, the Yukawa couplings g_H for all mesons H are removed by $Z = 1 - g_H^2 \tilde{\Pi}'(M^2) = 0$

- **Model parameters**: constituent quark masses, hadron size parameters, a universal infrared cut-off (totally **4+N+1** parameters for **N** hadrons → **1+5/N** per hadron)

Numerical results for Decay constants and Widths

- **Fixing** model parameters by fitting the ***electromagnetic decay widths*** and ***leptonic decay constants.***

M. A. Ivanov et al, Phys. Rev. D **85**, 034004 (2012).

- Fixed parameters:

$$\lambda = 0.181\,\text{GeV},$$
$$m_{ud} = 0.235\,\text{GeV},$$
$$m_s = 0.442\,\text{GeV},$$
$$m_c = 1.61\,\text{GeV},$$
$$m_b = 5.07\,\text{GeV}$$

TABLE III: The fitted values of the size parameters Λ_H in GeV.

π	K	D	D_s	B	B_s	B_c	η_c	η_b	
0.87	1.02	1.71	1.81	1.90	1.94	2.50	2.06	2.95	
ρ	ω	ϕ	J/ψ	K^*	D^*	D_s^*	B^*	B_s^*	Υ
0.61	0.50	0.91	1.93	0.75	1.51	1.71	1.76	1.71	2.96

- ***Electromagnetic decay widths:***

Process	Fit Values	Data [24]
$\pi^0 \to \gamma\gamma$	5.07×10^{-3}	$(7.7 \pm 0.4) \times 10^{-3}$
$\eta_c \to \gamma\gamma$	3.47	5.0 ± 0.4
$\rho^\pm \to \pi^\pm\gamma$	76.3	67 ± 7
$\omega \to \pi^0\gamma$	687	703 ± 25
$K^{*\pm} \to K^\pm\gamma$	57.7	50 ± 5
$K^{*0} \to K^0\gamma$	129	116 ± 10
$D^{*\pm} \to D^\pm\gamma$	0.59	1.5 ± 0.5
$J/\psi \to \eta_c\gamma$	1.90	1.58 ± 0.37

- ***Leptonic decay constants:***

	Fit Values	Data
f_π	128.4	130.4 ± 0.2
f_K	156.0	156.1 ± 0.8
f_D	206.7	206.7 ± 8.9
f_{D_s}	257.5	257.5 ± 6.1
f_B	189.7	192.8 ± 9.9
f_{B_s}	235.3	238.8 ± 9.5
f_{η_c}	386.6	438 ± 8
f_{B_c}	445.6	489 ± 5
f_{η_b}	609.1	801 ± 9

f_ρ	221.2	221 ± 1
f_ω	204.2	198 ± 2
f_ϕ	228.2	227 ± 2
$f_{J/\psi}$	415.0	415 ± 7
f_{K^*}	215.0	217 ± 7
f_{D^*}	223.0	245 ± 20
$f_{D_s^*}$	272.0	272 ± 26
f_{B^*}	196.0	196 ± 44
$f_{B_s^*}$	229.0	229 ± 46
f_Υ	661.3	715 ± 5

+ Agreement between our **fit values** and the **PDG data** is quite satisfactory.

+ The constituent quark masses and the values of $\boldsymbol{\Lambda_H}$ fall into the expected range.

+ The meson "size" ~1/ $\boldsymbol{\Lambda_H}$ shrinks as the mass grows.

Numerical results for Fermi coupling G

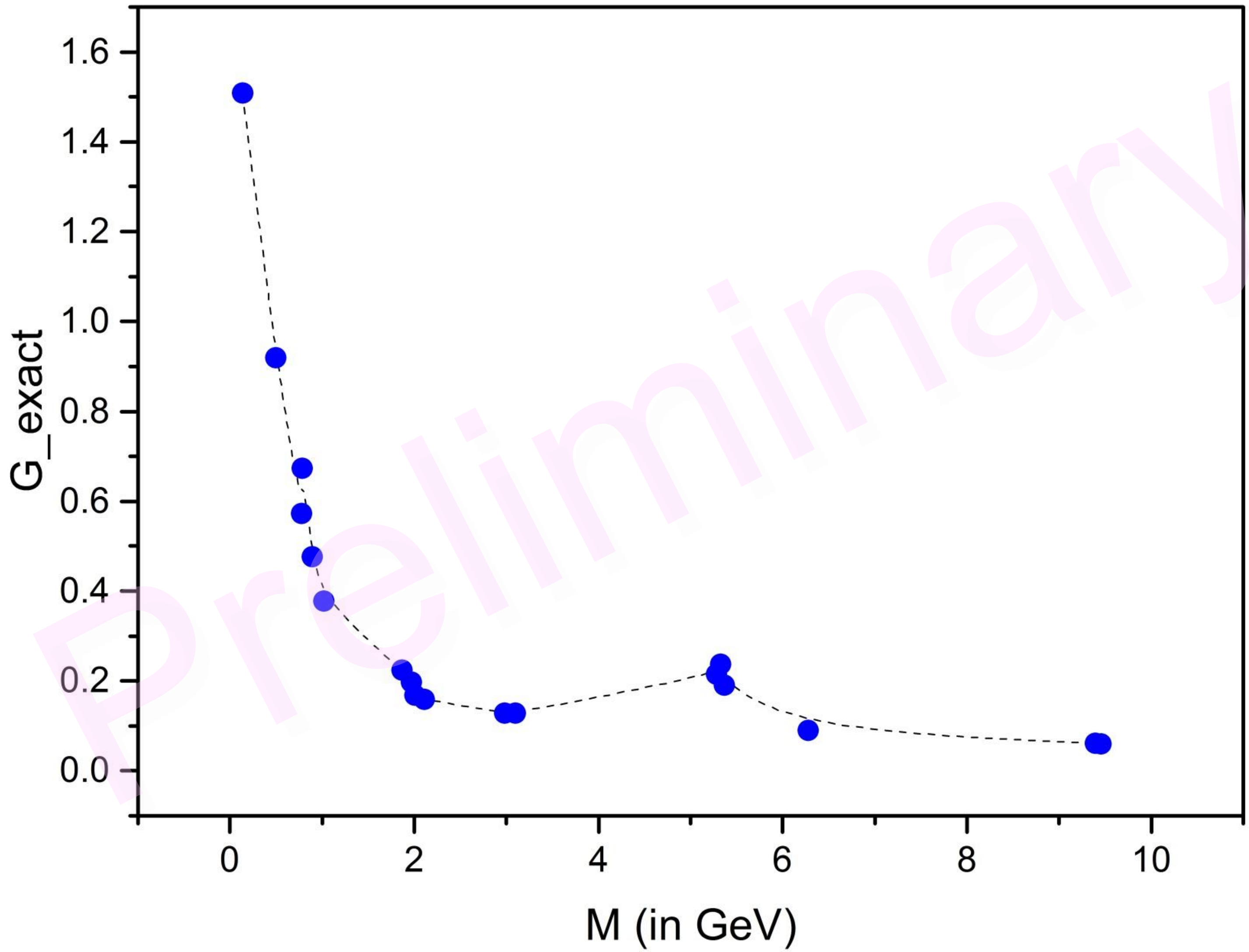
- Equation for the Fermi coupling:

$$G = 1/\tilde{\Pi}_H(M_{\exp}^2)$$

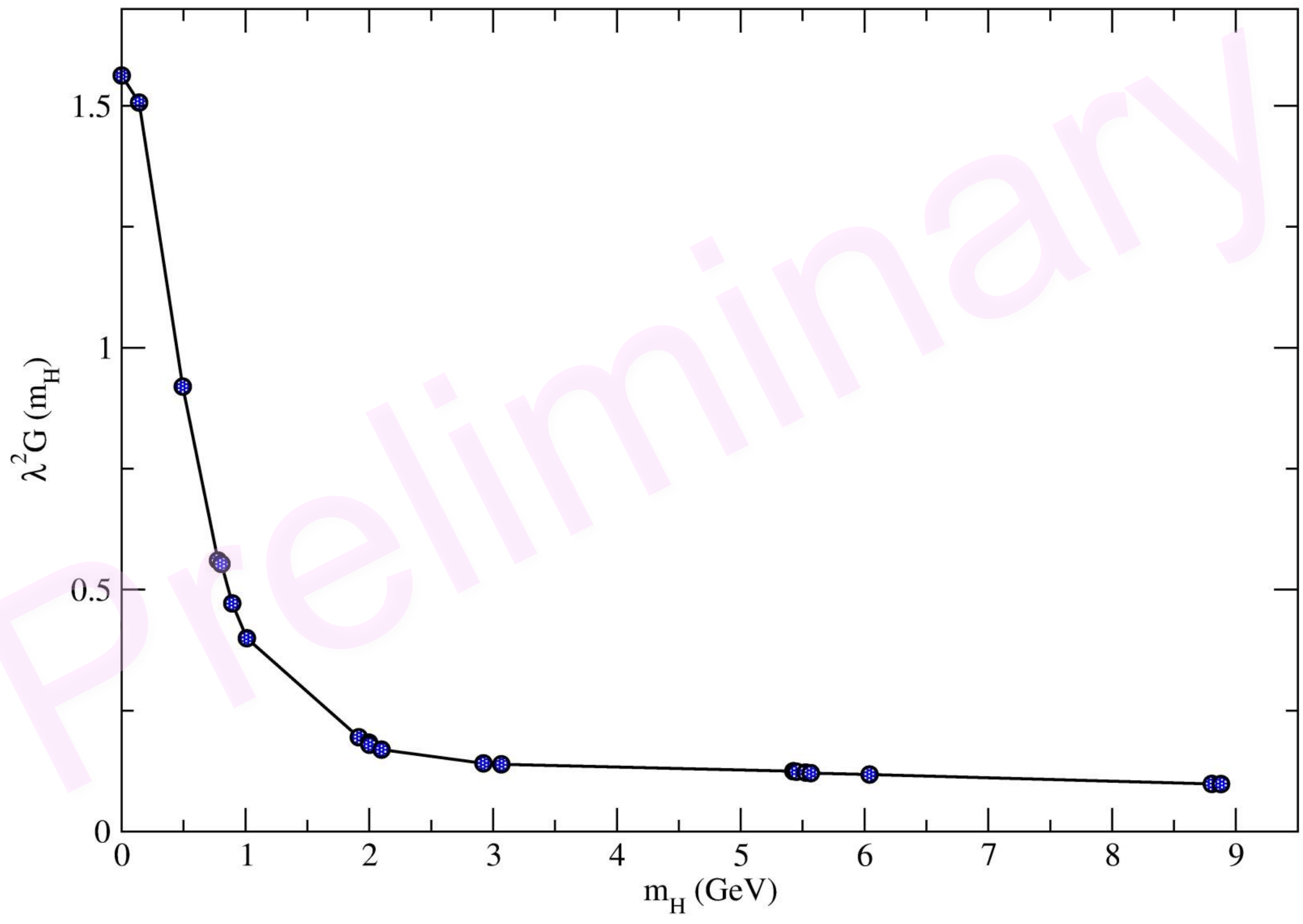
$$\tilde{\Pi}_H(M^2) \sim [\mathrm{GeV}^2] \quad \Rightarrow \quad \lambda^2 G(M) \sim [\text{dimensionless}]$$

$J^{PC} = 0^{-+}$	PDG (MeV)	λ^2 G	$J^{PC} = 1^{--}$	PDF (MeV)	λ^2 G
π	139.57	1.508	**ρ**	775.26	0.576
K	493.68	0.919	**ω**	782.65	0.673
D	1869.62	0.224	**K***	891.66	0.476
D_s	1968.50	0.197	**Φ**	1019.45	0.377
H_c	2983.7	0.128	**D***	2010.29	0.168
B	5279.26	0.215	**D*s**	2112.3	0.158
B_s	5366.77	0.191	**J/ψ**	3096.92	0.129
B_c	6274.5	0.0906	**B***	5325.2	0.237
η_b	9398.0	0.0612	**B*s**	5415.8	0.231
			ϒ	9460.3	0.0601

Plot of $\lambda^2 G$ by fitting meson physical masses



Plot of $\lambda^2 G$ by using a smoothing algorythm



Numerical results for meson masses

- Equation for Meson Mass:

$$1 - G_{smooth} \cdot \tilde{\Pi}_H(M^2) = 0$$

$J^{PC} = 0^{-+}$ | $J^{PC} = 1^{--}$

	PDG (MeV)	λ^2 G (smooth)	M (MeV)		PDG (MeV)	λ^2 G (smooth)	M (MeV)
π	139.57	1.507	141	**ρ**	775.26	0.560	778
K	493.68	0.920	493	**ω**	782.65	0.554	806
D	1869.62	0.195	1915	**K***	891.66	0.472	893
$\mathbf{D_s}$	1968.50	0.184	1998	**Φ**	1019.45	0.401	1011
$\mathbf{\eta_c}$	2983.7	0.141	2922	**D***	2010.29	0.180	2001
B	5279.26	0.125	5425	**D*s**	2112.3	0.170	2099
$\mathbf{B_s}$	5366.77	0.122	5524	**J/ψ**	3096.92	0.139	3067
$\mathbf{B_c}$	6274.5	0.118	6041	**B***	5325.2	0.124	5450
$\mathbf{\eta_b}$	9398.0	0.0986	8806	**B*s**	5415.8	0.121	5566
				ϒ	9460.3	0.0984	8880

Fermi coupling G: a comparison

- It is interesting to compare ***G*** with the effective QCD coupling $\boldsymbol{\alpha_s}$ obtained in the relativistic models with specific forms of analytically confined propagators.

$$L = -\frac{1}{4}\left(\partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g\, f^{abc} A_\mu^b A_\nu^c\right)^2 + \sum_f \left(\bar{q}_f^a \left[\gamma_\alpha \partial_\alpha - m_f + ig\,\gamma_\alpha t^C A_\alpha^C\right]^{ab} q_f^b\right)$$

$$\tilde{S}(\hat{p}) = \left(i\hat{p} + m\right) \cdot \int_0^{1/\Lambda^2} dt \exp\left\{-t \cdot (p^2 + m^2)\right\};$$

$$D(x) = \int_{\Lambda^2/4}^{\infty} ds\, e^{-s x^2} = \frac{e^{-x^2\Lambda^2/4}}{4\pi^2 x^2} \quad .$$

G.Ganbold,
Phys. Rev. D 79, 034034 (2009).
Phys. Part. Nucl. **43**, 79, (2012)
Phys. Part. Nucl. **45**, 10, (2014)

- In that models the four-quark nonlocal interaction is induced by one-gluon exchange between bi-quark currents. Since the confined gluon propagator has the dimension of **~ 1/GeV^2** , the resulting coupling $\boldsymbol{\alpha_s}$ is dimensionless.

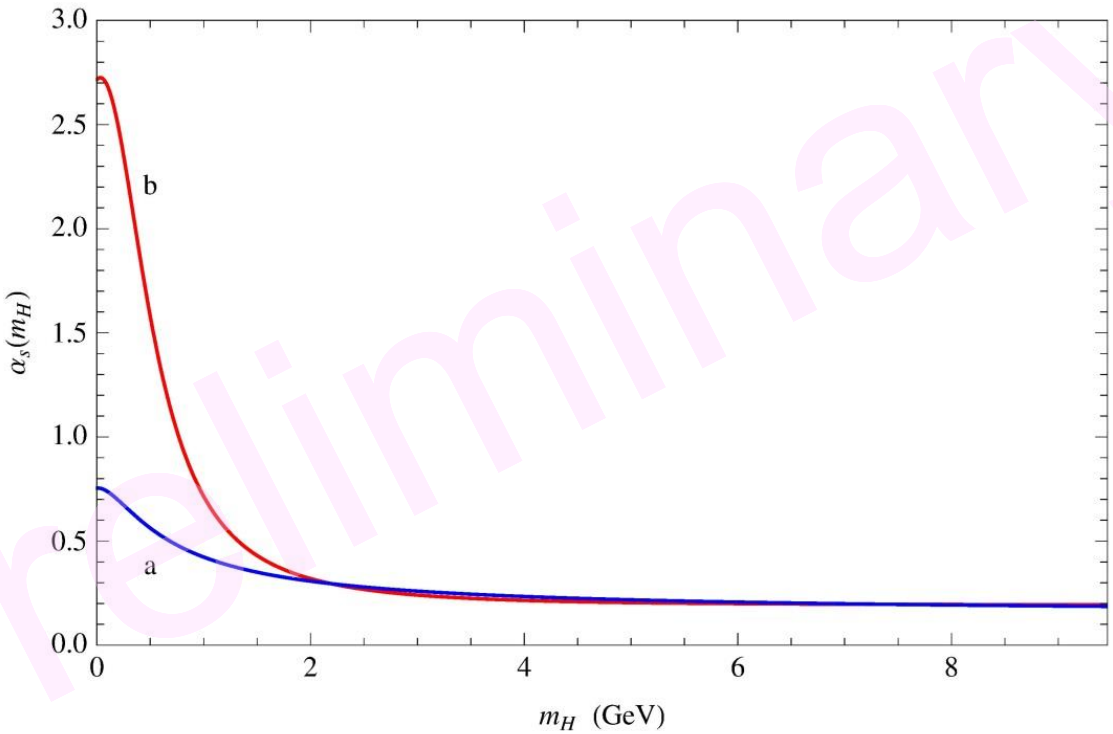
$$1 - \alpha_s \cdot \frac{8C_J}{3\pi^3} \int d^4k\, V_J(k) \cdot \Pi_J(p,k) \cdot V_J(-k) = 0; \qquad (p^2 = -M_J^2)$$

$$V_J(k) = \int dx \sqrt{D(x)}\, U_J(x)\, e^{ikx}$$

α_s (blue curve) in comparison with rescaled **~1.74 λ^2*G** (red curve)

$$\Lambda = 345 \text{ MeV}$$
$$m_{ud} = 193 \text{ MeV}$$
$$m_s = 293 \text{ MeV}$$
$$m_c = 1848 \text{ MeV}$$
$$m_b = 4693 \text{ MeV}$$

G.Ganbold,
Phys. Rev. D 81, 094008 (2010).



- Despite the different model origins and input parameter values, the behaviors of two curves are very similar each other in the region above ~ 2 GeV.

- Their values at origin are mostly determined by the confinement mechanisms realized in different ways. This could explain their different behaviors in the region below 2 GeV.

Summary and Outlook

- ♣ A brief sketch of an approach to the bound state problem in QFT based on the ***compositeness condition*** is represented.

- ♣ We have explicitly demonstrated that the four-fermion theory with the Fermi coupling ***G*** is equivalent to the Yukawa-type theory if,
  - the wave function renormalization constant in the Yukawa theory is equal to zero,
  - ***G*** is inversely proportional to the meson mass function calculated at physical mass.

- ♣ The mass spectrum of conventional mesons has been considered within the CCQ model. We updated the fit of model parameters and calculated ***G*** as a function of physical mass.

- ♣ A *smoothness criterion* has been suggested – just by varying the meson masses in such a way to obtain the smooth behavior of the Fermi coupling ***G***.
  The mass spectrum obtained in this manner is found to be in good agreement with the experimental data (*from **π(140)** up to **ϒ(9460)***.

- ♣ We have compared the behavior of ***G*** with the strong QCD coupling α_s calculated in the QCD-inspired models.

- ♣ The approach may be extended to other sections of hadron physics.