

# Analytic Structure of the Landau Gauge Gluon Propagators

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**HIC** | **FAIR**  
for  
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# Outline

Glueballs

Yang-Mills Dyson-Schwinger Equations

Complex Propagators

Summary and Perspectives

# Yang-Mills Greens Functions

Fundamental questions:

Gluon screening – Color confinement

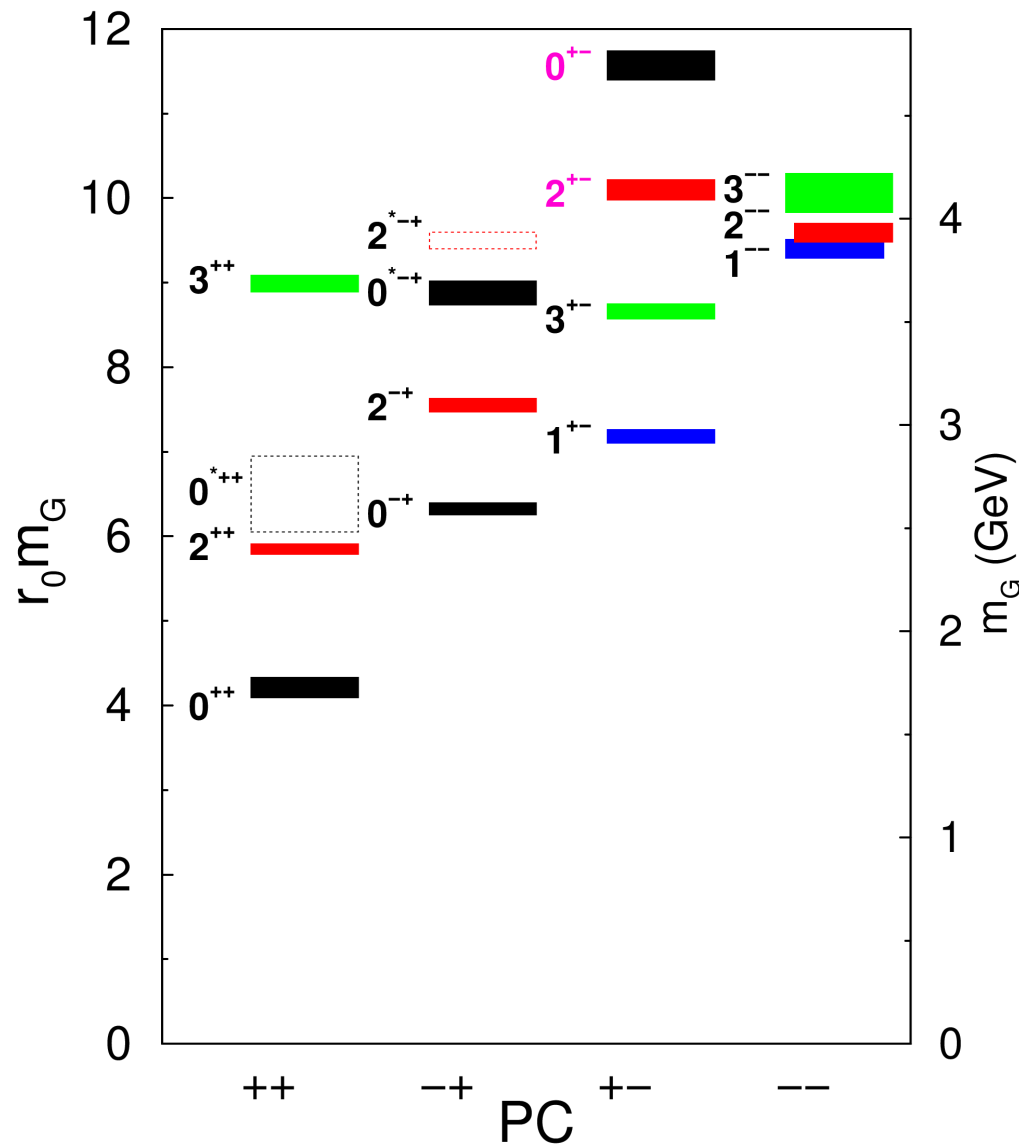
(soft) BRST breaking – scaling vs. decoupling

Practical questions:

Glueball spectroscopy

Glueballs

# Glueballs - Lattice



Quenched Spectrum:

[Bali, 1993],

[Morningstar, Peardon, 1999]

unquenched:

[Richards et al., 2010]

# Glueballs – other Methods

Coulomb gauge Hamiltonians

Potential models

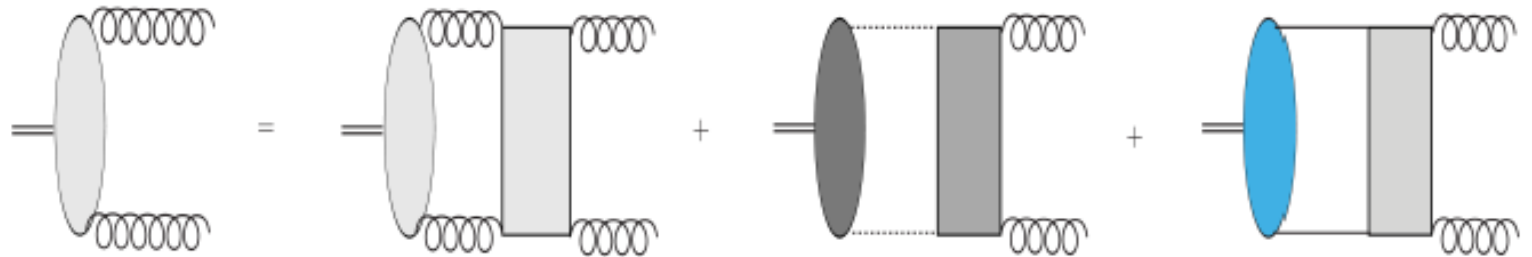
Light cone transverse Lattice

AdS/QCD

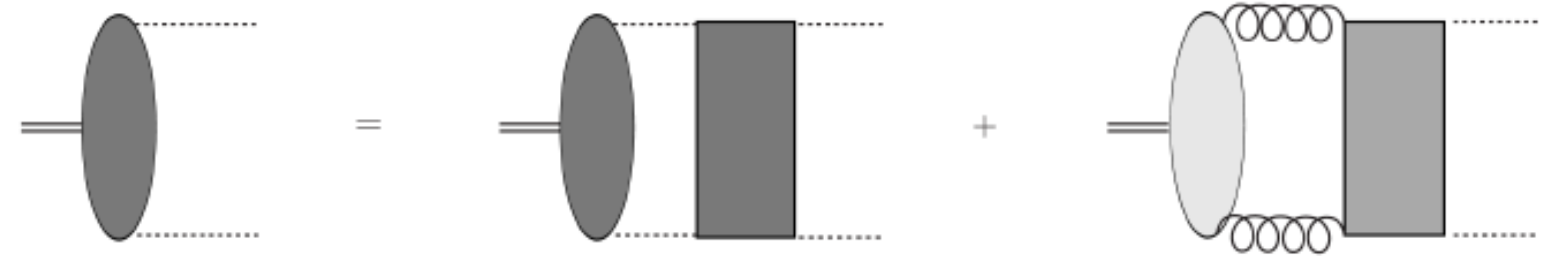
....

# Glueballs - Bethe Salpeter

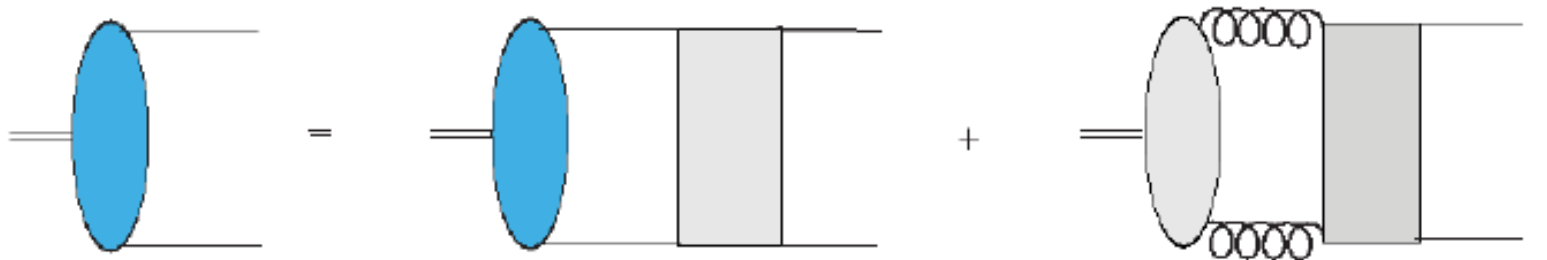
Glueball



Ghostball

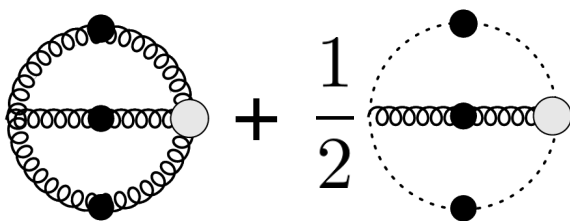


Meson



# 2PI Action

$$\Gamma[D, G] = \text{Tr} \ln D^{-1} + \text{Tr} D_0^{-1} D - 2\text{Tr} \ln G^{-1} - 2\text{Tr} G_0^{-1} G + \Gamma_2$$

$$\Gamma_2 = -\frac{1}{12} \text{Diagram 1} + \frac{1}{2} \text{Diagram 2}$$


Selfenergies and interaction kernels as  
functional derivatives

$$\text{DSE} \quad \Sigma = \frac{d\Gamma_2}{dD}$$

$$\text{BSE} \quad K = \frac{d^2\Gamma_2}{dD_i dD_j}$$





# Yang-Mills Dyson-Schwinger Equations

# Overview - DSEs

Full Green Function, Connected, 1PI Green Functions, ...

Dyson Schwinger Equations

Full

$$0 = \left( \frac{dS}{d\phi_i} \left[ \frac{d}{dJ} \right] + J_i \right) Z[J]$$

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Connected

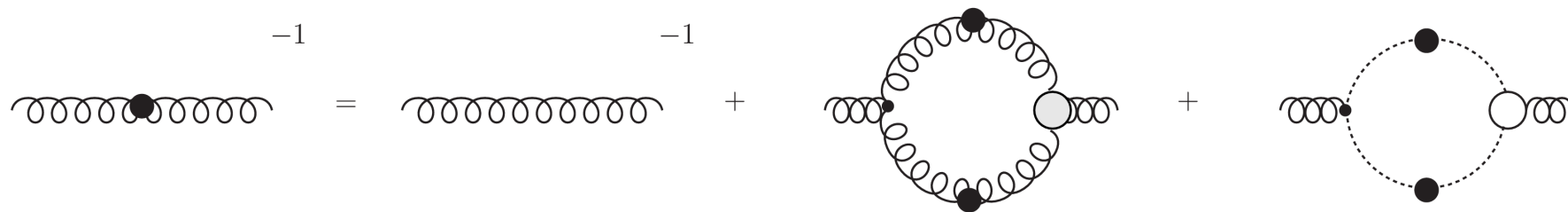
$$0 = \frac{dS}{d\phi_i} \left[ \frac{dW[J]}{dJ} + \frac{d}{dJ} \right] + J_i$$

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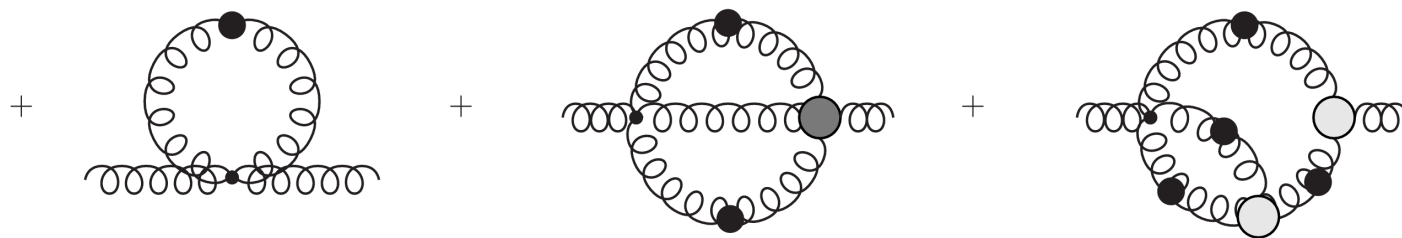
1 PI

$$\frac{d\Gamma[\phi]}{d\phi_i} = \frac{dS}{d\phi_i} \left[ \phi + W''[J] \frac{d}{d\phi} \right]$$

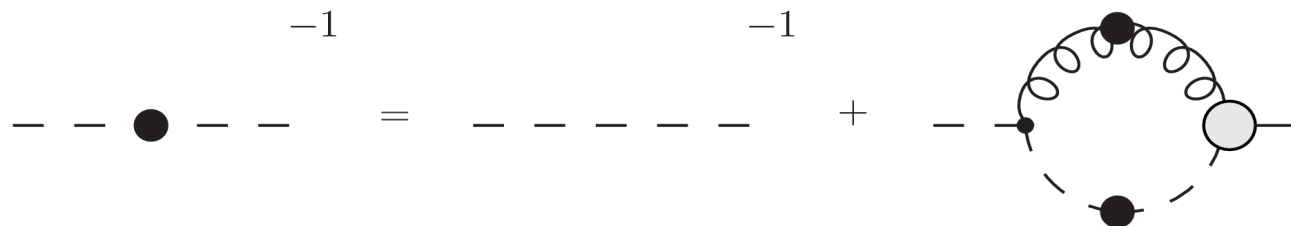
# Ghost Gluon System



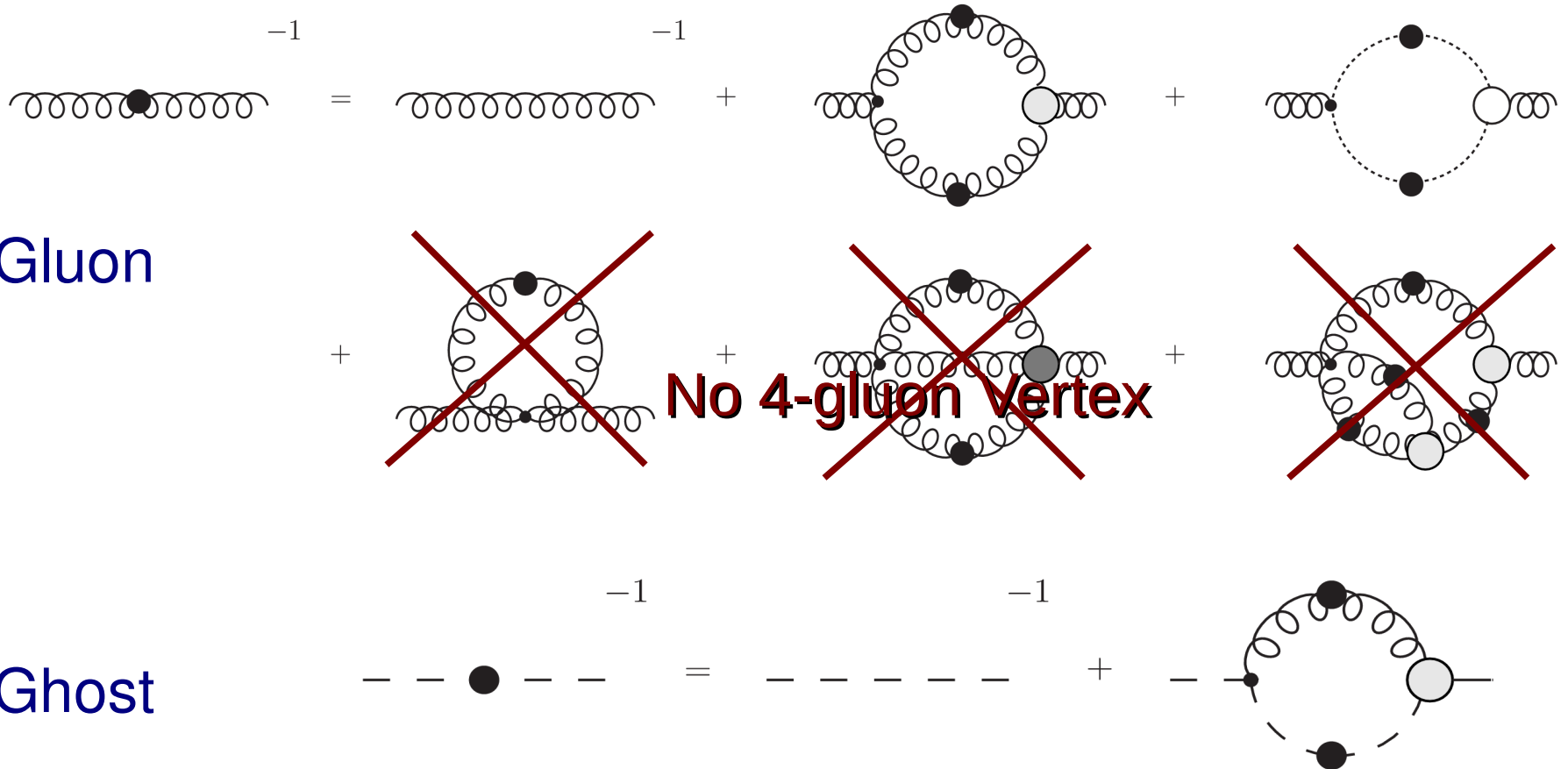
Gluon



Ghost

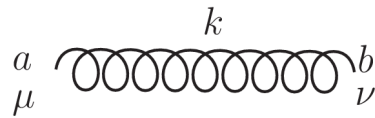


# Ghost Gluon System



# Ghost Gluon System

## Propagators

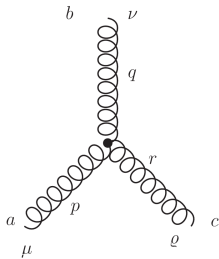


$$\frac{1}{k^2} \left[ \delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right] \delta_{ab}$$



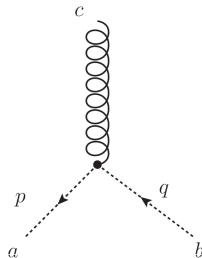
$$-\frac{1}{p^2} \delta_{ab}$$

## 3-Gluon Vertex



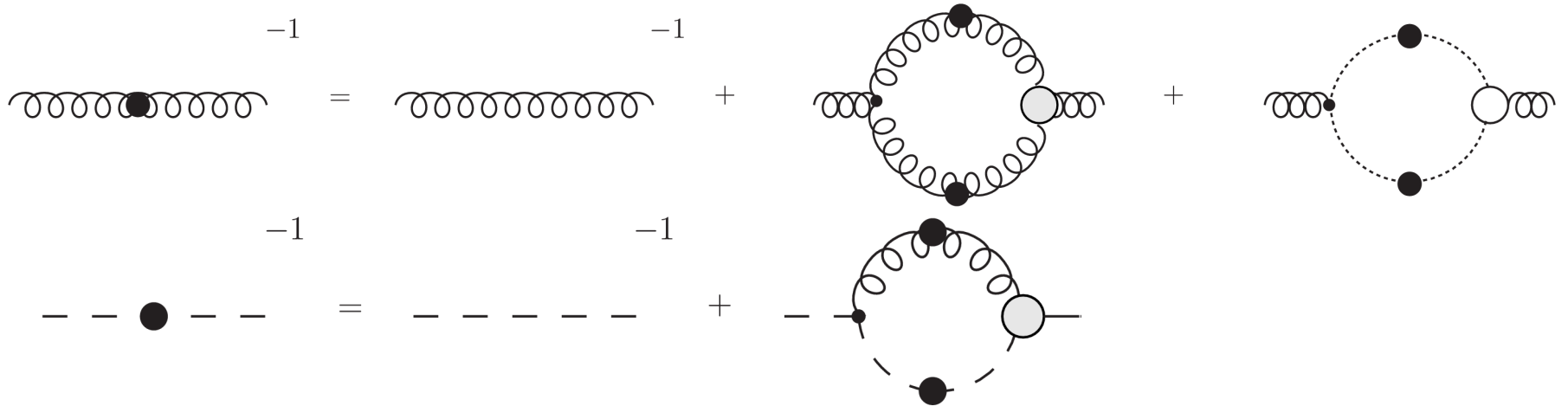
$$gf^{abc} [(r_\mu - q_\mu)g_{\nu\rho} + (p_\nu - r_\nu)g_{\mu\rho} + (q_\rho - p_\rho)g_{\mu\nu}]$$

## Ghost Gluon Vertex



$$igf^{abc} p_\mu$$

# Ghost Gluon System



**Gluon**

$$D_{\mu\nu}^{ab}(p^2) = \delta^{ab} \left( g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) \frac{Z(p^2)}{p^2}$$

**Ghost**

$$D^{ab}(p^2) = -\delta^{ab} \frac{G(p^2)}{p^2}$$

**Ghost Gluon Vertex**

$$\Gamma_\mu^{abc}(p, q, k) = \Gamma_\mu^0$$

**3 Gluon Vertex**

$$\Gamma_{\mu\nu\rho}^{abc}(p, q, k) = f^{abc} \tilde{\Gamma}_{\mu\nu\rho}(p, q, k)$$

# Vertex Ansatz

## 3-Gluon Ansatz

$$\tilde{\Gamma}_{\mu\nu\rho}(p, q) = \frac{1}{Z_1} \frac{G^{1-a/\delta-2a\delta}(p^2)}{Z^{1+a}(q^2)} \frac{G^{1-b/\delta-2b\delta}((p-q)^2)}{Z^{1+b}((p-q)^2)} \Gamma_{\mu\nu\rho}^0(p, q)$$

resummed perturbation theory in UV

$$G(x) = G(s) \left[ \omega \log \left( \frac{x}{s} \right) + 1 \right]^\delta$$

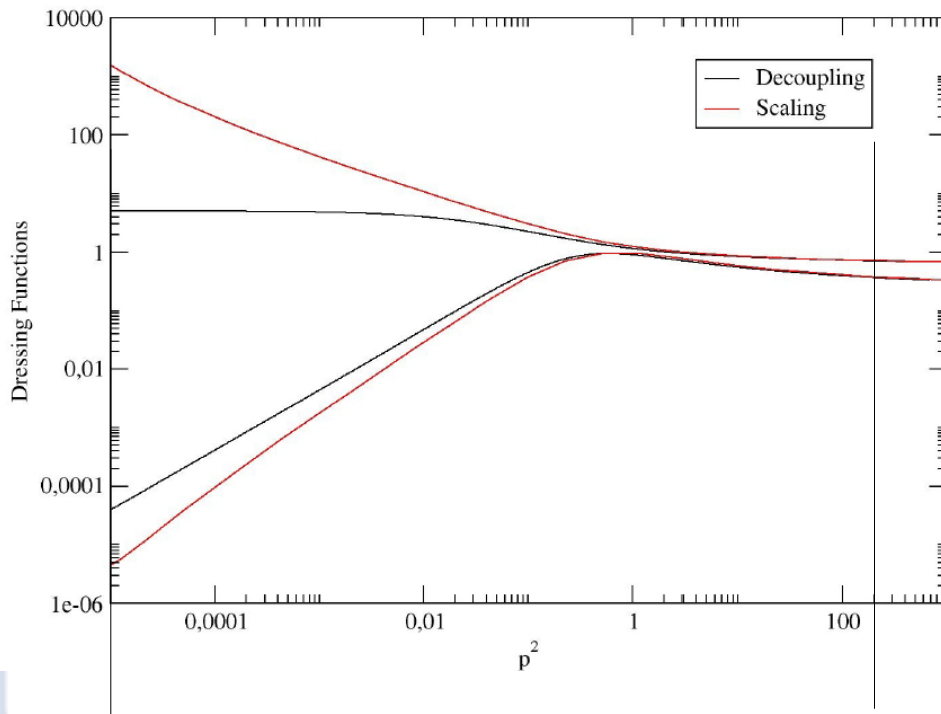
$$Z(x) = Z(s) \left[ \omega \log \left( \frac{x}{s} \right) + 1 \right]^\gamma$$

Equivalent to truncation with manifest transversality



# Euclidean Results

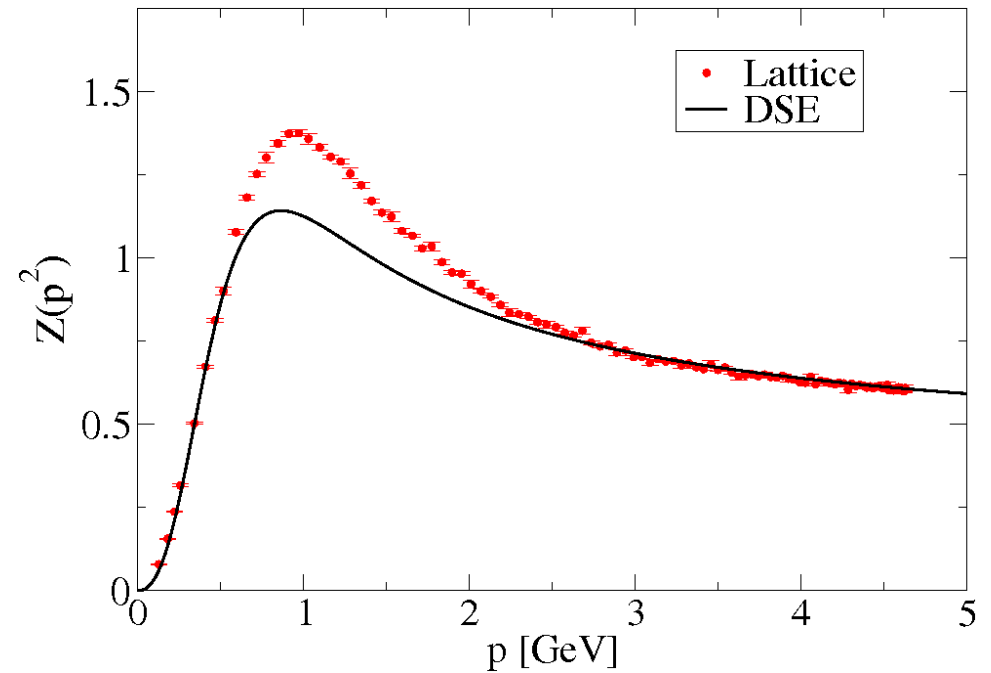
## Dyson Schwinger



Ghost

Gluon

## Dyson Schwinger vs Lattice



Sternbeck et al, PoSLAT2006, 076 (2006)  
Fischer, Maas, Pawlowski, Annals Phys. 324 (2009)

Sharp momentum cutoff + MOM scheme

# Collection of Vertices

Gluon Transversality

multiplicative Renormalizability

Bose symmetry of 3 Gluon Vertex

Additional information

Taylor's theorem for Ghost Gluon vertex

WTIs

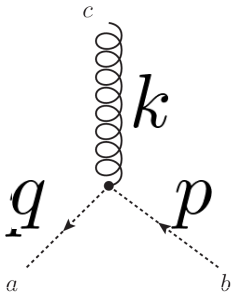
(resummed) Perturbation theory in UV

# Collection of Vertices

## Ghost-Gluon

$$igf^{abc} \left( q_\mu - k_\mu \frac{k \cdot q}{k^2} F_{IR} \right)$$

$$igf^{abc} \left( q_\mu - p_\mu \frac{k \cdot q}{k^2} F_{IR} \right)$$



## 3-Gluon

$$\Gamma_{\mu\nu\rho}^{(0)} \times \frac{1}{3} \left( \frac{G(k^2)}{Z(k^2)} + \frac{G(p^2)}{Z(p^2)} + \frac{G(q^2)}{Z(q^2)} \right)$$

$$\frac{1}{2} \left( \frac{G(q^2)}{Z(p^2)} + \frac{G(q^2)}{Z(k^2)} \right) g_{\mu\nu} (k - p)_\rho$$

+ cyclic permutations

$$\Gamma_{\mu\nu\rho}^{(0)} \times \frac{G^a(p^2 + q^2 + k^2 + c_{IR})}{Z^b(p^2 + q^2 + k^2 + c_{IR})}$$

# Complex Propagators

# Model Gluon Propagators

IR Gluon

Gribov-type propagators

$$D(p^2) = C \frac{p^2}{p^4 + b^2} \quad \rightarrow \text{purely imaginary poles}$$

Gribov-Stingl

$$D(p^2) = C \frac{p^2 + d}{(p^2 + a)^2 + b^2} \quad \rightarrow \text{complex conjugated poles}$$

Refined Gribov-Zwanziger

$$D(p^2) = C \frac{(p^2 + d)(p^2 + l)}{(p^4 + u^2 p^2 + t^2)(p^2 + v)} \quad \rightarrow \text{pairs of c.c. poles and real ones}$$

# Model Gluon Propagators

Gluon Propagator whole momentum range  
DSE fit propagator

$$(i) \quad \omega_I \frac{(p^2)^{2\kappa}}{(\Lambda_I^2)^{2\kappa} + (p^2)^{2\kappa}} \alpha^{-\gamma} (p^2)$$

$$(ii) \quad \omega_{II} \frac{(p^2)^{2\kappa}}{(\Lambda_{II}^2 + p^2)^{2\kappa}} \alpha^{-\gamma} (p^2)$$

→ branch cuts with singularities

# Analytic Continuation

Double  $\rightarrow$  complex<double>

branch cuts in kinematical kernels

complex Vertices – rational powers in 3-gluon vertex

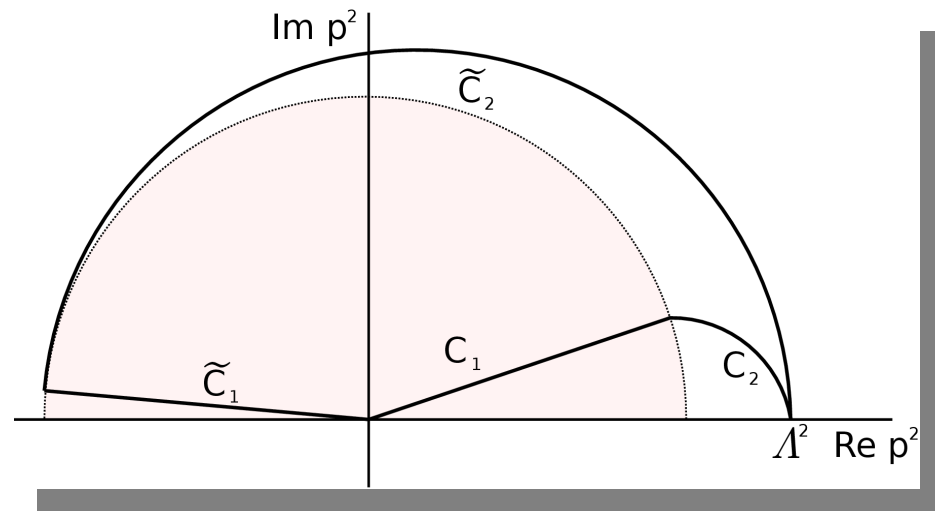
# Contour Method

Angular and momentum integration

$\frac{\sin^2 \theta}{z}$  term  $\rightarrow$  circular branch cuts

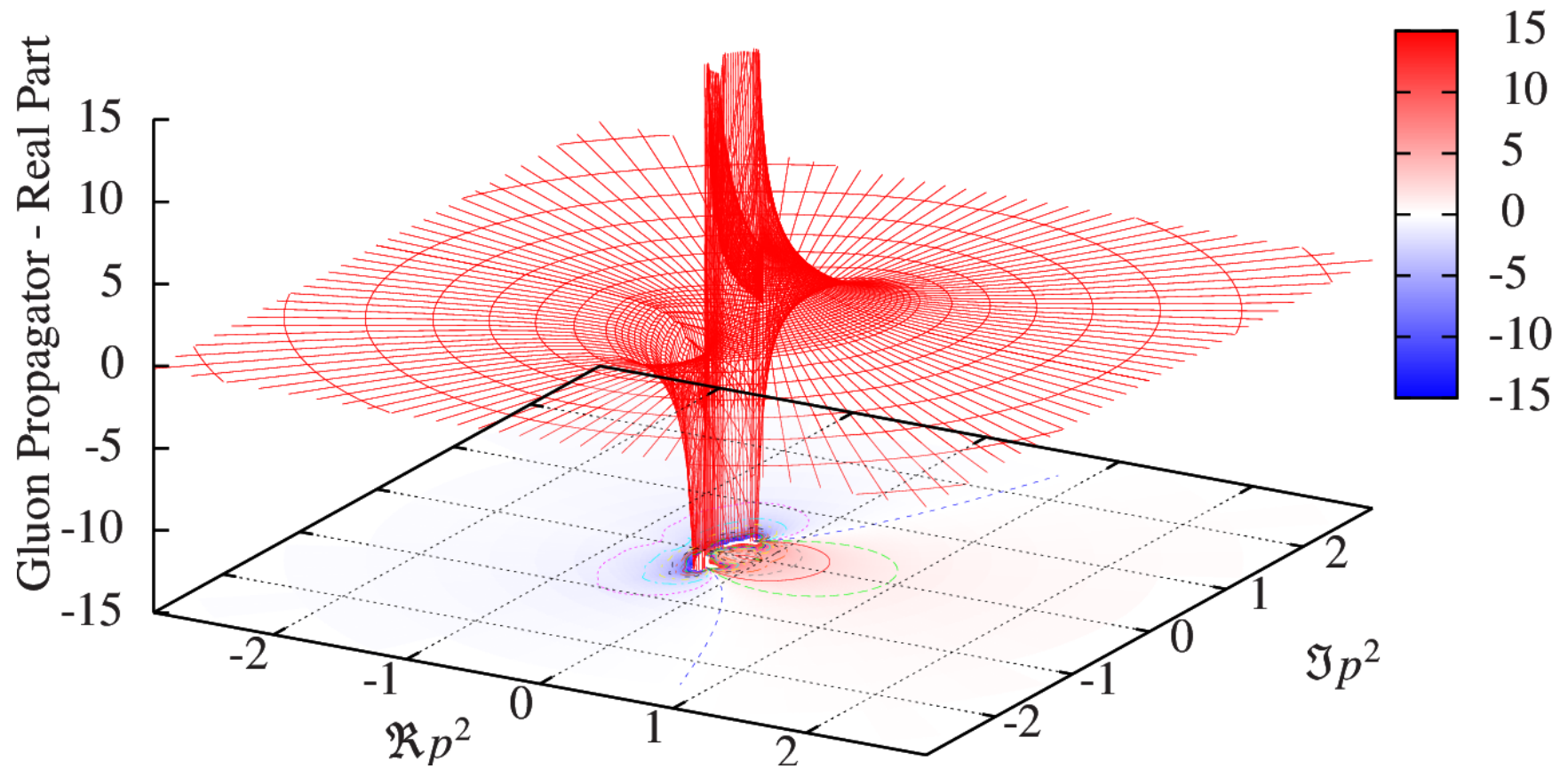
with  $z = p^2 + q^2 - 2\sqrt{p^2 q^2} \cos \theta$

All momenta  
complex

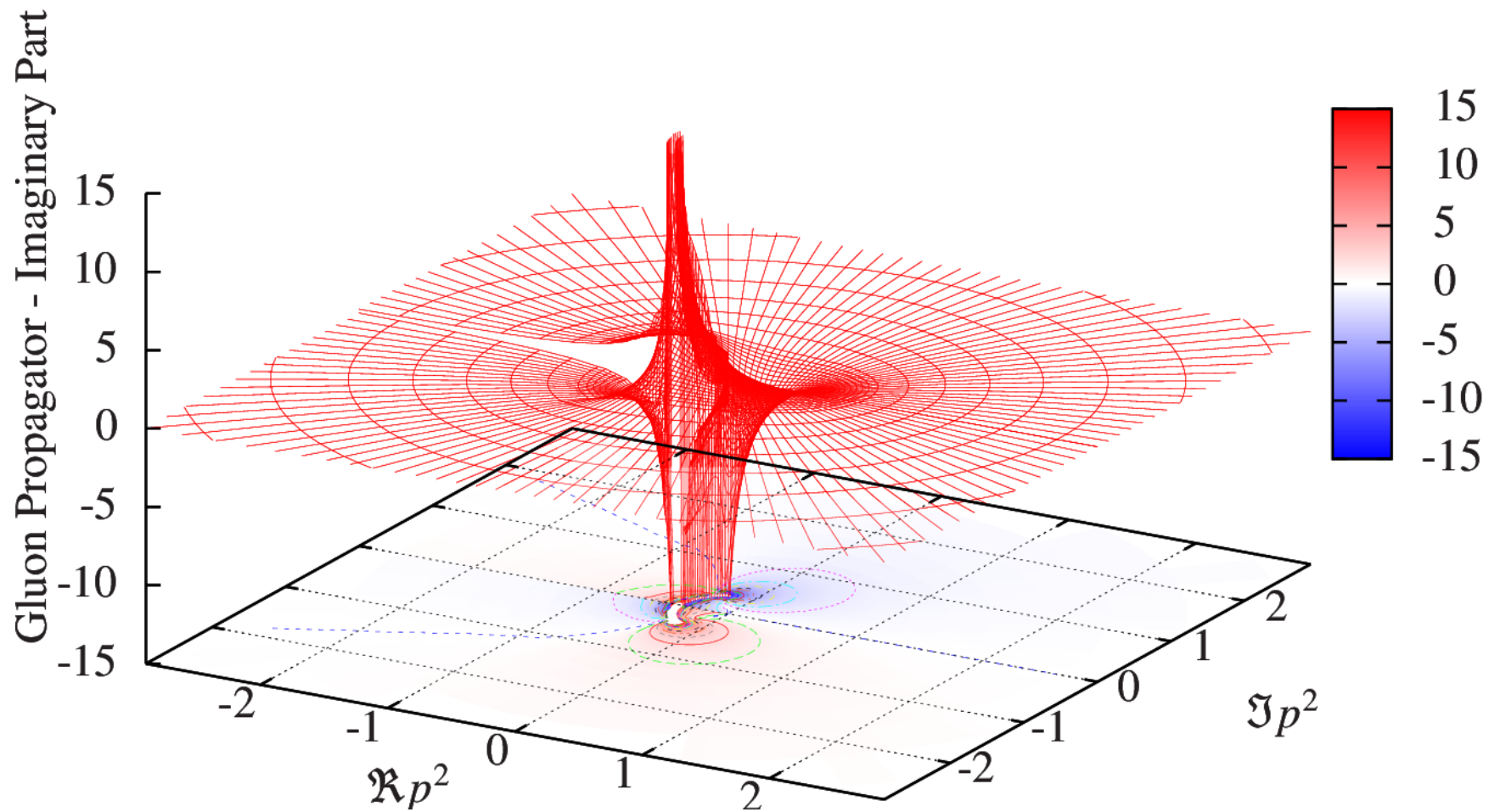




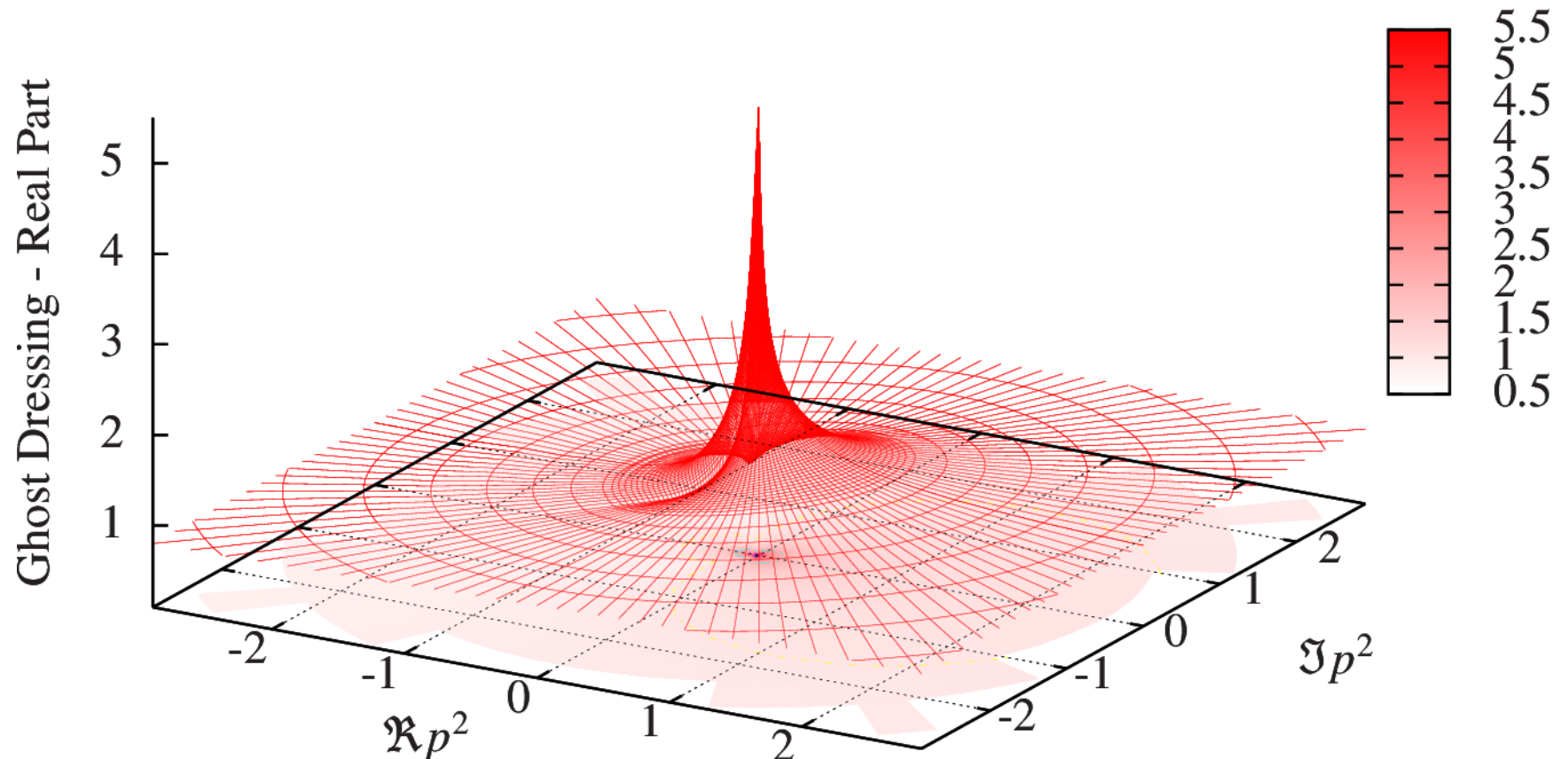
# Complex Structure Gluon



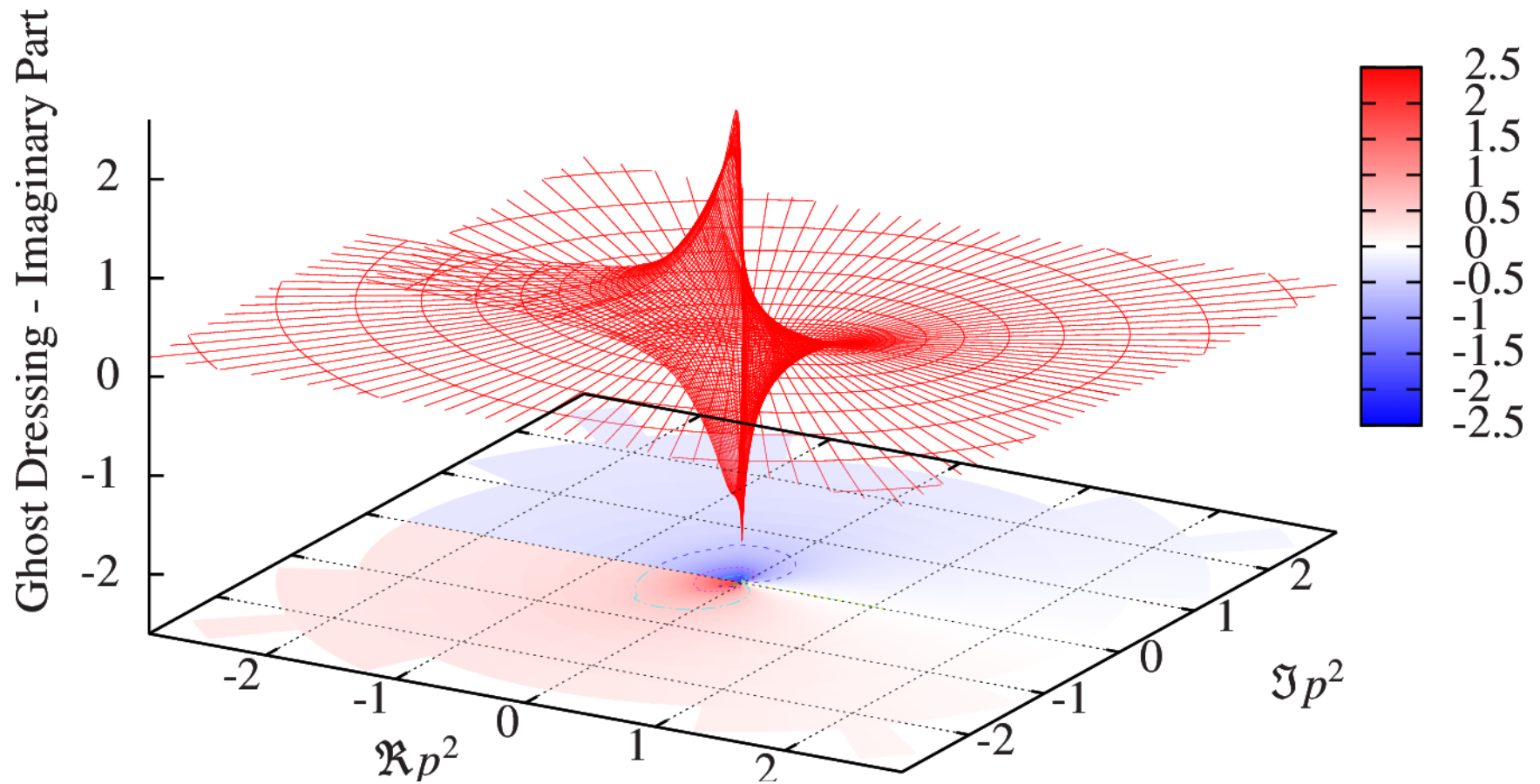
# Complex Structure Gluon



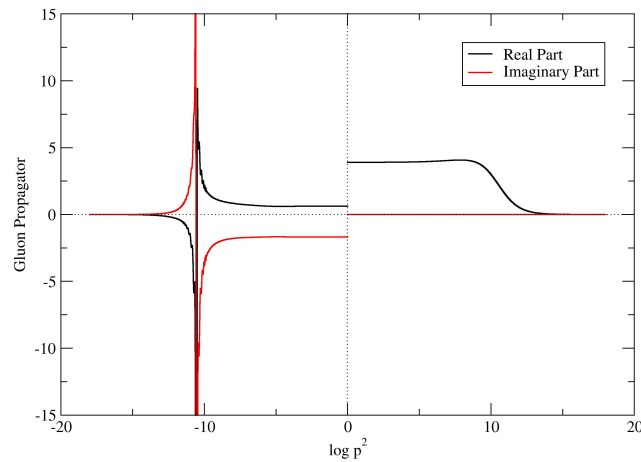
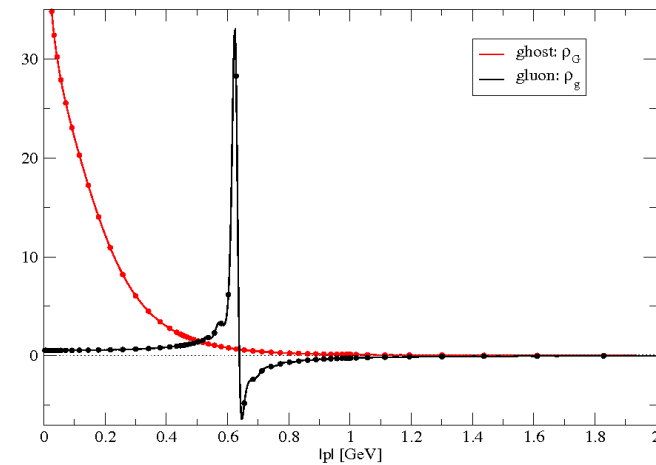
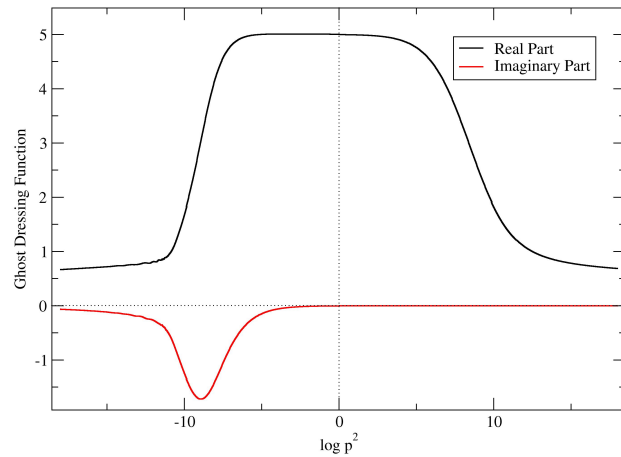
# Complex Structure Ghost



# Complex Structure Ghost



# Spectral Functions



Standard dispersion  
relation  $\rho(p^2) = -\Im D(p^2)/\pi$

Quasi Gluon

$$600 \text{ MeV} < m_g < 700 \text{ MeV}$$

Direct positivity  
violations

# Conclusions

Complex decoupling solution and bose symmetric vertex

Positivity violating spectral functions

No complex poles on first sheet

Branch cuts

# Perspectives

Complex scaling solution

different 3-Gluon and Ghost-Gluon Vertex constructions

quenched Glueball spectra and properties

coupling to Quarks