Непертурбативные проявления аксиальной аномалии в процессах квантовой хромодинамики

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Содержание

Axial anomaly

In QCD, for a given flavor q, the divergence of the axial current $J_{\mu 5}^{(q)} = \bar{q}\gamma_\mu\gamma_5 q$ acquires both electromagnetic and gluonic anomalous terms:

$$\partial_\mu J_{\mu 5}^{(q)} = m_q \bar{q}\gamma_5 q + \frac{e^2}{8\pi^2} e_q^2 N_c F\tilde{F} + \frac{\alpha_s}{4\pi} N_c G\tilde{G}, \tag{1}$$

An octet of axial currents

$$J_{\mu 5}^{(a)} = \sum_q \bar{q}\gamma_5\gamma_\mu \frac{\lambda^a}{\sqrt{2}} q$$

Singlet axial current $J_{\mu 5}^{(0)} = \frac{1}{\sqrt{3}}(\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d + \bar{s}\gamma_\mu\gamma_5 s)$:

$$\partial^\mu J_{\mu 5}^{(0)} = \frac{1}{\sqrt{3}}(m_u \overline{u}\gamma_5 u + m_d \overline{d}\gamma_5 d + m_s \overline{s}\gamma_5 s) + \frac{\alpha_{em}}{2\pi} C^{(0)} N_c F\tilde{F} + \frac{\sqrt{3}\alpha_s}{4\pi} N_c G\widetilde{G}, \tag{2}$$

The diagonal components of the octet of axial currents
$J_{\mu 5}^{(3)} = \frac{1}{\sqrt{2}}(\bar{u}\gamma_\mu\gamma_5 u - \bar{d}\gamma_\mu\gamma_5 d)$,
$J_{\mu 5}^{(8)} = \frac{1}{\sqrt{6}}(\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d - 2\bar{s}\gamma_\mu\gamma_5 s)$
acquire an electromagnetic anomalous term only:

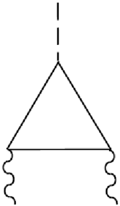
$$\partial^\mu J_{\mu 5}^{(3)} = \frac{1}{\sqrt{2}}(m_u\overline{u}\gamma_5 u - m_d\overline{d}\gamma_5 d) + \frac{\alpha_{em}}{2\pi}C^{(3)}N_c F\tilde{F}, \quad (3)$$

$$\partial^\mu J_{\mu 5}^{(8)} = \frac{1}{\sqrt{6}}(m_u\overline{u}\gamma_5 u + m_d\overline{d}\gamma_5 d - 2m_s\overline{s}\gamma_5 s) + \frac{\alpha_{em}}{2\pi}C^{(8)}N_c F\tilde{F}. \quad (4)$$

The electromagnetic charge factors $C^{(a)}$ are

$$\begin{aligned} C^{(3)} &= \frac{1}{\sqrt{2}}(e_u^2 - e_d^2) = \frac{1}{3\sqrt{2}}, \\ C^{(8)} &= \frac{1}{\sqrt{6}}(e_u^2 + e_d^2 - 2e_s^2) = \frac{1}{3\sqrt{6}}, \\ C^{(0)} &= \frac{1}{\sqrt{3}}(e_u^2 + e_d^2 + e_s^2) = \frac{2}{3\sqrt{3}}. \end{aligned} \quad (5)$$

Anomaly sum rule



The matrix element for the transition of the axial current $J_{\alpha 5}$ with momentum $p = k + q$ into two real or virtual photons with momenta k and q is:

$$T_{\alpha\mu\nu}(k,q) = \int d^4x d^4y e^{(ikx+iqy)} \langle 0| T\{J_{\alpha 5}(0) J_\mu(x) J_\nu(y)\}|0\rangle; \quad (6)$$

Kinematics:

$$k^2 = 0, Q^2 = -q^2$$

The VVA triangle graph amplitude can be presented as a tensor decomposition

$$
\begin{aligned}
T_{\alpha\mu\nu}(k,q) &= F_1\ \varepsilon_{\alpha\mu\nu\rho}k^\rho + F_2\ \varepsilon_{\alpha\mu\nu\rho}q^\rho \\
&+\ F_3\ k_\nu\varepsilon_{\alpha\mu\rho\sigma}k^\rho q^\sigma + F_4\ q_\nu\varepsilon_{\alpha\mu\rho\sigma}k^\rho q^\sigma \\
&+\ F_5\ k_\mu\varepsilon_{\alpha\nu\rho\sigma}k^\rho q^\sigma + F_6\ q_\mu\varepsilon_{\alpha\nu\rho\sigma}k^\rho q^\sigma,
\end{aligned} \tag{7}
$$

$F_j = F_j(p^2, k^2, q^2; m^2),\ p = k + q.$
Dispersive approach to axial anomaly leads to [Hořejší'85;Hořejší,Teryaev'95]:

$$
\int_{4m^2}^{\infty} A_3(s, Q^2; m^2)ds = \frac{1}{2\pi}N_c C^{(a)} \quad , a = 3, 8; \tag{8}
$$

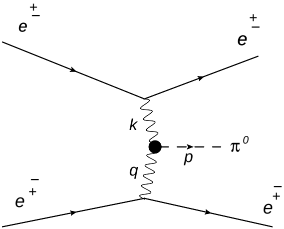
$$
A_3 \equiv \frac{1}{2}Im(F_3 - F_6),\ N_c = 3;
$$

$$
\begin{aligned}
C^{(3)} &= \tfrac{1}{\sqrt{2}}(e_u^2 - e_d^2) = \tfrac{1}{3\sqrt{2}}, \\
C^{(8)} &= \tfrac{1}{\sqrt{6}}(e_u^2 + e_d^2 - 2e_s^2) = \tfrac{1}{3\sqrt{6}},
\end{aligned} \tag{9}
$$

$$\int_{4m^2}^{\infty} A_3(s, Q^2; m^2) ds = \frac{1}{2\pi} N_c C^{(a)} \tag{10}$$

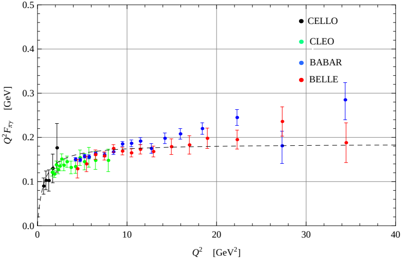
- Holds for any Q^2 and any m^2.
- It has neither α_s corrections (Adler-Bardeen theorem) nor nonperturbative corrections (t'Hooft's consistency principle).
- Exact nonperturbative relation – powerful tool.

Transition form factors



π^0 TFF: theoretical and experimental status

Pion transition form factor: available data



- The current experimental status of the pion transition form factor (TFF) $F_{\pi\gamma}$ is rather controversial.
- The measurements of the BABAR collaboration [Aubert et al. '09] show a steady rise of $Q^2 F_{\pi\gamma}$, surpassing the pQCD predicted asymptote $Q^2 F_{\pi\gamma} \to \sqrt{2} f_\pi$, $f_\pi = 130.7$ MeV at $Q^2 \simeq 10$ GeV^2 and questioning the collinear factorization.

π^0 TFF: theoretical and experimental status

- On the other hand, the recent BELLE data [Uehara et al. '12] do not show such striking behavior: although $Q^2 F_{\pi\gamma}$ reaches the pQCD asymptotic value, it does not manifest a further growth.
- The dispersive approach to axial anomaly allows to derive the *anomaly sum rule* (ASR) providing a handy tool to study the transition form factors even beyond the factorization hypothesis.
- In order to describe the BABAR data, ASR requires a new nonperturbative correction to the spectral density. This correction is absent in the local OPE and possibly originates from instantons or short strings [Chetyrkin,Zakharov '98].

ASR and meson contributions

Saturating the l.h.s. of the 3-point correlation function (6) with the resonances and singling out their contributions to ASR (10) we get the (infinite) sum of resonances with appropriate quantum numbers:

$$\pi \sum f_M^a F_{M\gamma} = \int_{4m^2}^{\infty} A_3(s, Q^2; m^2) ds = \frac{1}{2\pi} N_c C^{(a)}, \tag{11}$$

where the coupling (decay) constants f_M^a:

$$\langle 0 | J_{\alpha 5}^{(a)}(0) | M(p) \rangle = i p_\alpha f_M^a \ , \tag{12}$$

and form factors $F_{M\gamma}$ of the transitions $\gamma\gamma^* \to M$ are:

$$\int d^4 x e^{ikx} \langle M(p) | T\{J_\mu(x) J_\nu(0)\} | 0 \rangle = \epsilon_{\mu\nu\rho\sigma} k^\rho q^\sigma F_{M\gamma} \tag{13}$$

- Sum of finite number of resonances decreasing $F_{M\gamma}^{\mathrm{asymp}}(Q^2) \propto \frac{f_M}{Q^2}$- infinite number of states are needed to saturate ASR (collective effect). [Y.K.,A.Oganesian,O.Teryaev'10]

Isovector channel: π^0

$J^{(3)}_{\mu 5} = \frac{1}{\sqrt{2}}(\bar{u}\gamma_\mu\gamma_5 u - \bar{d}\gamma_\mu\gamma_5 d)$, $C^{(3)} = \frac{1}{\sqrt{2}}(e_u^2 - e_d^2) = \frac{1}{3\sqrt{2}}$.
For practical purposes let's use QHD and describe the higher resonances by continuum.

- π^0 + higher contributions ("continuum"):

$$\pi f_\pi F_{\pi\gamma}(Q^2) + \int_{s_0}^{\infty} A_3(s, Q^2; m^2) = \frac{1}{2\pi} N_c C^{(3)}. \tag{14}$$

The spectral density $A_3(s, Q^2; m^2)$ can be calculated from VVA triangle diagram:

$$A_3(s, Q^2; m^2) = \frac{1}{2\sqrt{2}\pi} \frac{1}{(Q^2 + s)^2} \left(Q^2 R + 2m^2 \ln \frac{1+R}{1-R} \right), \tag{15}$$

where $R(s, m) = \sqrt{1 - \frac{4m^2}{s}}$, m is a mass of quark. Then the pion TFF:

$$F_{\pi\gamma}(Q^2) = \frac{1}{2\sqrt{2}\pi^2 f_\pi} \frac{s_0}{s_0 + Q^2} (R_0 - \frac{2m^2}{s_0} \ln \frac{1+R_0}{1-R_0}), \tag{16}$$

$R_0 = R(s_0, m)$.

- $m = 0$:

$$F_{\pi\gamma}(Q^2) = \frac{1}{2\sqrt{2}\pi^2 f_\pi} \frac{s_0}{s_0 + Q^2} \tag{17}$$

Considering the limit $Q^2 \to \infty$ and relying the QCD factorization prediction for $Q^2 F_{\pi\gamma} = \sqrt{2} f_\pi$ $(\phi^{as}(x) = 6x(1-x))$

$$s_0 = 4\pi^2 f_\pi^2 = 0.67 GeV^2$$

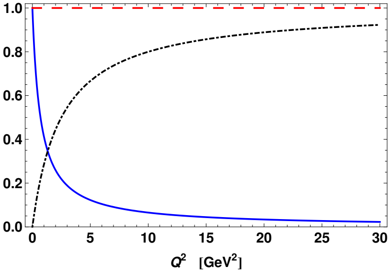
– fits perfectly the value extracted from SVZ (two-point) QCD sum rules $s_0 = 0.7 GeV^2$ [Shifman, Vainshtein, Zakharov'79].

– reproduces BL interpolation formula[Brodsky, Lepage'81]:

$$F_{\pi\gamma}^{\mathrm{BL}}(Q^2) = \frac{1}{2\sqrt{2}\pi^2 f_\pi} \frac{1}{1 + Q^2/\left(4\pi^2 f_\pi^2\right)}. \tag{18}$$

Derived from QHD [Radyushkin'96], now we related it to anomaly at all Q^2.

Corrections interplay



- The full integral is exact

$$\frac{1}{2\sqrt{2}\pi} = \int_0^\infty A_3(s, Q^2)ds = I_\pi + I_{cont}$$

- The continuum contribution $I_{cont} = \int_{s_0}^\infty A_3(s, Q^2)ds$ may have perturbative as well as power corrections.
- $\delta I_\pi = -\delta I_{cont}$: small relative correction to continuum – due to exactness of ASR – must be compensated by large relative correction to the pion contribution!

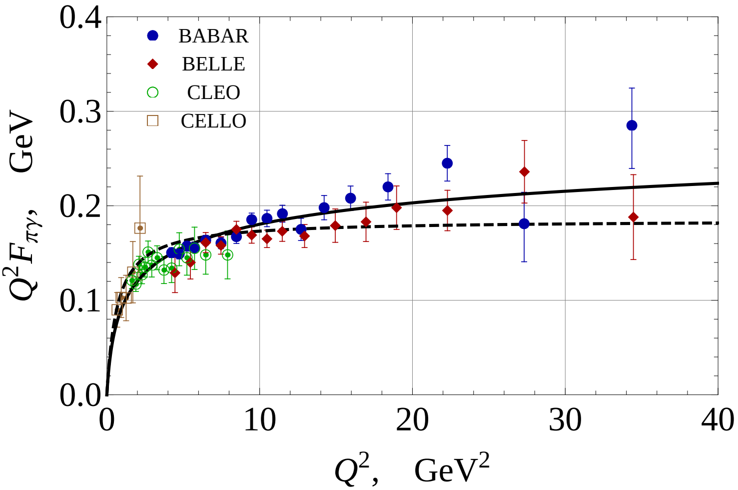
Possible corrections to A_3

- Perturbative two-loop corrections to spectral density A_3 are zero [Jegerlehner&Tarasov'06]
- Nonperturbative corrections to A_3 are possible: vacuum condensates, instantons, short strings.
- General requirements for the correction $\delta I = \int_{s_0}^{\infty} \delta A_3(s, Q^2) ds$:
  $\delta I = 0$
  - at $s_0 \to \infty$ (the continuum contribution vanishes),
  - at $s_0 \to 0$ (the full integral has no corrections),
  - at $Q^2 \to \infty$ (the perturbative theory works at large Q^2),
  - at $Q^2 \to 0$ (anomaly perfectly describes pion decay width).

$$\delta I = \frac{1}{2\sqrt{2}\pi} \frac{\lambda s_0 Q^2}{(s_0 + Q^2)^2} (\ln \frac{Q^2}{s_0} + \sigma), \tag{19}$$

$$\delta F_{\pi\gamma} = \frac{1}{\pi f_\pi} \delta I_\pi = \frac{1}{2\sqrt{2}\pi^2 f_\pi} \frac{\lambda s_0 Q^2}{(s_0 + Q^2)^2} (\ln \frac{Q^2}{s_0} + \sigma). \tag{20}$$

Correction vs. experimental data



CELLO+CLEO+BABAR: $\lambda = 0.14$, $\sigma = -2.36$, $\chi^2/n.d.f. = 0.94$

	$\frac{\chi^2}{n.d.f.}$	$\delta I \neq 0: \frac{\chi^2}{n.d.f.}$	λ	σ
CELLO+CLEO+BABAR+Belle	1.86	0.91	0.12	-2.50
CELLO+CLEO+Belle	1.01	0.46	0.07	-3.03
CELLO+CLEO+BABAR	2.29	0.94	0.14	-2.36
BABAR	3.61	0.99	0.20	-2.39
Belle	0.80	0.40	0.14	-2.86

- Although the BELLE data themselves may be described without the correction, but they do not also exclude its possibility. Unless the BABAR data will be disproved, the need for correction remains.

Octet channel (η,η')

$$J^{(8)}_{\alpha 5} = \frac{1}{\sqrt{6}}(\bar{u}\gamma_\alpha\gamma_5 u + \bar{d}\gamma_\alpha\gamma_5 d - 2\bar{s}\gamma_\alpha\gamma_5 s),$$

$$\int_{4m^2}^{\infty} A_3(s, Q^2; m^2)ds = \frac{1}{2\pi} N_c C^{(8)}, \tag{21}$$

$$C^{(8)} \equiv \frac{1}{\sqrt{6}}(e_u^2 + e_d^2 - 2e_s^2) = \frac{1}{3\sqrt{6}}$$

ASR in the octet channel:

$$f^8_\eta F_{\eta\gamma}(Q^2) + f^8_{\eta'} F_{\eta'\gamma}(Q^2) = \frac{1}{2\sqrt{6}\pi^2}\frac{s_0}{s_0 + Q^2} \tag{22}$$

- Significant mixing.
- η' decays into two real photons, so it should be taken into account explicitly along with η meson.

Large Q^2

[Anisovich,Melikhov,Nikonov'97; Feldmann,Kroll'98]

$$Q^2 F^{as}_{\eta\gamma} = 2(C^{(8)} f^8_\eta + C^{(0)} f^0_\eta) \int_0^1 \frac{\phi^{as}(x)}{x} dx, \tag{23}$$

$$Q^2 F^{as}_{\eta'\gamma} = 2(C^{(8)} f^8_{\eta'} + C^{(0)} f^0_{\eta'}) \int_0^1 \frac{\phi^{as}(x)}{x} dx, \tag{24}$$

$\phi^{as}(x) = 6x(1-x)$. Then ASR at $Q^2 \to \infty$:

$$4\pi^2((f^8_\eta)^2 + (f^8_{\eta'})^2 + 2\sqrt{2}[f^8_\eta f^0_\eta + f^8_{\eta'} f^0_{\eta'}]) = s_0. \tag{25}$$

$Q^2 = 0$

ASR takes the form:

$$f_\eta^8 F_{\eta\gamma}(0) + f_{\eta'}^8 F_{\eta'\gamma}(0) = \frac{1}{2\sqrt{6}\pi^2}, \qquad (26)$$

where

$$F_{M\gamma}(0) = \sqrt{\frac{4\Gamma_{M\to\gamma\gamma}}{\pi\alpha^2 m_M^3}}.$$

Mixing

PCAC relation for π^0

$$\partial_\mu J^{(3)}_{\mu 5} = f^{(3)}_\pi m^2_\pi \phi_\pi . \tag{27}$$

Generalization to the mixing system $\eta - \eta' - ...$:

[cf. Ioffe '79]

$$\partial_\mu \mathbf{J}_{\mu\mathbf{5}} = \mathbf{FM\Phi}, \tag{28}$$

where :

$$\mathbf{J}_{\mu\mathbf{5}} \equiv \begin{pmatrix} J^\alpha_{\mu 5} \\ J^\beta_{\mu 5} \end{pmatrix}, \mathbf{F} \equiv \begin{pmatrix} f^\alpha_\eta & f^\alpha_{\eta'} & f^\alpha_G & \cdots \\ f^\beta_\eta & f^\beta_{\eta'} & f^\beta_G & \cdots \end{pmatrix}, \mathbf{\Phi} \equiv \begin{pmatrix} \phi_\eta \\ \phi_{\eta'} \\ \phi_G \\ \vdots \end{pmatrix},$$

$$\mathbf{M} \equiv diag(m^2_\eta, m^2_{\eta'}, m^2_G, \cdots). \tag{29}$$

$$\langle 0 | J^{(a)}_{\alpha 5}(0) | M(p) \rangle = i p_\alpha f^a_M \ .$$

Mixing

Octet-singlet basis (of currents):

$$J^{(8)}_{\mu 5} = \frac{1}{\sqrt{6}}(\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d - 2\bar{s}\gamma_\mu\gamma_5 s),\ J^{(0)}_{\mu 5} = \frac{1}{\sqrt{3}}(\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d + \bar{s}\gamma_\mu\gamma_5 s). \tag{30}$$

For quark-flavor basis one explores the definitions of axial currents with decoupled light and strange quark composition:

$$J^q_{\mu 5} = \frac{1}{\sqrt{2}}(\bar{u}\gamma_\alpha\gamma_5 u + \bar{d}\gamma_\alpha\gamma_5 d),\ \ J^s_{\mu 5} = \bar{s}\gamma_\alpha\gamma_5 s, \tag{31}$$

$$\begin{pmatrix} J^8_{\mu 5} \\ J^0_{\mu 5} \end{pmatrix} = \mathbf{V}(\alpha)\begin{pmatrix} J^q_{\mu 5} \\ J^s_{\mu 5} \end{pmatrix},\ \mathbf{V}(\alpha) = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}, \tag{32}$$

where $\tan\alpha = \sqrt{2}$.

$$\widetilde{\mathbf{\Phi}} = \mathbf{U\Phi}, \quad \widetilde{\mathbf{\Phi}} \equiv \begin{pmatrix} \tilde{\phi}_\alpha \\ \tilde{\phi}_\beta \\ \tilde{\phi}_G \\ \vdots \end{pmatrix}. \qquad (33)$$

In terms of these fields, Eq. (28) can be rewritten as

$$\partial_\mu \mathbf{J}_{\mu 5} = \widetilde{\mathbf{F}}\widetilde{\mathbf{M}}\widetilde{\mathbf{\Phi}}, \qquad (34)$$

where $\widetilde{\mathbf{F}} = \mathbf{FU}$, $\widetilde{\mathbf{M}} = \mathbf{U^T M U}$.
In our notations the octet-singlet (quark-flavor) mixing scheme implies that the matrix $\widetilde{\mathbf{F}}$ has a (rectangular) diagonal form in the respective octet-singlet (quark-flavor) basis,

$$\widetilde{\mathbf{F}} = \begin{pmatrix} f_\alpha & 0 & 0 & \dots \\ 0 & f_\beta & 0 & \dots \end{pmatrix}. \qquad (35)$$

FU has a (rectangular) diagonal form (35) immediately follows that $\mathbf{FF^T}$ is a diagonal matrix.

So, imposing the mixing scheme is equivalent to imposing the constraint for the decay constants:

$$f_\eta^\alpha f_\eta^\beta + f_{\eta'}^\alpha f_{\eta'}^\beta + f_G^\alpha f_G^\beta + ... = 0. \tag{36}$$

- *Octet-singlet (SU(3)) mixing scheme*: $f_\eta^8 f_\eta^0 + f_{\eta'}^8 f_{\eta'}^0 = 0.$

$$\mathbf{F} = \left(\begin{array}{cc} f_8 \cos\theta & f_8 \sin\theta \\ -f_0 \sin\theta & f_0 \cos\theta \end{array} \right). \tag{37}$$

- *Quark-flavour mixing scheme*: [Feldmann,Kroll,Stech'97]

$$f_\eta^q f_\eta^s + f_{\eta'}^q f_{\eta'}^s = 0.$$

$$\mathbf{F_{qs}} = \left(\begin{array}{cc} f_q \cos\phi & f_q \sin\phi \\ -f_s \sin\phi & f_s \cos\phi \end{array} \right). \tag{38}$$

Duality: 2-point vs. 3-point correlator

The interplay of two- and three-point correlators was investigated for the case of isovector current and pion state in [Radyushkin'95] and the duality interval was expressed in terms of pion decay constant $f_\pi = 0.13\ GeV$:

$$s_3^\pi = 4\pi^2 f_\pi^2. \tag{39}$$

"$\eta + continuum$":

$$s_8^\eta = 4\pi^2 (f_\eta^8)^2. \tag{40}$$

"$\eta + \eta' + continuum$":

$$s_8^{\eta+\eta'} = 4\pi^2((f_\eta^8)^2 + (f_{\eta'}^8)^2). \tag{41}$$

At the same time from 3-point correlator:

$$s_8^{asr} = 4\pi^2((f_\eta^8)^2 + (f_{\eta'}^8)^2 + 2\sqrt{2}[f_\eta^8 f_\eta^0 + f_{\eta'}^8 f_{\eta'}^0]). \tag{42}$$

Additional constraint - $R_{J/\Psi}$.

The radiative decays $J/\Psi \to \eta(\eta')\gamma$ are dominated by non-perturbative gluonic matrix elements, and the ratio of the decay rates $R_{J/\Psi} = (\Gamma(J/\Psi) \to \eta'\gamma)/(\Gamma(J/\Psi) \to \eta\gamma)$ can be expressed as follows [Novikov'79]:

$$R_{J/\Psi} = \left| \frac{\langle 0 \mid G\widetilde{G} \mid \eta' \rangle}{\langle 0 \mid G\widetilde{G} \mid \eta \rangle} \right|^2 \left(\frac{p_{\eta'}}{p_\eta} \right)^3, \tag{43}$$

where $p_{\eta(\eta')} = M_{J/\Psi}(1 - m^2_{\eta(\eta')}/M^2_{J/\Psi})/2$.

$$\partial_\mu J^8_{\mu 5} = \frac{1}{\sqrt{6}}(m_u \overline{u}\gamma_5 u + m_d \overline{d}\gamma_5 d - 2m_s \overline{s}\gamma_5 s), \tag{44}$$

$$\partial_\mu J^0_{\mu 5} = \frac{1}{\sqrt{3}}(m_u \overline{u}\gamma_5 u + m_d \overline{d}\gamma_5 d + m_s \overline{s}\gamma_5 s) + \frac{1}{2\sqrt{3}} \frac{3\alpha_s}{4\pi} G\widetilde{G}. \tag{45}$$

$$R_{J/\Psi} = \left(\frac{f^8_{\eta'} + \sqrt{2} f^0_{\eta'}}{f^8_\eta + \sqrt{2} f^0_\eta} \right)^2 \left(\frac{m_{\eta'}}{m_{\eta'}} \right)^4 \left(\frac{p_{\eta'}}{p_\eta} \right)^3. \tag{46}$$

From experiment this ratio is: $R_{J/\Psi} = 4.67 \pm 0.15$ [PDG 2012].

$$f_\eta^8 F_{\eta\gamma}(Q^2) + f_{\eta'}^8 F_{\eta'\gamma}(Q^2) = \frac{1}{2\sqrt{6}\pi^2}\frac{s_0}{s_0 + Q^2}, \tag{47}$$

$$4\pi^2((f_\eta^8)^2 + (f_{\eta'}^8)^2 + 2\sqrt{2}[f_\eta^8 f_\eta^0 + f_{\eta'}^8 f_{\eta'}^0]) = s_0, \tag{48}$$

$$f_\eta^8 F_{\eta\gamma}(0) + f_{\eta'}^8 F_{\eta'\gamma}(0) = \frac{1}{2\sqrt{6}\pi^2}, F_{M\gamma}(0) = \sqrt{\frac{4\Gamma_{M\to\gamma\gamma}}{\pi\alpha^2 m_M^3}} \tag{49}$$

$$R_{J/\Psi} = \left(\frac{f_{\eta'}^8 + \sqrt{2}f_{\eta'}^0}{f_\eta^8 + \sqrt{2}f_\eta^0}\right)^2 \left(\frac{m_{\eta'}}{m_{\eta'}}\right)^4 \left(\frac{p_{\eta'}}{p_\eta}\right)^3. \tag{50}$$

TFFs of η and η': experiment

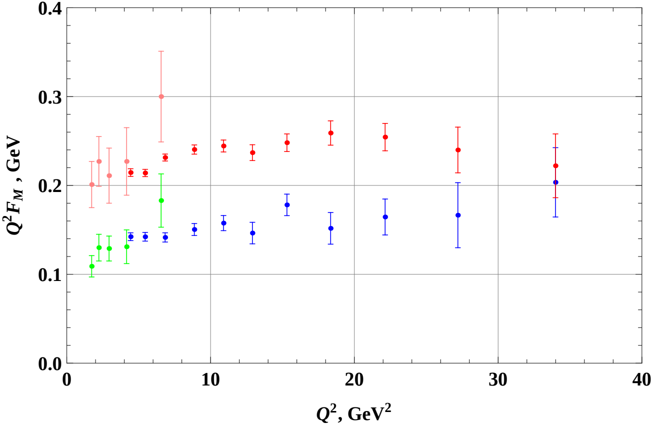
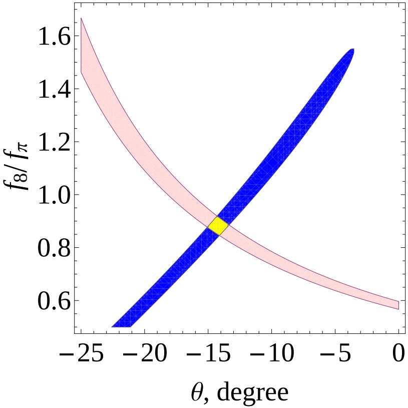


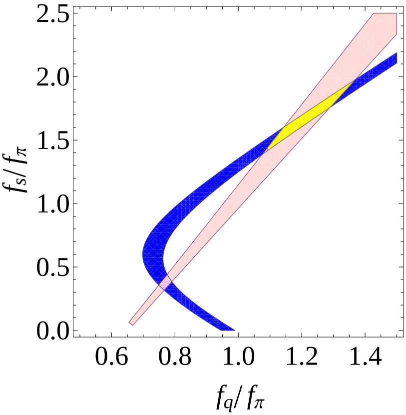
Рис. : Experimental data on transition form factors: η (CLEO-green, BABAR-blue), η' (CLEO-pink, BABAR-red)

Octet-siglet mixing scheme: parameters



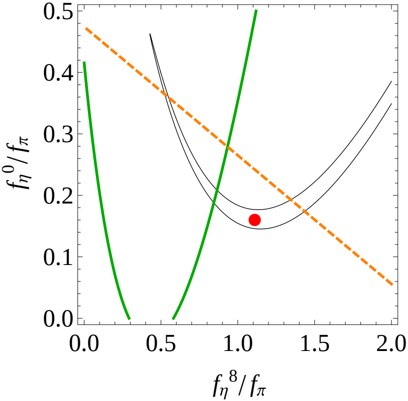
$$f_8/f_\pi = 0.88 \pm 0.04, \quad f_0/f_\pi = 0.81 \pm 0.07, \quad \theta = -14.2^\circ \pm 0.7^\circ$$

Quark-flavour mixing scheme: parameters



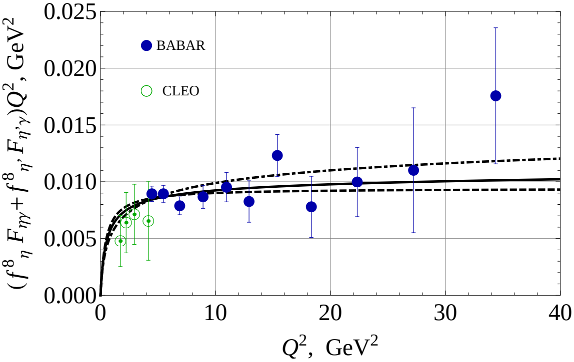
$$f_q/f_\pi = 1.20 \pm 0.15, \quad f_s/f_\pi = 1.65 \pm 0.25, \quad \phi = 38.1^\circ \pm 0.5^\circ$$

Mixing-scheme-independent determination



$$\mathbf{F} = \begin{pmatrix} f_\eta^8 & f_{\eta'}^8 \\ f_\eta^0 & f_{\eta'}^0 \end{pmatrix} = \begin{pmatrix} 1.11 & -0.42 \\ 0.16 & 1.04 \end{pmatrix} f_\pi \qquad (51)$$

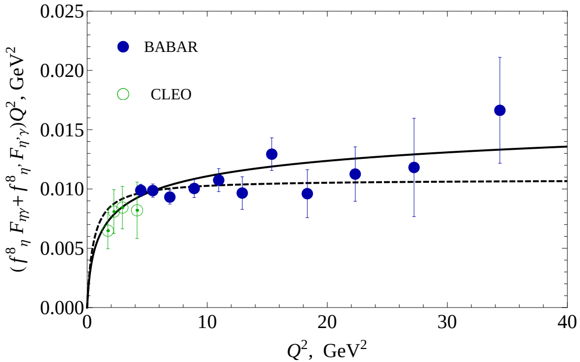
ASR in octet channel + correction: mixing-sceme-independent parameters



The similar correction in the octet channel leads to the ASR:

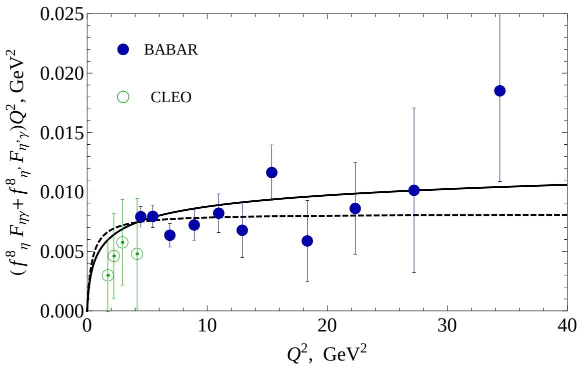
$$f^8_\eta F_{\eta\gamma}(Q^2) + f^8_{\eta'} F_{\eta'\gamma}(Q^2) = \frac{1}{2\sqrt{6}\pi^2}\frac{s_8}{s_8+Q^2}\left[1+\frac{\lambda Q^2}{s_8+Q^2}(\ln\frac{Q^2}{s_8}+\sigma)\right].$$

ASR in octet channel + correction: OS mixing scheme



$$f_8/f_\pi = 0.88, \quad f_0/f_\pi = 0.81, \quad \theta = -14.2^\circ.$$

ASR in octet channel + correction: QF mixing scheme



$$f_q/f_\pi = 1.20, \quad f_s/f_\pi = 1.65, \quad \phi = 38.1^\circ.$$

Заключение (результаты)

- Исходя из точного непертурбативного аномального правила сумм получена формула для переходного формфактора пиона. Она справедливая даже при нарушении КХД факторизации за счет поправки, выходящей за рамки операторного разложения. Проведен анализ современных экспериментальных данных показывающий, что они не исключают наличие такой поправки, хотя и не указывают однозначно на её существование. В случае, когда КХД факторизация выполняется, полученная формула обосновывает интерполяционную формулу Бродского-Лепажа.
- Метод кварк-адронной дуальности распостранен на случай сильного смешивания адронных состояний. Получен критерий для выбора схемы смешивания псевдоскалярных состояний, следующий из совпадения интервалов дуальности в двухточечных и трёхточечных корреляторах.
- Из аномального правила сумм в октетном канале получена связь между переходными формфакторами и константами распадов η и η' мезонов.Проведен анализ экспериментальных данных, позволяющий извлечь константы распадов в различных схемах смешивания. Установлено, что современная точность экспериментальных данных допускает наличие непертурбативной поправки того же порядка, что и в случае пиона.

Publications

- "Axial anomaly as a collective effect of meson spectrum"
  Yaroslav Klopot, Armen Oganesian, Oleg Teryaev,
  Phys.Lett. B695 (2011) 130
- "Axial anomaly and mixing: from real to highly virtual photons "
  Yaroslav Klopot, Armen Oganesian, Oleg Teryaev,
  Phys.Rev. D84 (2011) 051901
- Quark-hadron duality, axial anomaly and mixing"
  Yaroslav Klopot, Armen Oganesian, Oleg Teryaev,
  JETP Lett. 94 (2011) 729 [Письма ЖЭТФ 94, 791 (2011)]
- "Transition Form Factors and Mixing of Pseudoscalar Mesons from Anomaly Sum Rule"
  Yaroslav Klopot, Armen Oganesian, Oleg Teryaev,
  Phys.Rev. D87 (2013) 036013

Conference proceedings

- "Anomaly, mixing and transition form factors of pseudoscalar mesons "
  Yaroslav Klopot, Armen Oganesian, Oleg Teryaev,
  Nucl.Phys.Proc.Suppl. 219-220 (2011) 141
- "Axial Anomaly and Light Cone Distributions"
  Yaroslav Klopot, Armen Oganesian, Oleg Teryaev,
  Acta Phys. Polon. Supp. 6 (2013) 145
- "Axial anomaly, quark-hadron duality and transition form factors "
  Yaroslav Klopot, Armen Oganesian, Oleg Teryaev,
  Phys.Part.Nucl.Lett. 9 (2012) 769
- "Nonperturbative QCD and Transition Form Factors"
  Yaroslav Klopot, Armen Oganesian, Oleg Teryaev,
  Proceedings of Science, PoS(Baldin ISHEPP XXI)036 (2012)

Спасибо за внимание!

Backup

Bose symmetry $T_{\alpha\mu\nu}(k,q) = T_{\alpha\nu\mu}(q,k)$ implies:

$$\begin{aligned} F_1(k,q) &= -F_2(q,k), \\ F_3(k,q) &= -F_6(q,k), \\ F_4(k,q) &= -F_5(q,k). \end{aligned} \tag{52}$$

One can show also that

$$F_6(k,q) = -F_3(k,q)$$

vector Ward identities

$$k^\mu T_{\alpha\mu\nu} = 0, \qquad p^\nu T_{\alpha\mu\nu} = 0 \tag{53}$$

In terms of formfactors, the identities (53) read

$$\begin{aligned} F_1 &= k.q\, F_3 + q^2\, F_4 \\ F_2 &= k^2\, F_5 + k.q\, F_6 \end{aligned} \tag{54}$$

Anomalous axial-vector Ward identity for the amplitude (7) is [Adler'69]:

$$q^\alpha T_{\alpha\mu\nu}(k,q) = 2m T_{\mu\nu}(k,q) + \frac{1}{2\pi^2} \varepsilon_{\mu\nu\rho\sigma} k^\rho q^\sigma \tag{55}$$

Backup

$$T_{\mu\nu}(k,q) = G\ \varepsilon_{\mu\nu\rho\sigma} k^{\rho} q^{\sigma} \tag{56}$$

where G is the relevant form factor. In terms of form factors, eq.(55) reads

$$F_2 - F_1 = 2mG + \frac{1}{2\pi^2} \tag{57}$$

For the form factors F_3, F_4 and G one may write unsubtracted dispersion relations

$$F_j(p^2) = \frac{1}{\pi}\int_{4m^2}^{\infty} \frac{A_j(t)}{t - p^2} dt, \qquad j = 3, 4 \tag{58}$$

$$G(p^2) = \frac{1}{\pi}\int_{4m^2}^{\infty} \frac{B(t)}{t - p^2} dt$$

Backup

From (39) and (54) it is easy to see that for the considered kinematical configuration one has

$$F_2 - F_1 = (q^2 - p^2)F_3 - p^2 F_4 \tag{59}$$

Using now (58) and taking into account that the imaginary parts of the relevant formfactors satisfy non-anomalous Ward identities, in particular

$$(q^2 - t)A_3(t) - q^2 A_4(t) = 2mB(t) \tag{60}$$

one gets finally

$$F_2 - F_1 - 2mG = \frac{1}{\pi}\int_{4m^2}^{\infty} A_3(t)dt \tag{61}$$

Comparing eq.(61) with (57) one may thus observe that the occurrence of the axial anomaly is equivalent to a "sum rule"

$$\int_{4m^2}^{\infty} A_3(t, q^2; m^2)dt = \frac{1}{2\pi} \tag{62}$$

(which must hold for an arbitrary m and for any p^2 in the considered region).