Non-local PNJL models and quark matter in compact stars

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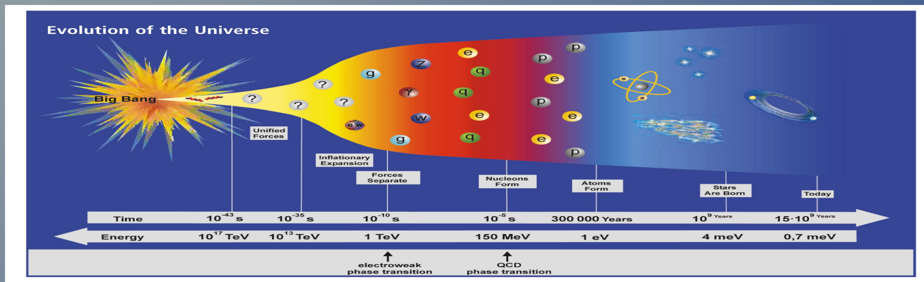
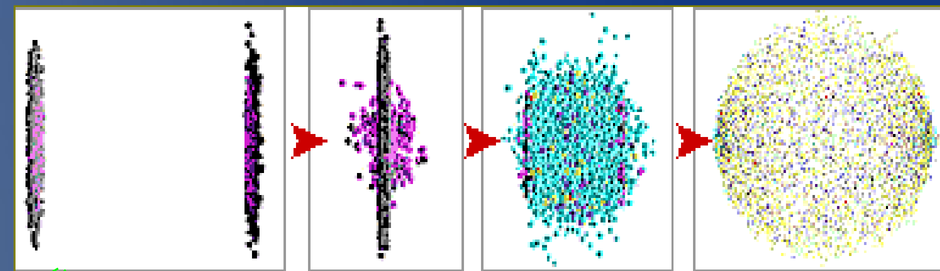
Outline

- Motivation: Heavy-Ion Collisions and Astrophysics
- The model and its parameterizations
- Thermodynamic properties and phase transitions
- Phase diagrams and critical endpoint (CEP)
- Equation of state for compact stars
- Masquerade problem and beyond meanfield
- Outlook: Beth-Uhlenbeck EoS

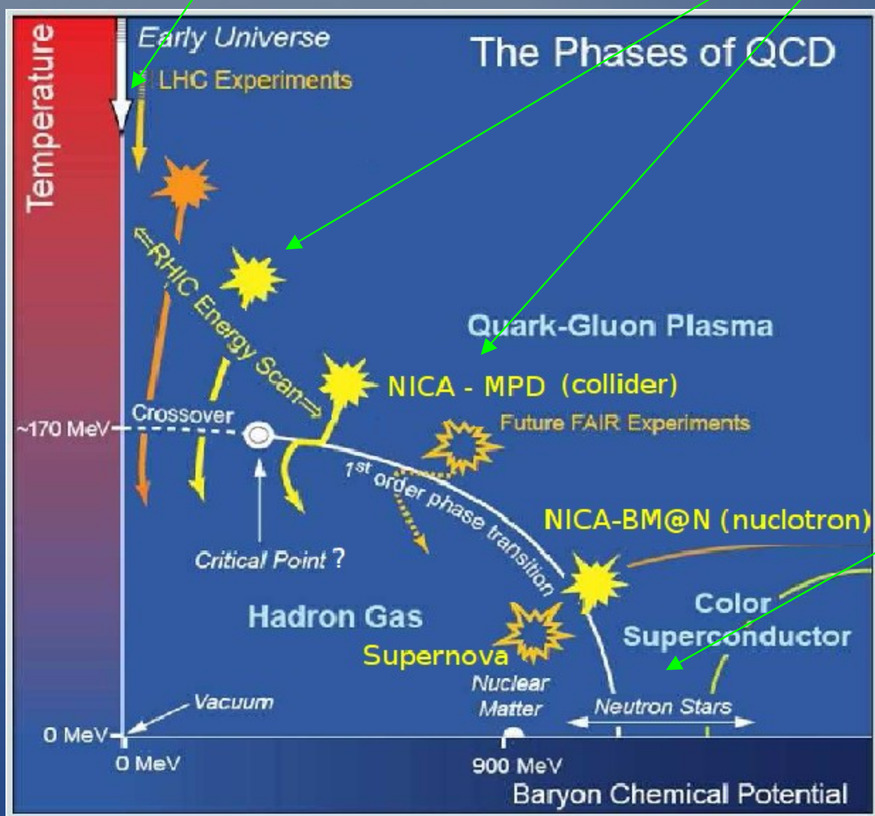
Motivation



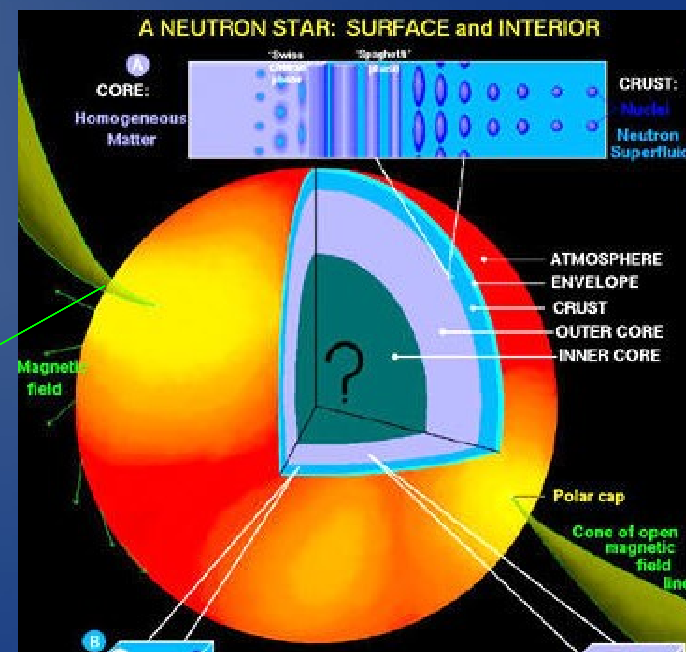
Cosmology



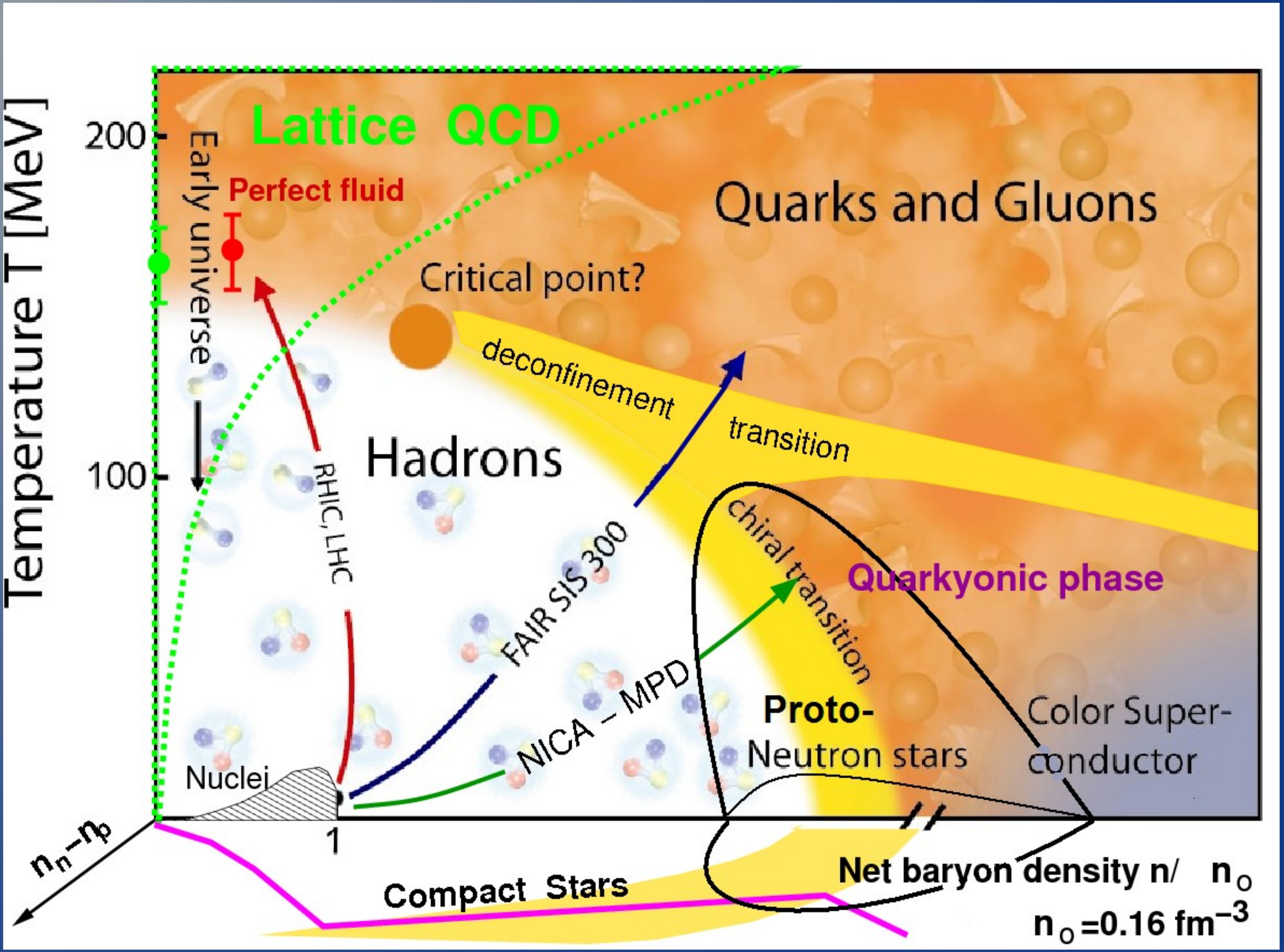
Heavy ion collisions (RHIC, LHC, FAIR, NICA ...)



QCD Phase Diagram?



Compact star astrophysics



Temperature T [MeV]
200
100
Lattice QCD
Early universe
Perfect fluid
Quarks and Gluons
Critical point?
deconfinement
transition
Hadrons
RHIC, LHC
FAIR SIS 300
NICA – MPD
chiral transition
Quarkyonic phase
Proto-
Neutron stars
Color Super-
conductor
Nuclei
1
$n_n - n_p$
Compact Stars
Net baryon density n/ n_o
n_o=0.16 fm^{-3}

QCD Theory and (local) NJL model

$$\mathcal{L}_{QCD} = \bar{q}(i\gamma_\mu D^\mu - m)q - \frac{1}{4}G^a_{\mu\nu}G_a^{\mu\nu}$$

$$D_\mu = \partial_\mu - ig\lambda_a A^a_\mu \quad ; \quad G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f_{abc} A^b_\mu A^c_\nu$$

-Asymptotic freedom (high energy): almost free quarks, interactions can be determined by using perturbation theory.

-Confinement (~ 1GeV): highly non linear, confined quarks. At this range of energies mesonic properties -such as masses, coupling and decay constants, mixing angles, etc- cannot be studied directly from continuum QCD.

$$\mathcal{L}_{NJL_{local}} = \bar{q}(i\partial\!\!\!/ - m)q + g[(\bar{q}q)^2 + (\bar{q}i\gamma_5\tau q)^2]$$

Non-local interactions are proposed to solve standard NJL problems:

- Absence of a confinement mechanism
- Regularization must be added to remove divergences

Non-local extended NJL models

$$L_{noLoc} = \bar{q}(p)\,(\not{p} - m)\,q(p')\,(2\pi)^4 \delta^{(4)}(p - p') + L_{int\_noLoc}$$

$$L_{int\_noLoc} = -\frac{G^2}{2}[\,j_\mu^a(p_1, p'_1) D_{\mu\nu}^{ab}(p_1, p_2, p'_1, p'_2)\, j_\mu^b(p_2, p'_2)]$$

Non-local color currents after Fierz transformations

$$j_\mu^a(p, p') = g(p, p')\bar{q}(p)\tau_a q(p')$$

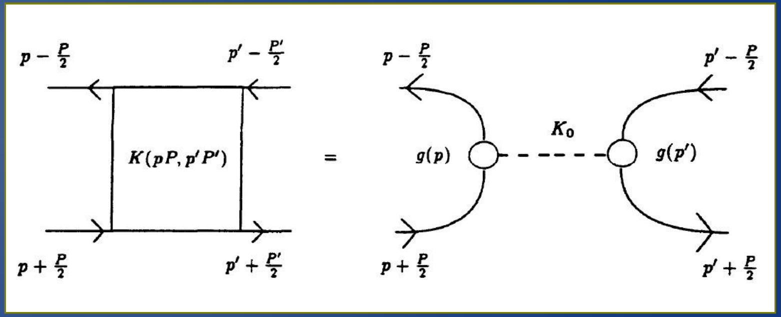
Gluon effective propagator:

$$D_{\mu\nu}^{ab}(p_1, p_2, p'_1, p'_2) = g_{\mu\nu}\delta^{ab} D(p_1, p_2, p'_1, p'_2)$$

Four point interaction Kernel

$$K(p_1, p_2, p'_1, p'_2) = g(p_1, p'_1)\, D(p_1, p_2, p'_1, p'_2)\, g(p_2, p'_2)$$

$$K(p, P; p', P') = -K_0 g(p) g(p') \delta_{P,P'}$$



The use of non-local interactions has the advantage that, with an adequate selection of the form factor(s), it could be achieved that the fermion propagator has no real mass poles. So the quarks don't appear as asymptotic states, which could be interpreted as realization of confinement in low-energy QCD.

D. Blaschke, S. Schmidt, Y. Kalinovsky. Phys. Rev. C 50, 435 (1994)

Non-local extended NJL model with WFR

$$S_E = \int d^4x \left\{ \bar{\psi}(x)(i\partial\!\!\!/ + m_q)\psi(x) - \frac{G_s}{2}\left[j_a(x) j_a(x) + j_P(x) j_P(x) \right] \right\}$$

Nonlocal currents: equivalent to separable form of gluon propagator

$$j_a(x) = \int d^4z \; g(z) \bar{\psi}(x+\tfrac{z}{2}) \Gamma_a \psi(x-\tfrac{z}{2}) \qquad \Gamma_a = (\mathbf{1}, i\gamma_5 \vec{\tau})$$

$$j_P(x) = \int d^4z \, f(z) \bar{\psi}(x+\tfrac{z}{2}) \frac{i\overleftrightarrow{\partial\!\!\!/}}{2\kappa_p} \psi(x-\tfrac{z}{2})$$

Bosonization (Hubbard-Stratonovich) and Mean Field Aproximation

$$S_E^{(MFA)} = -4N_c \int \frac{d^4p}{(2\pi)^4} \ln\left[\frac{(p)^2 + M^2(p)}{Z^2(p)} \right] + \frac{\sigma_1^2}{2G_S} + \frac{\kappa_p^2 \sigma_2^2}{2G_S}$$

$$Z(p) = (1 - \sigma_2 f(p))^{-1}$$

$$M(p) = Z(p)(m_q + \sigma_1 g(p))$$

Dynamical mass function M(p)

Wave function renormalization Z(p)

S. Noguera, N. N. Scoccola, Phys.Rev. **D78** (2008) 114002.

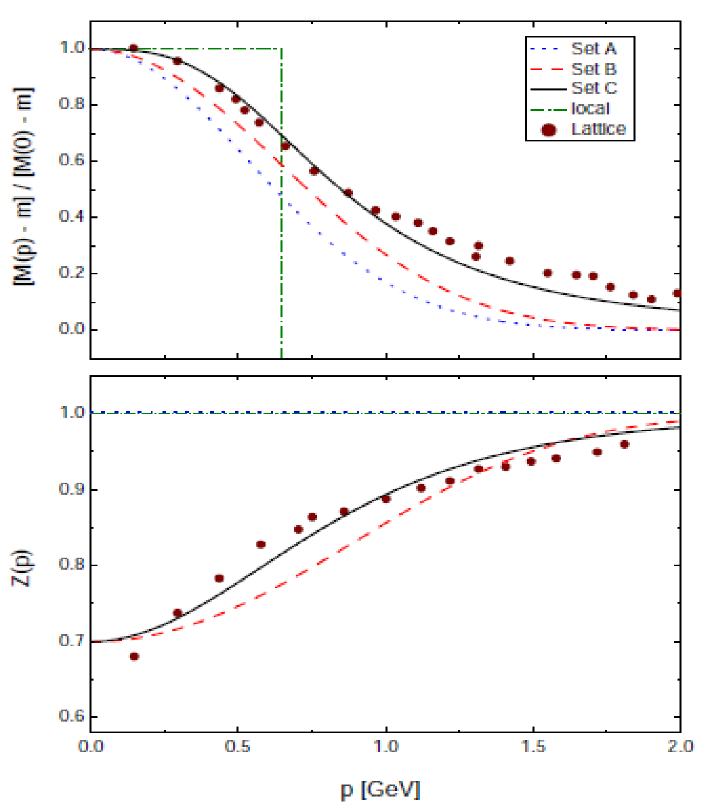
Non-local extended NJL model with WFR

Parameterization w/o WFR

Exponential (Set A)

$$g(p) = e^{-(p^2/\Lambda_0^2)}$$

$$f(p) = 0 \quad , \quad \sigma_2 = 0$$



M.B. Parapilly et al., PRD 73 (2006) 054504

Parameterizations with WFR

Exponential (Set B)

$$f(p) = e^{-(p^2/\Lambda_1^2)}$$

$$g(p) = e^{-(p^2/\Lambda_0^2)}$$

Lattice adjusted Lorentzian (Set C)

$$f(p) = \frac{1+\alpha_z}{1+\alpha_z f_z(p)} f_z(p)$$

$$g(p) = \frac{1+\alpha_z}{1+\alpha_z f_z(p)} \frac{\alpha_m f_m(p) - m_q \alpha_z f_z(p)}{\alpha_m - m_q \alpha_z}$$

$$f_m(p) = [1+(p^2/\Lambda_0^2)^{3/2}]^{-1} \quad , \quad f_z(p) = [1+(p^2/\Lambda_1^2)]^{-5/2}$$

$$\alpha_z = -0.3 \quad , \quad \alpha_m = 309\, MeV$$

S. Noguera, N. N. Scoccola, Phys.Rev. **D78** (2008) 114002

Nonlocal PNJL model: including the Polyakov loop

We incorporate the Polyakov loop using covariant derivative

$$D_\mu \equiv \partial_\mu - iA_\mu$$

Assuming that quarks move into a color gauge field

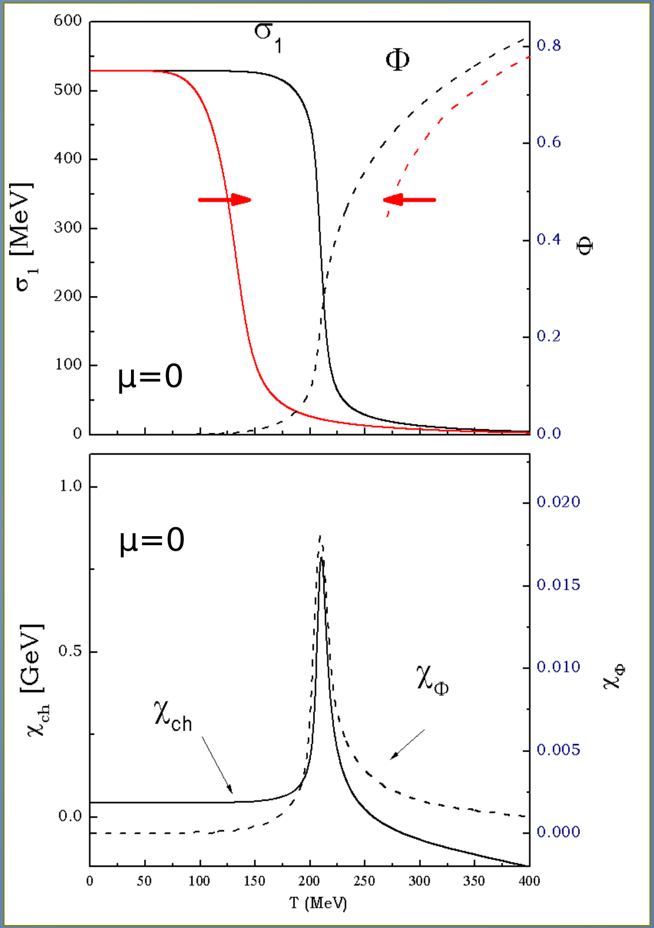
$$\phi = iA_0 = ig\delta_{\mu 0} G_a^\mu \lambda^a / 2$$

And the traced Polyakov loop (parameter of the confinement) results: $\Phi = \frac{1}{3}\mathrm{Tr}\ \exp(i\phi)$

Using the named Polyakov gauge, the matrix ϕ has diagonal representation: $\phi = \phi_3\lambda_3 + \phi_8\lambda_8$

In order to keep Ω_{MFA} real valued :

$$\phi_8 = 0 \ ; \quad \Phi = \tfrac{1}{3}[1 + 2\cos(\phi_3/T)]$$



D.B., M. Buballa, A. Radzhabov, M.K. Volkov, Phys. At. Nucl. **71** (2008) 1981; PRD **83** (2011) 116004

Non-local extended PNJL model at finite T and μ

Matsubara and imaginary time formalisms

$$p^2 \rightarrow \omega_p^2 = (\omega_n - i\mu)^2 + \vec{p}^{\,2} \quad \text{with} \quad \omega_n = (2n+1)\pi T$$

$$\int \frac{d^4 p}{(2\pi)^4} g(p) \rightarrow 2T \sum_{n=0}^{\infty} \int \frac{d^3 \vec{p}}{(2\pi)^3} g(\omega_n - i\mu, \vec{p})$$

Polyakov loop

$$\mu \rightarrow \mu_c = \mu - i\phi_c$$

$$N_c \rightarrow \sum_c \quad \text{with} \quad \phi_c = +\phi_3, 0, -\phi_3 \quad \text{for } r, g, b$$

$$\Omega_{MFA}(T,\mu) = -4T \sum_c \sum_n \int \frac{d^3 \vec{p}}{(2\pi)^3} \ln\left[\frac{\rho_{nc}^2 + M^2(\rho_{nc}^2)}{Z^2(\rho_{nc}^2)}\right] + \frac{\sigma_1^2}{2G_S} + \frac{\kappa_p^2 \sigma_2^2}{2G_S} + \mathcal{U}(\Phi, T)$$

$$\rho_{nc}^2 = \left[(2n+1)\pi T - i\mu + \phi_c\right]^2 + \vec{p}^{\,2}$$

$$\mathcal{U}(\Phi,T) = \left[-\tfrac{1}{2} a(T)\Phi^2 + b(T)\ln(1 - 6\Phi^2 + 8\Phi^3 - 3\Phi^4)\right] T^4$$

$a(T)$, $b(T)$ fitted to lattice QCD results.

S. Roessner, C. Ratti and W. Weise, Phys. Rev. D 75 (2007) 034007
G.A. Contrera, M. Orsaria, N.N. Scoccola, Phys. Rev. D 82 (2010) 054026

Regularization

Ω_{MFA} turns out to be divergent and needs to be regularized. We use

$$\Omega^{reg}_{MFA} = \Omega_{MFA} - \Omega_{free} + \Omega^{reg}_{free} + \Omega_0$$

where:
- Ω_{free} is obtained from Ω_{MFA} by setting $\sigma_1 = \sigma_2 = 0$
- Ω_0 is fixed by the condition $\Omega^{reg}_{MFA} = 0$ at $T = \mu = 0$
- Ω^{reg}_{free} is the regularized expression for the thermodynamical potential in the absence of fermion interactions. It is given by

$$\Omega^{reg}_{free} = -4T \int \frac{d^3\vec{p}}{(2\pi)^3} \sum_c \left[\ln\left(1 + \exp\left[-\frac{E_p + \mu + i\phi_c}{T}\right]\right) + \ln\left(1 + \exp\left[-\frac{E_p - \mu - i\phi_c}{T}\right]\right) \right]$$

with $E_p = \sqrt{\vec{p}^{\,2} + m^2}$

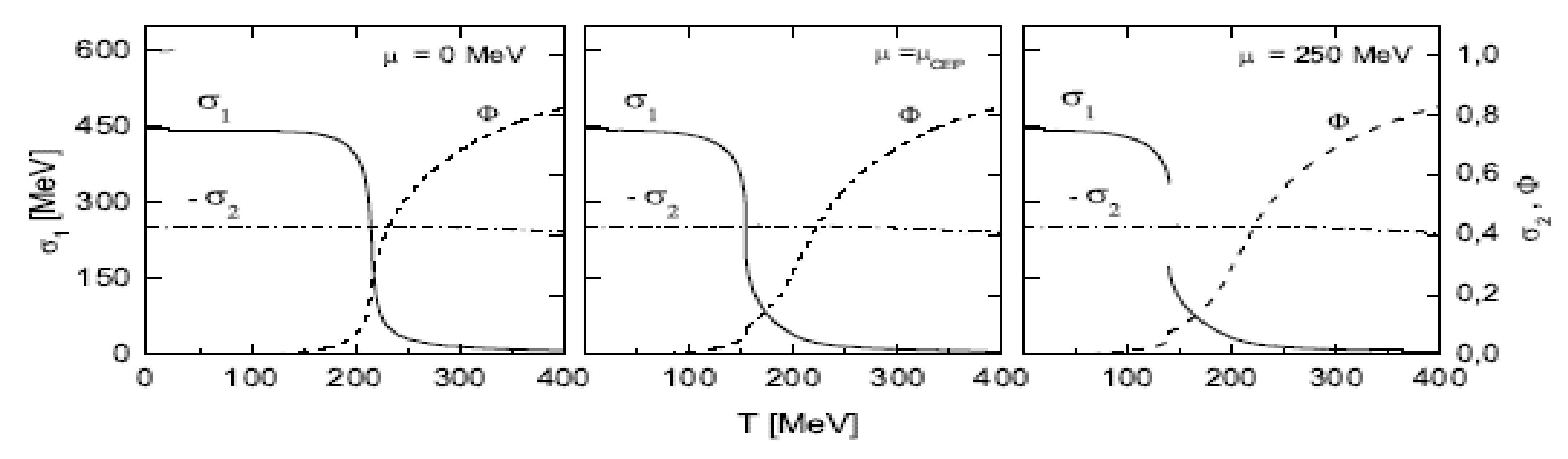
D.Gómez Dumm and N.N. Scoccola, Phys. Rev. C 72 (2005) 014909.

What could be determined with $\Omega^{reg}_{MFA}(T,\mu)$?

- Mean field values $\sigma_{1,2}$ and Φ at a given T and μ

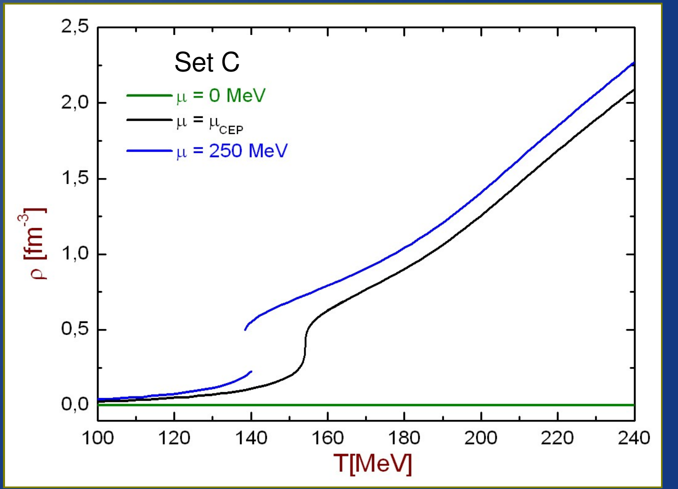
$$\left.\frac{\partial\Omega^{reg}_{MFA}}{\partial\sigma_1}\right|_{T,\mu} = \left.\frac{\partial\Omega^{reg}_{MFA}}{\partial\sigma_2}\right|_{T,\mu} = \left.\frac{\partial\Omega^{reg}_{MFA}}{\partial\Phi}\right|_{T,\mu} = 0$$

"gap" equations



- Quark condensate $\langle\bar{q}q\rangle$; quark density ρ

$$\langle\bar{q}q\rangle = \frac{\partial\Omega^{reg}_{MFA}}{\partial m} \quad ; \quad \rho = -\frac{\partial\Omega^{reg}_{MFA}}{\partial\mu}$$



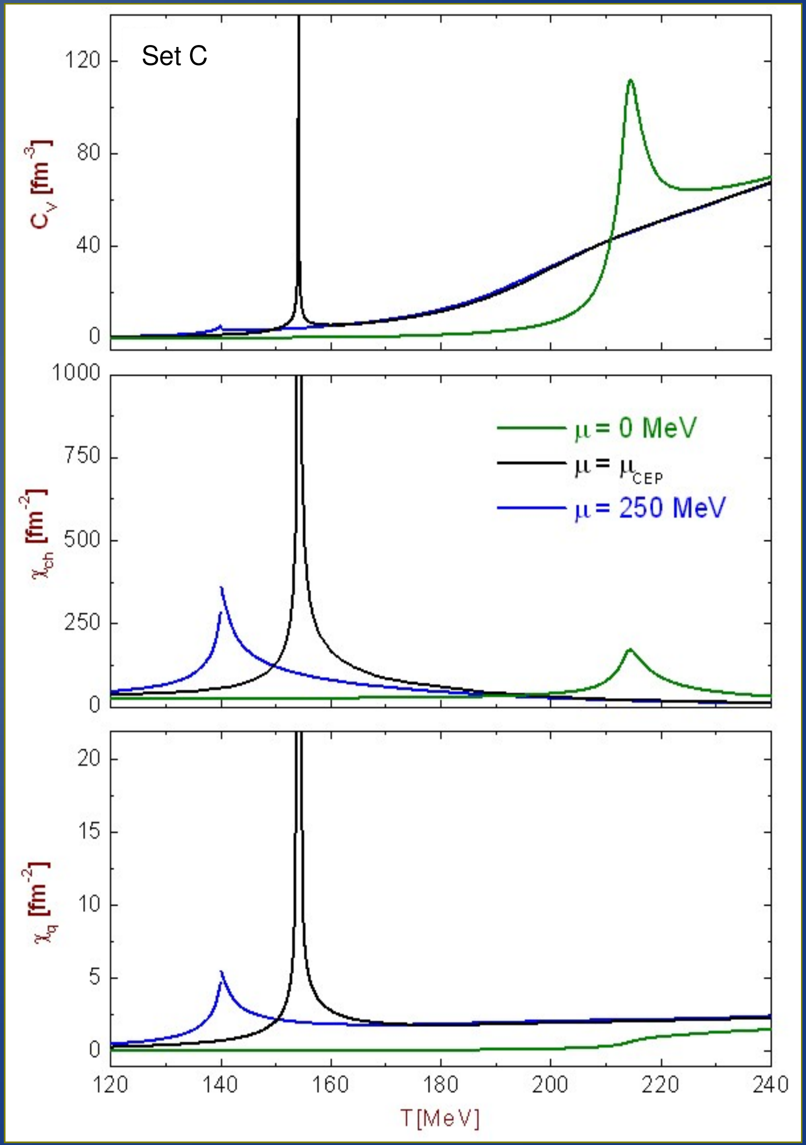
G. Contrera, M. Orsaria, N.N. Scoccola, PRD 82 (2010) 054026

What could be determined with $\Omega^{reg}_{MFA}(T,\mu)$?

- Specific heat: $c_v = -T\dfrac{\partial^2\Omega^{reg}_{MFA}}{\partial T^2}$

- Chiral susceptibility: $\chi_{ch} = \dfrac{\partial\langle\bar{q}q\rangle}{\partial m}$

- Quark number susceptibility: $\chi_q = \dfrac{\partial\rho}{\partial\mu}$



Chemical potential in the Polyakov-loop potentials

$T_0 = 208$ MeV which corresponds to 2 flavors case. Then, following Ref [1], we used also a polinomial ansatz for $\mathcal{U}$ given by

$$\mathcal{U}_2(\Phi, T) = \left[-\frac{b_2(T)}{2}\Phi^2 - \frac{b_3}{6}(\Phi^3 + \bar{\Phi}^3) + \frac{b_4}{4}\Phi^4 \right] T^4 \tag{10}$$

with the temperature-dependent coefficient

$$b_2(T) = a_0 + a_1 \left(\frac{T_0}{T}\right) + a_2 \left(\frac{T_0}{T}\right)^2 + a_3 \left(\frac{T_0}{T}\right)^3 \tag{11}$$

and the following set of parameters, $a_0 = 6.75$, $a_1 = -1.95$, $a_2 = 2.625$, $a_3 = -7.44$, $b_3 = 0.75$, and $b_4 = 7.5$.

In a more recent work [3] it has been proposed the following μ-dependent logarithmic potential:

$$\mathcal{U}_3(\Phi, T, \mu) = (a_0 T^4 + a_1 \mu^4 + a_2 T^2 \mu^2)\Phi^2 + a_3 T_0^4 \ln(1 - 6\Phi^2 + 8\Phi^3 - 3\Phi^4), \tag{12}$$

where the parameters are $a_0 = 1.85$, $a_1 = 1.44\text{x}10^{-3}$, $a_2 = 0.08$, $a_3 = 0.40$.

In the same way, we propose a polynomial potential which include this μ-dependence. So we have:

$$\mathcal{U}_4(\Phi, T) = \left[-\frac{b_2(T)}{2}\Phi^2 - \frac{b_3}{6}(\Phi^3 + \bar{\Phi}^3) + \frac{b_4}{4}\Phi^4 \right] T^4 \tag{13}$$

with the T-μ-dependent coefficient.

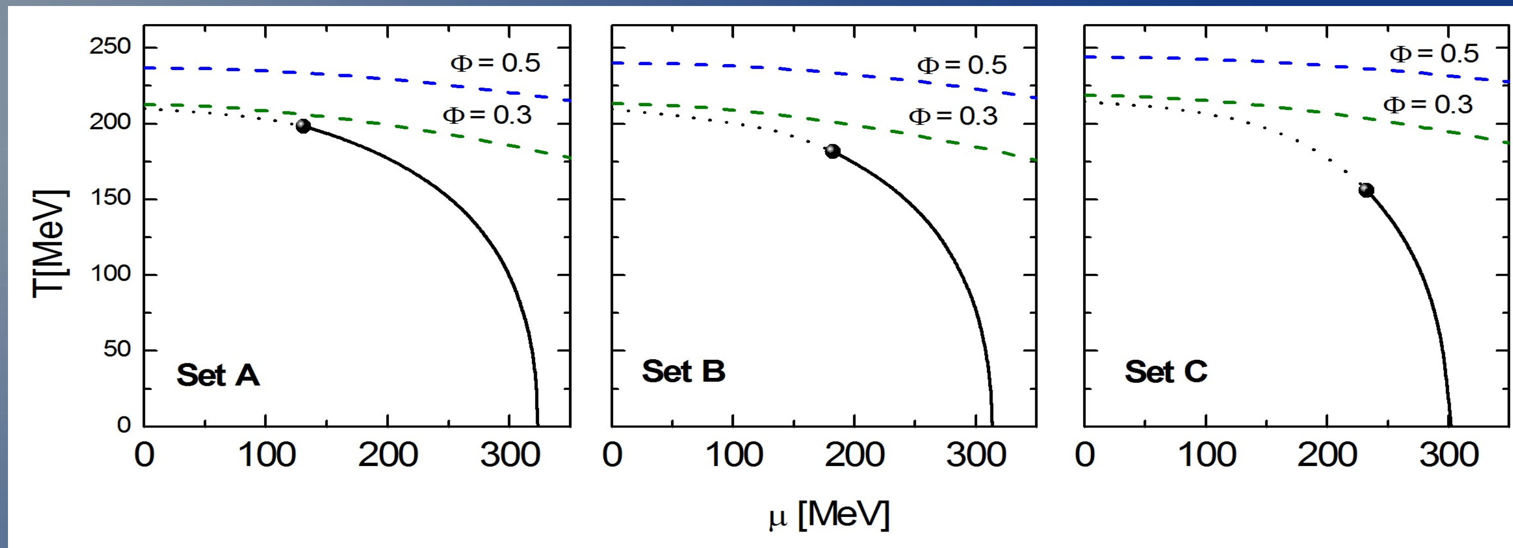
$$b_2(T, \mu) = [a_0 + c_2(\mu/T)^2 + c_4(\mu/T)^4] + a_1 \left(\frac{T_0}{T}\right) + a_2 \left(\frac{T_0}{T}\right)^2 + a_3 \left(\frac{T_0}{T}\right)^3 \tag{14}$$

and the same set of parameters as in the polinomial case, but adding the μ related ones $c_2 = 30/(7\pi^2) = 0.4342$ and $c_4 = 15/(7\pi^4) = 0.02199$

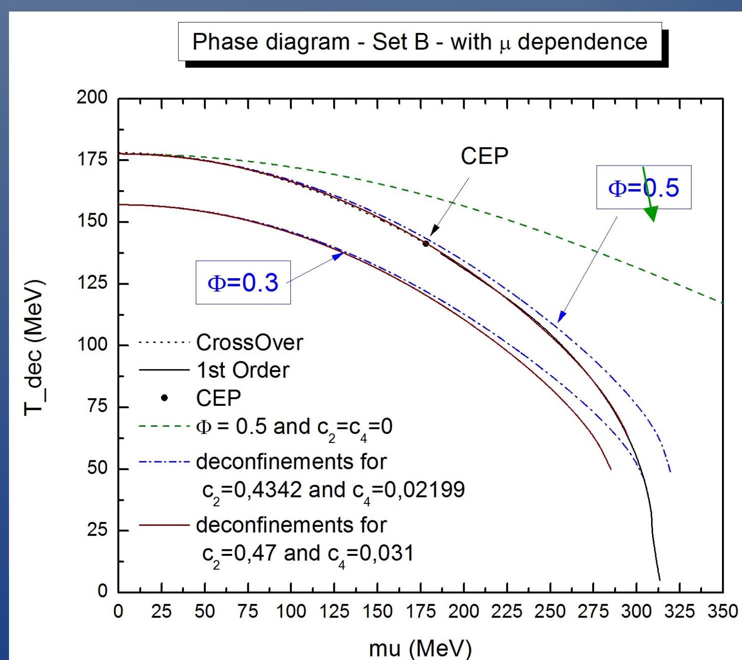
[1] C. Ratti, M. A. Thaler and W. Weise, Phys. Rev. D 73, 014019 (2006)
[3] V. A. Dexheimer and S. Schramm. Phys.Rev.C 81, 045201 (2010)

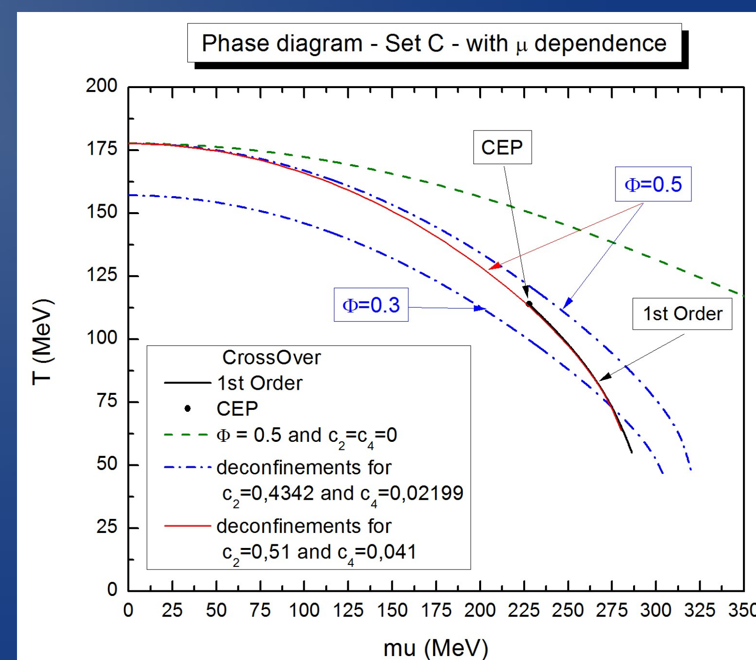
Fresh results: Phase diagrams w. Chiral & Polyakov lines

T_0=270 MeV
and
RRW potential

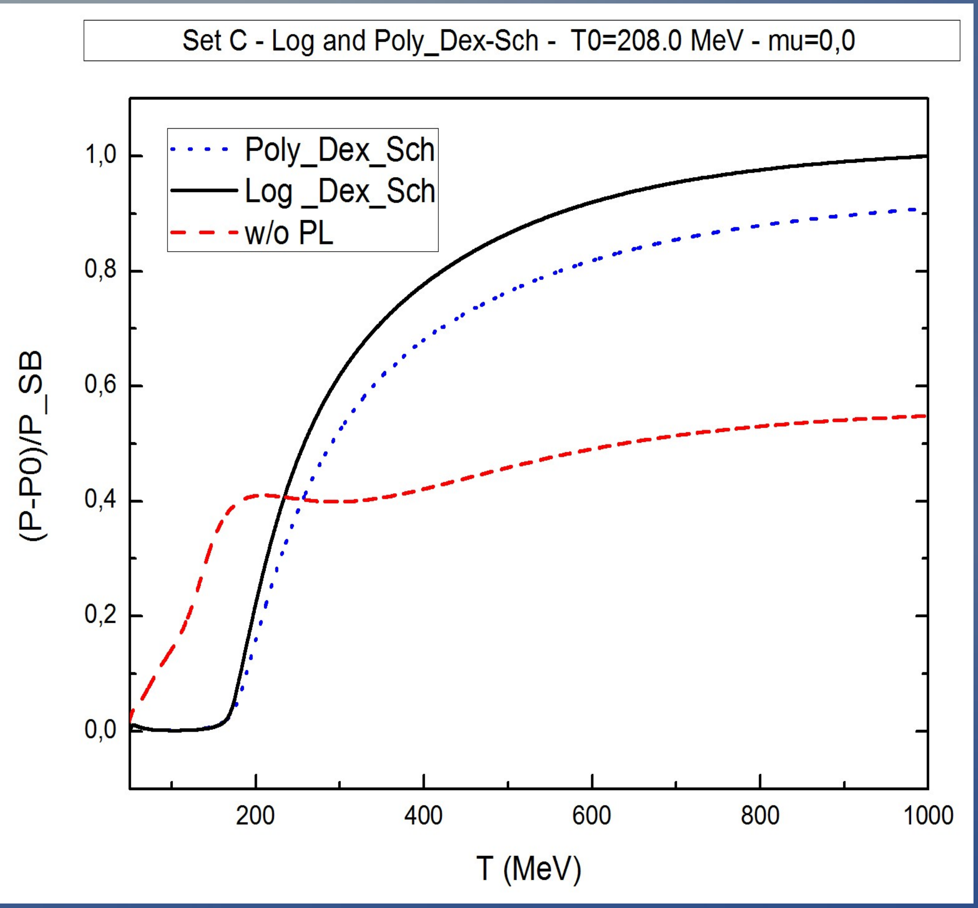


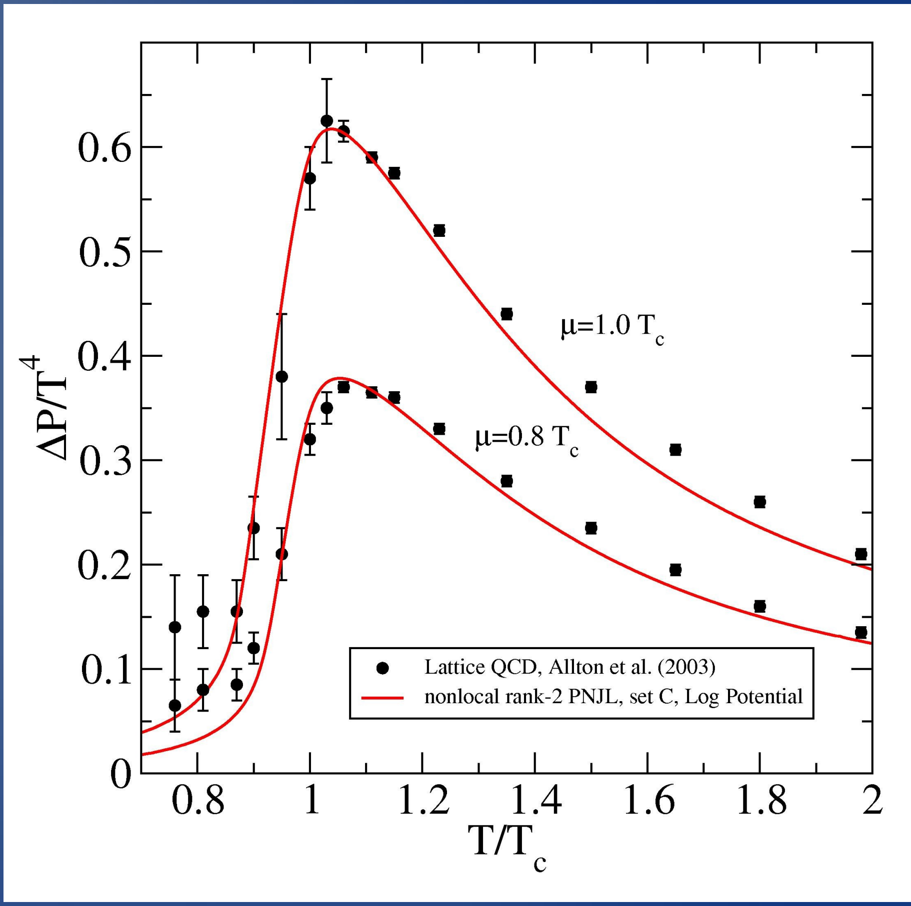
T_0=208 MeV
and
μ-dependent
polynomial
potential





Preliminary results: Pressure & Pressure difference





The pressure difference $\Delta P=P(T,\mu)-P(T,0)$ is very sensitive to the formfactor parametrization and in particular to the vector meson coupling which is important at nonzero μ.

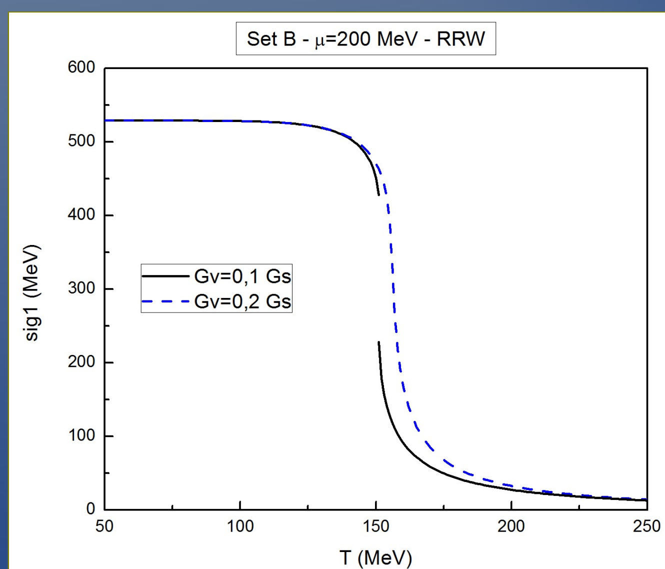
Development : vector interactions + color neutrality

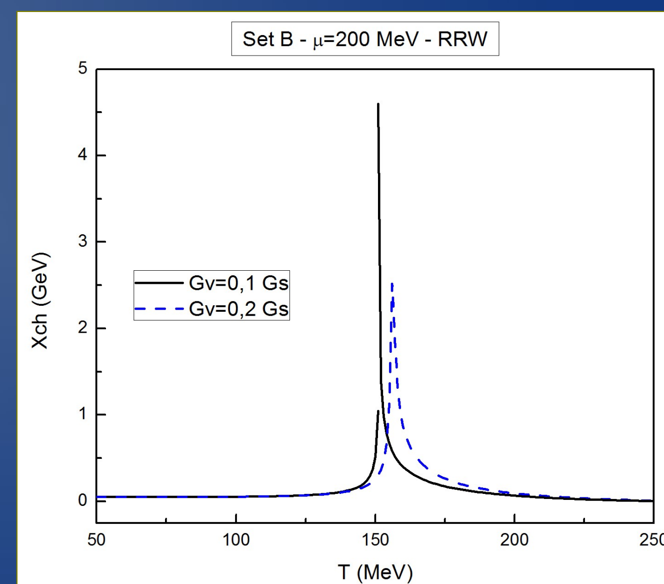
$$\Omega_{MFA}(T,\mu) = -\frac{4T}{\pi^2}\sum_c \sum_n \int \frac{d^3\vec{p}}{(2\pi)^3} \ln\left[\frac{q_{nc}^2 + M^2\left(\rho_{nc}^2\right)}{Z^2\left(\rho_{nc}^2\right)}\right] + \frac{\sigma_1^2}{2G_S} + \frac{\kappa_p^2\,\sigma_2^2}{2G_S} - \frac{\omega_V^2}{2G_V} + U\left(\Phi,\Phi^*,T\right)$$

$$\rho_{nc}^2 = [\omega_n - i\mu_c + \phi_c]^2 + \vec{p}^2 \longrightarrow \begin{array}{l} \phi_c = \lambda_3\phi_3 + \lambda_8\phi_8 \\ \mu_c = \mu I + \lambda_3\mu_3 + \lambda_8\mu_8 \end{array}$$

$$q_{nc}^2 = \left[\omega_n - i\mu_c^r + \phi_c\right]^2 + \vec{p}^2 \longrightarrow \mu_c^r = \mu_c - \omega_V\, g(\rho_{nc}^2)$$

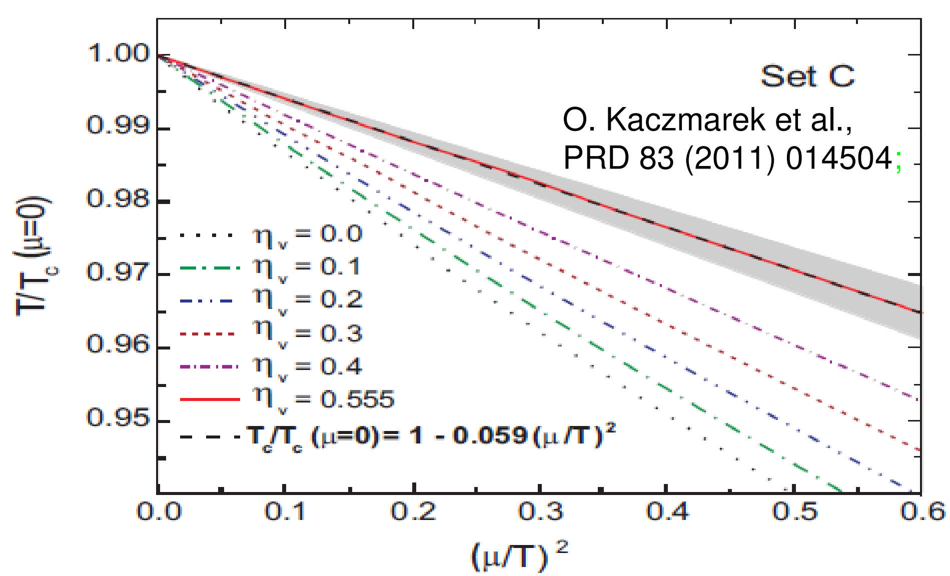
$$\begin{array}{l} \phi_8 = 0 \\ \mu_3 = 0 \\ \mu_8 = 0 \end{array} \longrightarrow$$

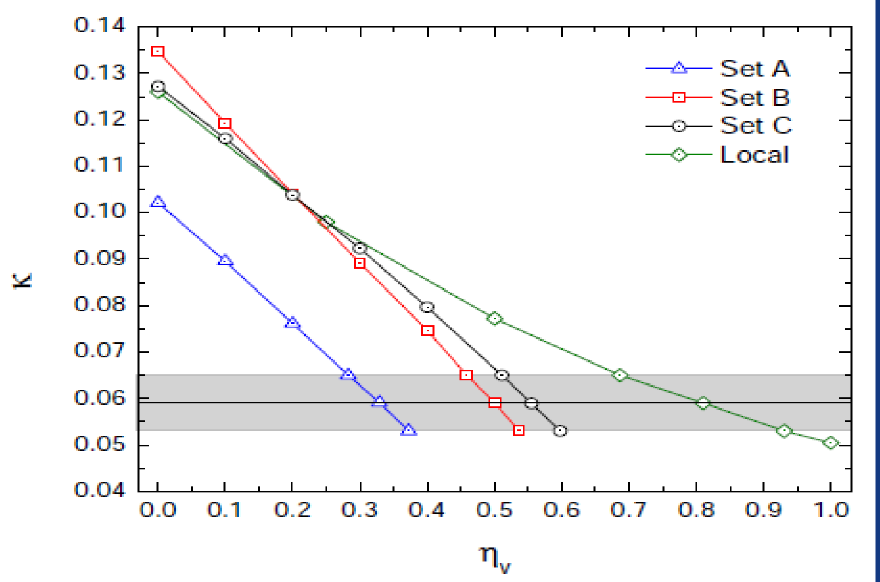


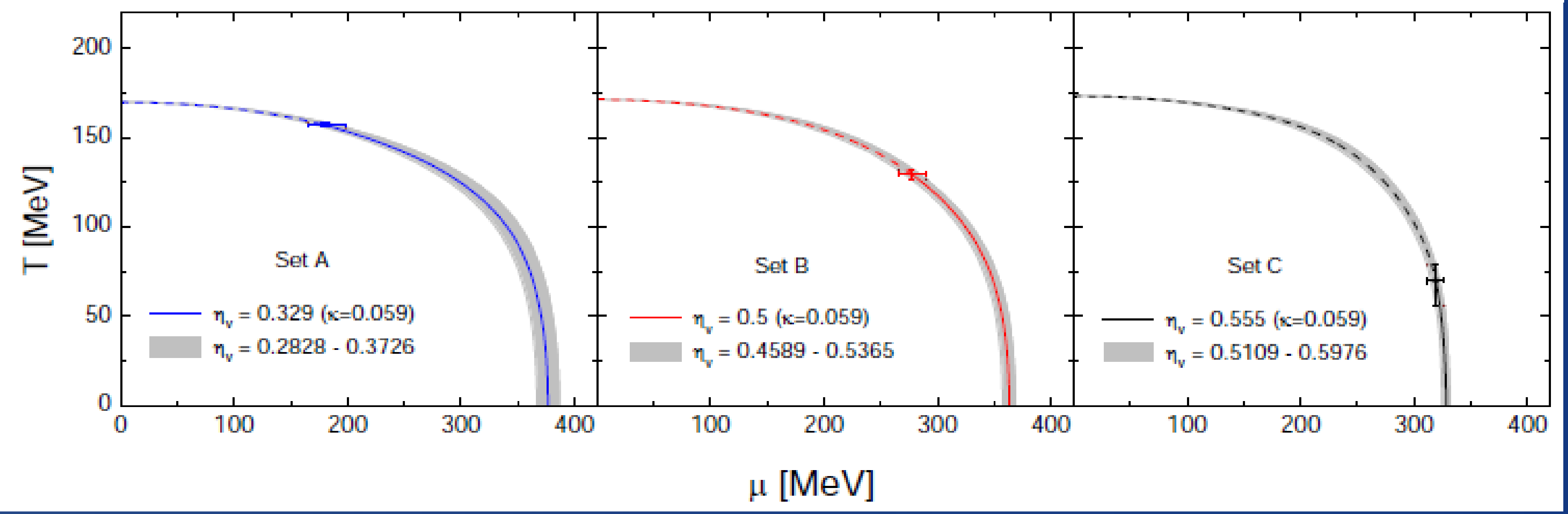


D. Gómez Dumm, D.B. Blaschke, A.G. Grunfeld, N.N. Scoccola, Phys.Rev.D **78**, 114021 (2008).
K. Fukushima, Phys.Rev.D **77**, 114028 (2008).
C. Sasaki, B. Friman, K. Redlich, Phys.Rev.D **75**, 054026(2007).

Vector coupling effects on Phase diagram and CEP

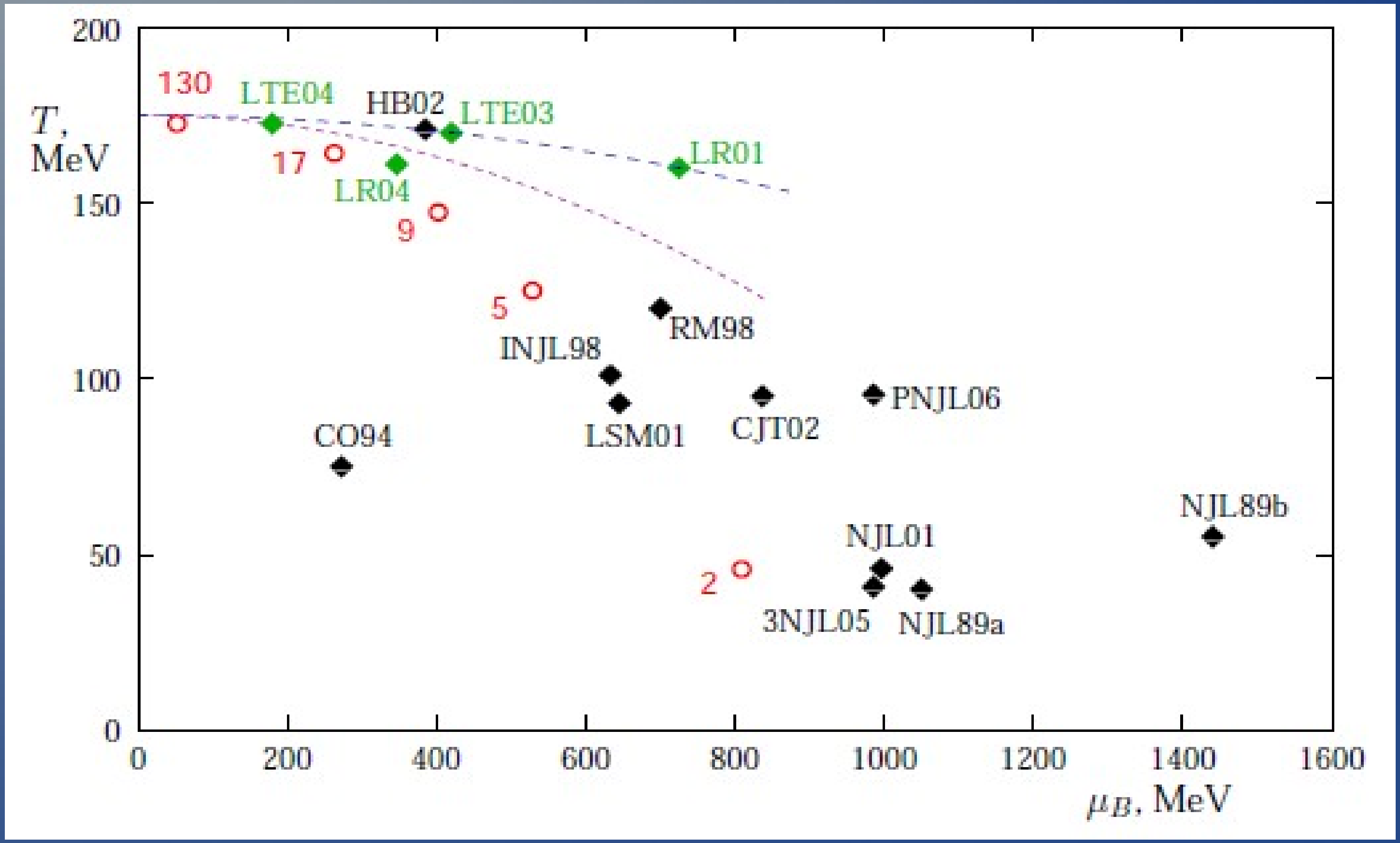






G. Contrera, G. Grunfeld, D.B., arXiv:1207.4890

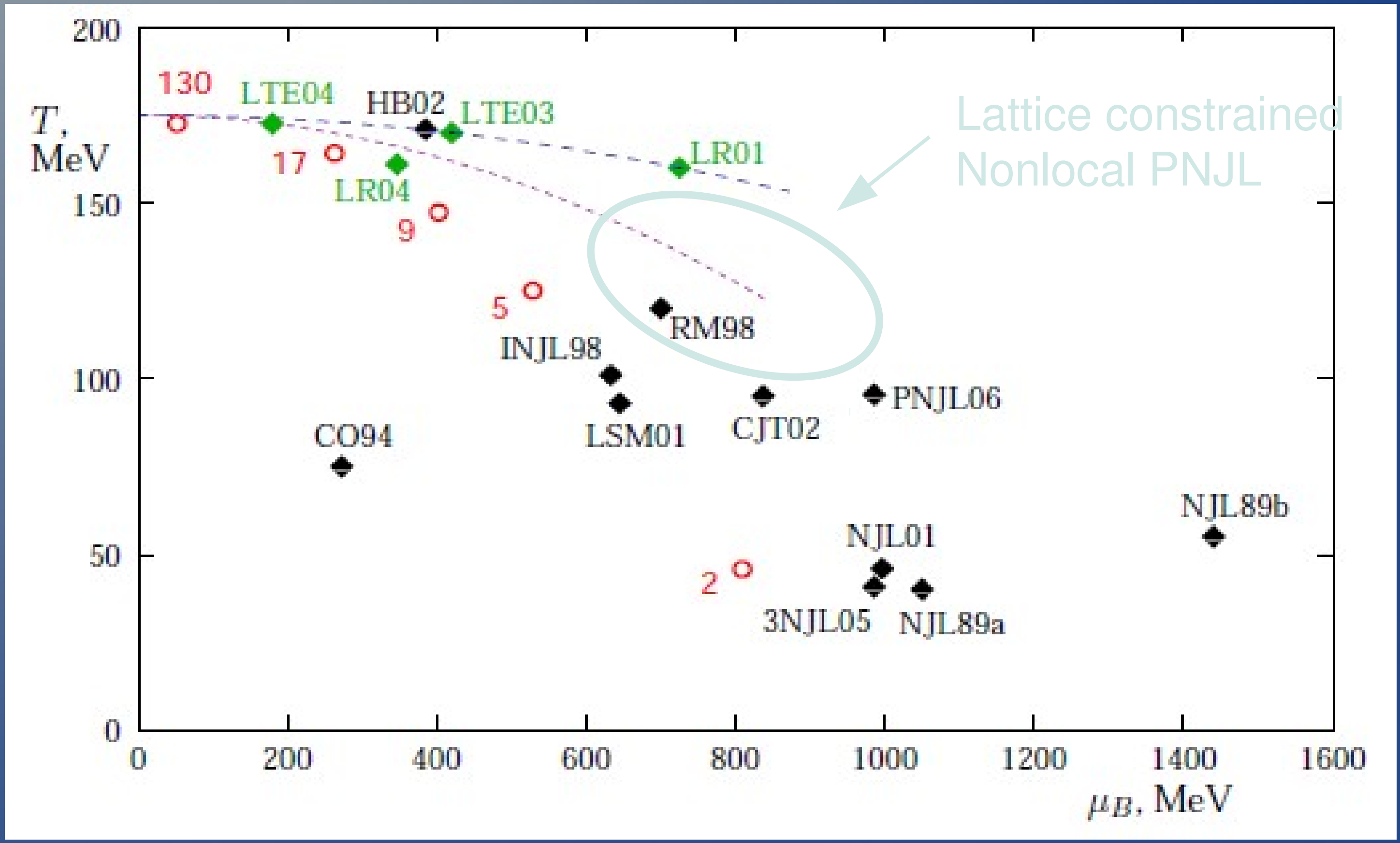
Constrain P-DSE models with lattice data: a solution for the Stephanov problem ?



M. Stephanov, PoS LAT2006, 024 (2006)

Unconstrained models predict a “skymap” of critical points in the QCD ph

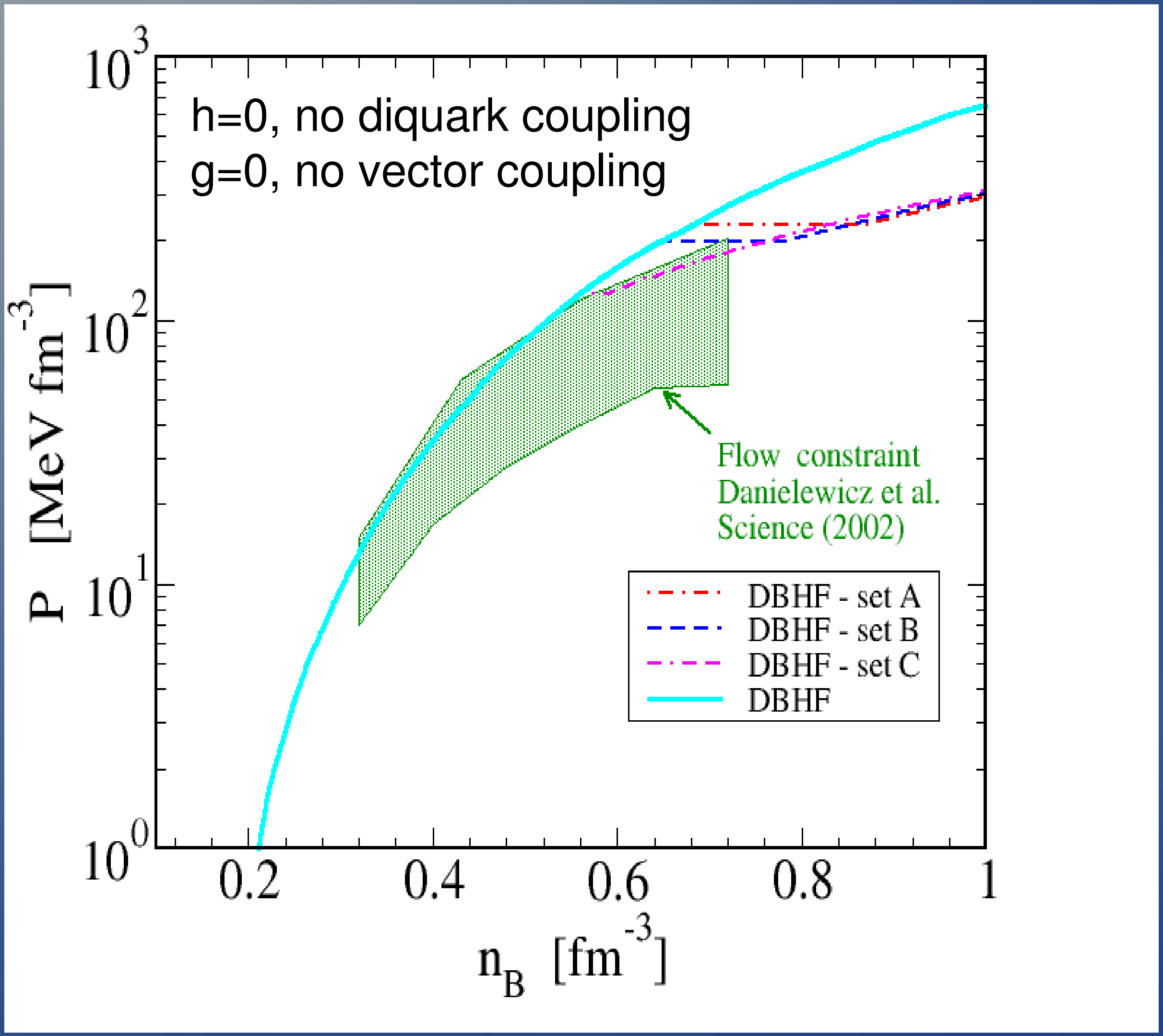
Constrain P-DSE models with lattice data:
a solution for the Stephanov problem ?

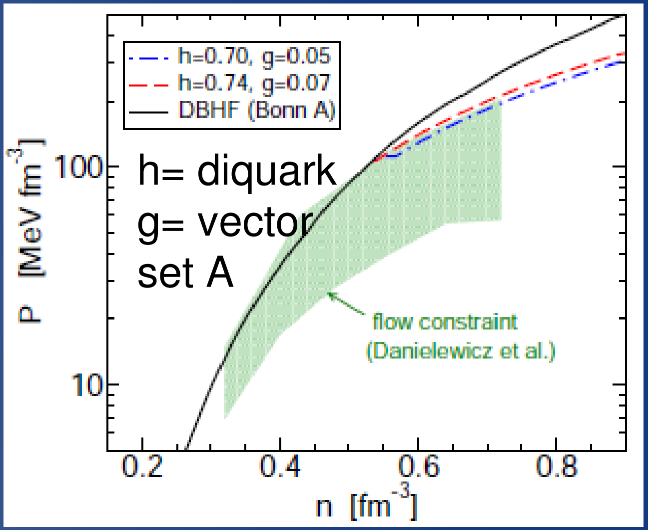


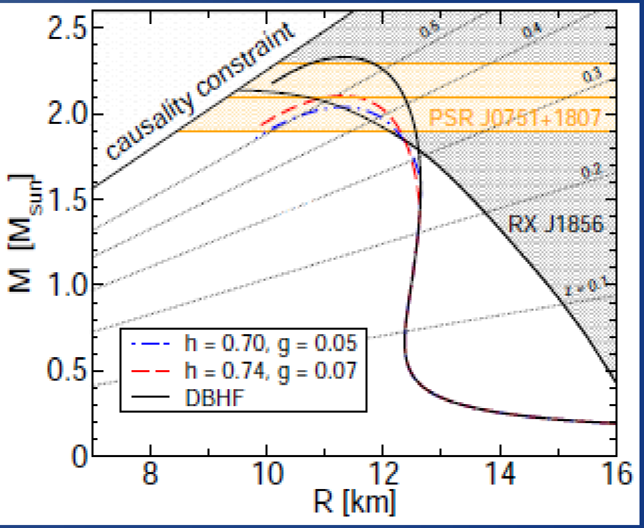
M. Stephanov, PoS LAT2006, 024 (2006)

Unconstrained models predict a “skymap” of critical points in the QCD ph
Lattice constrained Polyakov-DSE (nonlocal PNJL) models favor a much

T=0, Hybrid Equation of State



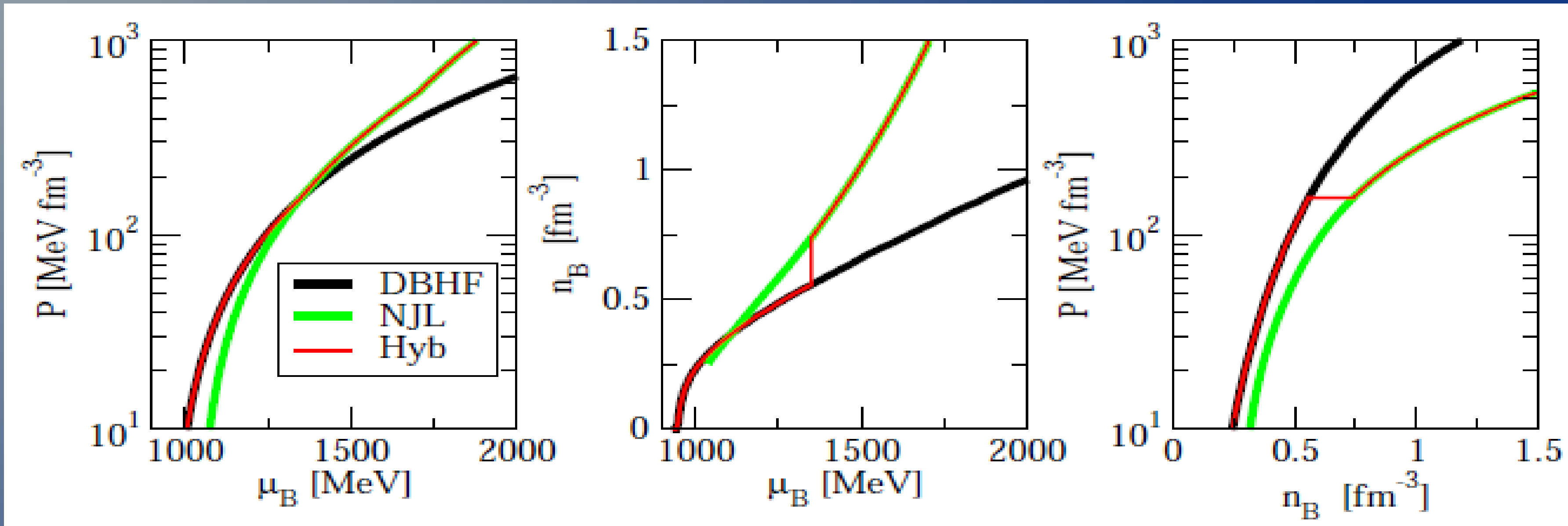




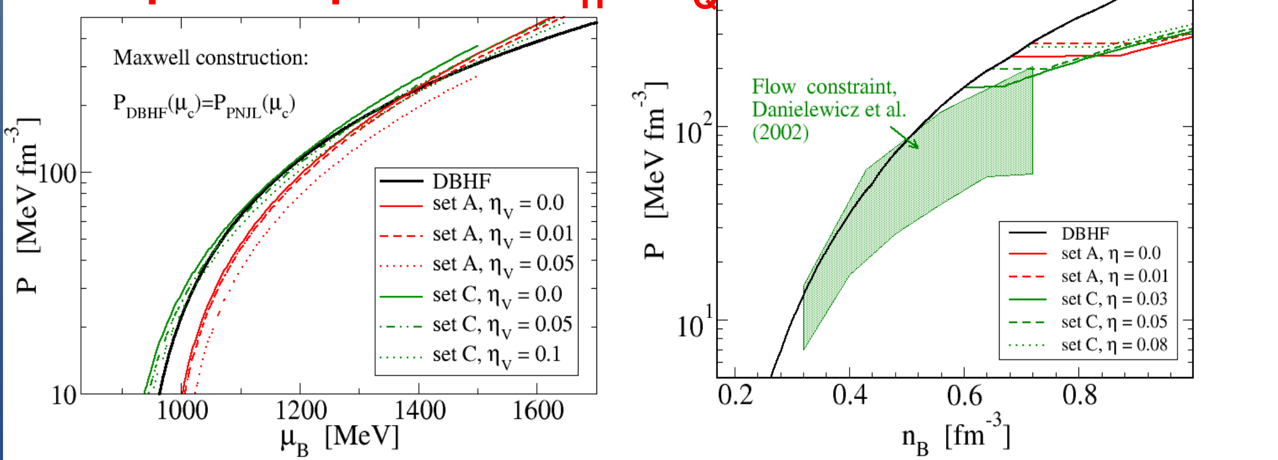
D.B., S. Benic, G. Contrera et al., in preparation (2012)

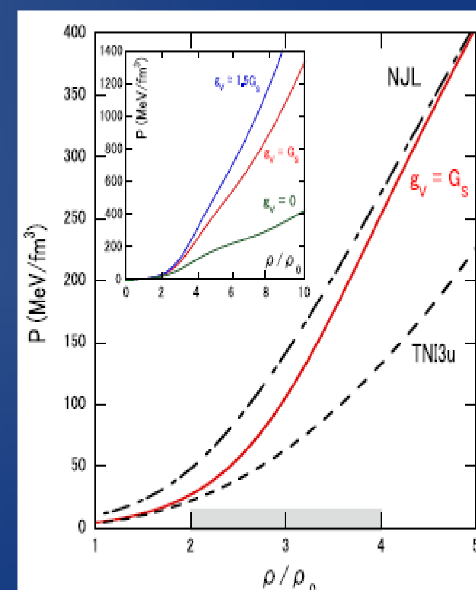
D.B., D. Gomez Dumm, A. Grunfeld, T. Klahn, N. Scoccola, PRC75 (2007)

Phase transition: Maxwell construction



Masquerade problem: $P_H \sim P_Q$





D.B., S. Benic, G. Contrera et al., in preparation (2012)

arxiv:1212.6803

PTEP

Prog. Theor. Exp. Phys. **2012**, 00000 (21 pages)
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Hadron-Quark Crossover and Massive Hybrid Stars

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On the basis of the percolation picture from the hadronic phase with hyperons to the quark phase with strangeness, we construct a new equation of state (EOS) with the pressure interpolated as a function of the baryon density. The maximum mass of neutron stars can exceed $2M_\odot$ if the following two conditions are satisfied; (i) the crossover from the hadronic matter to the quark matter takes place at around three times the normal nuclear matter density, and (ii) the quark matter is strongly interacting in the crossover region. This is in contrast to the conventional approach assuming the first order phase transition in which the EOS becomes always soft due to the presence of the quark matter at high density. Although the choice of the hadronic EOS does not affect the above conclusion on the maximum mass, the three-body force among nucleons and hyperons plays an essential role for the onset of the hyperon mixing and the cooling of neutron stars.

Subject Index Neutron stars, Nuclear matter aspects in nuclear astrophysics, Hadrons and quarks in nuclear matter, Quark matter

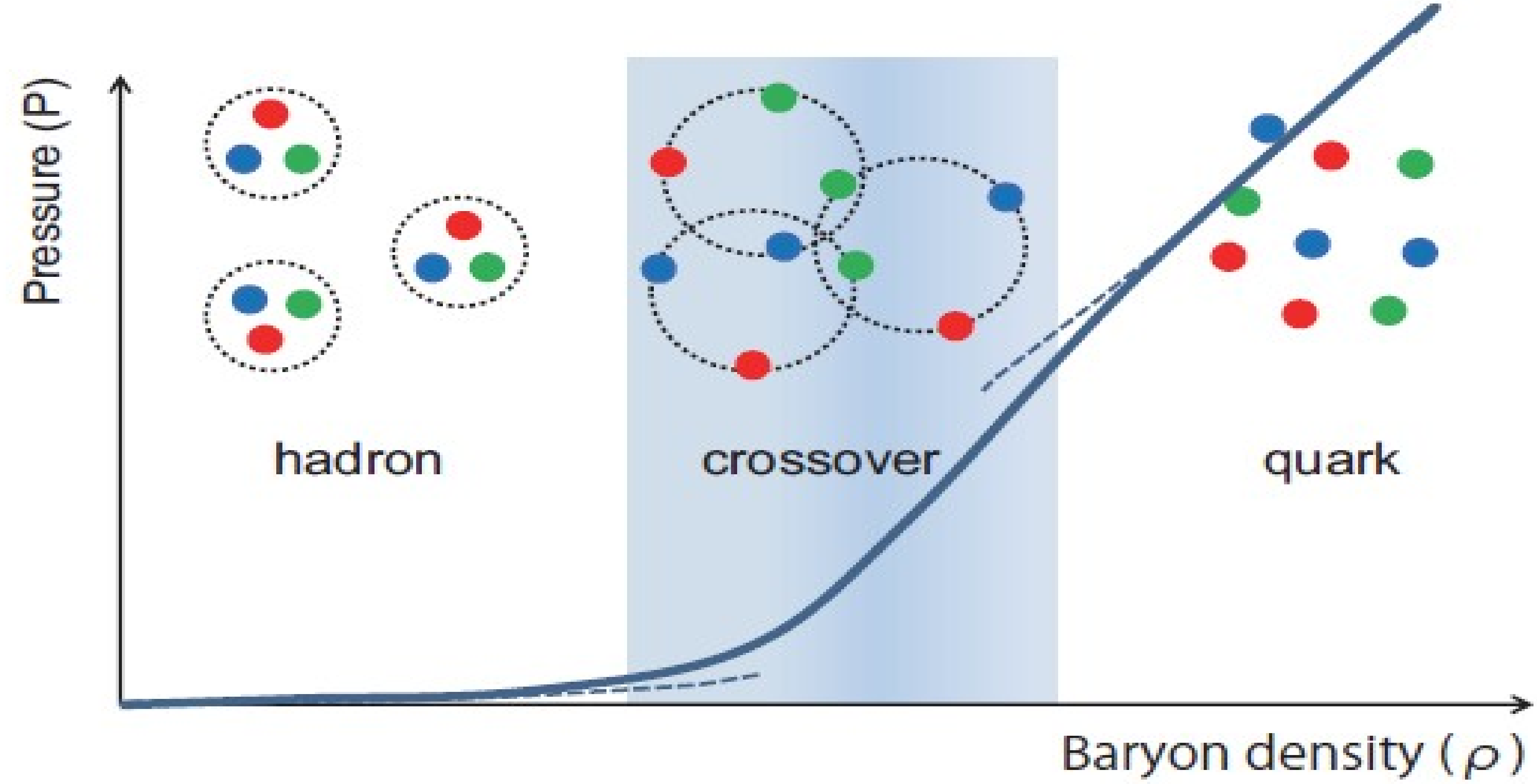
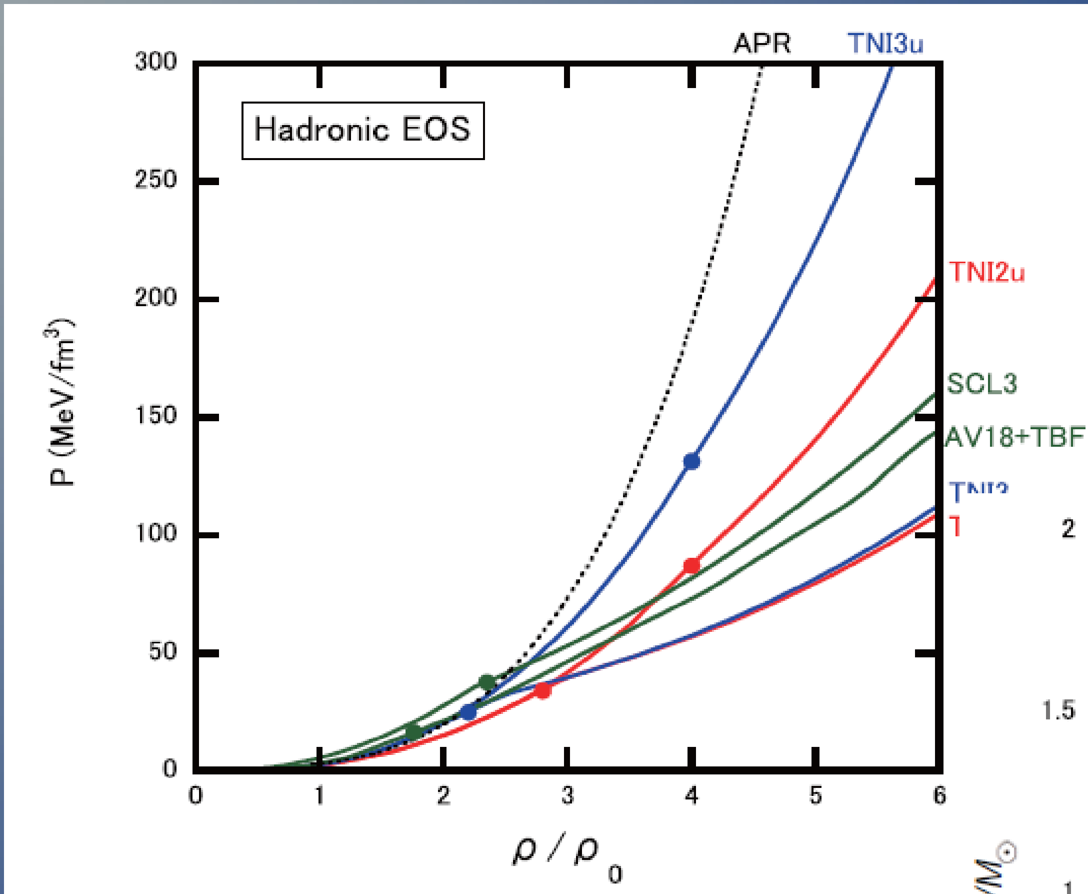
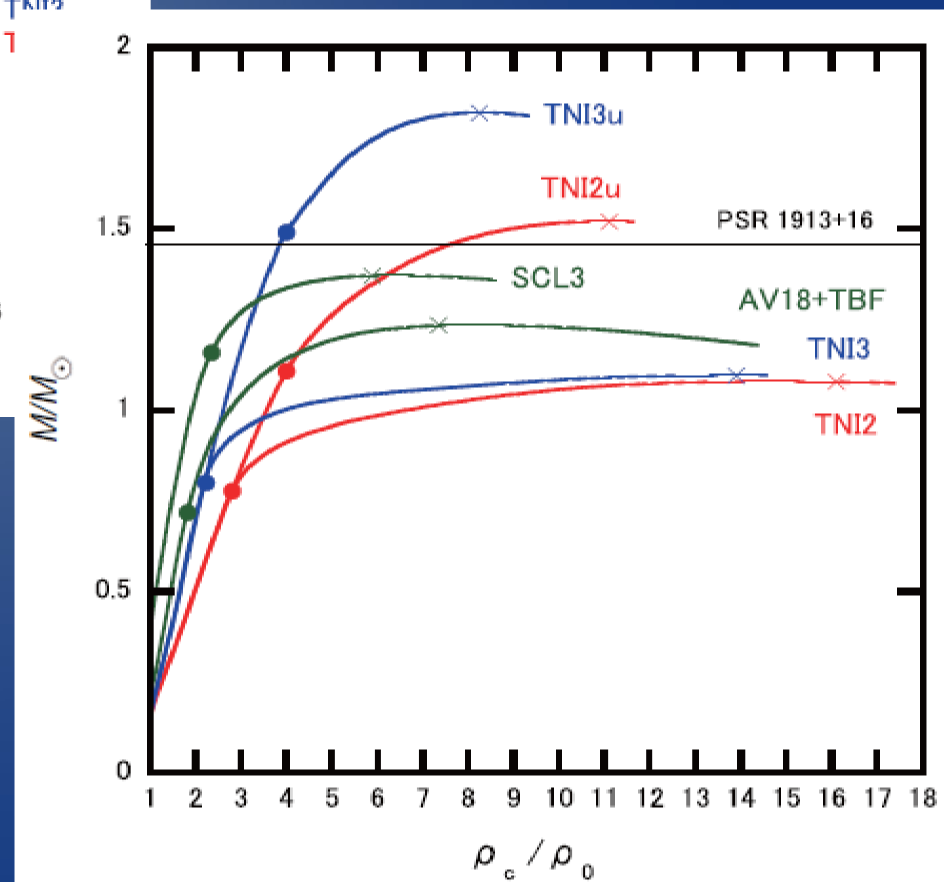


Fig. 1 Schematic picture of the QCD pressure (P) as a function of the baron density (ρ) under the assumption of the hadron-quark crossover. The crossover region where finite-size hadrons start to overlap and percolate is shown by the shaded area. The pressure calculated on the basis of the point-like hadrons (shown by the dashed line at low density) and that calculated on the basis of weakly interacting quarks (shown by the dashed line at high density) lose their validity in the crossover region, so that the naive use of the Gibbs conditions by extrapolating the dashed lines is not justified in general.



Hadronic EoS too soft !!

(Exception: APR)

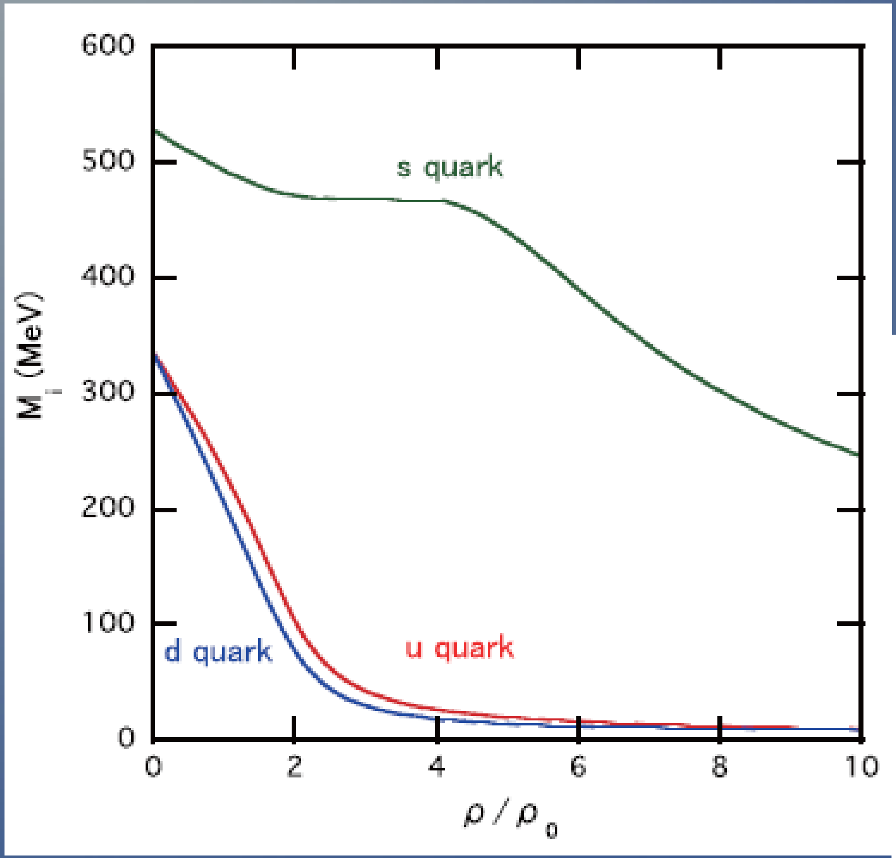


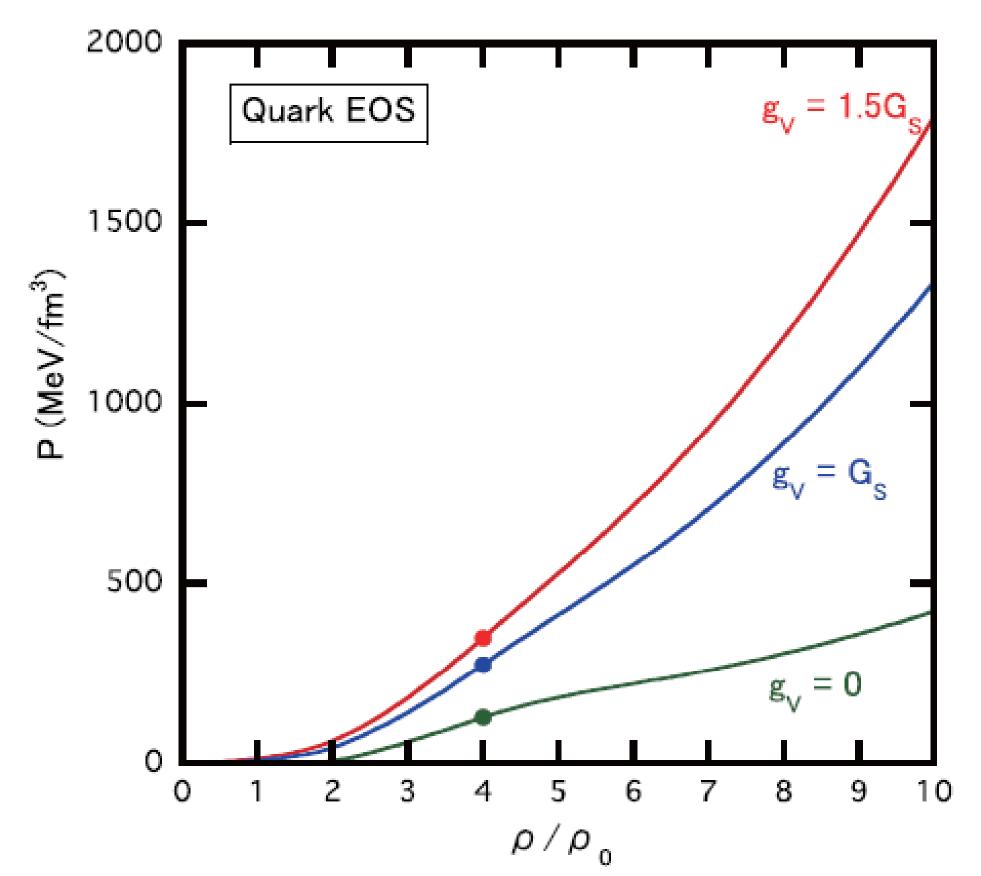
Observed maximum mass limit for Compact stars is not reached!

M=1.97 +/- 0.04 M_sun
(PSR J1614-2230)

Quark Matter EoS:

NJL model with vector coupling g_v





Hadron-Quark Matter "Crossover"

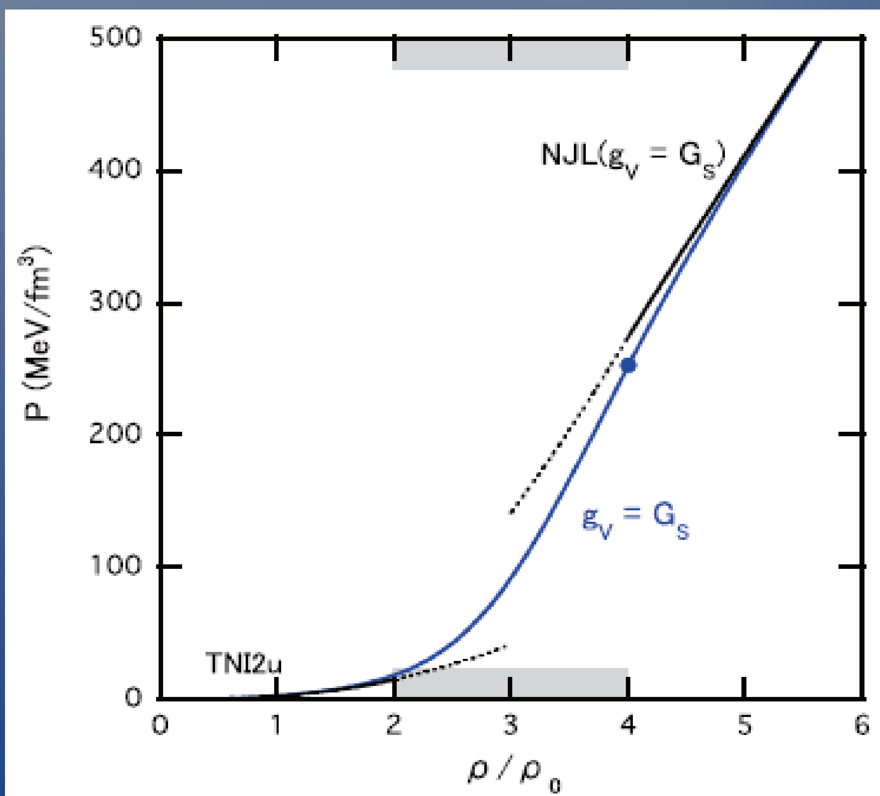
$$P(\rho) = P_H(\rho) f_-(\rho) + P_Q(\rho) f_+(\rho),$$

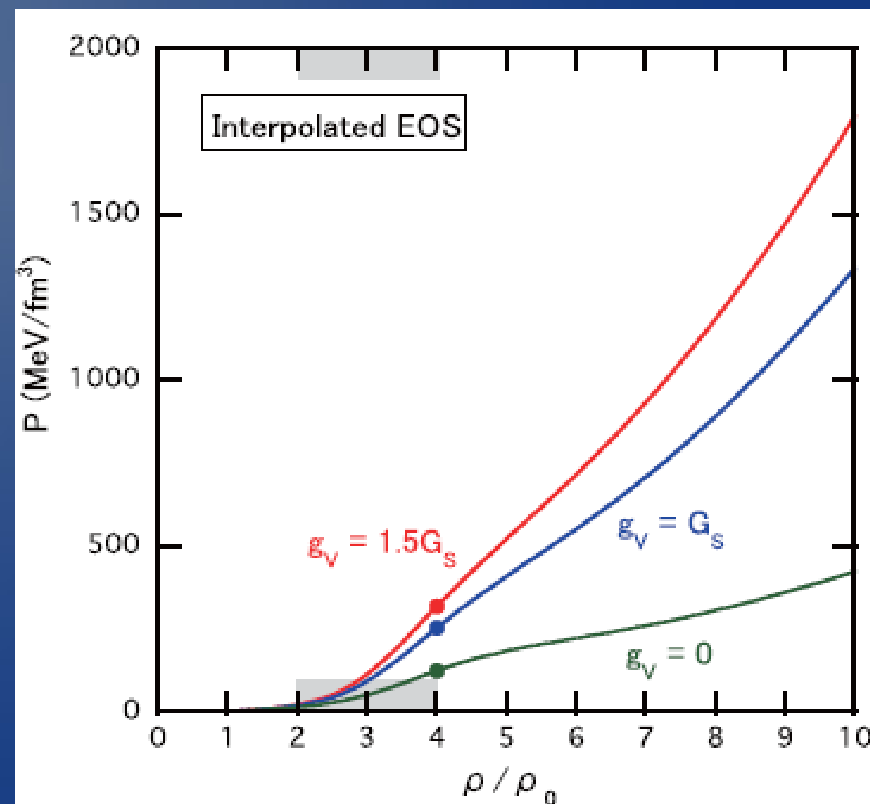
$$f_\pm(\rho) = \frac{1}{2}\left(1 \pm \tanh\left(\frac{\rho - \bar{\rho}}{\Gamma}\right)\right),$$

Correction term for thermodynamic consistency

$$\varepsilon(\rho) = \varepsilon_H(\rho) f_-(\rho) + \varepsilon_Q(\rho) f_+(\rho) + \Delta\varepsilon$$

$$\Delta\varepsilon = \rho \int_{\bar{\rho}}^{\rho} (\varepsilon_H(\rho') - \varepsilon_Q(\rho')) \frac{g(\rho')}{\rho'} d\rho'$$





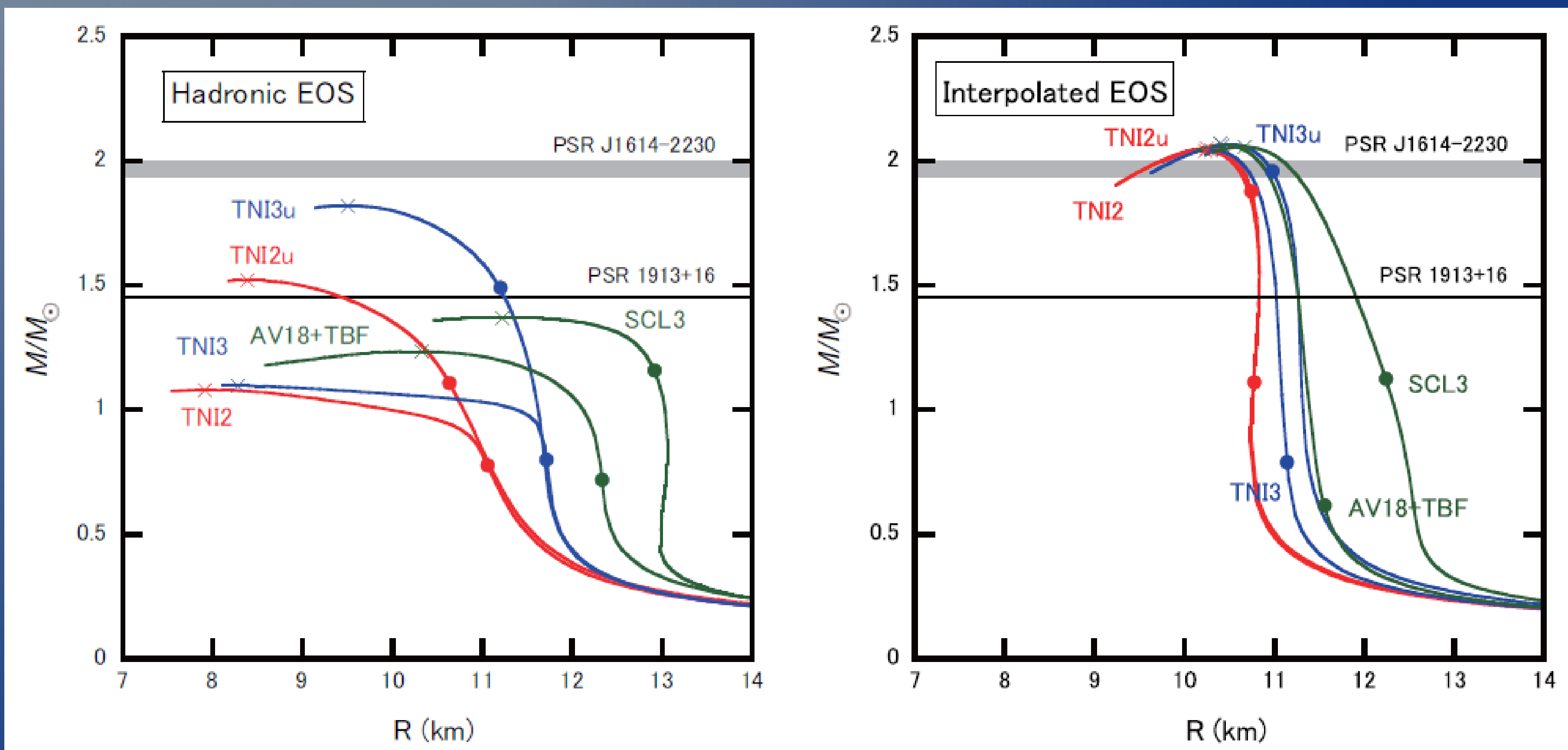
Massive hybrid stars due to stiffening of matter in crossover region

TOV Equations

P(e) ↔ M(R)

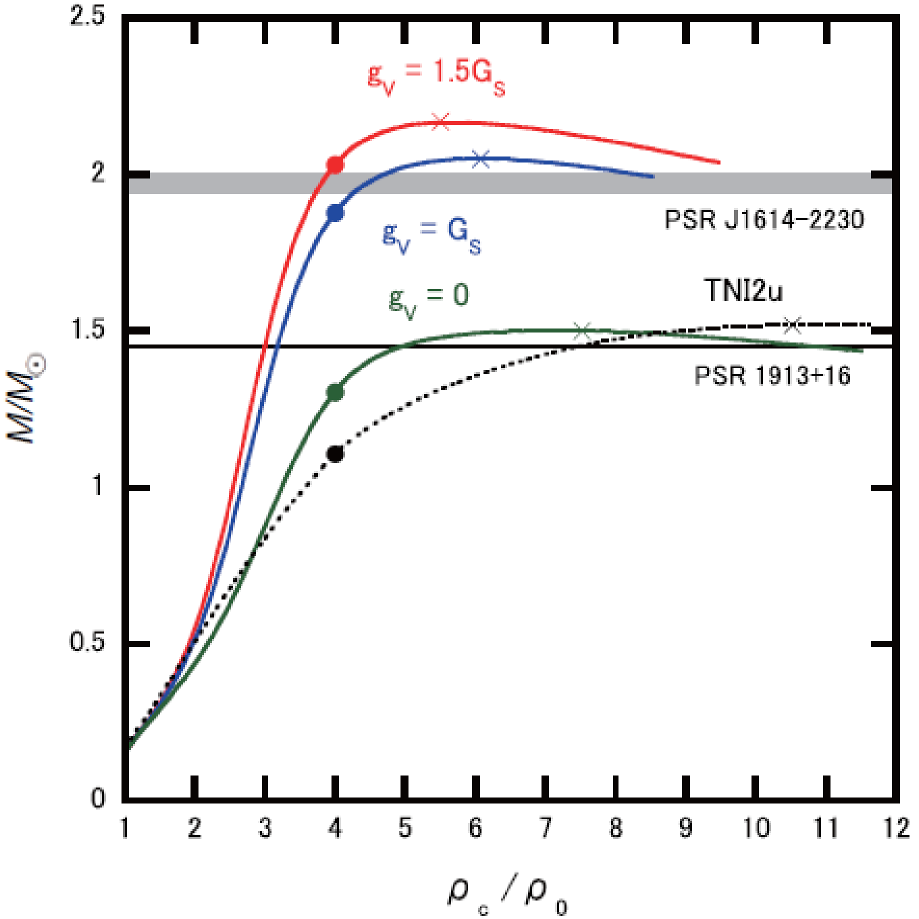
$$\frac{dP}{dr} = -\frac{G}{r^2}\left(M(r) + 4\pi P r^3\right)(\varepsilon + P)\left(1 - 2GM(r)/r\right)^{-1},$$

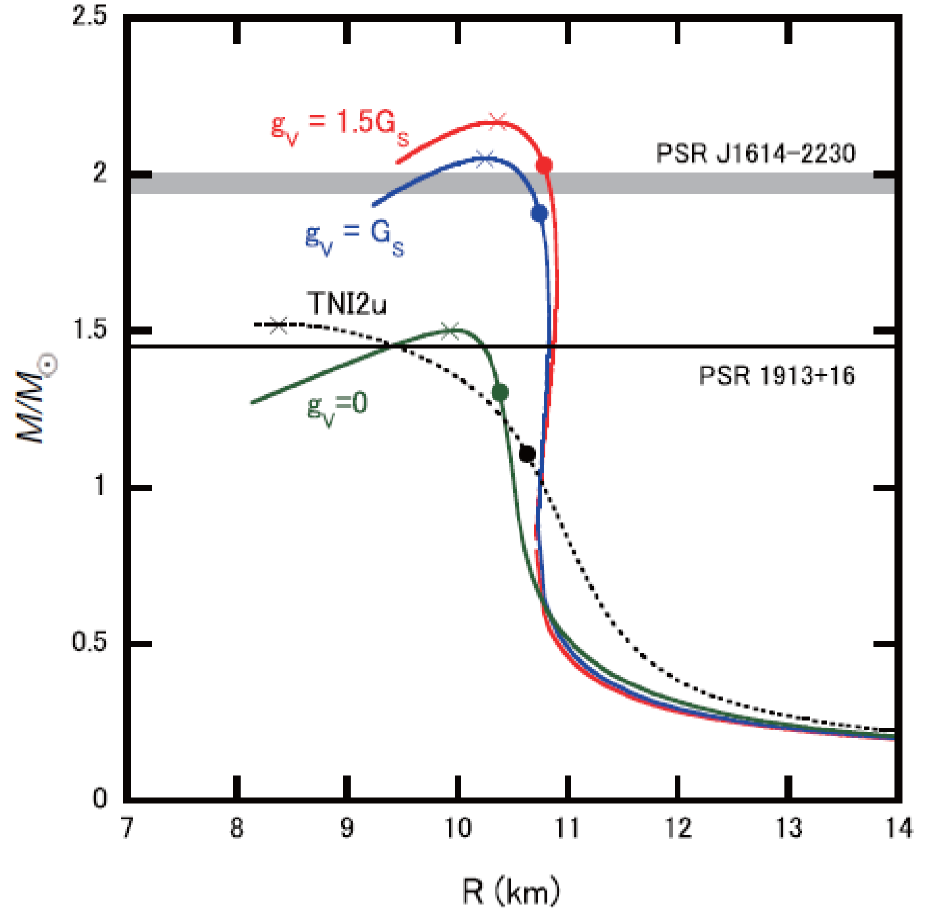
$$M(r) = \int_0^r 4\pi r'^2 \varepsilon(r') dr',$$



Strong vector coupling in Quark Matter is needed !

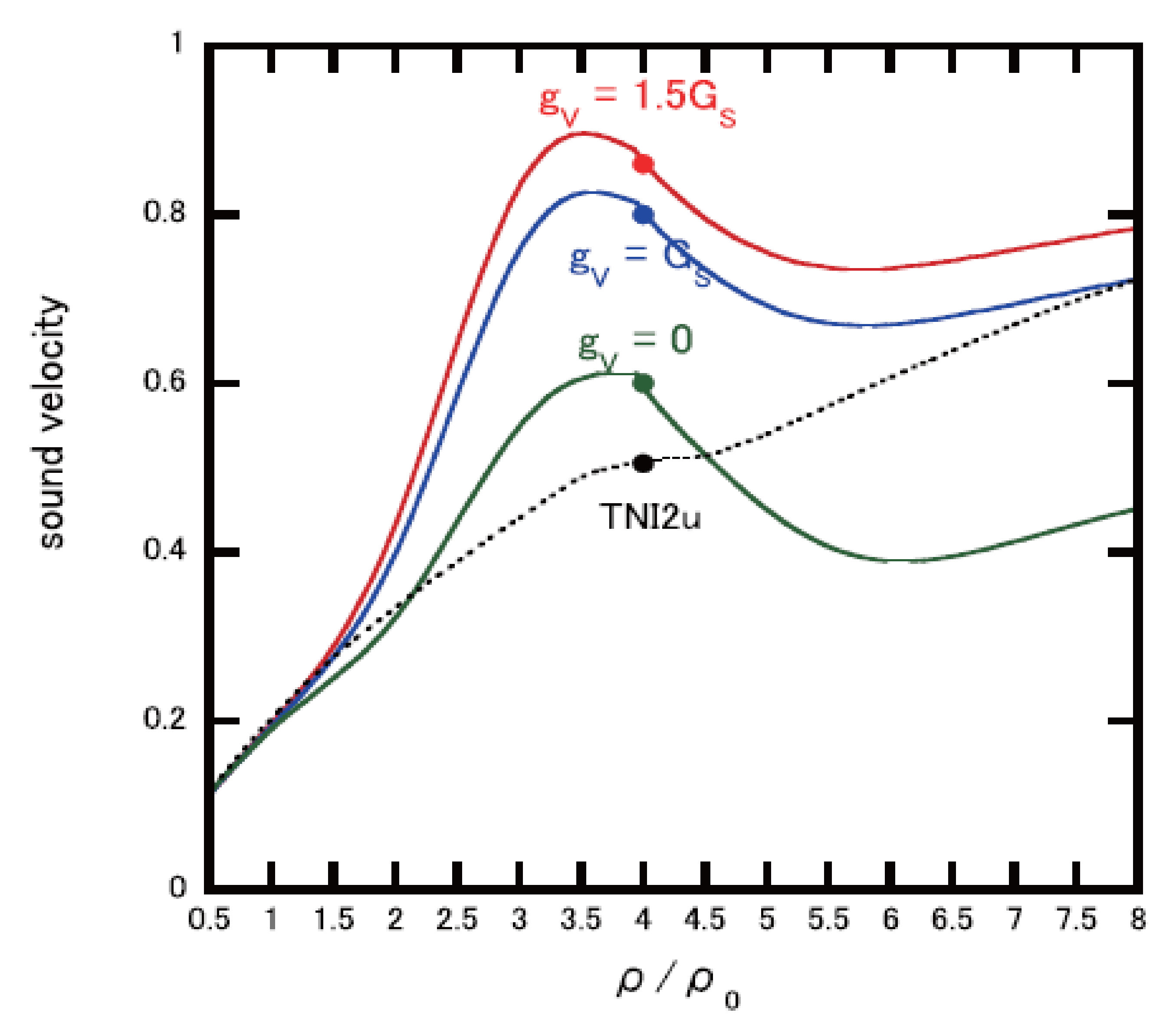
5.2. Dependence on Q-EOS





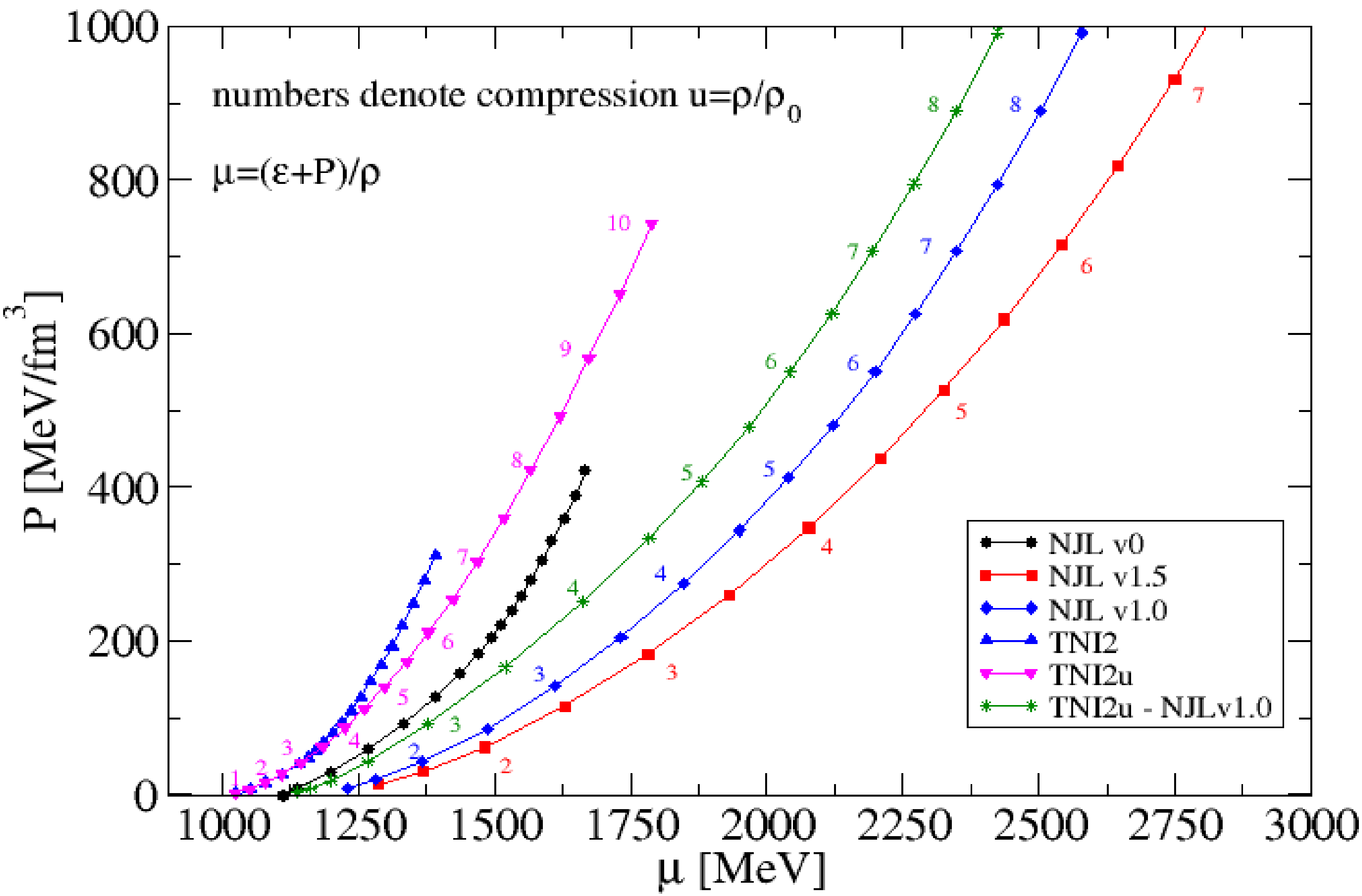
5.4. Sound velocity of interpolated EOS

One of the measures to quantify the stiffness of EOS is the sound velocity $v_s = \sqrt{dP/d\varepsilon}$.

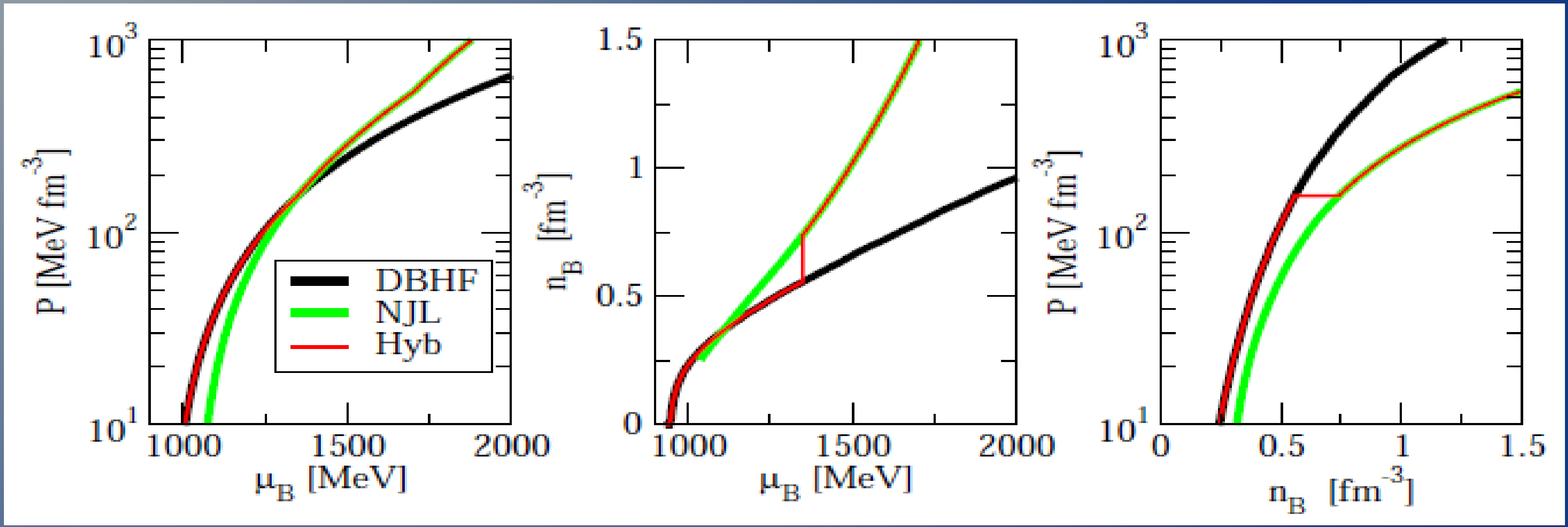


Very suspect: Stiffening of EoS in the Transition (Crossover) Region !!

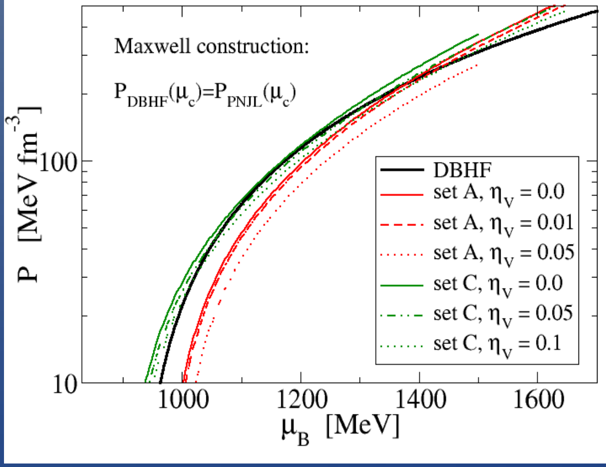
Pressure vs. chem. Potential for H-EoS, Q-EoS and hybrid EoS with “Crossover”

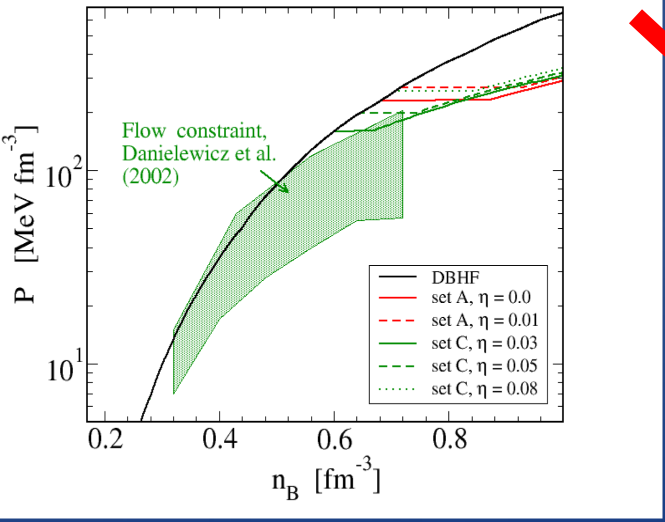


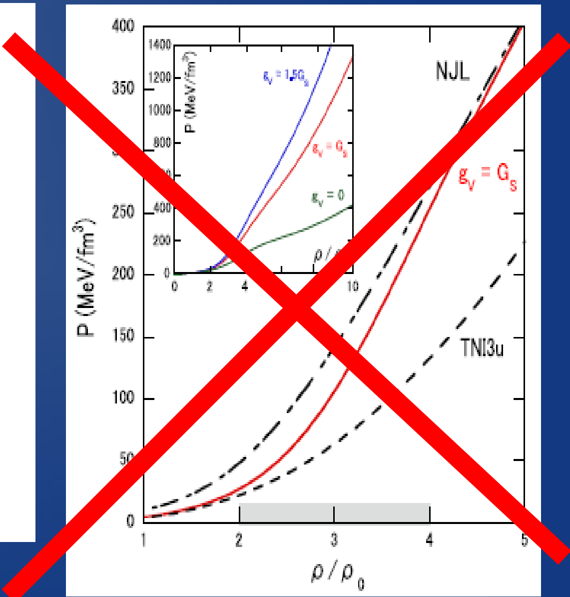
Phase transition: Maxwell construction



Always softens the EoS !

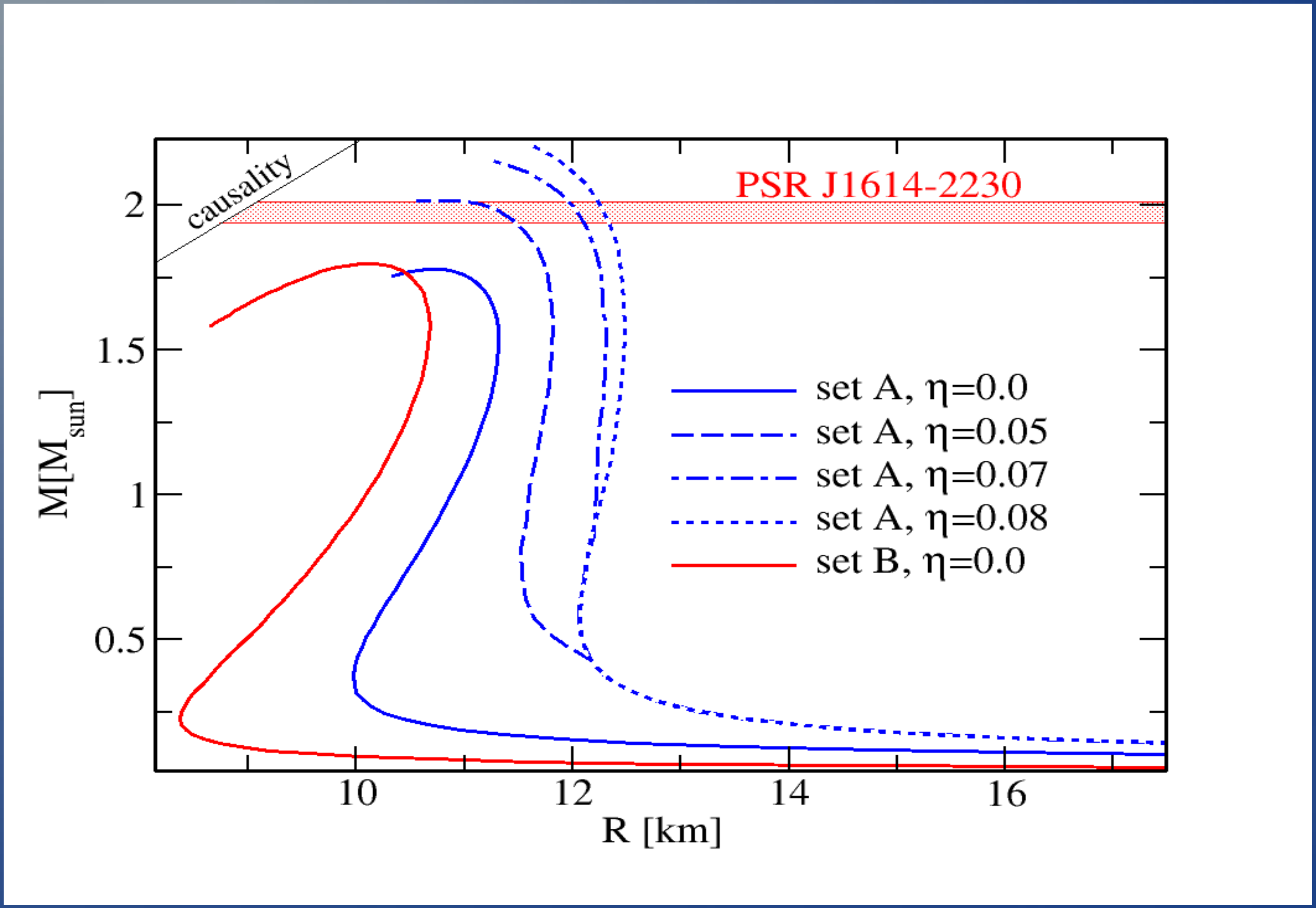






D.B., S. Benic, G. Contrera et al., in preparation (2012)

First results for hybrid stars



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Maximum mass of neutron stars with a quark core

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Hadron-quark phase transition in dense matter and neutron stars

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HADRONIC PHASE

$$\epsilon=\frac{1}{2}m_\omega^2\bar{\omega}_0^2+\frac{1}{2}m_\rho^2(\bar{\rho}_0^3)^2+\frac{1}{2}m_\sigma^2\bar{\sigma}^2+\frac{1}{3}bm_N(g_{\sigma N}\bar{\sigma})^3+\frac{1}{4}c(g_{\sigma N}\bar{\sigma})^4+\sum_i\epsilon_{\mathrm{FG}}(\bar{m}_i,\bar{\mu}_i)+\sum_l\epsilon_{\mathrm{FG}}(m_l,\mu_l),$$

$$P=\frac{1}{2}m_\omega^2\bar{\omega}_0^2+\frac{1}{2}m_\rho^2(\bar{\rho}_0^3)^2-\frac{1}{2}m_\sigma^2\bar{\sigma}^2-\frac{1}{3}bm_N(g_{\sigma N}\bar{\sigma})^3-\frac{1}{4}c(g_{\sigma N}\bar{\sigma})^4+\sum_i P_{\mathrm{FG}}(\bar{m}_i,\bar{\mu}_i)+\sum_l P_{\mathrm{FG}}(m_l,\mu_l)$$

QUARK PHASE

$$\epsilon_Q=\sum_q(\Omega_q+\mu_q\rho_q)+B,\qquad P_Q=-\sum_q\Omega_q-B,$$

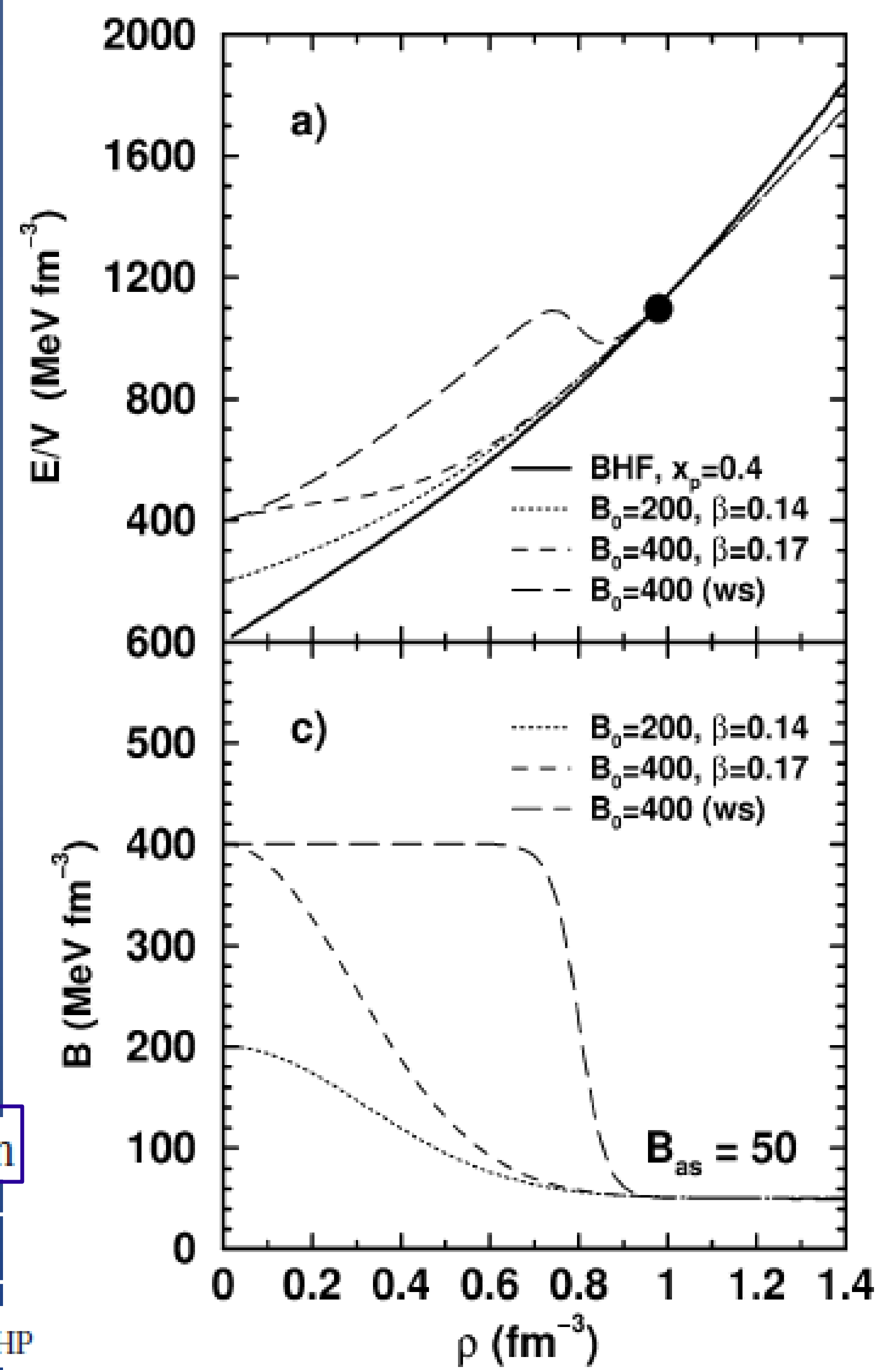
$$B(\rho)=B_{\mathrm{as}}+(B_0-B_{\mathrm{as}})\left[1+\exp\left(\frac{\rho-\bar{\rho}}{\rho_d}\right)\right]^{-1}$$

Phase transition in β-stable neutron star matter

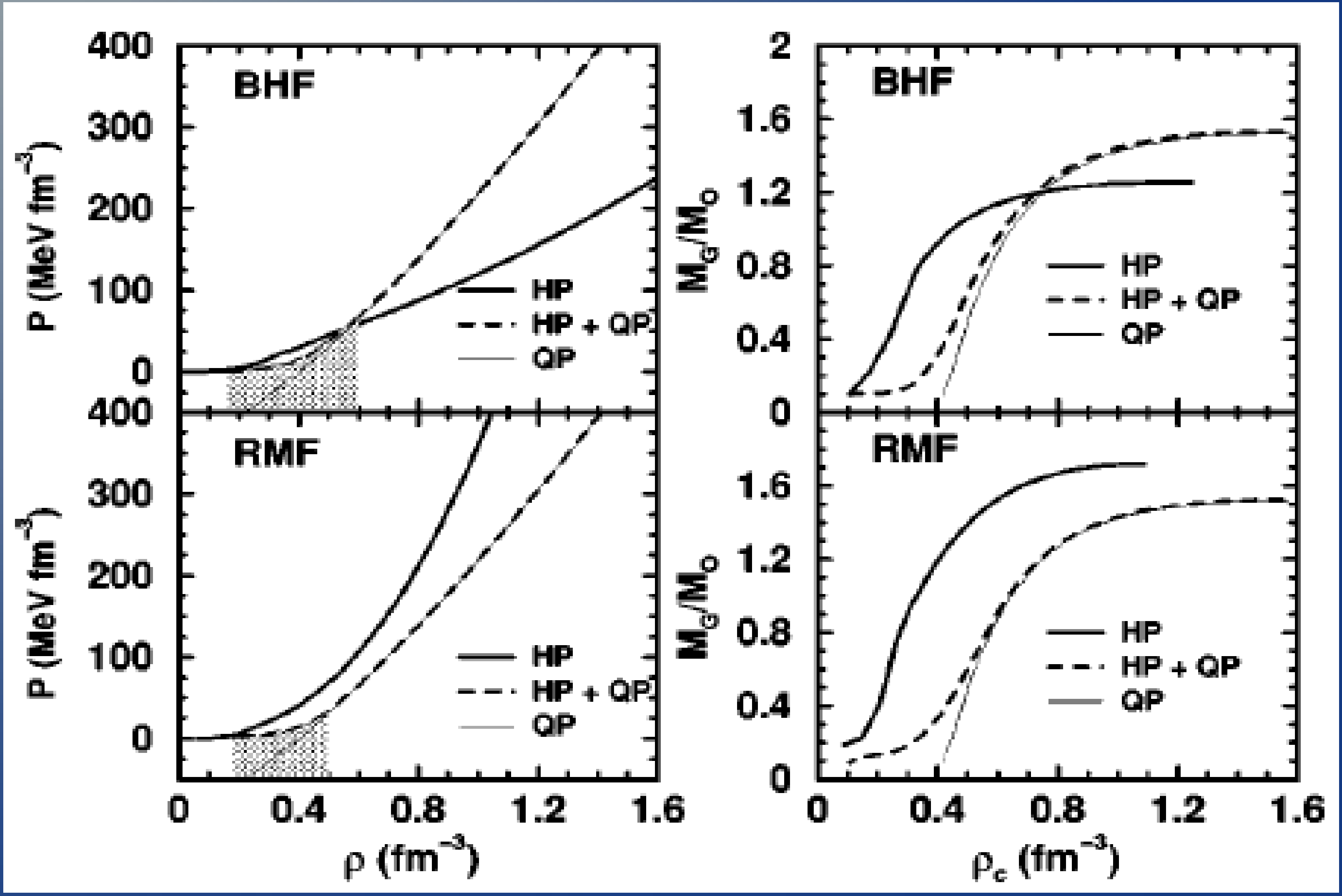
$$P_{\mathrm{HP}}(\mu_e,\mu_n)=P_{\mathrm{QP}}(\mu_e,\mu_n)=P_{\mathrm{MP}}$$ Gibbs condition

$$\chi\rho_c^{\mathrm{QP}}+(1-\chi)\rho_c^{\mathrm{HP}}=0.$$ **Global charge conservation**

$$\epsilon_{\mathrm{MP}}=\chi\epsilon_{\mathrm{QP}}+(1-\chi)\epsilon_{\mathrm{HP}},\qquad \rho_{\mathrm{MP}}=\chi\rho_{\mathrm{QP}}+(1-\chi)\rho_{\mathrm{HP}}$$

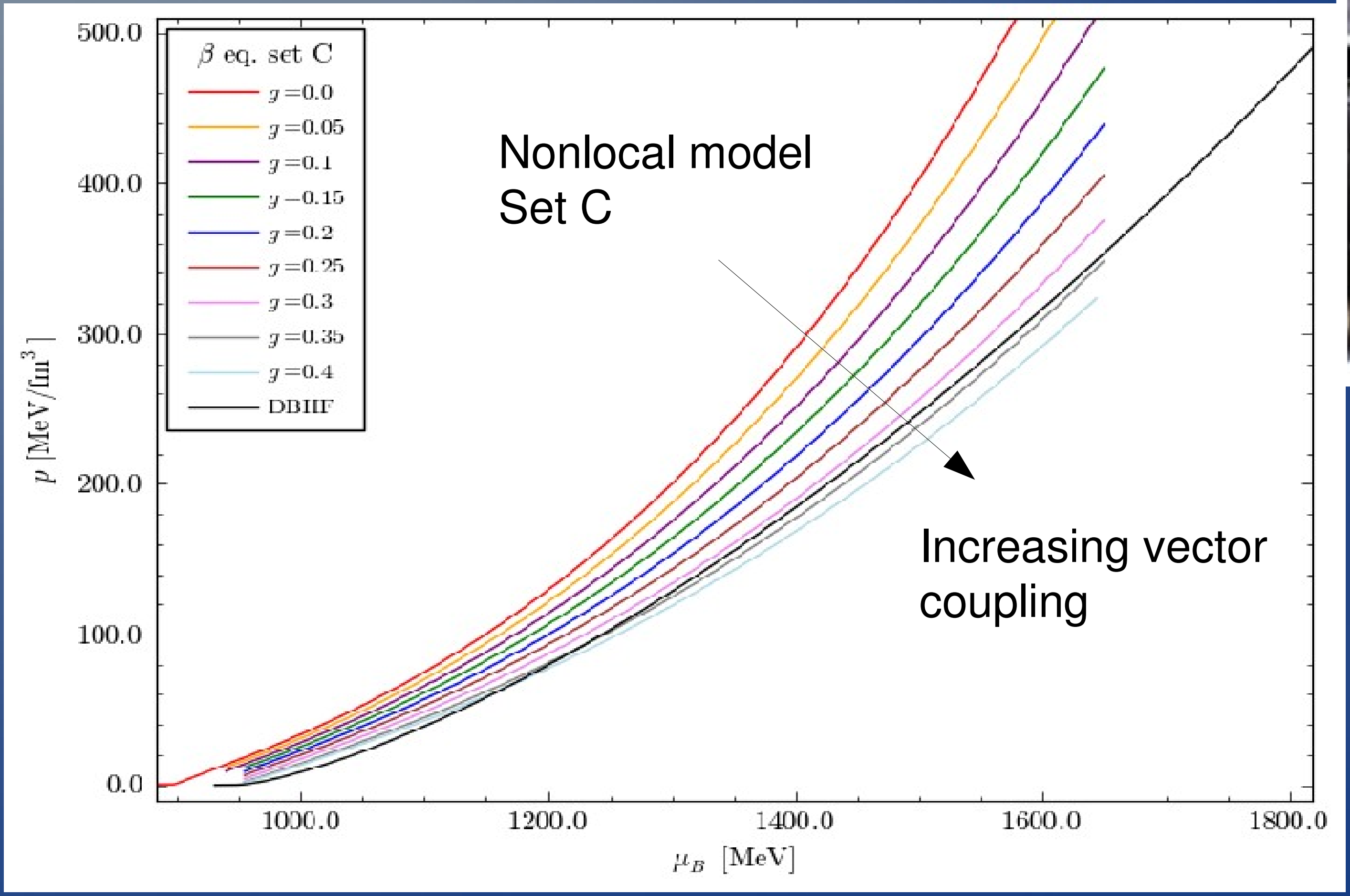


Gibbs Phase transition → Mixed phase, Softening the EoS; Quark Phase: stiff



An element of truth? “Melting” bags at high densities?

The gluon sector strikes back!
What did we hide in the vacuum renormalization?



S. Benic, DB, In preparation

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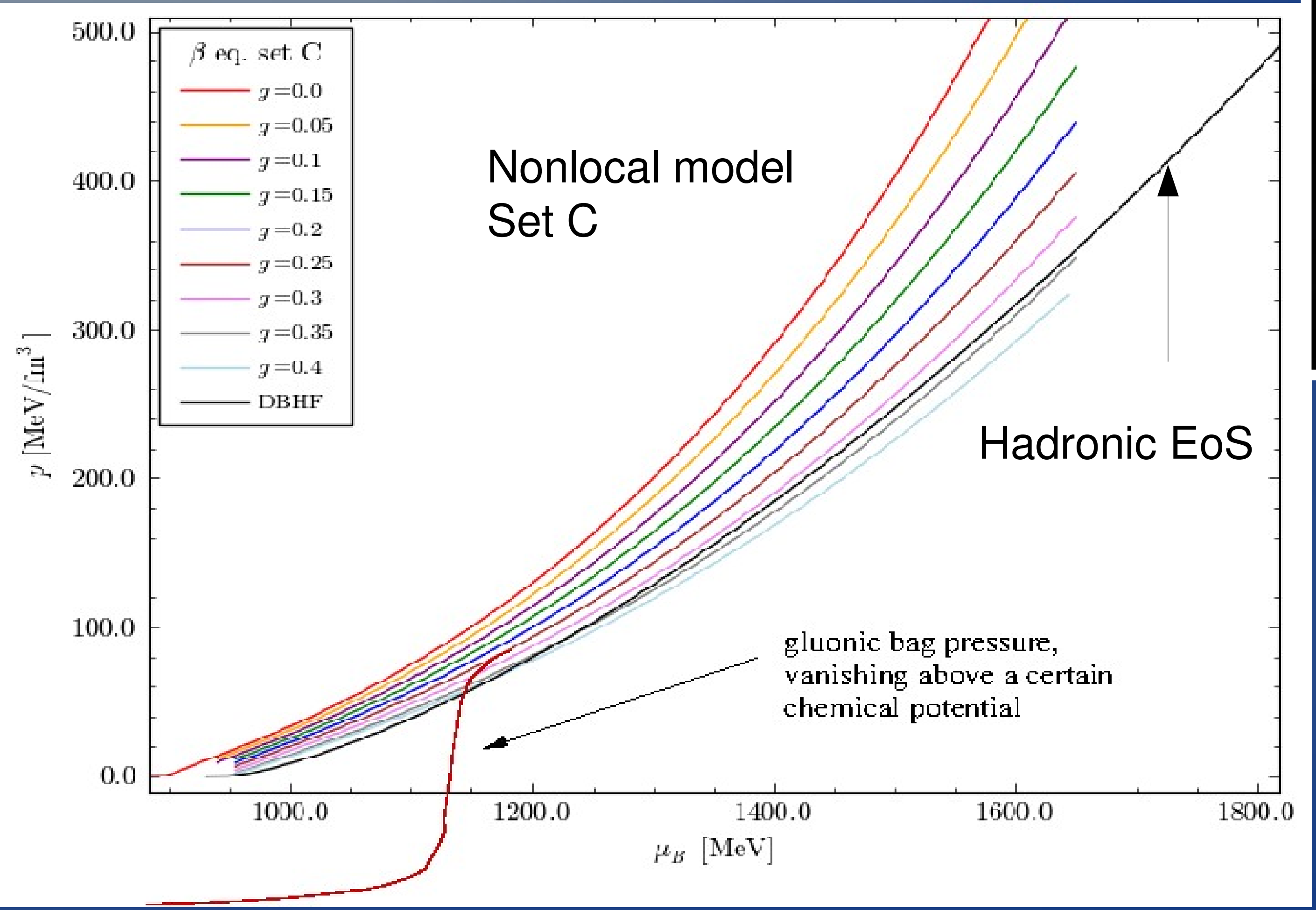
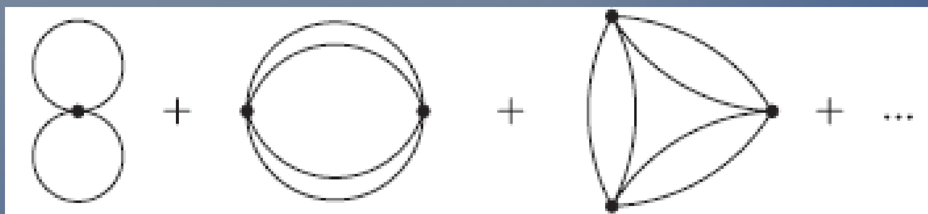


S. Benic, DB,
In preparation

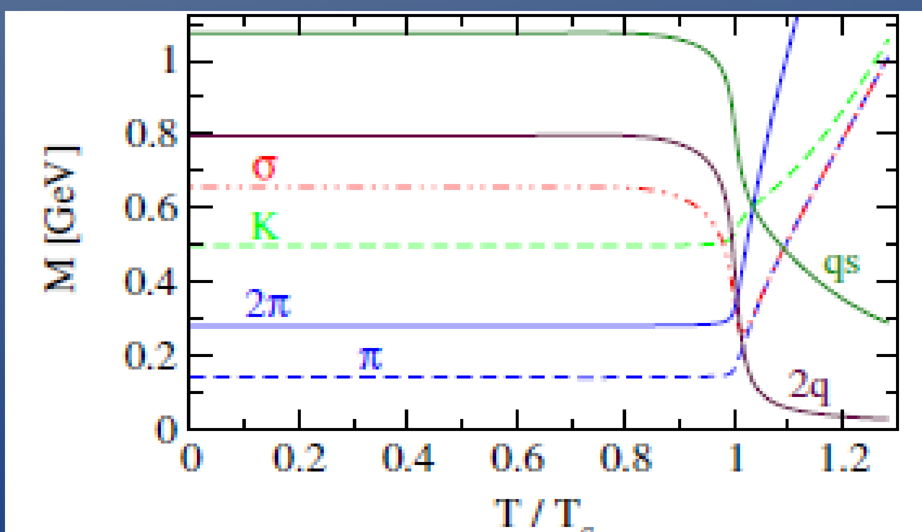
Development : Beyond mean Field - Mesons

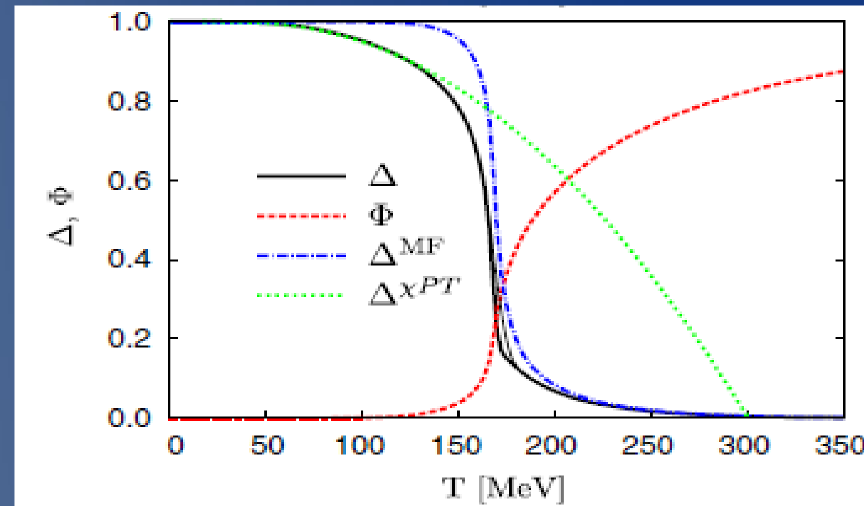
$$\Omega = i\,\mathrm{Tr}\ln(S^{-1}) + i\,\mathrm{Tr}(\Sigma S) + \Psi(S) + U(\Phi,\bar{\Phi}) - \Omega_0$$

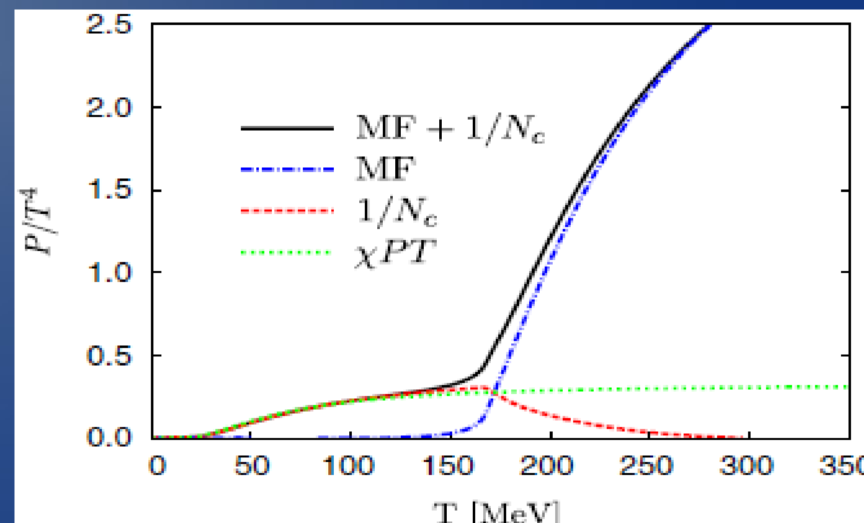
$$\Psi_{\mathrm{glasses}} = -\sum_{M=\pi,\sigma}\frac{G}{2}[-\mathrm{Tr}(\Gamma^M iS)]^2$$ **(MF)**



$$\Psi_{\mathrm{ring}} = -\sum_{M=\pi,\sigma}\frac{d_M}{2}\, i\,\mathrm{Tr}\ln[1 - G\Pi^M]$$ **(Mesons)**







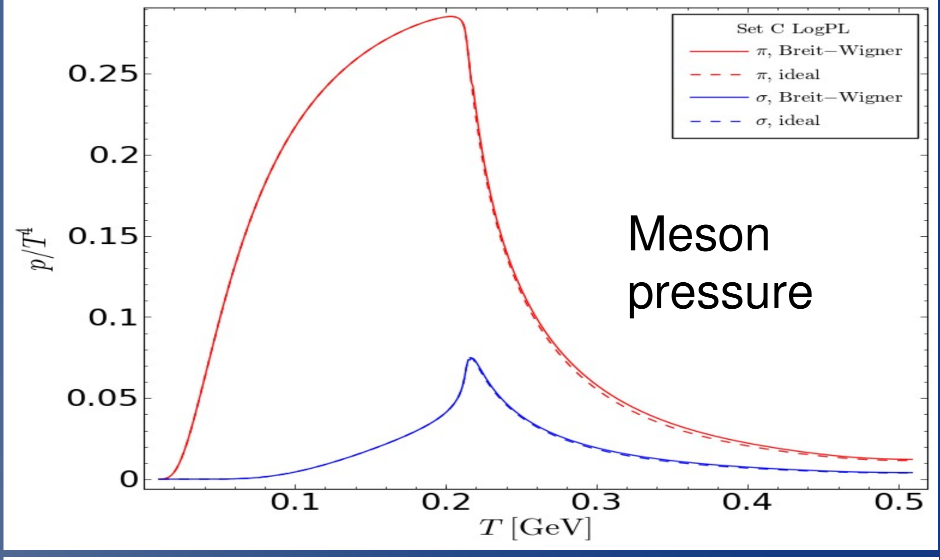
A. Radzhabov et al., PRD 83 (2011) 116004
D. Horvatic et al., PRD 84 (2011) 016005

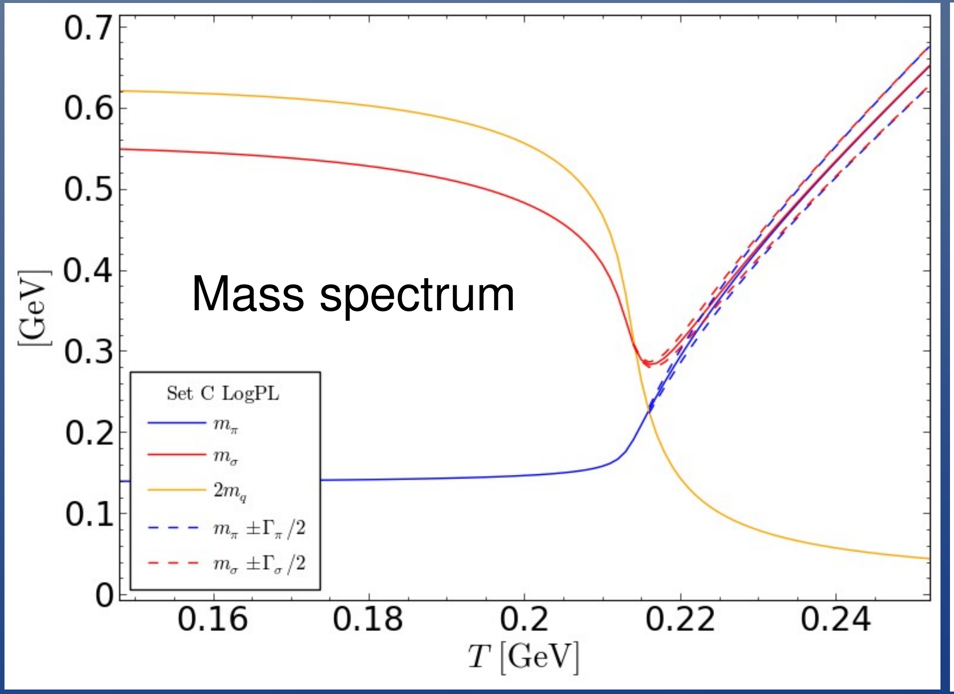
Beyond mean Field – Mesons, Beth-Uhlenbeck EoS

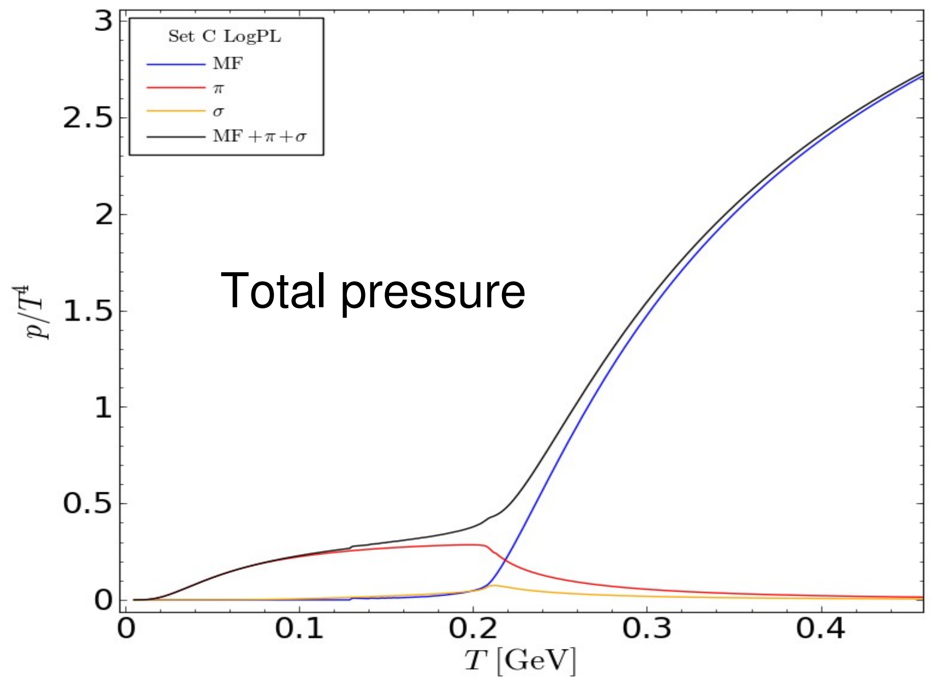
Local PNJL model, see:
A. Wergieluk, DB, Yu. Kalinovsky,
A. Friesen, arxiv:1212.5245 [nucl-th]

Beth-Uhlenbeck EoS: spectral function
Encodes Mott effect – mass and width

Nonlocal PNJL model, Set C param.,
S. Benic, DB, G. Contrera, in preparation







Conclusions - Outlook

- The Polyakov-DSE (nonlocal PNJL) model provides a model framework which is suitable to address lattice QCD data for:
  - quark propagator (vacuum and finite T, μ ; quark mass and flavor dependence, ...)
  - order parameters (chiral condensate, traced Polyakov loop, ...)
  - equation of state (dependences on T, μ, μ_I ; quark masses …)
  - meson properties (masses, spectral functions, correlation functions, …)

- It allows to extrapolate to the “Terra Incognita” of the phase diagram and predict its structure

- Microscopically based EoS for the exotic QCD degrees of freedom region (HIC, CS, SN)

Further development needed to include consistently the hadronic phase of QCD by going beyond MF → Beth-Uhlenbeck EoS