Relativistic effects in the processes of pair charmonium production at LHC

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Pair charmonium production

e^+e^- annihilation:

- K. Abe et al. (Belle Collaboration), Phys. Rev. Lett. **89**, 142001 (2002)

$$\sigma[e^+e^- \to J/\psi + \eta_c] \times \mathcal{B}_{\geq 4} = 33^{+7}_{-6} \pm 9 \text{ fb}$$

Theoretical predictions:

- E. Braaten and J. Lee, Phys. Rev. D 67, 054007 (2003); **72**, 099901(E) (2005)
- K.Y. Liu, Z. G. He, and K. T. Chao, Phys. Lett. B **557**, 45 (2003)

$$\sigma[e^+e^- \to J/\psi + \eta_c] = 3.78 \pm 1.26 \text{ fb}$$
$$\sigma[e^+e^- \to J/\psi + \eta_c] = 5.5 \text{ fb}$$

New experiments:

- K. Abe et al. (Belle Collaboration), Phys. Rev. D **70**, 071102 (2004)
- B. Aubert et al. (BABAR Collaboration), Phys. Rev. D **72**, 031101 (2005)

$$\sigma[e^+e^- \to J/\psi + \eta_c] \times \mathcal{B}_{>2} = 25.6 \pm 2.8 \pm 3.4 \text{ fb}$$
$$\sigma[e^+e^- \to J/\psi + \eta_c] \times \mathcal{B}_{>2} = 17.6 \pm 2.8 \pm 2.1 \text{ fb}$$

Pair charmonium production

Further theoretical efforts:

- Y.J. Zhang, Y.J. Gao, K.T. Chao, Phys. Rev. Lett. **96**, 092001 (2006)
  G.T. Bodwin, D. Kang, T. Kim, J.Lee, C. Yu, AIP Conf. Proc. **892**, 315 (2007)
  Z. G. He, Y. Fan, and K. T. Chao, Phys. Rev. D **75**, 074011 (2007)
- J.P. Ma, Z.G. Si, Phys. Rev. D **70**, 074007 (2004)
  A.E. Bondar, V.L. Chernyak, Phys. Lett. B **612**, 215 (2005)
  V.V. Braguta, A.K. Likhoded, A.V. Luchinsky, Phys. Rev. D **72**, 074019 (2005)
- D. Ebert, A.P. Martynenko, Phys. Rev. D **74**, 054008 (2006)
  D. Ebert, R.N. Faustov, V.O. Galkin, A.P. Martynenko,
  Phys. Lett. B **672**, 264 (2009)
  E.N. Elekina, A.P. Martynenko, Phys. Rev. D **81**, 054006 (2010)
  A.P. Martynenko, A.M. Trunin, arXiv:1106.2741
- G.T. Bodwin, J. Lee, C. Yu, Phys. Rev. D **77**, 094018 (2008)

$$\sigma_{[Bodwin,Lee,Yu]} = \mathbf{17.6}^{+8.1}_{-6.7}\ \text{fb}$$

nonrelativistic result, (fb)	relativistic corrections	QED	NLO α_s (+QED)	correlations of relativistic & NLO α_s
5.4	2.9	1.0	6.9	1.4

Pair charmonium production at LHC

LHCb experimentally measured value:

- R. Aaij *et al.* (LHCb Collaboration), Phys. Let. B **707**, 52 (2012)

$$\sigma^{exp}_{LHCb}[pp \to 2J/\psi + X] = 5.1 \pm 1.0 \pm 1.1 \text{ nb}\Big|_{\sqrt{S}=7 \text{ TeV}}$$

NRQCD predictions:

- R. Li, Y.-J. Zhang, K.-T. Chao, Phys. Rev. D **80**, 014020 (2009)
  S.P. Baranov, Phys. Rev. D **84**, 054012 (2011)
  A.V. Berezhnoy, A.K. Likhoded, A.V. Luchinsky, A.A. Novoselov, Phys. Rev. D **84**, 094023 (2011)

$$\sigma^{\rm NRQCD}_{\rm LO}[pp \to 2J/\psi + X] = 4.1 \pm 1.2 \text{ fb } (+1.5 \div 2 \text{ fb})$$

Relativistic corrections:

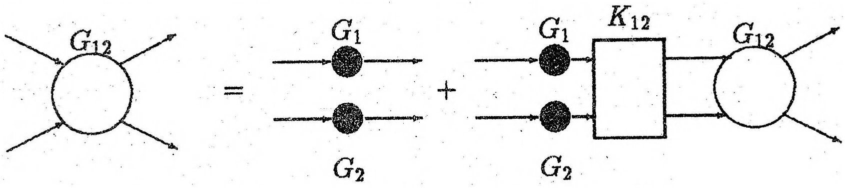
- A.P. Martynenko, A.M. Trunin, Phys. Rev. D **86**, 094003 (2012)
- Y.-J. Li, G.-Z. Xu, K.-Y. Liu, Y.-J. Zhang, J. High Energy Phys. **07** (2013) 051.

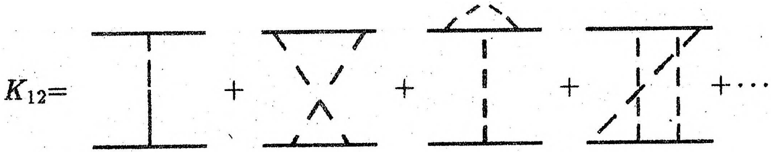
Quasipotential approach to relativistic quark model

Bethe-Salpeter equation:

- E.E. Salpeter and H.A. Bethe, Phys. Rev. **82**, 309 (1951); **84**, 1232 (1951)

$$G_{12}(x_1,x_2;y_1,y_2) = G_1(x_1-y_1)G_2(x_2-y_2) + \int dz_1 dz_2 dz_1' dz_2' \, G_1(x_1-z_1)G_2(x_2-z_2)K_{12}(z_1,z_2;z_1',z_2')G_{12}(z_1',z_2';y_1,y_2)$$





Quasipotential approach to relativistic quark model

Bethe-Salpeter equation:

$$(\not{p}_1 - m_1)(\not{p}_2 - m_2)\psi_P(p) = i\int \frac{d^4q}{(2\pi)^4} K_{12}(p,q;P)\psi_P(q),$$

$\psi_P(x_1,x_2) = \langle 0|T\{\psi_1(x_1)\psi_2(x_2)\}|P\rangle$ — Bethe-Salpeter amplitude or wave function, $x_1^0 \neq x_2^0$:

«...a proton today and an electron yesterday do not constitute a hydrogen atom»
A. Eddington

Logunov-Tavkhelidze equation:

- A.A. Logunov and A.N. Tavkhelidze, Nuovo Cimento **29**, 380 (1963)
- V.G. Kadyshevsky, Nucl. Phys. **B** 6, 125 (1968)
- C. Itzykson, V.G. Kadyshevsky, I. T. Todorov, Phys. Rev. D **1**, 2823 (1970)
- R.N. Faustov, Teor. Mat. Fiz **3**, 240 (1970)

$$\left[M - \sqrt{\mathbf{p}^2 + m_1^2} - \sqrt{\mathbf{p}^2 + m_1^2}\right]\psi^{(+)}(\mathbf{p}) = \int \frac{d\mathbf{q}}{(2\pi)^3} V(\mathbf{p},\mathbf{q};M)\,\psi^{(+)}(\mathbf{q})$$

Quasipotential approach to relativistic quark model

Logunov-Tavkhelidze equation in "rationalized" form:

- R.N. Faustov and A.P. Martynenko, Teor. Mat. Fiz **64**, 179 (1985)

$$\left[\frac{b^2(M)}{2\mu_R} - \frac{\mathbf{p}^2}{2\mu_R}\right]\psi^{(+)}(\mathbf{p}) = \int \frac{d\mathbf{q}}{(2\pi)^3} V(\mathbf{p},\mathbf{q};M)\,\psi^{(+)}(\mathbf{q}),$$

$$b^2(M) = \mathbf{p^2}\big|_{\text{on shell}} = \frac{1}{4M^2}\left[M^2-(m_1+m_2)^2\right]\left[M^2-(m_1-m_2)^2\right],$$

$\mu_R = \frac{1}{4M^3}\left[M^4-(m_1^2-m_2^2)^2\right]$ — relativistic reduced mass.

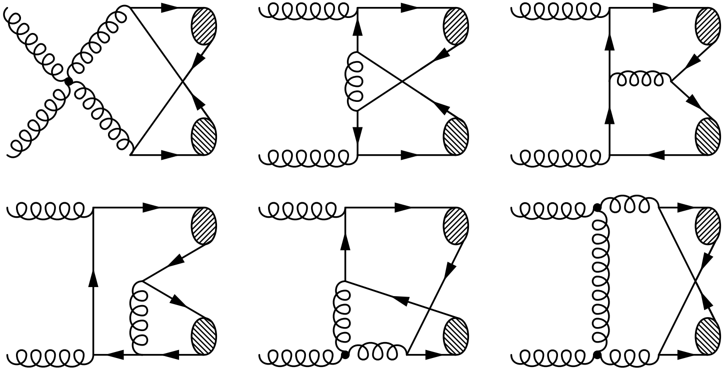
Quasipotential construction:

- R.N. Faustov, Fiz. El. Chast. Atom. Yad. **3**, 238 (1972)
- D. Ebert, V.O. Galkin, R.N. Faustov, Phys. Rev. D **57**, 5663 (1998)
- D. Ebert, R.N. Faustov, V.O. Galkin, Phys. Rev. D **72**, 034026 (2005)
- V.A. Matveev, V.I. Savrin, A.N. Sissakian, A.N. Tavkhelidze, Teor. Mat. Fiz **132**, 267 (2002)

31 LO α_s CSM gluon fusion diagrams

$$d\sigma[pp \to 2J/\psi + X] = \int dx_1 dx_2\, f_{g/p}(x_1) f_{g/p}(x_2)\, d\sigma[gg \to 2J/\psi]. \quad (9)$$

$$\mathcal{M}[gg \to 2J/\psi] = \int \frac{d\mathbf{p}}{(2\pi)^3} \int \frac{d\mathbf{q}}{(2\pi)^3}\, \bar{\Psi}(p, P)\bar{\Psi}(q, Q) \otimes \mathcal{T}(p_1, p_2; q_1, q_2), \quad (10)$$



Production amplitude

$$\mathcal{M}[gg \to 2J/\psi](k_1, k_2, P, Q) = \frac{1}{9} M_{J/\psi}\, \pi^2 \alpha_s^2 \int \frac{d\mathbf{p}}{(2\pi)^3} \int \frac{d\mathbf{q}}{(2\pi)^3}\, \mathrm{Tr}\, \mathfrak{M},$$

$$\mathfrak{M} = \mathcal{D}_1 \gamma_\beta \bar{\Psi}_{q,Q} \Gamma_1^\beta \bar{\Psi}_{p,P}\, \hat{\varepsilon}_2 \frac{m - \hat{k}_2 + \hat{p}_1}{(k_2 - p_1)^2 - m^2} +$$

$$\mathcal{D}_2 \gamma_\beta \bar{\Psi}_{q,Q} \Gamma_2^\beta \bar{\Psi}_{p,P}\, \hat{\varepsilon}_1 \frac{m - \hat{k}_1 + \hat{p}_1}{(k_1 - p_1)^2 - m^2} + \mathcal{D}_3 \bar{\Psi}_{q,Q} \Gamma_3^\beta \bar{\Psi}_{p,P}\, \gamma_\beta +$$

$$\mathcal{D}_4 \bar{\Psi}_{p,P} \Gamma_4^\beta \bar{\Psi}_{q,Q}\, \gamma_\beta + \mathcal{D}_1 \bar{\Psi}_{q,Q} \Gamma_5^\beta \bar{\Psi}_{p,P}\, \gamma_\beta \frac{m + \hat{k}_2 - \hat{q}_1}{(k_2 - q_1)^2 - m^2} \hat{\varepsilon}_2 +$$

$$\mathcal{D}_2 \bar{\Psi}_{q,Q} \Gamma_6^\beta \bar{\Psi}_{p,P}\, \gamma_\beta \frac{m + \hat{k}_1 - \hat{q}_1}{(k_1 - q_1)^2 - m^2} \hat{\varepsilon}_1, \tag{11}$$

$k_{1,2} = x_{1,2}\sqrt{S}/2\,(1, 0, 0, \pm 1)$ — the initial gluon four-momenta;
P, Q — the total four-momenta of outcoming charmonia;
$p = L_P(0, \mathbf{p})$, $q = L_Q(0, \mathbf{q})$ — the relative four-momenta of (anti)quarks.
$\varepsilon_{1,2}$ — the polarization vectors of initial gluons.

Vertex functions

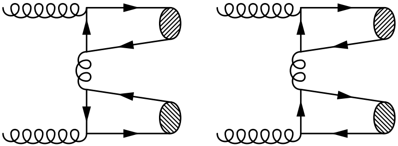
$$\Gamma_1^\beta = \hat{\varepsilon}_1 \frac{m-\hat{k}_1+\hat{q}_2}{(k_1-q_2)^2-m^2}\gamma^\beta - 8\,\gamma^\beta \frac{m+\hat{k}_1-\hat{p}_2}{(k_1-p_2)^2-m^2}\hat{\varepsilon}_1,$$

$$\begin{aligned}\Gamma_3^\beta = {}& \hat{\varepsilon}_1 \frac{m-\hat{k}_1+\hat{q}_2}{(k_1-q_2)^2-m^2}\left[\gamma^\beta \frac{m+\hat{k}_2-\hat{p}_2}{(k_2-p_2)^2-m^2}\hat{\varepsilon}_2 - 8\,\hat{\varepsilon}_2 \frac{m-\hat{p}_1-\hat{p}_2-\hat{q}_1}{(p_1+p_2+q_1)^2-m^2}\gamma^\beta\right]+\\
& \hat{\varepsilon}_2 \frac{m-\hat{k}_2+\hat{q}_2}{(k_2-q_2)^2-m^2}\left[\gamma^\beta \frac{m+\hat{k}_1-\hat{p}_2}{(k_1-p_2)^2-m^2}\hat{\varepsilon}_1 - 8\,\hat{\varepsilon}_1 \frac{m-\hat{p}_1-\hat{p}_2-\hat{q}_1}{(p_1+p_2+q_1)^2-m^2}\gamma^\beta\right]-\\
& 8\,\gamma^\beta \frac{m+\hat{p}_1+\hat{q}_1+\hat{q}_2}{(p_1+q_1+q_2)^2-m^2}\left[\hat{\varepsilon}_2 \frac{m+\hat{k}_1-\hat{p}_2}{(k_1-p_2)^2-m^2}\hat{\varepsilon}_1 + \hat{\varepsilon}_1 \frac{m+\hat{k}_2-\hat{p}_2}{(k_2-p_2)^2-m^2}\hat{\varepsilon}_2\right]+\\
& 18\,\gamma_\alpha\left[\mathcal{D}_1 \frac{m-\hat{k}_1+\hat{q}_2}{(k_1-q_2)^2-m^2}\varepsilon_1^\alpha\gamma_\mu \mathfrak{E}_2^{\beta\mu}(p_1+q_1) - \mathcal{D}_1 \frac{m+\hat{k}_1-\hat{p}_2}{(k_1-p_2)^2-m^2}\hat{\varepsilon}_1 \mathfrak{E}_2^{\beta\alpha}(p_1+q_1)+\right.\\
& \left.\mathcal{D}_2 \frac{m-\hat{k}_2+\hat{q}_2}{(k_2-q_2)^2-m^2}\varepsilon_2^\alpha\gamma_\mu \mathfrak{E}_1^{\beta\mu}(p_1+q_1) - \mathcal{D}_2 \frac{m+\hat{k}_2-\hat{p}_2}{(k_2-p_2)^2-m^2}\hat{\varepsilon}_2 \mathfrak{E}_1^{\beta\alpha}(p_1+q_1)\right],\end{aligned} \tag{12}$$

$$\mathfrak{E}_{1,2}^{\alpha\beta}(x) = \frac{1}{2}\left(2\,x\varepsilon_{1,2}\,g^{\alpha\beta} - (k_{1,2}^\beta + x^\beta)\varepsilon_{1,2}^\alpha + (2k_{1,2}^\alpha - x^\alpha)\varepsilon_{1,2}^\beta\right),$$

$\mathcal{D}_{1,2}^{-1} = (k_2 - p_{1,2} - q_{1,2})^2$,
$\mathcal{D}_{3,4}^{-1} = (p_{1,2} + q_{1,2})^2$ — inverse denominators of gluon propagators.

Pair η_c production: 8 additional diagrams



$$\mathcal{M}[gg \to \eta_c\eta_c](k_1, k_2, P, Q) = \frac{1}{9} M\pi^2\alpha_s^2 \int \frac{d\mathbf{p}}{(2\pi)^3} \int \frac{d\mathbf{q}}{(2\pi)^3} \left[\mathrm{Tr}\,\mathfrak{M} + 3\Delta\mathfrak{M}\right], \tag{13}$$

$$\begin{aligned}
\Delta\mathfrak{M} = \frac{1}{t}\,\mathrm{Tr}\left[\hat{\varepsilon}_1 \frac{m - \hat{k}_1 + \hat{p}_1}{(k_1 - p_1)^2 - m^2}\gamma_\beta \bar{\Psi}(p, P) + \gamma_\beta \frac{m + \hat{k}_1 - \hat{p}_2}{(k_1 - p_2)^2 - m^2}\hat{\varepsilon}_1 \bar{\Psi}(p, P)\right] \times \\
\mathrm{Tr}\left[\hat{\varepsilon}_2 \frac{m - \hat{k}_2 + \hat{q}_2}{(k_2 - q_2)^2 - m^2}\gamma^\beta \bar{\Psi}(q, Q) + \gamma^\beta \frac{m + \hat{k}_2 - \hat{q}_1}{(k_2 - q_1)^2 - m^2}\hat{\varepsilon}_2 \bar{\Psi}(q, Q)\right] + \\
\frac{1}{2M^2 - s - t}\,\mathrm{Tr}\left[\hat{\varepsilon}_1 \frac{m - \hat{k}_1 + \hat{q}_2}{(k_1 - q_2)^2 - m^2}\gamma_\beta \bar{\Psi}(q, Q) + \gamma_\beta \frac{m + \hat{k}_1 - \hat{q}_1}{(k_1 - q_1)^2 - m^2}\hat{\varepsilon}_1 \bar{\Psi}(q, Q)\right] \times \\
\mathrm{Tr}\left[\hat{\varepsilon}_2 \frac{m - \hat{k}_2 + \hat{p}_1}{(k_2 - p_1)^2 - m^2}\gamma^\beta \bar{\Psi}(p, P) + \gamma^\beta \frac{m + \hat{k}_2 - \hat{p}_2}{(k_2 - p_2)^2 - m^2}\hat{\varepsilon}_2 \bar{\Psi}(p, P)\right].
\end{aligned} \tag{14}$$

Transformation of relativistic wave functions

Quasipotential wave functions are calculated in the meson rest frame and then transformed to the reference frames moving with the four-momenta $P(Q)$:

$$\bar{\Psi}_{p,P} = \frac{\bar{\Psi}_0^{J/\psi}(\mathbf{p})}{\left[\frac{\epsilon(p)}{m}\frac{\epsilon(p)+m}{2m}\right]}\left[\frac{\hat{v}_1-1}{2}+\hat{v}_1\frac{\mathbf{p}^2}{2m(\epsilon(p)+m)}-\frac{\hat{p}}{2m}\right]\times$$
$$\hat{\varepsilon}^*_P(P,S_z)\,(1+\hat{v}_1)\left[\frac{\hat{v}_1+1}{2}+\hat{v}_1\frac{\mathbf{p}^2}{2m(\epsilon(p)+m)}+\frac{\hat{p}}{2m}\right], \qquad (15)$$

$$\bar{\Psi}_{q,Q} = \frac{\bar{\Psi}_0^{J/\psi}(\mathbf{q})}{\left[\frac{\epsilon(q)}{m}\frac{\epsilon(q)+m}{2m}\right]}\left[\frac{\hat{v}_2-1}{2}+\hat{v}_2\frac{\mathbf{q}^2}{2m(\epsilon(q)+m)}+\frac{\hat{q}}{2m}\right]\times$$
$$\hat{\varepsilon}^*_Q(Q,S_z)\,(1+\hat{v}_2)\left[\frac{\hat{v}_2+1}{2}+\hat{v}_2\frac{\mathbf{q}^2}{2m(\epsilon(q)+m)}-\frac{\hat{q}}{2m}\right]. \qquad (16)$$

$v_1=\frac{P}{M_{J/\psi}}$, $v_2=\frac{Q}{M_{J/\psi}}$;
$\epsilon(p)=\sqrt{m^2+\mathbf{p}^2}$;
m — c-quark mass.

$\varepsilon_{P,Q}$ — polarizations of outcoming charmonia with four-momenta P(Q).

Expansion of quark and gluon propagators

$$\frac{1}{(p_1+q_1)^2} = \frac{4}{s} - \frac{16}{s^2}\left[(p+q)^2 + pQ + qP\right] + \cdots\,,$$
$$\frac{1}{(k_2-q_2)^2-m^2} = \frac{2}{t-M^2} - \frac{4}{(t-M^2)^2}\left[q^2 + 2\,qk_2\right] + \cdots\,, \tag{17}$$

where $s = x_1 x_2 S$ and $t = (P-k_1)^2 = (Q-k_2)^2$ — the Mandelstam variables for the gluonic subprocess $gg \to 2J/\psi$.

$$4M^2 \le s, \quad \left|t + \frac{s}{2} - M^2\right| \le \frac{s}{2}\sqrt{1 - \frac{4M^2}{s}}. \tag{18}$$

In the case of the most unfavourable values of the variables $x_{1,2}$ and t the expansion parameters in (17) can be roughly assessed as $2p^2/M^2$ and $2q^2/M^2$.

$$\int \frac{\Psi_0^{\mathcal{S}}(\mathbf{p})}{\left[\frac{\epsilon(p)}{m}\frac{\epsilon(p)+m}{2m}\right]}\frac{d\mathbf{p}}{(2\pi)^3} = \frac{1}{\sqrt{2}\,\pi}\int_0^\infty \frac{p^2 R_{\mathcal{S}}(p)}{\left[\frac{\epsilon(p)}{m}\frac{\epsilon(p)+m}{2m}\right]}dp,$$
$$\int p_\mu p_\nu\,\frac{\Psi_0^{\mathcal{S}}(\mathbf{p})}{\left[\frac{\epsilon(p)}{m}\frac{\epsilon(p)+m}{2m}\right]}\frac{d\mathbf{p}}{(2\pi)^3} = -\frac{1}{3\sqrt{2}\,\pi}(g_{\mu\nu} - v_{1\mu}v_{1\nu})\int_0^\infty \frac{p^4 R_{\mathcal{S}}(p)}{\left[\frac{\epsilon(p)}{m}\frac{\epsilon(p)+m}{2m}\right]}dp. \tag{19}$$

Effective relativistic Hamiltonian

$$H = H_0 + \Delta U_1 + \Delta U_2 + \Delta U_3, \tag{20}$$

$$H_0 = 2\sqrt{\mathbf{p}^2 + m^2} - 2m - \frac{C_F\tilde{\alpha}_s}{r} + Ar + B, \tag{21}$$

$$\Delta U_1(r) = -\frac{C_F\alpha_s^2}{4\pi r}\left[2\beta_0 \ln(\mu r) + a_1 + 2\gamma_E\beta_0\right],$$

$$\Delta U_2(r) = -\frac{C_F\alpha_s}{2m^2 r}\left[\mathbf{p}^2 + \frac{\mathbf{r}(\mathbf{r}\mathbf{p})\mathbf{p}}{r^2}\right] + \frac{\pi C_F\alpha_s}{m^2}\delta(\mathbf{r}) + \frac{3C_F\alpha_s}{2m^2 r^3}(\mathbf{S}\mathbf{L}) -$$
$$\frac{C_F\alpha_s}{2m^2}\left[\frac{\mathbf{S}^2}{r^3} - 3\frac{(\mathbf{S}\mathbf{r})^2}{r^5} - \frac{4\pi}{3}(2\mathbf{S}^2 - 3)\delta(\mathbf{r})\right] - \frac{C_A C_F\alpha_s^2}{2mr^2},$$

$$\Delta U_3(r) = f_V\Big[\frac{A}{2m^2 r}\Big(1 + \frac{8}{3}\mathbf{S}_1\mathbf{S}_2\Big) + \frac{3A}{2m^2 r}\mathbf{L}\mathbf{S} + \frac{A}{3m^2 r}\Big(\frac{3}{r^2}(\mathbf{S}_1\mathbf{r})(\mathbf{S}_2\mathbf{r}) - \mathbf{S}_1\mathbf{S}_2\Big)\Big] -$$
$$(1 - f_V)\frac{A}{2m^2 r}\mathbf{L}\mathbf{S}, \tag{22}$$

$$C_A = 3, \quad C_F = 4/3, \quad a_1 = \frac{31}{3} - \frac{10}{9}n_f, \quad \beta_0 = 11 - \frac{2}{3}n_f$$
$$A = 0.18\ \text{GeV}^2, \quad B = -0.16\ \text{GeV}, \quad \alpha_s(m^2) \approx 0.314, \quad \tilde{\alpha}_s(m^2) \approx 0.242,$$
$$\Lambda = 0.168\ \text{GeV}, \quad m = 1.55\ \text{GeV}.$$

Effective relativistic Hamiltonian

Rationalization of the kinetic energy term:

$$T = 2\sqrt{\mathbf{p}^2 + m^2} = 2\frac{\mathbf{p}^2 + m^2}{\sqrt{\mathbf{p}^2 + m^2}} \approx \frac{\mathbf{p}^2}{\tilde{m}} + \frac{2m^2}{\tilde{E}},$$
$$\tilde{m} = \frac{\tilde{E}}{2} = \frac{1}{2}\sqrt{\mathbf{p}_{eff}^2 + m^2}. \quad (23)$$

Comparison between the numerical and experimental charmonium masses and parameters of the model:

charmonium state	$n^{2S+1}L_J$	f_V	$\mathbf{p}_{eff}^2$, GeV^2	b, GeV	M^{num}, GeV	M^{PDG}, GeV
J/ψ	1^3S_1	0.9	0.48	1.5	3.090	3.097
η_c	1^1S_0				2.995	2.984
χ_{c0}	1^3P_0	0.64	0.52	–	3.427	3.415
χ_{c1}	1^3P_1				3.501	3.511
χ_{c2}	1^3P_2				3.567	3.556
h_c	1^1P_1				3.513	3.525

- J. Beringer *et al.* (Particle Data Group), Phys. Rev. D **86**, 010001 (2012).

The differential cross-section

$$\frac{d\sigma}{dt}[gg \to 2J/\psi](t,s) = \frac{\pi M^2 \alpha_s^4}{9216\, s^2}\, |\tilde{R}(0)|^4 \sum_{i=0}^{3} \omega_i F^{(i)}(t,s) =$$
$$\frac{\pi M^2 \alpha_s^4}{9216\, s^2}\, |\tilde{R}(0)|^4 \left(F^{(0)}(t,s) + \sum_{i=1}^{3} \omega_i F^{(i)}(t,s) \right), \tag{24}$$

$$\omega_0 = 1, \quad \omega_1 = \frac{I_1}{I_0}, \quad \omega_2 = \frac{I_2}{I_0}, \quad \omega_3 = \omega_1^2. \tag{25}$$

The relativistic radial wave function at the origin:

$$\tilde{R}(0) = \sqrt{\frac{2}{\pi}}\, I_0 = \sqrt{\frac{2}{\pi}} \int\limits_0^\infty R(p) p^2 dp. \tag{26}$$

$$I_{1,2} = \int\limits_0^m \frac{m + \epsilon(p)}{2\epsilon(p)} \left(\frac{m - \epsilon(p)}{m + \epsilon(p)} \right)^{1,2} R(p) p^2 dp. \tag{27}$$

The definition of the relativistic parameters $I_{1,2}$ contains cutoff $\Lambda = m$ due to uncertainty of the relativistic wave function in the region $p \gtrsim m$.

Numerical results

Our results (CTEQ5L pdf):

$$\sigma_{nonrel}^{2<y<4.5} = 6.65 \text{ nb}, \qquad \sigma_{rel}^{2<y<4.5} = 5.04 \text{ nb}.$$

Experimentally measured value ($2 < y_{P,Q} < 4.5$):

$$\sigma_{exp}^{LHCb}[pp \to 2J/\psi + X] = 5.1 \pm 1.0 \pm 1.1 \text{ nb}.$$

NRQCD predictions:

$$\sigma_{NRQCD}^{2<y<4.5} = 4.1 \text{ nb}.$$

- A.V. Berezhnoy, A.K. Likhoded, A.V. Luchinsky, and A.A. Novoselov, Phys. Rev. D **84**, 094023 (2011).

Double parton scattering contribution:

$$\sigma_{DPS}^{LHCb} = 1.5 \div 2 \text{ nb}.$$

- S.P. Baranov, A.M. Snigirev, and N.P. Zotov, Phys. Lett. B **705**, 116 (2011);
- A. Novoselov, arXiv:1106.2184.

Relativistic corrections within NRQCD approach:

- Y.-J. Li, G.-Z. Xu, K.-Y. Liu, Y.-J. Zhang, J. High Energy Phys. **07** (2013) 051.

$$\sigma_{\text{NRQCD}}^{\text{rel.}} = 5.18 - \mathbf{0.02} = 5.16 \text{ nb}.$$

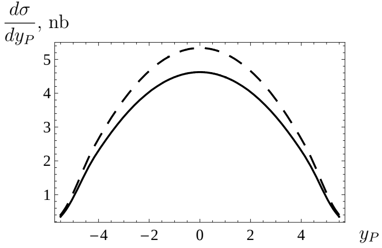
Numerical results

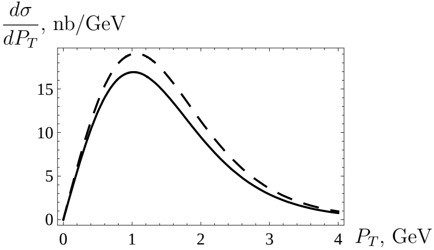
Energy $\sqrt{S}$	Meson pair	σ(total), nb		$\sigma(2 < y_{P,Q} < 4.5)$, nb	
		CTEQ5L	CTEQ6L1	CTEQ5L	CTEQ6L1
$\sqrt{S} = 7$ TeV	$J/\psi\, J/\psi$, rel.	30.5	23.4	5.0	3.8
	$J/\psi\, J/\psi$, nonrel.	40.2	30.9	6.7	5.1
	$\eta_c\, \eta_c$, rel.	161.0	136.0	6.8	5.1
	$\eta_c\, \eta_c$, nonrel.	98.3	84.0	2.7	2.0
$\sqrt{S} = 14$ TeV	$J/\psi\, J/\psi$, rel.	54.1	41.7	9.4	6.7
	$J/\psi\, J/\psi$, nonrel.	71.6	55.2	12.4	8.9
	$\eta_c\, \eta_c$, rel.	328.2	270.6	12.5	8.8
	$\eta_c\, \eta_c$, nonrel.	203.4	165.3	5.0	3.5

Numerical values for the integral parameters entering cross section:

charmonium state	$R(0)$, GeV$^{3/2}$	$\tilde{R}(0)$, GeV$^{3/2}$	ω_1	ω_2
J/ψ	0.91	0.81	-0.036	0.0032
η_c		1.08	-0.032	0.0031

Numerical results





Different sources of relativistic corrections (nb):

pair	full nonrel.	+ relativistic mass	+ relativistic wave function	+ amplitude expansion
$J/\psi J/\psi$ $2 < y_{P,Q} < 4.5$	6.65	6.65	4.36 ($\times$0.66)	5.04 ($\times$1.16)
$\eta_c \eta_c$ $2 < y_{P,Q} < 4.5$	1.85	2.67 ($\times$1.44)	5.45 ($\times$2.04)	6.84 ($\times$1.26)
$\eta_c \eta_c$ no y-cuts	74.2	98.3 ($\times$1.32)	200.7 ($\times$2.04)	161.0 ($\times$0.80)

Thank you for attention!