The decays $\rho^- \to \eta\pi^-$ and $\tau^- \to \eta(\eta')\pi^-\nu$ in the NJL model

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Motivation

Studying the decays defined by the light quark mass difference can provide new information about chiral symmetry breaking mechanism.

It is possible to discover new anomalies in the weak interactions and clarify the nature of the a_0 scalar meson.

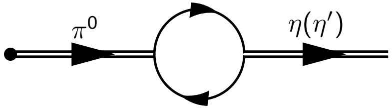
Current experimental limits

$$
\begin{aligned}
\mathcal{B}(\rho^- \to \eta\pi^-) &< 6 \cdot 10^{-3} \\
\mathcal{B}(\tau^- \to \eta\pi^-\nu) &< 0.99 \cdot 10^{-4} \\
\mathcal{B}(\tau^- \to \eta'\pi^-\nu) &< 7.2 \cdot 10^{-7}
\end{aligned}
$$

Historical overview

- S. Tisserant and T. N. Truong, "$\tau \to \delta\nu$ decay induces by light quark mass difference", Phys. Lett. B **115**, 264 (1982).
- A. Bramon, S. Narison and A. Pich,"The $\tau \to \nu_\tau \eta\pi$ Process In And Beyond Qcd", Phys. Lett. B **196**, 543 (1987).
- H. Neufeld and H. Rupertsberger,"Isospin breaking in chiral perturbation theory and the decays $\eta \to \pi l\nu$ and $\tau \to \eta\pi\nu$", Z. Phys. C **68**, 91 (1995).
- S. Nussinov and A. Soffer, "Estimate of the branching fraction $\tau \to \eta\pi\nu_\tau$, the $a_0(980)$, and non-standard weak interactions", Phys. Rev. D **78**, 033006 (2008)
- S. Nussinov and A. Soffer, "Estimate of the Branching Fraction of $\tau \to \pi\eta'\nu_\tau$", Phys. Rev. D **80**, 033010 (2009)
- N. Paver and Riazuddin, "On meson dominance in the 'second class' $\tau \to \eta\pi\nu_\tau$ decay", Phys. Rev. D **82**, 057301 (2010)
- N. Paver and Riazuddin, "On the branching ratio of the 'second class' $\tau \to \eta'\pi\nu_\tau$ decay", Phys. Rev. D **84**, 017302 (2011)

The $\pi^0 - \eta(\eta')$ transitions are described by the given diagram



The amplitude of the transition $\pi^0 - \eta(\eta')$ has the form

$$\epsilon_{\pi\eta(\eta')} = 2g_\pi^2((2I_1(m_d) + m^2_{\eta(\eta')}I_2(m_d)) - (2I_1(m_u) + m^2_{\eta(\eta')}I_2(m_u)))\frac{\epsilon_{\eta(\eta')}}{m_\pi^2 - m^2_{\eta(\eta')}}$$

$$m_d - m_u \approx 3.7 \text{ MeV}$$
$$\epsilon_\eta = \sin\bar{\theta} \text{ for } \eta \text{ meson}$$
$$\epsilon_{\eta'} = \cos\bar{\theta} \text{ for } \eta' \text{ meson}$$
$$\text{where } \bar{\theta} = -54^\circ$$

The I_1 and I_2 are divergent integrals describe quark loops and g_π is constant defined from Goldberger – Treiman relation

$$\begin{aligned} g_\pi &= \frac{m_u}{F_\pi} \\ I_1(m) &= -i\frac{N_c}{(2\pi)^4}\int^{\Lambda_4}\frac{\mathrm{d}^4k}{(m^2-k^2)} = \frac{N_c}{(4\pi)^2}\left[\Lambda_4 - m^2\log\left(\frac{\Lambda_4^2}{m^2}+1\right)\right] \\ I_2(m) &= -i\frac{N_c}{(2\pi)^4}\int^{\Lambda_4}\frac{\mathrm{d}^4k}{(m^2-k^2)^2} = \frac{N_c}{(4\pi)^2}\left[\log\left(\frac{\Lambda_4^2}{m^2}+1\right) - \left(1+\frac{m^2}{\Lambda_4^2}\right)^{-1}\right] \end{aligned}$$

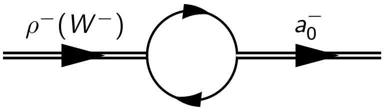
$$N_c = 3, \quad m_u = 280 \text{ MeV}, \quad F_\pi = 93 \text{ MeV}, \quad \Lambda_4 = 1250 \text{ MeV}.$$

$\pi^0 - \eta(\eta')$ TRANSITIONS

Obtained estimates coincide with estimates given by Paver & Riazuddin

$\epsilon_{\pi\eta}^{PR}$	$\epsilon_{\pi\eta}^{NJL}$	$\epsilon_{\pi\eta'}^{PR}$	$\epsilon_{\pi\eta'}^{NJL}$
$1.34 \cdot 10^{-2}$	$1.55 \cdot 10^{-2}$	$(3 \pm 1) \cdot 10^{-3}$	$6.79 \cdot 10^{-3}$

The $\pi^0 - \eta(\eta')$ transitions are described by the given diagram



The transition $\rho^- - a_0^-$ takes the form

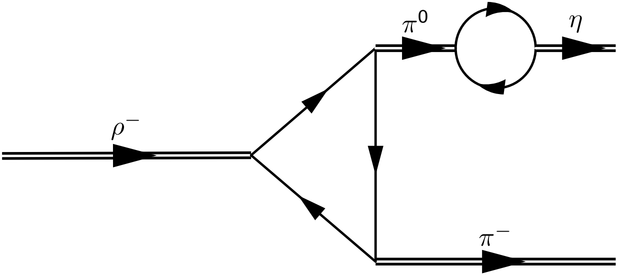
$$\frac{\sqrt{6}}{2}(m_d - m_u)p^\mu \rho_\mu^- a_0^-$$

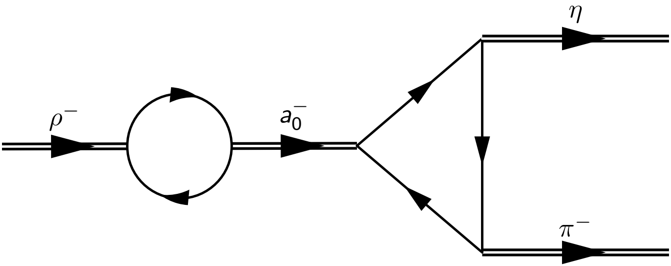
For the W^- boson we get

$$\frac{\sqrt{3}g_{EW}|V_{ud}|}{4g_\rho}(m_d - m_u)p^\mu W_\mu^- a_0^-$$

where $g_\rho = 6.14$ and p is a vector boson momentum

THE DECAY $\rho^- \to \eta\pi^-$





THE DECAY $\rho^- \to \eta\pi^-$

The first diagram describes the amplitude which contains the $\pi^0 - \eta$ transitions in the final state

$$T_1 = g_\rho \epsilon_{\pi\eta}(p_-^\mu - p_0^\mu)\rho_\mu^- \eta\pi^- \ ,$$

The second diagram describes the amplitude containing the intermediate a_0^- meson

$$T_2 = 2Zg_\rho \frac{m_u(m_d - m_u)}{m_{a_0}^2 - m_\rho^2} \epsilon_\eta p^\mu \rho_\mu^- \eta\pi^- \ ,$$

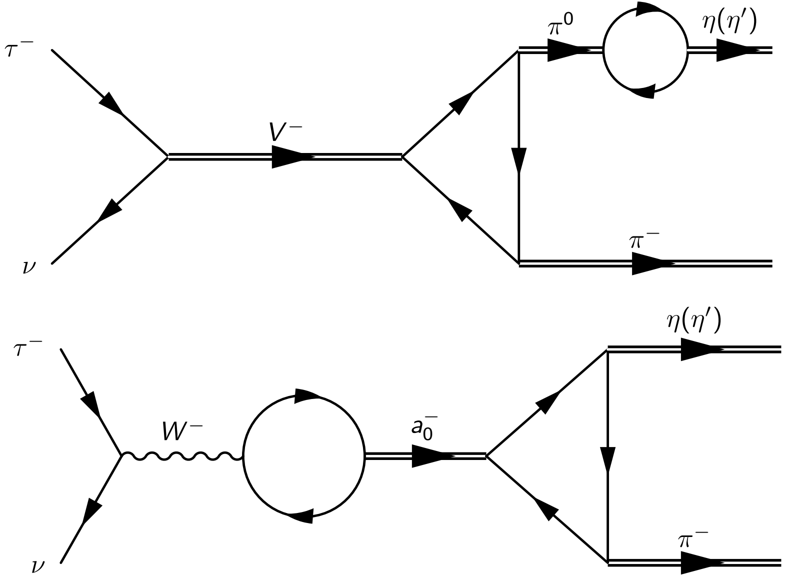
This amplitude contains the amplitude of the $a_0^- \to \eta\pi^-$ decay

$$\frac{4}{\sqrt{6}} Zg_\rho m_u \epsilon_\eta a_0^- \eta\pi^-$$

For branching fractions we get

$$
\begin{aligned}
\mathcal{B}_1 &= \epsilon^2_{\pi\eta} \frac{\Lambda^{3/2}(m_\rho^2, m_\eta^2, m_\pi^2)}{\Lambda^{3/2}(m_\rho^2, m_\pi^2, m_\pi^2)} = 1.78 \cdot 10^{-5} \\
\mathcal{B}_2 &= 4Z^2 \sin^2\bar{\theta} \left(\frac{m_u(m_d - m_u)}{m_{a_0}^2 - m_\rho^2} \right)^2 \frac{\Lambda^{3/2}(m_\rho^2, m_\eta^2, m_\pi^2)}{\Lambda^{3/2}(m_\rho^2, m_\pi^2, m_\pi^2)} = 0.33 \cdot 10^{-5} \\
\mathcal{B}_{NS} &= 1.4 \cdot 10^{-5} \simeq \mathcal{B}_1 \quad \text{(Nussinov \& Soffer)}
\end{aligned}
$$

THE DECAYS $\tau^- \to \eta(\eta')\pi^-$



The decays $\tau^- \to \eta(\eta')\pi^-$

The amplitude with $\pi^0 - \eta(\eta')$ transitions in final state

$$\epsilon_{\pi\eta(\eta')} m_\rho^2 \left((1 - \frac{i\sqrt{q^2}\Gamma_\rho(p^2)}{m_\rho^2}) BW_\rho(p^2) + \beta \frac{p^2}{m_\rho^2} BW_{\rho'}(p^2) \right) (p^\mu_{\pi^-} - p^\mu_{\eta(\eta')}) l_\mu \pi^- \eta(\eta')$$

The amplitude with intermediate a_0^- meson

$$2Z m_u (m_d - m_u) \epsilon_{\eta(\eta')} BW_{a_0}(p^2) p^\mu l_\mu \pi^- \eta(\eta')$$

For branching fractions we get

$$\begin{aligned} \mathcal{B}_V^{\pi\eta} &= 4.35 \cdot 10^{-6} \\ \mathcal{B}_S^{\pi\eta} &= 0.38 \cdot 10^{-6} \\ \mathcal{B}_V^{\pi\eta'} &= 1.11 \cdot 10^{-8} \\ \mathcal{B}_S^{\pi\eta'} &= 1.98 \cdot 10^{-8} \end{aligned}$$

We can compare our results with ones obtained in previous works

	$\mathcal{B}_V^{\pi\eta} \times 10^{-6}$	$\mathcal{B}_S^{\pi\eta} \times 10^{-6}$	$\mathcal{B}_{tot}^{\pi\eta} \times 10^{-6}$
NJL	4.35	0.38	4.72
PR	$1.58 \div 5.70$	$10.7 \div 65.9$	$\simeq 26$
NS	3.6	~ 10	$3 \div 10$

	$\mathcal{B}_V^{\pi\eta'} \times 10^{-8}$	$\mathcal{B}_S^{\pi\eta'} \times 10^{-8}$	$\mathcal{B}_{tot}^{\pi\eta'} \times 10^{-8}$
NJL	1.11	1.98	3.09
PR	$0.14 \div 3.4$	$6 \div 18$	–
NS	$< 2 + 8$	$< 10 + (20 \div 120)$	< 140

Conclusions

We can get decay width and branching fraction for $\tau \to a_0 \nu$

$$\Gamma = \frac{G_F^2 |V_{ud}|^2 m_\tau^3}{16\pi} \left(\frac{\sqrt{6}}{2} \frac{m_d - m_u}{g_\rho} \right)^2 \left(1 - \frac{m_{a_0}^2}{m_\tau^2} \right)^2$$

$$\begin{aligned} \mathcal{B}^{NJL} &= 3.28 \cdot 10^{-6} \\ \mathcal{B}^{NS} &= 8 \cdot 10^{-6} \quad \text{(Nussinov \& Soffer)} \\ \mathcal{B}^{TT} &= 16 \cdot 10^{-6} \quad \text{(Tisserant \& Truong)} \end{aligned}$$

Conclusions

The branching fraction for $\tau^- \to a_0^- \nu$ confirms relevancy of our expression for the vertex $\tau a_0 \nu$. For the vertex $a_0^- \to \eta\pi^-$ also was used well-known expression. It contradicts with the ansatz used by Bramon, Narison, Pich (1987) and Paver & Riazuddin (2010)

$$\frac{\epsilon_{\pi\eta} M_R^2}{M_R^2 - p^2 - iM_R\Gamma_R(p^2)}$$

On the other side, if we use this ansatz for vector to scalar transition and calculate $\rho^- \to \eta\pi^-$ with this ansatz then we get

$$\mathcal{B} \sim \epsilon_{\pi\eta}^2 \left(\frac{m_{a_0}^2}{m_{a_0}^2 - m_\rho^2}\right)^2 \frac{\Lambda^{3/2}(m_\rho^2, m_\eta^2, m_\pi^2)}{\Lambda^{3/2}(m_\rho^2, m_\pi^2, m_\pi^2)} \sim 10^{-3}$$

This limit can be reached in the experiment in the near future.

Thanks for your attention