On the field theoretical electron-proton scattering amplitude in the Coulomb and Lorentz gauges

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CONTENT

Why do we need the rescattering equations for the elastic *ep* scattering reactions and for the Hydrogen:

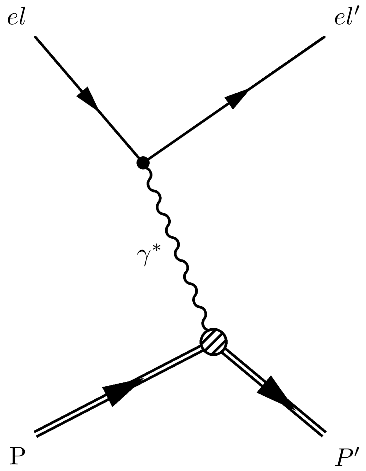


Figure 1: **One photon exchange V_{OPE}.**

Leading order terms in the ep amplitude $\mathcal{A}$

$$\mathcal{A} \simeq V_{OPE} \sim \frac{\alpha^2}{(\sqrt{M^2+\mathbf{p'}^2} - \sqrt{M^2+\mathbf{p}^2})^2 - (\mathbf{p'} - \mathbf{p}}$$

$\alpha^2 = \frac{1}{137.068}$ Next of leading order terms

$$\mathcal{A} - V_{OPE} \simeq \mathcal{A}(\alpha^4) \simeq 0.75\% \quad V_{OPE} \quad ?$$

ep (Hydrogen) discrete states:

M. Eades et all; Phys. Rep. 342 (2001) 66.

S. G. Karschenboim; Phys. Rep. 422 (2005) 1.

G. T. Bodwin, D. R. Yennie and M. A. Gregorio; Rev. Mod. Phys. 57 (1985) 723.

Hydrogen energies E_{njl}:

♣ Units: n-energy number: Rydberg constant $\mathcal{R} = 14.93eV$:

$$E_{n'jl} - E_{njl} \sim \mathcal{R}(\frac{1}{n'^2} - \frac{1}{n^2});$$

Coulomb interaction $n = l + 1 + n_C$; : $\mathcal{R} = 2\pi^2 e^4 m_e/h^3$

♣ Correction $\sim \alpha^4$:

Fine structure — (j-dependence):

$$\delta E_{njl}(j) \sim 10GHz; \qquad 1GHz = 4.1057(10)^{-6}eV$$

Radiation and relativistic corrections, Lamb shift:

$$\delta E_{njl}(Lamb) = 1057MHz \sim \alpha^4 < \mathbf{r^2} >; \quad 1MHz = 4.1057(10)^{-9}eV$$

$< \mathbf{r^2} >$ classical Interpretation: fluctuation of the proton magnetic field

$< \mathbf{r^2} >$ quantum field-theoretical Interpretation: proton size parameter

♣ Superfine structure (spin-spin interaction):

$$\delta E_{njl}(\sigma_1.\sigma_2) \sim 100MHz; \quad \sim \alpha^4$$

$\Updownarrow$ High precision experimental data $\Updownarrow$ Applications: Spectroscopy, Laser Physic, Astrophysics, Solid state Physics,...

Field-theoretical approach: Energy levels E_{njl}: 3D quasipotential equation (Gross: one on shell nucleon in $\gamma^* NN$)

$$\mathcal{A} = V + V G_o \mathcal{A} \equiv V + V G_o V + V G_o V G_o V G_o V + \ldots$$

$$\mathcal{A} = V|\Psi> \qquad (H_o + V)|\Psi> = E|\Psi>$$

$$G_o = \frac{1}{E - H_o + i\epsilon};$$

$$E_{njl} \simeq < \Psi^o | H_o + V + V G_o V + \ldots | \Psi^o >$$

Perturbation series: $\Psi^o >$ is the full Coulomb-type wave function.

Desired result: unified approach and approximation by calculation of the discrete and continuous *ep* interaction.

ep scattering in the low ($< 200 MeV$ and $3-5 GeV$ energy region.

⊙ Two photon exchange models

J. Arrington, P. G. Blunden and W. Melnitchouk, Prog. Part. Nucl. Phys. **66** (2011) 782.

N. Kivel and M. Vanderhaeghen, arXiv 1212 0683, 2012.

C. E. Perdrisat, V. Punjabi and Vanderhaeghen, Prog. Part. Nucl. Phys. 59 (2007) 694.

J. Arrington, W. Melnitchouk and J.A. Tjon. Phys. Rev. **C76** (2007) 035205.

⊙ Two photon exchange models as the next of the leading order corrections:

Low energy region ($E < 200 MeV$:

$$\mathcal{A} - V_{OPE} \simeq \mathcal{A}(\alpha^4) \simeq 0.75\% \;\; V_{OPE}\, is\, valid$$

Exception: Δ-resonance degrees of freedom. It is necessary to introduce the special electromagnetic interaction for Δ-s

⊙ Two photon exchange models as the next of the leading order corrections:

Energy region $1-2$ GeV:

Experimental result of ratio of the cross sections of the electron-proton (ep) and positron-proton (e^+p) cross sections $Q^2 > 1GeV^2$

$$\alpha^2(e^+p) = -\alpha^2(ep)$$

$$\mathcal{A}(e^+p) = -V(\alpha^2) + V(\alpha^2)G_oV(\alpha^2) - ...$$

$$1.05 \leq \frac{\sigma(e^+p)}{\sigma(ep)} \leq 1.1$$

Next order corrections: $5-10\%$. Larger than

$$\mathcal{A}-V_{OPE} \simeq \mathcal{A}(\alpha^4) \simeq 0.75\% \;\; V_{OPE}\, is\, valid$$

Big experimental (systematic) errors.

⊙ Difficulties of two photon exchange models Arrington, Melnitchouk & Tjon. PR **C76** 2007.

1. Ambiguities of 3D reductions of 4D Bethe-Salpeter equation.
2. In the Bethe-Salpeter equations and their quasipotential reductions one has the multi-variable form factors

Gross quasipotential equation with 4 el.-mag. form-factors which are 2-variable functions

$$\Gamma_{N\to\gamma^*N^*}(Gross) = e\overline{u}(\mathbf{p'_N})[(\gamma^\mu F_1(t,(p'_N)^2) - \frac{i\sigma_{\mu\nu}(p'_N - p_N)^\nu}{2M}F_2(t,(p'_N)^2)) + \frac{i\gamma_\nu {p'_N}^\nu - M}{2M}(\gamma^\mu F_3(t,(p'_N)^2) - \frac{i\sigma_{\mu\nu}(p'_N - p_N)^\nu}{2M}F_4(t,(p'_N)^2))]u(\mathbf{p_N})$$

Other quasipontial equations and the Bethe-Salpeter equation requires 8 el.-mag. form factors which are functions of the 3 variable.

$\odot$ Difficulties of two photon exchange models Arrington, Melnitchouk & Tjon. PR **C76** 2007.

Problems with separation of the quark and hadron degrees of freedom. Unitary condition for the electron+ hadron system. Double counting.

Suggested 3D field-theoretical equations:

1. Is free from ambiguities of 3D reductions of 4D Bethe-Salpeter equation.
2. Automatically satisfies unitary condition. Quark and hadron degrees of freedom are separated
3. INPUT el.-mag. vertex with on mass shell nucleons

$$< \mathbf{p'_N}|J^{\mu}(0)|\mathbf{p_N} >= e\overline{u}(\mathbf{p'_N})(\gamma^{\mu}F_1(t) - \frac{i\sigma_{\mu\nu}(p'_N - p_N)^{\nu}}{2m_N}F_2(t))u(\mathbf{p_N})$$

Troubles of the field-theoretical approach to the ep scattering:

♣ Divergence of the perturbation series in QED

$$A = V + VG_oA \equiv V + VG_oV + VG_oVG_oVG_oV + \ldots$$

$$G_o = \frac{1}{E - H_o + i\epsilon}$$

Methods of the asymptotic expansions:

F.J.Dyson, Phys. Rev 85 (1952)

C. Itzykson and J.B. Zuber. "Quantum Field theory". 1980 ch. 9.4.

S. Weinberg. "The Quantum theory of fields," 2001, ch. 20.7 L.N. Lipatov)

I.M.Suslov, JETP 100 (2005) :
For the equation $A = V + VG_oA$

$$A = (1 - VG_o)^{-1}V$$

Existence of $(1 - VG_o)^{-1}$ for divergent perturbation series $V + VG_oV + VG_oVG_oVG_oV + \ldots$

Troubles of the field-theoretical approach to the ep scattering:

♣ ♣ If components of $A_\mu(x)$ are independent and quantized, then in the Lorentz gauge is necessary the Gupta-Bleuler indefinite metric.

$$< \psi|\frac{\partial A_\mu(x)}{\partial x_\mu}|\psi >= 0$$

$$A_o(x) \Longrightarrow i a_o(x)$$

$$[a_o(x) - a_3(x)]|0_L >= 0$$

And set other conditions V. M. Dubovik & S.V. Shabanov JPA (1990)

For Heisenberg interacted fields $A_\mu(x)$

Short list of my related publications:

[I] *A. I. MACHAVARIANI, A. J. BUCHMANN, AMAND FAESSLER and G. A. EMELYANENKO* Annals of Physics **253**(1997)149.

[II] *A. I. MACHAVARIANI/ and/ AMAND/ FAESSLER* Nuclear Physics **A646** (2002) 231.

[III] *A. I. MACHAVARIANI* Phys. Letters **B540**(2002) 81.

[IV] *A. I. MACHAVARIANI and AMAND FAESSLER;* Annals of Phys.**409**(2004)p.49-92.

[V] *A. I. MACHAVARIANI and AMAND FAESSLER;* Phys. Review **C72** (2005) 024002.

[VI] *A. I. MACHAVARIANI and AMAND FAESSLER;* Journal of Physics G: Nucl. Part. Phys. **37** (2010) 075004 and **38** (2011) 35002

NUMERICAL CALCULATIONS:
πN, NN and πd phase shifts and cross sections.
Cross sections of the ep and γp bremstrahlung in the Δ resonance region and in the Born approximation.

- $\mathcal{S}$-matrix reduction formulas and the 3D equations. $p'_o = \sqrt{m_e^2 + \mathbf{p}'^2} \quad p_o = \sqrt{m_e^2 + \mathbf{p}^2}$

$$\mathcal{S}_{e'N',eN} = < out; \mathbf{p}'_{e'}, \mathbf{p}'_N | \mathbf{p}_e, \mathbf{p}_N; in >$$

$$\mathcal{S}_{e'N',eN} = < out; \mathbf{p}'_N | b_{\mathbf{p}'e'}(out) | \mathbf{p}_e, \mathbf{p}_N; in >$$

$$b_{\mathbf{p}e}(out) = \overline{u}(\mathbf{p}_e)\gamma_o \int d^3p_e e^{ip_e^o x_o - i\mathbf{p_e x}} \psi_e^{out}(x);$$

$$(i\gamma_\mu \frac{\partial}{\partial x_\mu} - m_e)\psi_e^{out}(x) = 0$$

$$b_{\mathbf{p}e}(x_o) = \overline{u}(\mathbf{p}_e)\gamma_o \int d^3x e^{ip_e^o x_o - i\mathbf{p_e x}} \psi_e(x) :$$

$$(i\gamma_\mu \frac{\partial}{\partial x_\mu} - m_e)\psi_e(x) = \eta(x)$$

$$b_{\mathbf{p}'e}(out) = b_{\mathbf{p}'e}(in) + \overline{u}(\mathbf{p}'_e) \int d^4x e^{ip'_e x} \theta(x_o)\eta(x)$$

$$\mathcal{S}_{e'N',eN} =< in; \mathbf{p'}_e, \mathbf{p'}_N | \mathbf{p}_e, \mathbf{p}_N; in >$$
$$+i(2\pi)^4 \delta(p_e + p_N - p'_e - p'_N) \mathcal{A}_{e'N',eN}$$

$$\mathcal{A}_{e'N',eN} =< out; \mathbf{p'}_N | \eta_{\mathbf{p'_e}}(0) | \mathbf{p}_e, \mathbf{p}_N; in >$$

Both protons and e are on mass shell in $\mathcal{A}_{e'N',eN}$.

$$\eta_{\mathbf{p'_e}}(x) = \overline{u}(\mathbf{p'}_e)\eta(x)$$

$$b^+_{\mathbf{p}e}(in) = b^+_{\mathbf{p}_e}(0) - \int d^4x e^{ip'_e x} \theta(-x_o) \overline{\eta}(x) u(\mathbf{p}_e)$$

$$\mathcal{A}_{e'N',eN} =< \mathbf{p'}_N | \{\eta_{\mathbf{p'_e}}(0), b^+_{\mathbf{p_e}}(0)\} | \mathbf{p}_N > -$$
$$i \int d^4x e^{-ip_e x} \theta(-x_o) < \mathbf{p'}_N | \{\eta_{\mathbf{p'_e}}(0), \overline{\eta}_{\mathbf{p_e}}(x))\} | \mathbf{p}_N >$$

$$\mathcal{A}_{e'N',eN} =< \mathbf{p'}_N|\{\eta_{\mathbf{p'_e}}(0), b^+_{\mathbf{p_e}}(0)\}|\mathbf{p}_N > -$$

$$i\int d^4xe^{-ip_ex}\theta(-x_o) < \mathbf{p'}_N|\{\eta_{\mathbf{p'_e}}(0), \overline{\eta}_{\mathbf{p_e}}(x))\}|\mathbf{p}_N >$$

Both protons are on mass shell in $\mathcal{A}_{e+p\rightarrow e'+p'}$.

$$\eta_{\mathbf{p'_e}}(x) = \overline{u}(\mathbf{p'}_e)\eta(x)$$

4D Bethe-Salpeter equation for the full 4D Green function $G_{e+p\rightarrow e'+p'}$

$$G_{e+p\rightarrow e'+p'} = g_o^{free} + \mathcal{V}^{BS} g_o^{free} G_{e+p\rightarrow e'+p'}$$

$$G_{e+p\rightarrow e'+p'} =< 0|\mathbf{T}(\psi_{e'}(0)\psi_{N'}(y)\overline{\psi}_e(x)\psi_N(z))|0 >$$

where all four particles from "in" and "out" states are extracted. electrons & protons are off shell

$\mathcal{V}^{BS}$ is sum of all Feynman diagrams without intermediate $e + p$ states.

$$\mathcal{A}_{e'N',eN} =< \mathbf{p'}_N|\{\eta_{\mathbf{p'_e}}(0), b^+_{\mathbf{p_e}}(0)\}|\mathbf{p}_N > -$$

$$i\int d^4x e^{-ip_e x}\theta(-x_o) < \mathbf{p'}_N|\{\eta_{\mathbf{p'_e}}(0), \overline{\eta}_{\mathbf{p_e}}(x))\}|\mathbf{p}_N >$$

Both protons are on mass shell in $\mathcal{A}_{e+p\to e'+p'}$.

$$\sum_n |n; in >< in; n| = 1$$

$$\mathcal{A}_{e'N',eN} =< \mathbf{p'}_N|\{\eta_{\mathbf{p'_e}}(0), b^+_{\mathbf{p_e}}(0)\}|\mathbf{p}_N >$$

$$+\sum_n \mathcal{A}_{e'N',n}\frac{(2\pi)^3\delta(\mathbf{P_n}-\mathbf{p_e}-\mathbf{p_N})}{E_{\mathbf{p_e}}+E_{\mathbf{p_N}}-P^o_n+i\epsilon}\mathcal{A}^+_{n,eN}$$

$$+\sum_m \mathcal{A}_{N',em}\frac{(2\pi)^3\delta(\mathbf{P_m}+\mathbf{p_e}-\mathbf{p'_N})}{E_{\mathbf{p_e}}-E_{\mathbf{p'_N}}+P^o_m}\mathcal{A}^+_{N,e'm}$$

$$\mathcal{A}_{e'N',n} =< \mathbf{p'}_N|\eta_{\mathbf{p'_e}}(0)|n; in >; \quad n = H, ep, ep\gamma, \ldots$$

$$\mathcal{A}^+_{N,e'm} =< in; m|\eta_{\mathbf{p'_e}}(0)|\mathbf{p}_N; in >; \quad m = \overline{e}p, \overline{e}\gamma p\ldots$$

$$E_{\mathbf{p'_e}} = \sqrt{m_e^2+\mathbf{p'}^2_e}; \qquad E_{\mathbf{p'_N}} = \sqrt{M^2+\mathbf{p'}^2_e}$$

$$\mathcal{A}_{e'N',n} =< \mathbf{p'}_N|\eta_{\mathbf{p'_e}}(0)|n; in >; \quad n = H, ep, ep\gamma, ...$$

Connected and Disconnected parts (cluster decomposition)

$$\mathcal{A}_{e'N',n} =< \mathbf{p'}_N|\eta_{\mathbf{p'_e}}(0)|n; in >_C$$
$$+ < \mathbf{p'}_N|\mathbf{p_N}" >< 0|\eta_{\mathbf{p'_e}}(0)|n'; in >_C \quad n' = e\gamma, ...$$

$$\sum_n \mathcal{A}_{e'N',n} \frac{(2\pi)^3 \delta(\mathbf{P_n} - \mathbf{p_e} - \mathbf{p_N})}{E_{\mathbf{p_e}} + E_{\mathbf{p_N}} - P_n^o + i\epsilon} \mathcal{A}^+_{n,eN}$$

$$\updownarrow \qquad \updownarrow \qquad \updownarrow$$

$$\sum_{n'} \mathcal{A}_{e',n'} \frac{(2\pi)^3 \delta(\mathbf{P_{n'}} + \mathbf{p'_N} - \mathbf{p_e} - \mathbf{p_N})}{E_{\mathbf{p_e}} + E_{\mathbf{p_N}} - P_{n'}^o - E_{\mathbf{p'_e}}} \mathcal{A}^+_{N'n',eN}$$

$$\mathcal{A}_{e'N',eN} =< \mathbf{p'}_N|\{\eta_{\mathbf{p'_e}}(0), b^+_{\mathbf{p_e}}(0)\}|\mathbf{p}_N >$$

$$\sum_n \mathcal{A}_{e'N',n} G^S_o(n) \mathcal{A}^+_{n,eN} + crossing_{(N',e',N,e)}$$

$n = H, eN.\gamma eN, \ldots$ s-channel on shell particle exchange

crossing $\Longleftrightarrow u, \overline{s}, \overline{u}, \ldots$-channel exchange

$$\mathcal{A}_{e'N',eN} - \mathcal{A}^+_{e'N',eN} =$$

$$\sum_n \mathcal{A}_{e'N',n}[G^S_o(n) - G^{S^+}_o(n)]\mathcal{A}^+_{n,eN}$$

- General field theoretical 3D unitary condition for $ep \to ep$ with any number of the intermediate particles n.

- Generalized Chew-Low type equations for the ep scattering

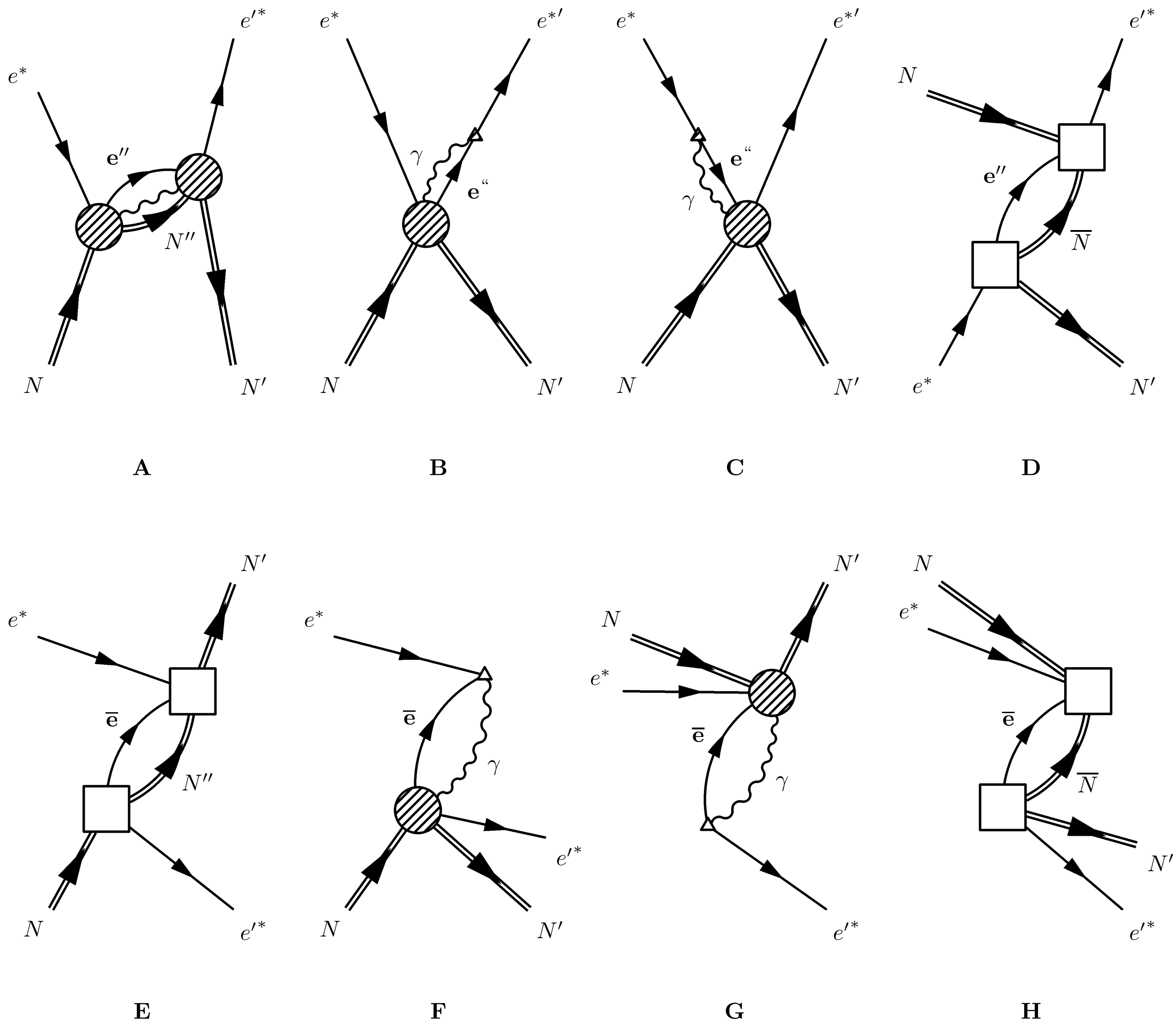
- Matrix representation of the Bogoljubov-Medvedev-Polivanov equations

- Spectral decomposition of $\mathcal{A}_{e+p\to e'+p'}$ over the complete set asymptotic states

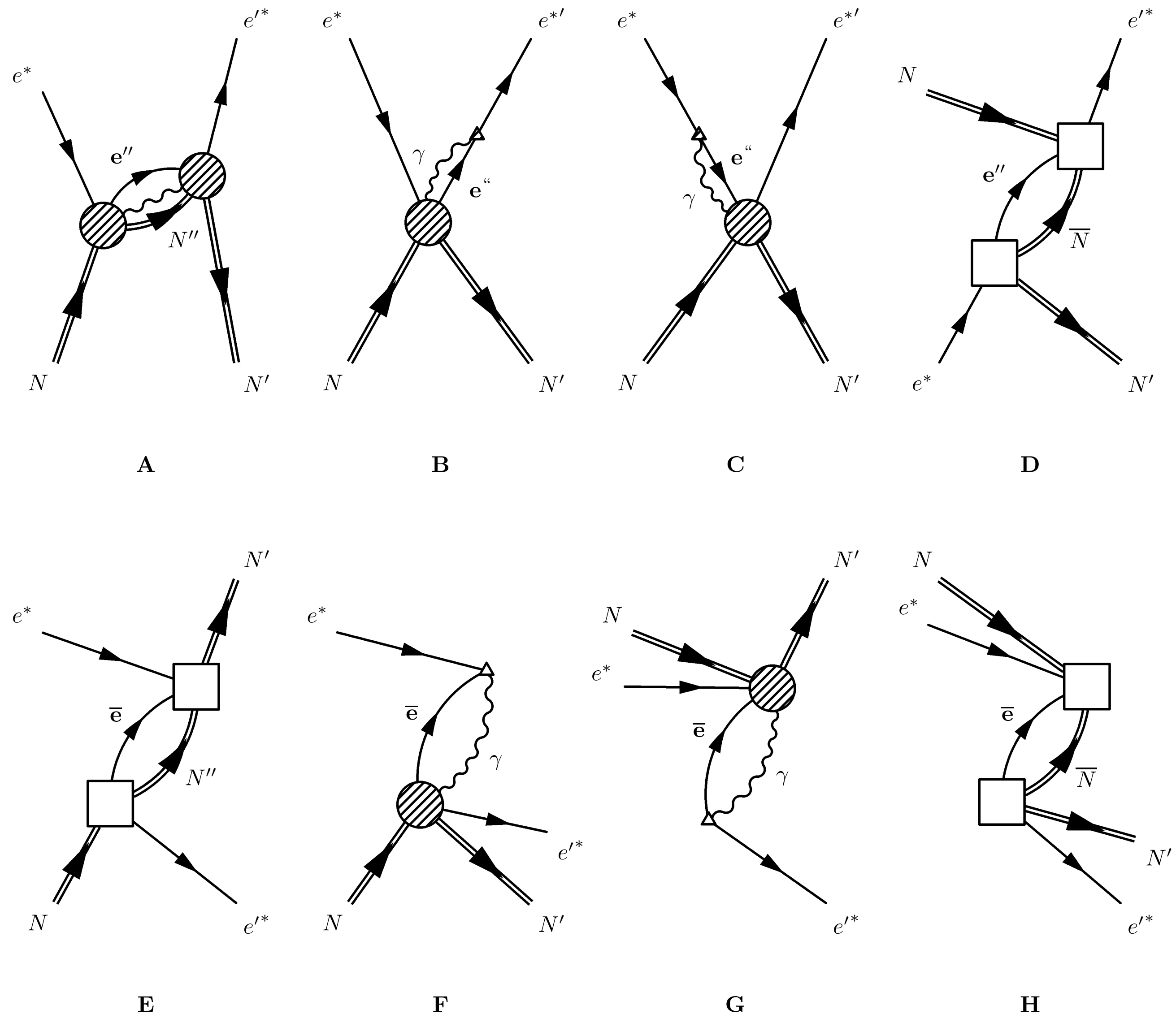
Two-body equation for the $ep-ep$ amplitude

$$\mathcal{A}_{e'N',eN} =< \mathbf{p'_N}|\{\eta_{\mathbf{p'_e}}(0), b^+_{\mathbf{p_e}}(0)\}|\mathbf{p_N} > +\mathcal{W}_{e'N',eN}$$
$$+ \sum_{H} \mathcal{A}_{e'N',H} \frac{(2\pi)^3\delta(\mathbf{P_H} - \mathbf{p_e} - \mathbf{p_N})}{E_{\mathbf{p_e}} + E_{\mathbf{p_N}} - P^o_H} \mathcal{A}^+_{H,eN}$$
$$+ \sum_{n=e"N"} \mathcal{A}_{e'N',e"N"} \frac{(2\pi)^3\delta(\mathbf{P_n} - \mathbf{p_e} - \mathbf{p_N})}{E_{\mathbf{p_e}} + E_{\mathbf{p_N}} - P^o_n + i\epsilon} \mathcal{A}^+_{n,eN}$$

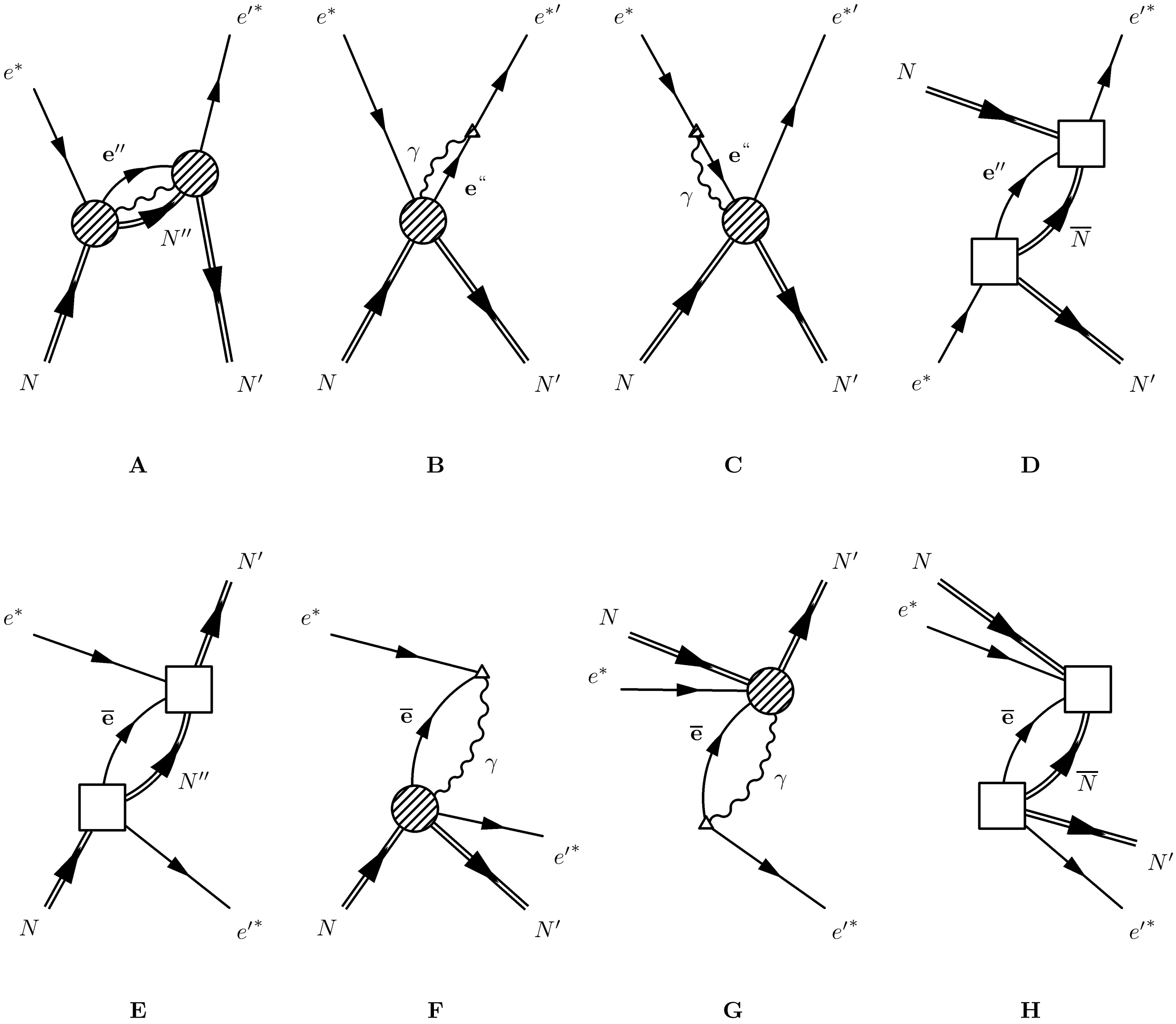
$\mathcal{W}_{e'N',eN}$ **includes all on mass shell exchange terms excepting** H, ep **exchange in the** s **channel**



Potential $\mathcal{W}_{e+p\to e'+p'}$ with the off mass shell external electrons: (A) s-channel γeN exchange. (B), (C) Parts with the intermediate amplitude $ep - ep\gamma$ (D), (H) Z-diagrams with intermediate antinucleon (E), (F), (G) Anti-electron exchange parts



$\mathcal{W}_{e+p\to e'+p'}$ **with the off mass shell external electrons: • nucleons are on mass shell. • Proton vertex correction are included in $\gamma^* pp$ vertex.(Equal-time term). • Self-energy terms do not appear. • electron vertex correction are included in (B) and (C).**



Potential $\mathcal{W}_{e+p\to e'+p'}$ with the off mass shell external electrons: the time-ordered 3D diagrams. Complete set of the next of the leading order terms $\sim \alpha^4$.

Linearization:

$$\mathcal{W}_{e'N',eN} = \mathrm{A}_{e'N',eN} + (E_{\mathbf{p'_e}} + E_{\mathbf{p'_N}})\mathrm{B}_{e'N',eN}$$

Linear energy depending potential

$$\mathcal{U}_{e'N',eN}(E) =< \mathbf{p'}_N|\{\eta_{\mathbf{p'_e}}(0), b^{+}_{\mathbf{p_e}}(0)\}|\mathbf{p}_N > + \mathrm{A}_{e'N',eN} + E\ \mathrm{B}_{e'N',eN}$$

Final 3D relativistic Lippmann-Schwinger-type equation

$$\mathcal{T}_{e'N',eN}(E) = \mathcal{U}_{e'N',eN}(E) + \sum_{e"N"} \mathcal{U}_{e'N',e"N"}(E) g_o(E) \mathcal{T}_{e"N",eN}(E)$$

$$g_o = \frac{(2\pi)^3 \delta(\mathbf{p'_e} + \mathbf{p'_N} - \mathbf{p_e} - \mathbf{p_N})}{E_{\mathbf{p_e}} + E_{\mathbf{p_N}} - E + i\epsilon}$$

$$\mathcal{T}_{e'N',eN}(E = E_{\mathbf{p'_e}} + E_{\mathbf{p'_N}}) = \mathcal{A}_{e'N',eN} = < out; \mathbf{p'}_N|\eta_{\mathbf{p'_e}}(0)|\mathbf{p}_e, \mathbf{p}_N; in >

Equal-time commutators and One Photon Exchange (OPE) Naive canonical quantization of

the four independent photon components

$$Y_{e'N',eN} =< out; \mathbf{p'}_N|\{\eta_{\mathbf{p'_e}}(0), b^+_{\mathbf{p_e}}(0)\}|\mathbf{p}_N; in >$$

$$[\overset{o}{A}_\mu(x_o, \mathbf{x}), A_\nu(x_o, \mathbf{y})] = ig_{\mu\nu}\delta(\mathbf{x}-\mathbf{y})$$

$$\{\psi_\alpha(x_o, \mathbf{x}), \psi^+_\beta(x_o, \mathbf{y})\} = \delta_{\alpha\beta}\delta(\mathbf{x}-\mathbf{y})$$

$$b^+_{\mathbf{p}_e}(x_o) = \int d^3x e^{-ip_e x}\overline{\psi}_e(x)\gamma_o u(\mathbf{p_e})$$

$$\eta_e(x) = e\gamma^\mu\psi_e(x)A_\mu(x)$$

$$Y_{e'N',eN} = e\overline{u}(\mathbf{p'_e})\gamma_\mu u(\mathbf{p_e}) < out; \mathbf{p'}_N|A^\mu(0)|\mathbf{p}_N; in$$

$$J^\mu(x) = \Box A^\mu(x) = e\overline{\psi}_e(x)\gamma^\mu\psi_e(x) + e\overline{\psi}_N(x)\gamma^\mu\psi_N($$

$$Y_{e'N',eN} = e\overline{u}(\mathbf{p'_e})\gamma_\mu u(\mathbf{p_e})\frac{< out; \mathbf{p'}_N|J^\mu(0)|\mathbf{p}_N; in}{t_N}$$

$$Y_{e'N',eN} = e\overline{u}(\mathbf{p'_e})\gamma_\mu u(\mathbf{p_e})\frac{< out; \mathbf{p}'_N|J^\mu(0)|\mathbf{p}_N; in}{t_N}$$

$$t_N = ({p^o_N}' - p^o_N)^2 - (\mathbf{p'_N} - \mathbf{p_N})^2 \text{ OPE term}$$

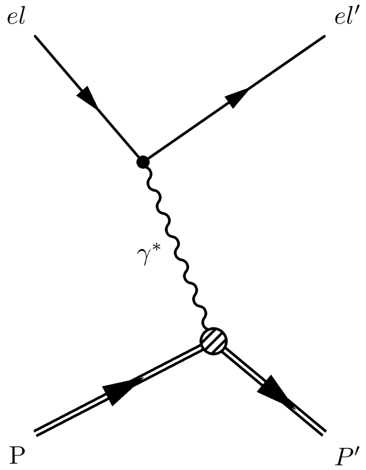


Figure 2: **One photon exchange** $V_{OPE} \equiv Y_{e'N',eN}$.

Gupta-Bleuer indefinite metric, additional conditions,....

Coulomb gauge

$$\frac{\partial A_i^C(x)}{\partial x_i} = 0; \qquad\qquad i = 1, 2, 3$$

$$(i\gamma^\mu \frac{\partial}{\partial x^\mu} - m_e)\psi_e(x) = \eta(x) = e\gamma^\mu A_\mu^C(x)\psi_e(x)$$

$$\Box_x A^C{}_i(x) = J_i^{tr}(x) = J_i(x) - \frac{\partial}{\partial x^i}\frac{\partial J^k(x)}{\partial x^k}$$

Poisson eq. (Nonlocality $\rightarrow$ Coulomb energy)

$$-\Delta A^C{}_o(x) \equiv -\frac{\partial}{\partial x^i}\frac{\partial A^C{}_o(x)}{\partial x_i} = J_i^o(x)$$

$$A_o^C(x) = \int \frac{d\mathbf{x}' J_o(x_o, \mathbf{x}')}{4\pi|\mathbf{x} - \mathbf{x}'|}$$

$A_o^C(x)$ is defined via $J_o(x) = e\overline{\psi}(x)\gamma_o\psi(x)$

$$[\frac{\partial A_i^C(x_o, \mathbf{x})}{\partial x_o}, A_j^C(x_o, \mathbf{y})] =$$

$$\delta_{ij}\delta(\mathbf{x} - \mathbf{y}) - \frac{1}{\Delta}\frac{\partial^2}{\partial x_i \partial y_j}\frac{1}{|\mathbf{x} - \mathbf{y}|}$$

$$Y_{e'N',eN} =< out; \mathbf{p'}_N|\{\eta_{\mathbf{p'_e}}(0), b^+_{\mathbf{p_e}}(0)\}|\mathbf{p}_N; in >$$

$$Y_{e'N',eN} = Y^C_I + Y^C_{II}$$

OPE in Coulomb gauge:

$$Y^C_I = \frac{e\overline{u}(\mathbf{p'_e})\gamma^o u(\mathbf{p_e})}{-(\mathbf{p'_N} - \mathbf{p_N})^2} < \mathbf{p'}_N|J^{tr}_o(0)|\mathbf{p}_N >$$

$$-e\overline{u}(\mathbf{p'_e})\gamma^i u(\mathbf{p_e})\frac{1}{t_N} < \mathbf{p'}_N|J^{tr}_i(0)|\mathbf{p}_N >$$

Second nonlocal part is generated by $[A^C_{\mu=o}(0), \psi^+(0$

$$Y^C_{II} = -e\overline{u}(\mathbf{p'_e})\gamma^o \int \frac{d\mathbf{x'}}{4\pi|\mathbf{x'}|}$$

$$< \mathbf{p'}_N|\psi^+_e(0, \mathbf{x'})\psi_e(0, 0)|\mathbf{p_N} > u(\mathbf{p_e})$$

Y^C_{II} is generated by the Poisson relation i.e. definition of $A^C_o(x)$ via $J_o(x)$.

This follows from the Electro-static (Coulomb) interaction Nonlocal interaction

Y^C_{II} is next of the leading order over α^2

Lorentz gauge

$$\frac{\partial A^L_\mu(x)}{\partial x_\mu} = 0; \qquad \mu = 0, 1, 2, 3$$

$$\Box_x A^{L}{}^{tr}_i(x) = J_i(x) = e\overline{\Psi}^L_e(x)\gamma_i\Psi^L_e(x); \quad i = 1, 2$$

$$A^L{}_3(x) = \Box_x^{-1} J_3(x); \quad J_3(x) = e\overline{\Psi}^L_e(x)\gamma_3\Psi^L_e(x)$$

$$A^L{}_o(x) = \Box_x^{-1} J_o(x); \quad J_o(x) = e\overline{\Psi}^L_e(x)\gamma_o\Psi^L_e(x)$$

$$(A^L)^{tr}_i(x) = (A^L)_i(x) - \frac{\partial}{\partial x^i}\frac{\partial (A^L)^k(x)}{\partial x^k}$$

$$(A^L)^l_i(x) = \frac{\partial}{\partial x^i}\frac{\partial (A^L)^k(x)}{\partial x^k}$$

$$[\frac{\partial (A^L)^{tr}_i(x_o, \mathbf{x})}{\partial x_o}, (A^L)^{tr}_j(x_o, \mathbf{y})] =$$

$$\delta_{ij}\delta(\mathbf{x} - \mathbf{y}) - \frac{1}{\Delta}\frac{\partial^2}{\partial x_i \partial y_j}\frac{1}{|\mathbf{x} - \mathbf{y}|}$$

Relationship between Lorentz and Coulomb gauges

$$(i\gamma^\mu \frac{\partial}{\partial x^\mu} - m_e)\psi_e^L(x) = e\gamma^\mu A_\mu^L(x)\psi_e^L(x)$$

$$\updownarrow \qquad\qquad\qquad \updownarrow \qquad\qquad\qquad \updownarrow$$

$$(i\gamma^\mu \frac{\partial}{\partial x^\mu} - m_e)\psi_e^C(x) == e\gamma^\mu A_\mu^C(x)\psi_e^C(x)$$

$$\psi_e^C(x) = e^{ie\lambda(x)}\psi_e^L(x)$$

$$A_\mu^C(x) = e^{-ie\lambda(x)}A_\mu^L(x)e^{ie\lambda(x)} + \frac{\partial\lambda(x)}{\partial x_\mu}$$

$$e^{-ie\lambda(x)}A_\mu^L(x)e^{ie\lambda(x)} = A_\mu^L(x) + ie[A_\mu^L, \lambda]$$

$$+ie[ie[A_\mu^L, \lambda], \lambda] + ... \equiv A_\mu^L(x) + \mathcal{D}(A_\mu^L, \lambda)$$

If λ is determined through the relations

$$\mathcal{D}(A_\mu^L, \lambda) + \frac{\partial\lambda(x)}{\partial x_\mu} = +\frac{1}{\Delta}\frac{\partial}{\partial x_\mu}\frac{\partial \mathsf{a}_o(x)}{\partial x_o} = 0$$

then

$$A_\mu^C(x) = A^L{}_\mu(x) - \frac{1}{\Delta}\frac{\partial}{\partial x_\mu}\frac{\partial A^L{}_o(x)}{\partial x_o}$$

$$-\Delta A_o^C = \Box A^L{}_o = J_o(x)$$

$$\frac{\partial A_i^C(x)}{\partial x_i} = 0$$

$A_i^C(x)$ is the photon field in the Coulomb gauge

$$< \mathbf{p}'_N|A^L{}_{\mu=1,2}(0)|\mathbf{p}_N> = < \mathbf{p}'_N|A^C{}_{\mu=1,2}(0)|\mathbf{p}_N>$$

$$< \mathbf{p}'_N|A^L{}_o(0)|\mathbf{p}_N> = \frac{t_N < \mathbf{p}'_N|A^C{}_o(0)|\mathbf{p}_N>}{-(\mathbf{p'_N} - \mathbf{p_N})^2}$$

The equal time canonical commutators are not gauge invariant

$$\{\Psi^{C+}(x_o, \mathbf{x}), \Psi^{C}(x_o, \mathbf{y})\} = \delta(\mathbf{x} - \mathbf{y}) + \mathcal{C}(x, \mathbf{y})$$

$$\begin{aligned}\mathcal{C}(x, \mathbf{y}) = \Psi^{L+}(x)[e^{-i\lambda(x)}, e^{i\lambda(x_o.\mathbf{y})}]\Psi^{L}(x_o, \mathbf{y}) \\ +[\Psi^{L+}(x), e^{i\lambda(x_o.\mathbf{y})}]e^{-i\lambda(x_o.\mathbf{y})}\Psi^{L}(x_o, \mathbf{y}) \\ +e^{i\lambda(x_o.\mathbf{y})}[\Psi^{L}(x_o, \mathbf{y}), e^{-i\lambda(x_o.\mathbf{y})}]\Psi^{L+}(x)\end{aligned}$$

Relations in the Coulomb gauge are more transparent and compact than in the Lorentz gauge.

The electro-static (Coulomb) interactions are exactly separated in the quantization of the transverse phhoton fields

Nucleon as three quark bound (cluster) state
R. Haag, Phys. Rev. **112** (1958)
K. Nishijima, Phys. Rev. **111** (1958)
W. Zimmermann, Nuovo Cim. 10 (1958)
K. Huang and H. A. Weldon, Phys. Rev. D11 (1975) 257.

Construction of the cluster (bound) state asymptotic crearion annihilation) operator

$$\mathcal{B}^{in(out)}(\mathbf{p}) = \lim_{X^0 \to -\infty(+\infty)}^{weekly} \mathcal{B}_{\mathbf{p}}(X^0),$$

$$\mathcal{B}_{\mathbf{p}}(X^0) = \int d^3\mathbf{X} \exp{(ipX)} \overline{u}(\mathbf{p}) \gamma_0 \Upsilon_p(X)$$

Jacobi four-coordinates

$$\rho_{12} = x_1 - x_2$$

$$\rho_3 = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} - x_3$$

$$X = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3}$$

,

$$\Upsilon_p(X) = \int d^4\rho_{12} d^4\rho_3 \chi_p^\dagger(X = 0, \rho_{12}, \rho_3)$$

$$T(q_1(x_1) q_2(x_2) q_3(x_3)).$$

$$\chi_p^\dagger(x_1, x_2, x_3) = < \mathbf{p}_N | T(q_1(x_1) q_2(x_2) q_3(x_3)) | 0 >$$

The leading term in the formulations with off shell nucleons and on shell electrons

$$Y_{e'N',eN} =< out; \mathbf{p'}_e|\{J_{\mathbf{p'_N}}(0), B^+_{\mathbf{p_N}}(0)\}|\mathbf{p}_e; in >$$

$$J_{\mathbf{p'_N}}(X) = (i\gamma_\mu \frac{\partial}{\partial X_\mu} - m_N)\Upsilon_p(X)$$

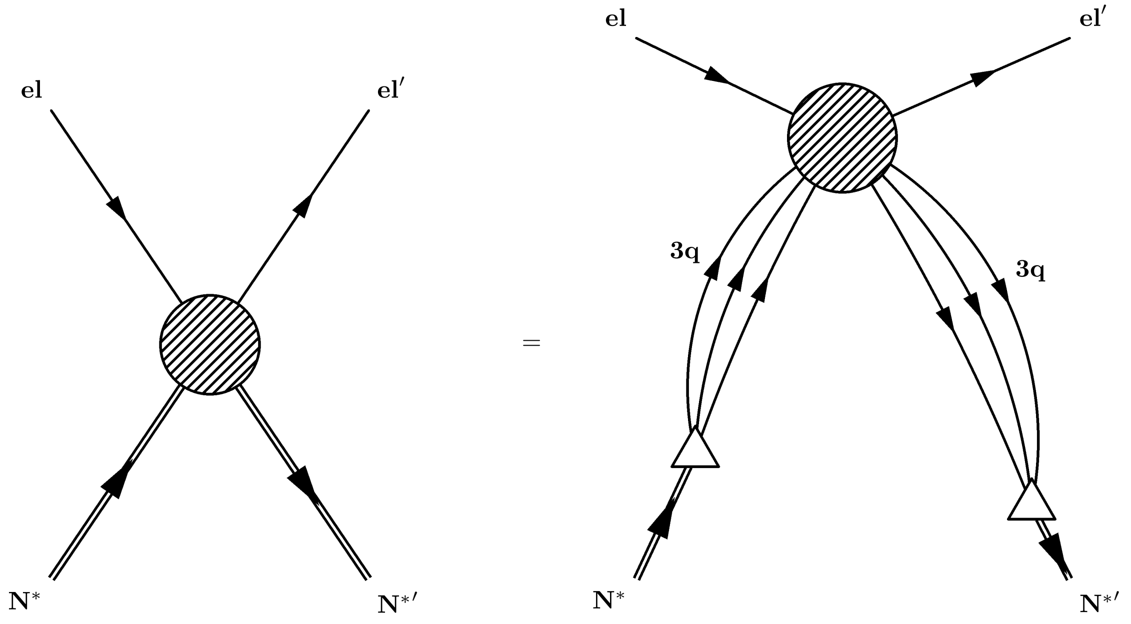


Figure 3: *ep* **scattering amplitude with off mass shell nucleons and on shell electrons. Other kind set of the unitary condition**

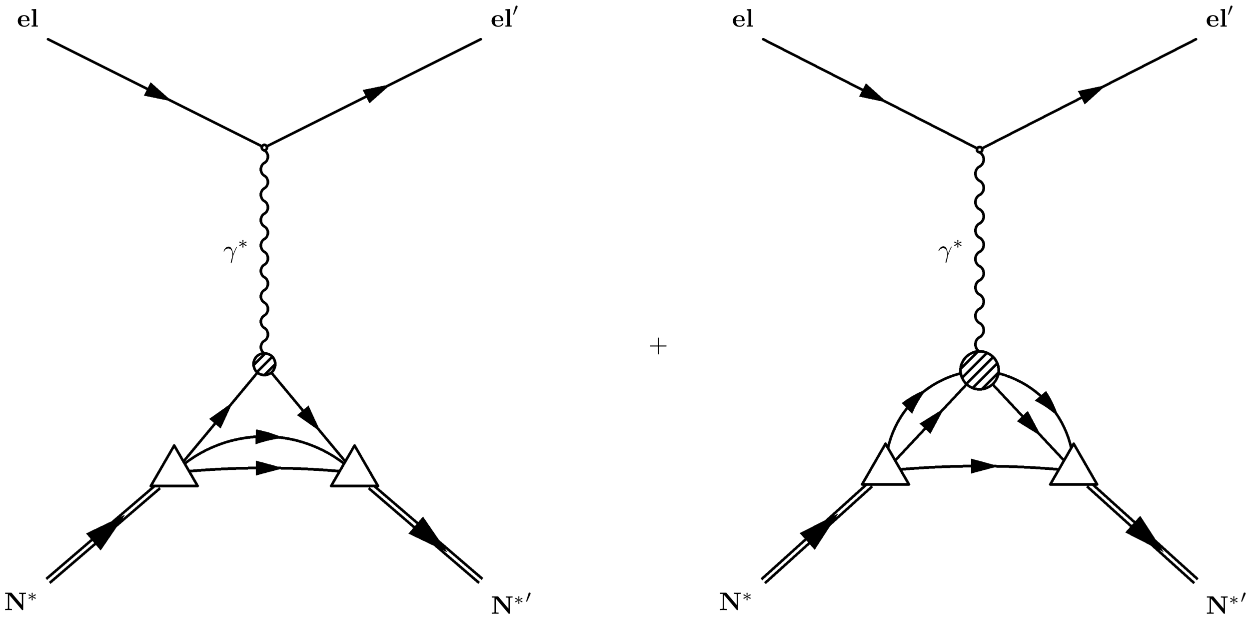


Figure 4: **The leading terms in *ep* scattering amplitude calculated in the canonical equal-time commutation relations.**

One can reformulate this approach completely in the 3D form, where

$$\chi_p^\dagger(x_1,x_2,x_3) =< \mathbf{p}_N|T(q_1(x_1)q_2(x_2)q_3(x_3))|0>$$

$$\updownarrow \qquad \updownarrow \qquad \updownarrow$$

$$< \mathbf{p}_N|j_1(0)|\mathbf{p}_{q1}\mathbf{p}_{q2}; in >$$

$$j_1(x) = (i\gamma_\mu \frac{\partial}{\partial x_\mu} - m_{q1})q_1(x)$$

- Propagation of the quark and gluons in the intermediate states does not contribute into the unitary conditions with hadrons and leptons.
- Unitarity in the hadron sector ensure separation of the quark and hadron degrees of freedom
- Unitarity allows to avoid the double-counting
- The three-quark cluster does not contribute in the leading order diagrams.
- The form of the 3D equations with and without quarks are the same.
- 3D equation with off mass shell nucleons and with off mass shell electrons presents different kind of the unitarity conditions.

Conclusion and outlook

1. New three dimensional field theoretical approach for the unified description of the Hydrogen-like systems and the lepton-nucleon scattering is suggested.

2. This formulation is free from the ambiguities of the Bethe-Salpeter equations which arise by 3D reduction of these equations.

3. Unlike to the Bethe-Salpeter equations and their quasipotential reductions, the potential of the present equation is constructed from the one variable form factors.

4. Present 3D relativistic equations are obtained after analytic linearization of the unitarity conditions with an arbitrary n on mass shell intermediate states

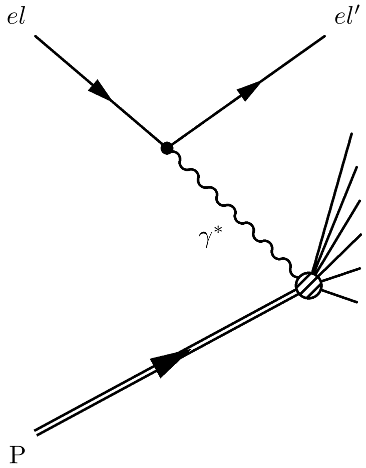


Figure 5: **Inclusive electron-proton scattering in the leading one-photon exchange approximation**

• Suggested equations can be extended for the inclusive ep oand pp scattering
• The present approach allows estimate next of the leading order corrections keeping the first principles (Unitarity, causality, gauge invariance etc.
• In the present approach are exactly included the most important couplings of the elastic ep scattering with the $ep-\gamma ep$ reaction and the positron degrees of freedom.