QCD in terms of gauge invariant dynamical degrees of freedom[1]

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- H.-P. P., *$SU(2)$ Yang-Mills quantum mechanics of spatially constant fields*, Phys. Lett. B **648** (2007) 97-106.
- H.-P. P., *Expansion of the Yang-Mills Hamiltonian in spatial derivatives and glueball spectrum*, Phys. Lett. B **685** (2010) 353-364.
- H.-P. P., *SU(2) Dirac-Yang-Mills quantum mechanics of spatially constant quark and gluon fields*, Phys. Lett. B **700** (2011) 265-276.
- H.-P. P., *Unconstrained Hamiltonian formulation of low energy SU(3) Yang-Mills quantum theory* , arXiv: 1205.2237v1 [hep-th] (2012).

- Gribov ambiguity (1978) $\rightarrow$ Attempt of an exact resolution of the Gauss-laws to have an QCD Hamiltonian at low energy (a.o. Jackiw+Goldstone, Faddeev, T.D.Lee)
- Unconstrained Hamiltonian of 2-color QCD
- Derivative expansion
- Spectrum: Role of fermions, Renormalisation
- Extension to SU(3)

Aim: Alternative nonperturbative formulation of QCD

Constrained QCD

The QCD action

$$\mathcal{S}[A,\psi,\overline{\psi}] := \int d^4x \left[-\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \overline{\psi}\left(i\gamma^\mu D_\mu - m\right)\psi\right]$$

$$F^a_{\mu\nu} := \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f_{abc} A^b_\mu A^c_\nu \ , \quad a = 1,..,8$$

$$D_\mu := \partial_\mu - igA^a_\mu \tau_a/2$$

is invariant under the $SU(3)$ gauge transformations $U[\omega(x)] \equiv \exp(i\omega_a\tau_a/2)$

$$\psi^\omega(x) = U[\omega(x)]\ \psi(x)$$

$$A^\omega_{a\mu}(x)\tau_a/2 = U[\omega(x)]\left(A_{a\mu}(x)\tau_a/2 + \frac{i}{g}\partial_\mu\right)U^{-1}[\omega(x)]$$

chromoelectric : $\quad E^a_i \equiv F^a_{i0} \quad$ and chromomagnetic $\quad B^a_i \equiv \frac{1}{2}\epsilon_{ijk}F^a_{jk}$

$\Pi_{ai} = -E_{ai}$ momenta can. conj. to the spatial A_{ai} $\rightarrow$ canonical Hamiltonian

$$H_C = \int d^3x \left[\frac{1}{2}E^2_{ai} + \frac{1}{2}B^2_{ai}(A) - gA_{ai}\, j_{ia}(\psi) + \overline{\psi}\left(\gamma_i\partial_i + m\right)\psi \right.$$
$$\left. - gA_{a0}\left(D_i(A)_{ab}E_{bi} - \rho_a(\psi)\right)\right]$$

with the covariant derivative $D_i(A)_{ab} \equiv \delta_{ab}\partial_i - gf_{abc}A_{ci}$

Constrained Quantisation

Exploit the time dependence of the gauge transformations to put

$$A_{a0} = 0 \ , \qquad a = 1, .., 8 \qquad \text{(Weyl gauge)}$$

The dynamical vaiables A_{ai}, $-E_{ai}$, $\psi_{\alpha r}$ and $\psi^*_{\alpha r}$ are quantized in the Schrödinger functional approach imposing the equal-time (anti-)CR, e.g. $-E_{ai} = -i\partial/\partial A_{ai}$. The physical states Φ

$$\begin{aligned} H\Phi &= \int d^3x \left[\frac{1}{2}E^2_{ai} + \frac{1}{2}B^2_{ai}[A] - A_{ai}\, j_{ia}(\psi) + \overline{\psi}\,(\gamma_i\partial_i + m)\,\psi\right]\Phi = E\Phi \ , \\ G_a(x)\Phi &= \left[D_i(A)_{ab}E_{bi} - \rho_a(\psi)\right]\Phi = 0 \ , \quad a = 1, .., 8 \ . \end{aligned}$$

The Gauss law operators G_a are the generators of the residual time independent gauge transformations, satisfying $[G_a, H] = 0$ and $[G_a, G_b] = i f_{abc} G_c$.

Angular momentum operators $\quad [J_i, H] = 0$

$$J_i = \int d^3x \left[-\epsilon_{ijk}A_{aj}E_{ak} + \Sigma_i(\psi) + \text{orbital parts}\right] \ , \quad i = 1, 2, 3 \ ,$$

The matrix element of an operator O is given in the Cartesian form

$$\langle \Phi'|O|\Phi\rangle \ \propto \int dA\ d\overline{\psi}\ d\psi\ \Phi'^*(A, \overline{\psi}, \psi)\, O\, \Phi(A, \overline{\psi}, \psi) \ .$$

For $SU(3)$ Yang-Mills QM of spat.const.gluon fields: P. Weisz and V. Ziemann (1986)

Point trafo to new set of adapted coordinates,
$A_{ai}, \psi_\alpha \to$ 3 gauge angles q_j, the pos. definite symmetric 3×3 matrix S, and new ψ'_β

$$A_{ai}(q,S) = O_{ak}(q)\, S_{ki} - \frac{1}{2g}\epsilon_{abc}\left(O(q)\,\partial_i O^T(q)\right)_{bc}, \quad \psi_\alpha(q,\psi') = U_{\alpha\beta}(q)\,\psi'_\beta$$

orthog. $O(q)$ and unitary $U(q)$ related via $\quad O_{ab}(q) = \frac{1}{2}\mathrm{Tr}\left(U^{-1}(q)\tau_a U(q)\tau_b\right)$.

Generalisation of the (unique) polar decomposition of A and corresponds to

$$\chi_i(A) = \epsilon_{ijk}A_{jk} = 0 \quad \text{("symmetric gauge")}.$$

Preserving the CCR $->$ old canonical momenta in terms of the new variables

$$-E_{ai}(q,S,p,P) = O_{ak}(q)\left[P_{ki} + \epsilon_{kil}{}^*D^{-1}_{ls}(S)\left(\Omega^{-1}_{sj}(q)p_j + \rho_s(\psi') + D_n(S)_{sm}P_{mn}\right)\right]$$

$$\Rightarrow \qquad G_a\Phi \equiv O_{ak}(q)\Omega^{-1}_{ki}(q)p_i\Phi = 0 \quad \Leftrightarrow \quad \frac{\delta}{\delta q_i}\Phi = 0 \quad \text{(Abelianisation)}$$

$$\text{Ang. mom. op.} \quad J_i = \int d^3x \left[-2\epsilon_{ijk}S_{mj}P_{mk} + \Sigma_i(\psi') + \rho_i(\psi') + \text{orbital parts}\right]$$

$\to$ S colorless spin 0,2 glueball field, ψ' colorless reduced quark fields of spin 0,1
Reduction: Color $\to$ Spin (unusual spin-statistics relation specific to SU(2) !)

Physical quantum Hamiltonian of 2-color QCD in symmetric gauge

The correctly ordered physical quantum Hamiltonian (Christ and Lee 1980) in terms of the physical variables $S_{ik}(\mathbf{x})$ and the can. conj. $P_{ik}(\mathbf{x}) \equiv -i\delta/\delta S_{ik}(\mathbf{x})$ reads

$$H(S,P) = \frac{1}{2}\mathcal{J}^{-1}\int d^3\mathbf{x}\; P_{ai}\; \mathcal{J} P_{ai} + \frac{1}{2}\int d^3\mathbf{x}\left[B^2_{ai}(S) - S_{ai}\, j_{ia}(\psi') + \overline{\psi}'\,(\gamma_i\partial_i + m)\,\psi'\right]$$
$$-\mathcal{J}^{-1}\int d^3\mathbf{x}\int d^3\mathbf{y}\Big\{\Big(D_i(S)_{ma}P_{im} + \rho_a(\psi')\Big)(\mathbf{x})\mathcal{J}$$
$$\langle \mathbf{x}\; a|{}^*D^{-2}(S)|\mathbf{y}\; b\rangle \;\; \Big(D_j(S)_{bn}P_{nj} + \rho_b(\psi')\Big)(\mathbf{y})\Big\}$$

with the FP operator

$${}^*D_{kl}(S) \equiv \epsilon_{kmi}D_i(S)_{ml} = \epsilon_{kli}\partial_i - g(S_{kl} - \delta_{kl}\mathrm{tr}S)$$

and the Jacobian $\mathcal{J} \equiv \det|{}^*D|$
The matrix element of a physical operator O is given by

$$\langle\Psi'|O|\Psi\rangle \;\propto \int_{\mathrm{S\ pos.def.}}\int_{\overline{\psi}',\psi'}\prod_{\mathbf{x}}\Big[dS(\mathbf{x})d\overline{\psi}'(\mathbf{x})d\psi'(\mathbf{x})\Big]\mathcal{J}\Psi'^*[S,\overline{\psi}',\psi']O\Psi[S,\overline{\psi}',\psi']$$

The inverse of the FP operator and hence the physical Hamiltonian can be expanded in the number of spatial derivatives $\equiv$ expansion in $\lambda = g^{-2/3}$

Coarse graining (1): Equivalence to an expansion in $\lambda = g^{-2/3}$

Introduce UV cutoff a: infinite spatial lattice of granulas $G(\mathbf{n}, a)$ at $\mathbf{x} = a\mathbf{n}$ ($\mathbf{n} \in Z^3$) and averaged variables

$$S(\mathbf{n}) := \frac{1}{a^3} \int_{G(\mathbf{n},a)} d\mathbf{x}\ S(\mathbf{x})$$

and the discretised spatial derivatives.
Expansion of the Hamiltonian in $\lambda = g^{-2/3}$

$$H = \frac{g^{2/3}}{a} \left[\mathcal{H}_0 + \lambda \sum_\alpha \mathcal{V}_\alpha^{(\partial)} + \lambda^2 \left(\sum_\beta \mathcal{V}_\beta^{(\Delta)} + \sum_\gamma \mathcal{V}_\gamma^{(\partial\partial\neq\Delta)} \right) + \mathcal{O}(\lambda^3) \right]$$

The "free" Hamiltonian

$$\mathcal{H}_0 \quad = \quad \sum_{\mathbf{n}} \mathcal{H}_0^{QM}(\mathbf{n})$$

is the sum of the Hamiltonians of $SU(2)$-Yang-Mills quantum mechanics of constant fields in each box. The interaction terms

$$\mathcal{V}^{(\partial)}, \mathcal{V}^{(\Delta)}, ..$$

lead to interactions between the granulas.

Derivative Expansion (2): Zeroth order Hamiltonian

Intrinsic system $S = R^T(\alpha,\beta,\gamma)\ \mathrm{diag}(\phi_1,\phi_2,\phi_3)R(\alpha,\beta,\gamma)$ with Jac. $\prod_{i<j}(\phi_i-\phi_j)$
Zeroth order Hamiltonian

$$H = \frac{g^{2/3}}{V^{1/3}}\left[\mathcal{H}^G+\mathcal{H}^D+\mathcal{H}^C\right]+\frac{1}{2}m\left[\left(\widetilde{u}_L^{(0)\dagger}\widetilde{v}_R^{(0)}+\sum_{i=1}^{3}\widetilde{u}_L^{(i)\dagger}\widetilde{v}_R^{(i)}\right)+h.c.\right]$$

$$\mathcal{H}^G := \frac{1}{2}\sum_{ijk}^{\text{cyclic}}\left(-\frac{\partial^2}{\partial\phi_i^2}-\frac{2}{\phi_i^2-\phi_j^2}\left(\phi_i\frac{\partial}{\partial\phi_i}-\phi_j\frac{\partial}{\partial\phi_j}\right)+(\xi_i-\widetilde{J}_i^Q)^2\frac{\phi_j^2+\phi_k^2}{(\phi_j^2-\phi_k^2)^2}+\phi_j^2\phi_k^2\right),$$

$$\mathcal{H}^D := \frac{1}{2}(\phi_1+\phi_2+\phi_3)\left(\widetilde{N}_L^{(0)}-\widetilde{N}_R^{(0)}\right)+\frac{1}{2}\sum_{ijk}^{\text{cyclic}}(\phi_i-(\phi_j+\phi_k))\left(\widetilde{N}_L^{(i)}-\widetilde{N}_R^{(i)}\right)\ ,$$

$$\mathcal{H}^C := \sum_{ijk}^{\text{cyclic}}\frac{\widetilde{\rho}_i(\xi_i-\widetilde{J}_i^Q+\widetilde{\rho}_i)}{(\phi_j+\phi_k)^2}\ ,$$

and the total spin $\qquad J_i = R_{ij}(\chi)\ \xi_j\ , \qquad [J_i,H]=0$

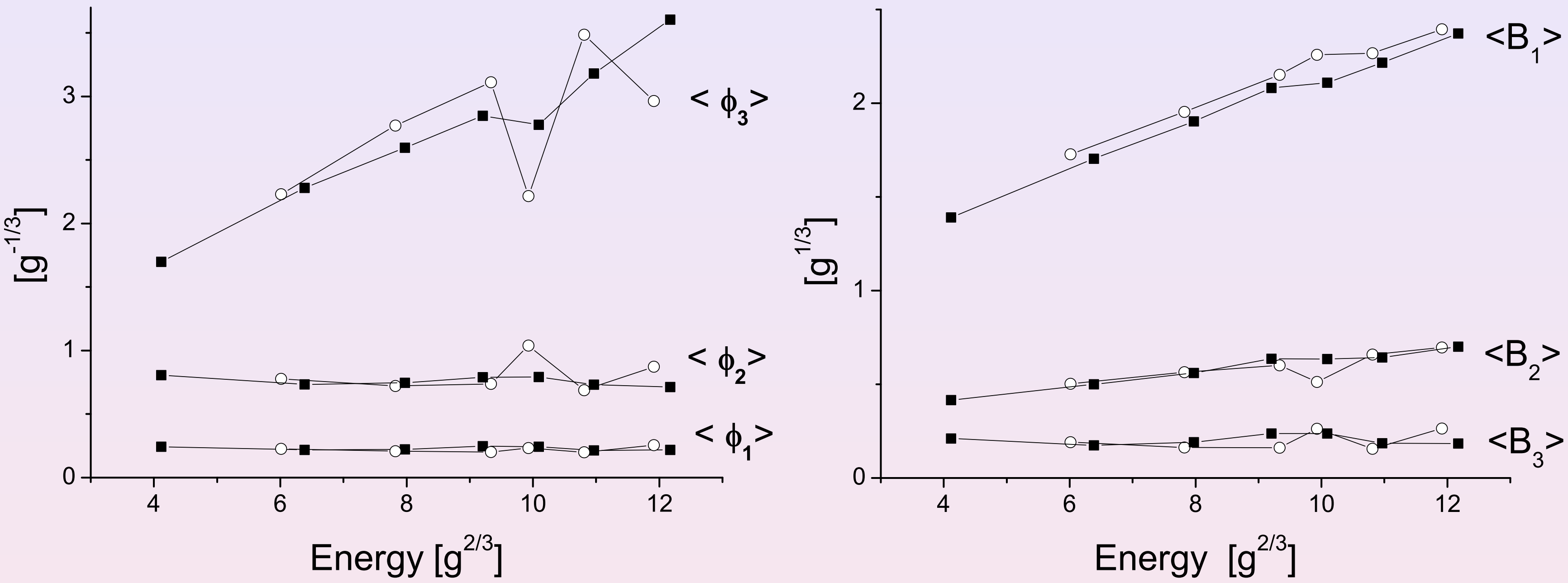
in terms of the intrinsic spin ξ_i satisfying $[J_i,\xi_j]=0$ and $[\xi_i,\xi_j]=-i\epsilon_{ijk}\xi_k$

magn.pot. $B^2=\phi_2^2\phi_3^2+\phi_3^2\phi_1^2+\phi_1^2\phi_2^2$ has 0-valleys $\text{"}\phi_1=\phi_2=0,\quad \phi_3 \text{ arbitrary"}$

The matrix elements become

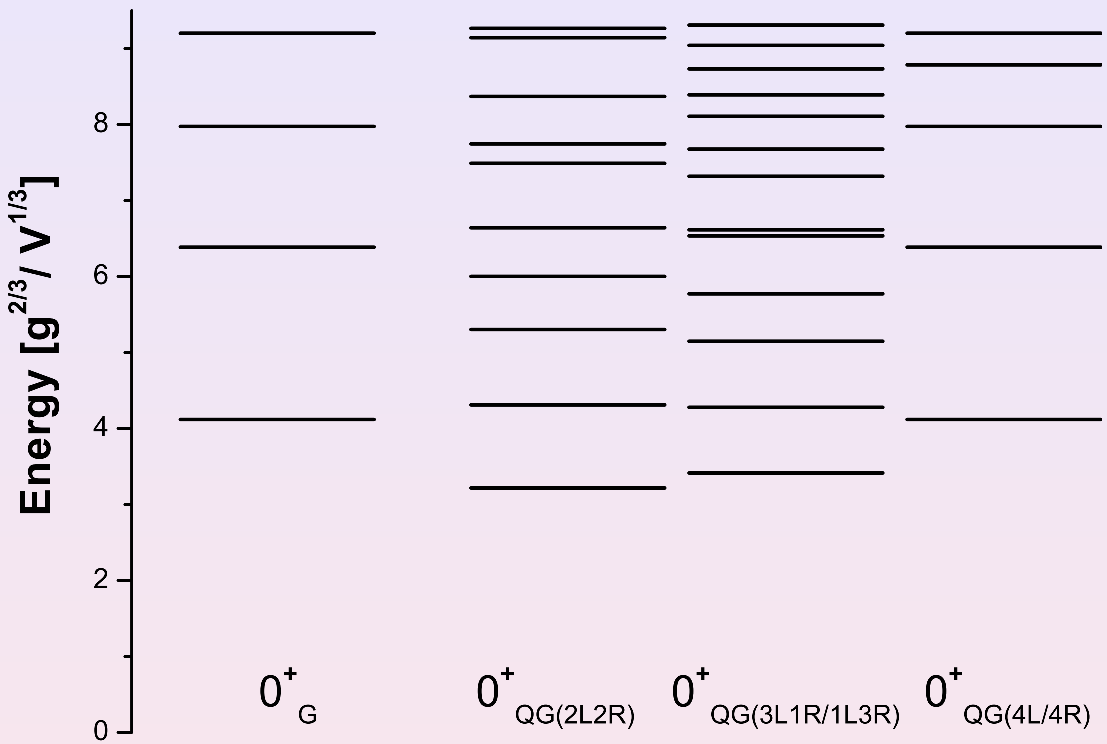
$$\langle\Phi_1|\mathcal{O}|\Phi_2\rangle=\int d\alpha\sin\beta d\beta d\gamma\int_{0<\phi_1<\phi_2<\phi_3}d\phi_1 d\phi_2 d\phi_3\ (\phi_1^2-\phi_2^2)(\phi_2^2-\phi_3^2)(\phi_3^2-\phi_1^2)\int d\overline{\psi}'d\psi'\,\Phi_1^*\mathcal{O}\Phi_2\ .$$

$\langle \phi_i \rangle$ and $\langle B_i \rangle \equiv \langle g\phi_j\phi_k \rangle$ expectation values for $0^+$and 2^+



$\langle \phi_3 \rangle$ is raising with increasing excitation, whereas $\langle \phi_1 \rangle$ and $\langle \phi_2 \rangle$ are practically constant. $\langle B_3 \rangle$ is practically constant with increasing excitation, whereas $\langle B_1 \rangle$ and $\langle B_2 \rangle$ are raising.

0^+ energy spectrum for the pure-gluon and the quark-gluon cases



The energies of the quark-gluon ground state and the sigma-antisigma excitation are lower than that of the lowest pure-gluon state !

Vacuum: $4.11\ (\text{pure}-\text{glueball}) \leftrightarrow 3.22\ (5.63, -2.43, 0.02)\ (\text{quark}-\text{glueball})$

1st and 2nd order pert. theory in $\lambda = g^{-2/3}$ give the result (for the $(+)$ b.c.)

$$E^{+}_{\rm vac} = \mathcal{N}\frac{g^{2/3}}{a}\left[4.1167 + 29.894\lambda^2 + \mathcal{O}(\lambda^3)\right] \ ,$$

for the energy of the interacting glueball vacuum, and

$$E^{(0)+}_{1}(k) - E^{+}_{\rm vac} = \left[\ 2.270 + 13.511\lambda^2 + \mathcal{O}(\lambda^3)\right]\frac{g^{2/3}}{a} + 0.488\frac{a}{g^{2/3}}k^2 + \mathcal{O}((a^2k^2)^2) \ ,$$

for the energy spectrum of the interacting spin-0 glueball.

Lorentz invariance : $E = \sqrt{M^2 + k^2} \simeq M + \frac{1}{2M}\ k^2 \quad \rightarrow \quad \widetilde{c}^{(i)} = 1/[2\mu_i]$

$\longrightarrow$ Consider $J = L + S$ states:

Glueball mass and coupling constant renormalisation in the IR

Consider the physical mass

$$M = \frac{g_0^{2/3}}{a}\left[\mu + c g_0^{-4/3}\right]$$

Demanding its independence of box size a, one obtains

$$\gamma(g_0) \equiv a\frac{d}{da}g_0(a) = \frac{3}{2}g_0\frac{\mu + c g_0^{-4/3}}{\mu - c g_0^{-4/3}}$$

vanishes for $\quad g_0 = 0 \quad \text{(pert. fixed point)} \quad \text{or} \quad g_0^{4/3} = -c/\mu \quad (\text{IR fixed point, if } c < 0)$

My (incomplete) result $c_1^{(0)}/\mu_1^{(0)} = 5.95(1.34)$ suggests, that no IR fixed points exist.

$$\text{for } c > 0 : \quad g_0^{2/3}(Ma) = \frac{Ma}{2\mu} + \sqrt{\left(\frac{Ma}{2\mu}\right)^2 - \frac{c}{\mu}}\,, \qquad a > a_c := 2\sqrt{c\mu}/M$$

critical coupling $g_0^2|_c = 14.52\ (1.55)$ and

$$\text{for } M \sim 1.6 \text{ GeV} : \quad a_c \sim 1.4 \text{ fm } (0.9 \text{ fm})\ .$$

Symmetric gauge for SU(3)

Use idea of *minimal embedding* of $su(2)$ in $su(3)$ by Kihlberg + Marnelius (1982)

$$\tau_1 := \lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad \tau_2 := -\lambda_5 = \begin{pmatrix} 0 & 0 & i \\ 0 & 0 & 0 \\ -i & 0 & 0 \end{pmatrix} \quad \tau_3 := \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\tau_4 := \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad \tau_5 := \lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \tau_6 := \lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\tau_7 := \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \tau_8 := \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

The corresponding non-trivial non-vanishing structure constants $[\frac{\tau_a}{2}, \frac{\tau_b}{2}] = i c_{abc} \frac{\tau_c}{2}$,
have at least one index $\in \{1, 2, 3\}$

"symmetric gauge" for SU(3) : $\quad \chi_a(A) = \sum_{b=1}^{8} \sum_{i=1}^{3} c_{abi} A_{bi} = 0 \ , \quad a = 1, ..., 8$

Symmetric gauge for SU(3): Unconstrained representation

Carrying out the coordinate transformation (generalized polar decomposition)

$$A_{ak}\left(q_1,..,q_8,\widehat{S}\right) = O_{a\hat{a}}(q)\,\widehat{S}_{\hat{a}k} - \frac{1}{2g}c_{abc}\left(O(q)\,\partial_k O^T(q)\right)_{bc},$$

$$\psi_\alpha\left(q_1,..,q_8,\psi^{RS}\right) = U_{\alpha\hat{\beta}}(q)\,\psi^{RS}_{\hat{\beta}}$$

$$\widehat{S}_{\hat{a}k} \equiv \begin{pmatrix} S_{ik} \\ \overline{S}_{Ak} \end{pmatrix} = \left(\begin{array}{ccc} & S_{ik}\ \text{pos. def.} & \\ \hline W_0 & X_3-W_3 & X_2+W_2 \\ X_3+W_3 & W_0 & X_1-W_1 \\ X_2-W_2 & X_1+W_1 & W_0 \\ -\frac{\sqrt{3}}{2}Y_1-\frac{1}{2}W_1 & \frac{\sqrt{3}}{2}Y_2-\frac{1}{2}W_2 & W_3 \\ -\frac{\sqrt{3}}{2}W_1-\frac{1}{2}Y_1 & \frac{\sqrt{3}}{2}W_2-\frac{1}{2}Y_2 & Y_3 \end{array}\right), \quad c_{\hat{a}\hat{b}k}\widehat{S}_{\hat{b}k} = 0$$

exists and unique : $\widehat{S}_{\hat{a}i}\widehat{S}_{\hat{a}j} = A_{ai}A_{aj}$ (6) $\qquad d_{\hat{a}\hat{b}\hat{c}}\widehat{S}_{\hat{a}i}\widehat{S}_{\hat{b}j}\widehat{S}_{\hat{c}k} = d_{abc}A_{ai}A_{bj}A_{ck}$ (10)

reduced gluons (glueballs): Spin 0,1,2,3 $\quad$ reduced quarks: Spin $3/2$ Rarita-Schwinger

Reduction: Color $\rightarrow$ Spin, consequ.for Spin-Physics? $\Delta^{++}(3/2) : (3/2, +1/2, -1/2)$?

Rotate into the intrinsic frame of submatrix S representing the embedded $su(2)$

$$\widehat{S} = \left(\begin{array}{c} S \\ \hline \overline{S} \end{array}\right) = \left(\begin{array}{c|c} R(\alpha,\beta,\gamma) & 0 \\ \hline 0 & D^{(2)}(\alpha,\beta,\gamma) \end{array}\right) \left(\begin{array}{c} \mathrm{diag}(\phi_1,\phi_2,\phi_3) \\ \hline \overline{S}(X_i \to x_i \\ Y_i \to y_i \\ W_i \to w_i) \end{array}\right) \left(R^T(\alpha,\beta,\gamma) \right)$$

The magnetic potential V_{magn} has the zero-energy valleys ("constant Abelian fields")

$$B^2 = 0 \quad : \quad \phi_3 \text{ and } y_3 \text{ arbitrary} \quad \wedge \quad \text{all others zero}$$

At the bottom of the valleys the string-interaction becomes diagonal

$$\mathcal{H}^D_{\mathrm{diag}} = \frac{1}{2}\widetilde{\psi}_L^{(1,\frac{1}{2})\dagger}\left[(\phi_3\lambda_3 + y_3\lambda_8)\otimes\sigma_3\right]\widetilde{\psi}_L^{(1,\frac{1}{2})} - \frac{1}{2}\widetilde{\psi}_R^{(\frac{1}{2},1)\dagger}\left[\sigma_3\otimes(\phi_3\lambda_3 + y_3\lambda_8)\right]\widetilde{\psi}_R^{(\frac{1}{2},1)}$$

Faddeev-Popov operator for ymmetric gauge for SU(3)

$$\gamma_{\hat{a}\hat{b}} = c_{\hat{a}\hat{c}i} D_i(S)_{\hat{c}\hat{b}} = c_{\hat{a}\hat{c}i}\left(\delta_{\hat{b}\hat{c}}\partial_i - c_{\hat{b}\hat{c}\hat{d}}\widehat{S}_{\hat{d}i}\right) = -c_{\hat{a}\hat{c}i}c_{\hat{b}\hat{c}\hat{d}}\widehat{S}_{\hat{d}i} + c_{\hat{a}\hat{b}i}\partial_i$$

Explicit form of the intrinsic $\widetilde{\gamma}$,

$$\left(\begin{array}{ccc|ccc|cc}
\phi_2+\phi_3 & 0 & 0 & & & & & \\
0 & \phi_3+\phi_1 & 0 & \multicolumn{5}{c}{-2\overline{S}^T(-\frac{3}{2}v, w)} \\
0 & 0 & \phi_1+\phi_2 & & & & & \\
\hline
 & & & 4\phi_1+\phi_2+\phi_3 & 0 & 0 & 0 & 0 \\
 & & & 0 & \phi_1+4\phi_2+\phi_3 & 0 & 0 & 0 \\
\multicolumn{3}{c|}{-2\overline{S}\left(-\frac{3}{2}v, w\right)} & 0 & 0 & \phi_1+\phi_2+4\phi_3 & 0 & 0 \\
\cline{4-8}
 & & & 0 & 0 & 0 & \phi_1+\phi_2+4\phi_3 & -\sqrt{3}(\phi_1-\phi_2) \\
 & & & 0 & 0 & 0 & -\sqrt{3}(\phi_1-\phi_2) & 3(\phi_1+\phi_2)
\end{array}\right)$$

In contrast to the $SU(2)$ case, transition to the intrinsic system does not completely diagonalize γ.

Symmetric gauge for SU(3): 1 spatial dimension

In one spatial dimension the symmetric gauge for SU(3) reduces to

$$A^{(1d)} = \begin{pmatrix} 0 & 0 & A_{13} \\ 0 & 0 & A_{23} \\ 0 & 0 & A_{33} \\ 0 & 0 & A_{43} \\ 0 & 0 & A_{53} \\ 0 & 0 & A_{63} \\ 0 & 0 & A_{73} \\ 0 & 0 & A_{83} \end{pmatrix} \quad \to \quad S^{(1d)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \phi_3 \\ \hline 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_3 \end{pmatrix}$$

which consistently reduces the above equs. for S for given A_3 to

$$\phi_3^2 + y_3^2 \quad = \quad A_{a3}A_{a3} \quad \wedge \quad \phi_3^2\, y_3 - 3\, y_3^3 \quad = \quad d_{abc}A_{a3}A_{b3}A_{c3}$$

with 6 solutions separated by zero-lines of the FP-determinant ("Gribov-horizons").
Exactly one solution exists in the "fundamental domain"
$0 < \phi_3 < \infty \ \wedge \ \phi_3/\sqrt{3} < y_3 < \infty$, and we can replace

$$\int_{-\infty}^{+\infty} \prod_{a=1}^{8} dA_{a3} \quad \to \quad \int_0^\infty d\phi_3 \int_{\phi_3/\sqrt{3}}^\infty dy_3\ \phi_3^2\, \left(\phi_3^2 - 3y_3^2\right)^2 \propto \int_0^\infty r dr \int_{\pi/6}^{\pi/2} d\psi \cos^2(3\psi)$$

Symmetric gauge for SU(3): 2 spatial dimensions

For two spatial dimensions, one can show that (putting $W_1 \equiv X_1, W_2 \equiv -X_2$)

$$A^{(2d)} = \begin{pmatrix} A_{11} & A_{12} & 0 \\ A_{21} & A_{22} & 0 \\ A_{31} & A_{32} & 0 \\ A_{41} & A_{42} & 0 \\ A_{51} & A_{52} & 0 \\ A_{61} & A_{62} & 0 \\ A_{71} & A_{72} & 0 \\ A_{81} & A_{82} & 0 \end{pmatrix} \to \widehat{S}^{(2d)}_{\text{intr}} = \left(\begin{array}{ccc} \phi_1 & 0 & 0 \\ 0 & \phi_2 & 0 \\ 0 & 0 & 0 \\ \hline 0 & x_3 & 0 \\ x_3 & 0 & 0 \\ 2x_2 & 2x_1 & 0 \\ -\frac{\sqrt{3}}{2}y_1 - \frac{1}{2}x_1 & \frac{\sqrt{3}}{2}y_2 + \frac{1}{2}x_2 & 0 \\ -\frac{\sqrt{3}}{2}y_1 + \frac{1}{2}x_1 & -\frac{\sqrt{3}}{2}y_2 + \frac{1}{2}x_2 & 0 \end{array}\right)$$

consistently reduces the above equs. for S to a system of 7 equs. for 8 physical fields (incl. rot.-angle γ), which, adding as an 8th equ. $(d_{\hat{a}\hat{b}\hat{c}}\widehat{S}_{\hat{b}1}\widehat{S}_{\hat{c}2})^2 = (d_{abc}A_{b1}A_{c2})^2$, can be solved numerically for randomly generated $A^{(2d)}$, again yielding solutions separated by horizons. Restricting to a fundamental domain

$$\int_{-\infty}^{+\infty} \prod_{a,b=1}^{8} dA_{a1} dA_{b2} \to \int_{0<\phi_1<\phi_2<\infty} d\phi_1 d\phi_2 (\phi_2 - \phi_1) \int_{R_1(\phi_1,\phi_2)} dx_1 dx_2 dx_3 \int_{R_2(x_1,x_2,x_3,\phi_1,\phi_2)} dy_1 dy_2 \; \mathcal{J}$$

Due to the difficulty of the FP-determinant, I have, however, not yet succeeded in a satisfactory description of the regions R_1 and R_2.

For 3 dimensions, I have found several solutions of the S-equations numerically for a randomly generated A, but to write the corresponding unconstrained integral over a fundamental domain is a difficult, but I think solvable, future task.

Conclusions

- Using a canonical transformation of the dynamical variables, which Abelianises the non-Abelian Gauss-law constraints to be implemented, a reformulation of QCD in terms of gauge invariant dynamical variables can be achieved.
- Using minimal embedding, the symmetric gauge $\epsilon_{ijk}A_{jk} = 0$ for $SU(2)$ can be generalized to the corresponding $SU(3)$ symmetric gauge $c_{abi}A_{bi} = 0$.
- The exact implementation of the Gauss laws reduces the colored spin-1 gluons and spin-1/2 quarks to unconstrained colorless spin-0, spin-1, spin-2 and spin-3 glueball fields and colorless Rarita-Schwinger fields respectively.
- The obtained physical Hamiltonian admits a systematic strong-coupling expansion in powers of $\lambda = g^{-2/3}$, equivalent to an expansion in the number of spatial derivatives. The strong coupling expansion in $g^{-2/3}$ for large box volumes is similar to Lueschers weak coupling expansion in $g^{2/3}$ applicable for small boxes.
- The leading-order term $\longrightarrow$ non-interacting hybrid-glueballs, low-lying masses can be calculated with high accuracy by solving the Schrödinger-equation of Dirac-Yang-Mills QM of spatially constant fields (at the moment only for the unphysical, but technically much simpler 2-color case).
- Higher-order terms in $\lambda \longrightarrow$ interactions between the hybrid-glueballs can be taken into account systematically, using perturbation theory in λ, and quite accurate results can in principle be obtained for the energy-momentum relation of glueballs. It allows for the study of the difficult questions of Lorentz invariance and coupling constant renormalisation in the IR.
- The conversion of color to spin in the reduction process might allow for interesting possible insights into low energy Spin-Physics.
- Gauge reduced approach is difficult (due to the complicated Jacobian), but possible and direct. It should be a useful alternative to lattice calculations.
- The investigation can be extended to flux-tubes (string-tension).