A new conception in describing the Drell-Yan processes

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Talk Plan

Motivation

A set of evolution equations for correlators of densities of quark and gluons is considered. Approximate solutions is obtained in frames of gluon and quark dominance. A new formulation of the cross sections of Drell-Yan process is suggested. Differential cross sections for the QCD sub-processes of type $2 \to 2$ are obtained. Sub-process $gb \to tH^-$ as well considered.

Motivation: General Formalism

The quark parton model of Feynman (R. P. Feynman, *Photon-hadron interactions* (Benjamin, New-York, 1972).) provides the simple description of deep inelastic phenomena as well as Drell-Yan processes. Theoretical justify of it was done in terms of asymptotically free gauge theories (H. D. Politzer, Phys. Rept. **14**, 129 (1974)). Drell-Yan picture based on factorization of contributions from small and large distances was justified in papers of Collins (J. C. Collins, D. E. Soper, and G. F. Sterman, Adv. Ser. Direct. High Energy Phys. **5**, 1 (1988), arXiv:hep-ph/0409313). Deviation from the naive Bjorken scaling of the structure functions DIS was recognized to be broken by so called "large logarithms"-the logarithms of the ratio of momentum squared $Q^2 = -q^2$ (virtualities) of particles deep off mass shell to their masses. Reasons of appearing such a logarithms in QED was clarified in methods of quasi-real photons and electrons (C. F. von Weizsäcker, Z. Phys. **88**, 612 (1934), E. J. Williams, Phys. Rev. **45**, 729 (1934), P. Kessler, Nuovo Cimento, X. Ser. **17**, 809 (1960), V. N. Baier, V. S. Fadin, and V. A. Khoze, Nucl. Phys. **B65**, 381 (1973)). Keeping in mind the contributions of higher orders of perturbation theory,in the leading logarithmical approximation description of processes with large virtualities can as well be formulated in parton language with some definite dependence on Q^2 of partons (quark,gluons) densities–structure functions, $q^i(x,t)$, $G(x,t)$.

The evolution equations of Altarelli–Parisi (AP) (G. Altarelli and G. Parisi, Nucl. Phys. **B126**, 298 (1977)), for these densities have a form

$$\frac{dq^i(x,t)}{dt} = \frac{\alpha_s(t)}{2\pi}\int\limits_x^1 \frac{dy}{y}\left[q^i(y,t)\,P_{qq}\left(\frac{x}{y}\right) + G(y,t)\,P_{iG}\left(\frac{x}{y}\right)\right], \qquad (1)$$

$$t = \ln\left(\frac{Q^2}{m^2}\right),$$

$$\frac{dG(x,t)}{dt} = \frac{\alpha_s(t)}{2\pi}\int\limits_x^1 \frac{dy}{y}\left[\sum_{i=1}^{2N_f} q^i(y,t)\,P_{Gq^i}\left(\frac{x}{y}\right) + G(y,t)\,P_{GG}\left(\frac{x}{y}\right)\right],$$

$$i = u, d, s, c, b, t.$$

Motivation: General Formalism

They describe the dependence of quarks $q(x,t)$ and gluons $G(x,t)$ densities with the energy fraction x inside the proton on a scale of Q^2 (where $Q^2 > 0$). Here m is the suitable normalization point $m \sim Q_0 \sim M_p \approx 1$ GeV, $\alpha_s(t)$ is the QCD coupling constant on the scale Q^2 and $P_{ij}(x)$ are the AP equation kernels:

$$P_{qq}(z) = C_F \left(\left(\frac{1+z^2}{(1-z)}_+ \right) + \frac{3}{2}\delta(1-z) \right),$$

$$P_{Gq} = C_F \frac{1+(1-z)^2}{z}; P_{qG} = \frac{1}{2}[z^2 + (1-z)^2];$$

$$P_{GG}(z) = 2C_V \left(\frac{1-z}{z} + \left(\frac{z}{(1-z)_+} \right) + z(1-z) + \frac{11}{12}\delta(z-1) \right),$$

with $C_F = \frac{N^2-1}{2N}$ and $C_V = 2N$ for the color group $SU(N)$. These quantities satisfy the following properties:

$$\int_0^1 dz\ P_{qq}(z) = 0, \qquad \int_0^1 dz\ z\ P_{GG}(z) = 0. \qquad (2)$$

Motivation: General Formalism

It's useful to remind here the statistical interpretation of AP equations in terms of densities (L. N. Lipatov, Sov. J. Nucl. Phys. **20**, 94 (1975), [Yad.Fiz.20:181-198,1974]) by means of a set of correlation functions, satisfying the system of statistical equations (renormalization group equation). It was a success of the numerous applications of APL set of equations, working with two densities q, G of quarks and gluons into a proton.
Problems associated with processes with large multiplicity (E. Kokoulina, Acta Phys. Polon. **B35**, 295 (2004), arXiv:hep-ph/0401223), however require some generalization of traditional approach, introducing the correlation functions (A. P. Bukhvostov, G. V. Frolov,L. N. Lipatov and E. A. Kuraev, Nucl. Phys. B258(1985),601).
Namely let us introduce $D^q(x,t), D^g(x,t), D^{\bar{q}}(x,t)$- densities of quark, gluon, anti-quark into the initial quark and the similar quantities for the initial anti-quark (see Fig. 1).
Besides let introduce $G^q(x,t), G^g(x,t), G^{\bar{q}}(x,t)$-the similar densities for the initial gluon.
In complete analogy to the case of AL equations one can obtain the evolution equations for these set of densities. They are presented in Appendix A.

Motivation: General Formalism

When neglecting the presence of densities of anti-quarks in gluon and quark, the combinations $(G^g + D^g)(x,t) = G(x,t)$ and $(G^g + D^g)(x,t) = q(x,t)$ can be shown (see Appendix A) to obey the equations of AP. One of successful phenomenological model-based on the dominant role of gluon distribution G^g in describing the processes with high hadron multiplicities (E. Kokoulina, Acta Phys. Polon. **B35**, 295 (2004), arXiv:hep-ph/0401223).
Solving the equation for G^g by iteration method we see that regeneration of gluon density in the channel $G^g \to D^q \to G^g$ is associated with small factor

$$K_0 = \left(\frac{C_F}{2C_V}\right)^2 = \left(\frac{4/3}{6}\right)^2 \approx 0.05. \quad (3)$$

Terms of such a magnitude can be neglected thus determining the accuracy of the approximation. Alternatively it can be included as a some contribution to K -factor.

Motivation: General Formalism

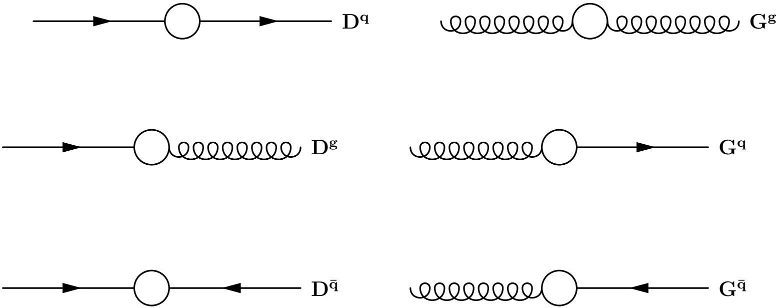

Figure: Definition of correlator densities.

Motivation: General Formalism

Let now identify the nonsinglet structure function $q - \bar{q}$ with D^q - a quark dominance density. We introduce the gluon dominance density G^g, satisfying the equations:

$$D^q(x, \beta_q) = \delta(x-1) + \int\limits_{m^2}^{s} \frac{\alpha_s(t)}{2\pi}\frac{dt}{t}\int\limits_{x}^{1}\frac{dy}{y}P_{qq}\left(\frac{x}{y}\right)D^q(y), \beta_{qt}, \quad (4)$$

$$xG^g(x, \beta_g) = \delta(x-1) + \int\limits_{m^2}^{s} \frac{\alpha_s(t)}{2\pi}\frac{dt}{t}\int\limits_{x}^{1}\frac{dy}{y}P_{GG}\left(\frac{x}{y}\right)yD_g(()\, y, \beta_{gt}, \quad (5)$$

where

$$\beta_q = C_F\frac{\alpha_s}{2\pi}\left(\ln\left(\frac{s}{m^2}\right)-1\right); \beta_{qt} = C_F\frac{\alpha_s}{2\pi}\left(\ln\left(\frac{t}{m^2}\right)-1\right);$$
$$\beta_g = 2C_V\frac{\alpha_s}{2\pi}\left(\ln\left(\frac{s}{m^2}\right)-1\right); \beta_{gt} = 2C_V\frac{\alpha_s}{2\pi}\left(\ln\left(\frac{t}{m^2}\right)-1\right). \quad (6)$$

Motivation: General Formalism

Using the solutions of the homogeneous equations for quark non-singlet density D^q and G^g: (we use the method similar to one developed in frames QED in (E. A. Kuraev and V. S. Fadin, Sov. J. Nucl. Phys. **41**, 466 (1985)) (see Eq. (11) and Eq. (20) in (E. A. Kuraev and V. S. Fadin, Sov. J. Nucl. Phys. **41**, 466 (1985))):

$$D^q(x,\beta_q) = 2\beta_q(1-x)^{2\beta_q-1}\left(1+\frac{3}{4}\beta_q\right) - \beta_q(1+x) + O\left(\beta_q^2\right),$$

$$xG^g(x,\beta_g) = 2\beta_g(1-x)^{2\beta_g-1}\left(1+\frac{11}{12}\beta_g\right) + \beta_g\left(-2x+x^2(1-x)\right) + O\left(\beta_g^2\right). \quad (7)$$

These solutions satisfy the properties (2), i.e.:

$$\int_0^1 dx\ D^q(x,\beta_q) = 1, \qquad \int_0^1 dx\ xG^g(x,\beta_g) = 1. \quad (8)$$

Motivation: General Formalism

Below we consider the Drell-Yan process in collision of protons with the hard subprocess $a + b \to F^{ab}$ (where a and b are partons and F^{ab} is some final state produced by them) which is the part of more complicated process $p + p \to jet_1 + jet_2 + F^{ab}$. Thus we choose scale of order $Q^2 = s$ (i.e. $\alpha_s\left(Q^2\right) = \alpha_s$) where $\sqrt{s} = 2E$ is the total energy of process in the center of mass frame.
For inclusive experiments in pp collisions the Drell-Yan form of cross section take place:

$$E^2 \frac{d\sigma_{pp\to F_{AB}+X}}{d\Omega} = \int\limits_0^1 \frac{dx_1 dx_2}{x_1 x_2} \theta(x_1 x_2 - z_{th}) \int\limits_{z_{th}}^{x_1 x_2} dz \int\limits_z^{x_1} \frac{dy}{y} \psi^{AB}(z, y, \cos\theta)$$
$$\frac{1}{(a + b\cos\theta)^2} \left[W^q(x_1) D^A(y, \beta_q) K_q + W^g(x_1) G^A(y, \beta_g) K_g \right]$$
$$\left[W^q(x_2) D^A(\frac{z}{y}, \beta_q) K_q + W^g(x_2) G^A(\frac{z}{y}, \beta_g) K_g \right] + (A \leftrightarrow B). \quad (9)$$

Motivation: General Formalism

Here $W^a(x)$ is the probability to find a parton of sort a with energy fraction x inside proton with small virtuality (module of it's momentum square of order of $1GeV^2$), these quantities are given in (Martin A. D. et al. Eur.Phys.J. C63 (2009)189-285). These quantities are presented in Appendix C. D^A, G^A are the densities of parton A inside the parton q, g. The quantity

$$\psi^{AB} = s\frac{d\sigma^{AB\to F_{AB}}}{d\Omega_c} \tag{10}$$

describe the hard sub-process, the relevant expression is given below.
The substitutions must be performed to express kinematical invariants in center of mass of sub-process through the ones in center of mass of initial particles:

$$s = 4E^2 z, t = -2E_z^2(1-\cos\theta_c), u = -s-t;$$
$$\cos\theta_c = \frac{b + a\cos\theta}{a + b\cos\theta}; \quad d\Omega_c = d\Omega\left(\frac{1}{b + a\cos\theta}\right)^2, d\Omega = 2\pi\sin\theta d\theta,$$
$$a = y + \frac{z}{y}, b = y - \frac{z}{y}. \tag{11}$$

Polar angle θ_c is the angle between 3-vectors $\theta_c = \vec{p}_1\vec{p}_3$ in cm frame of sub-process. Polar angle θ is the angle between the axes of the initial beams with the direction of one of particles from the created state F_{AB}.
K_q-factor associated with quark density have a form $K_q = 1 + \frac{\alpha_s}{2\pi}k_q$ with (J. Kodaira and L. Trentadue, Phys. Lett. **B112**, 66 (1982))

$$k_q = \frac{1}{2}C_V\left(\frac{67}{18} - \frac{\pi^2}{6}\right) - \frac{5n_f}{18} \approx 1,5. \tag{12}$$

K_g-factor associated with gluon density have a form $K_g = 1 + \frac{1}{2}K_0$, with K_0 given above.
Solution of evolution equations for D^q, G^g are given above. Solution for $D^g, D^{\bar{q}}, G^q = G^{\bar{q}}$ are presented in Appendix B. Density correlators of type $D^{q'}$ which describe the density of presence inside a quark q' in quark q and $D^{\bar{q}'}$ -density of anti-quark $\bar{q}'$ into quark q as well are discussed in Appendix B.
The quantity $z_{th} = s_{th}/s$ determine the threshold invariant mass of subprocess s_{th} which takes into account the detection of particles created in subprocess F_{AB}.

Motivation: Application to Definite Subprocess

Below we consider two types os subprocesses.
Typical QCD sub-processes of type $2 \to 2$ (Journal of Physics G (Particle data group) v 33 (2006),p325)

$$a(p_1) + b(p_2) \to c(p_3) + d(p_4), s = (p_1 + p_2)^2, t = (p_1 - p_3)^2, u = (p_1 - p_4)^2, \\ p_i^2 = 0, \quad (13)$$

with gluons or light quarks production have one can use the expresionsfor ψ: (center of mass reference frame $\vec{p}_1 + \vec{p}_2 = 0$ implied)

$$\psi^{q\bar{q}\to q\bar{q}} = s\frac{d\sigma(q\bar{q} \to q\bar{q})}{d\Omega_c} = \frac{\alpha_s^2}{9}\left[\frac{t^2+u^2}{s^2} + \frac{s^2+u^2}{t^2} - \frac{2u^2}{3st}\right],$$
$$\psi^{qq\to qq} = s\frac{d\sigma(qq \to qq)}{d\Omega_c} = \frac{\alpha_s^2}{9}\left[\frac{t^2+s^2}{u^2} + \frac{u^2+s^2}{t^2} - \frac{2s^2}{3ut}\right], \quad (14)$$

Motivation: Application to Definite Subprocess

for identical quarks;

$$\psi^{q\bar{q}'\to q\bar{q}'} = s\frac{d\sigma(q\bar{q}'\to q\bar{q}')}{d\Omega_c} = \frac{\alpha_s^2}{9}\frac{t^2+u^2}{s^2};$$
$$\psi^{qq'\to qq'} = s\frac{d\sigma(qq'\to qq')}{d\Omega_c} = \frac{\alpha_s^2}{9}\frac{s^2+u^2}{t^2}, \tag{15}$$

for quarks of different flavors. As well

$$\psi^{q\bar{q}\to gg} = s\frac{d\sigma(q\bar{q}\to gg)}{d\Omega_c} = \frac{8\alpha_s^2}{27}(t^2+u^2)(\frac{1}{tu}-\frac{9}{4s^2}),$$
$$\psi^{gg\to q\bar{q}} = s\frac{d\sigma(gg\to q\bar{q})}{d\Omega_c} = \frac{\alpha_s^2}{24}(t^2+u^2)(\frac{1}{tu}-\frac{9}{4s^2}),$$
$$\psi^{gg\to gg} = s\frac{d\sigma(gg\to gg)}{d\Omega_c} = \frac{9\alpha_s^2}{8}(3-\frac{ut}{s^2}-\frac{su}{t^2}-\frac{st}{u^2});$$
$$\psi^{qg\to qg} = s\frac{d\sigma(qg\to qg)}{d\Omega_c} = \frac{\alpha_s^2}{9}(s^2+u^2)(-\frac{1}{su}+\frac{9}{4t^2}). \tag{16}$$

Motivation: Application to Definite Subprocess

In Fig. 4 the function $T_{gg} = E^2 \frac{d\sigma^{pp\to gg+X}}{d\Omega}$ is presented.
Consider the important application to the process $p + p \to t + H^- +$ jets (N. Kidonakis, JHEP **05**, 011 (2005), arXiv:hep-ph/0412422; S.-h. Zhu, Phys. Rev. **D67**, 075006 (2003), arXiv:hep-ph/0112109),

$$\psi(bg \to tH^-) = 4\pi s^2 \frac{d\sigma_B^{bg\to tH^-}}{dt} = \sigma_0 \left[\frac{s+t-M_{H^-}^2}{2s} - \right. \\ - \frac{m_t^2\left(u - M_{H^-}^2\right) + M_{H^-}^2\left(t - m_t^2\right) + s\left(u - m_t^2\right)}{s\left(u - m_t^2\right)} - \\ \left. \frac{m_t^2\left(u - M_{H^-}^2 - s/2\right) + su/2}{\left(u - m_t^2\right)^2} \right], \qquad (17)$$

and σ_0 is

$$\sigma_0 = \frac{\pi^2 \alpha \alpha_s \left[m_b^2 \tan^2\beta + m_t^2 \cot^2\beta\right]}{3\, M_W^2 \sin^2\theta_W}, \qquad (18)$$

Motivation: Application to Definite Subprocess

where θ_W is the Weinberg angle and β is the parameter of MSSM model. For $\tan\beta = 40$ and $\alpha_s = 0.1$ we have $\sigma_0 \approx 0.06$.
Functions $E^2 d\sigma_{pp\to F_i+X}/d\Omega_{F_i}$ are presented in Fig. 2–4 Figure 2 shows the quantity F_H:

$$F_H(\theta) = \frac{1}{\sigma_0} E^2 \frac{d\sigma_{pp\to tH^-+X}}{d\Omega}, \quad (19)$$

which is built for few values of charges Higgs boson mass M_H. The total cross section of this process is proportional to the quantity T_H:

$$T_H(s) = \int\limits_0^\pi d\theta F_H(\theta), \quad (20)$$

and is presented in Fig. 3.

Motivation: Application to Definite Subprocess

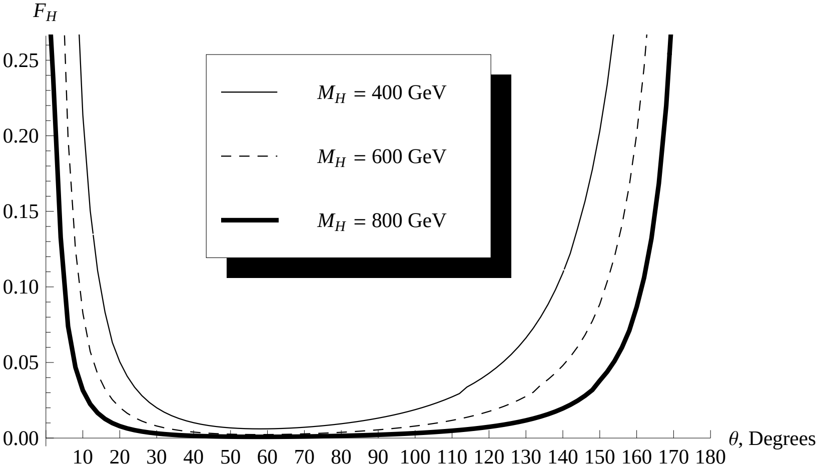

Figure: The angular dependence of quantity F_H defined in (19).

Motivation: Application to Definite Subprocess

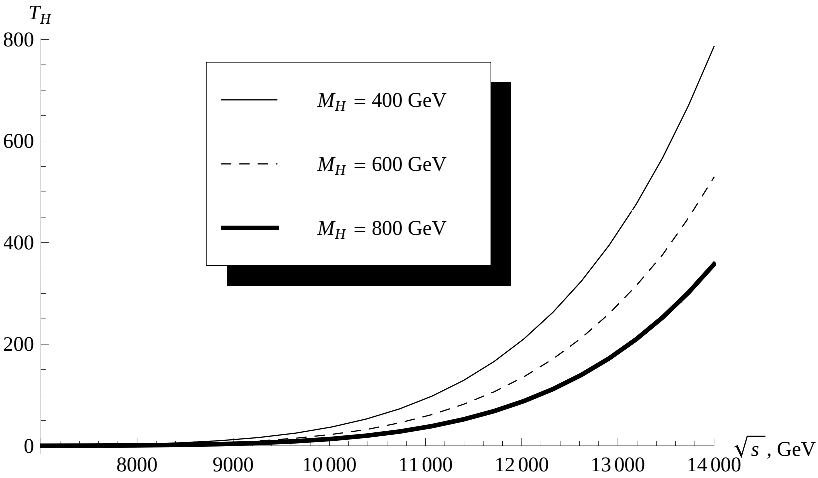

Figure: The quantity T_H defined in (20) as function of total invariant mass s.

Motivation: Application to Definite Subprocess

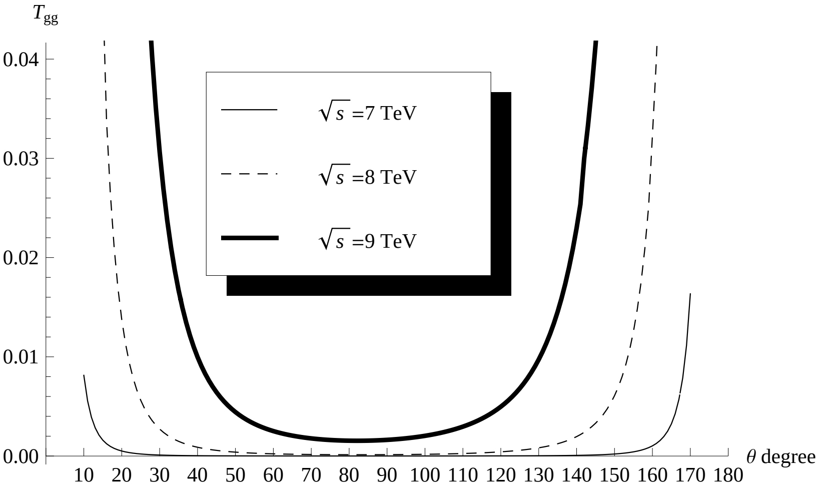

Figure: The angular dependence of quantity T_{gg} defined in (16), for the sub-processes $q\bar{q} \to gg$ and $gg \to gg$.

Motivation: Conclusion

In this paper we discuss some modification of the method of taking into account the QCD leading logarithm radiative corrections based on the Structure Functions approach. Modification consists construction of set of evolution equations for density of parton of sort a in the initial quark $D^a(x, \beta_q)$ and density of parton of sort b in the initial gluon $G^b(x, \beta_g)$. This set of equations is solved in quark and gluon dominance approximation $D^q >> D^a, a \neq q$ and $G^g >> G^a, a \neq g$. This approximation can be improved for the accuracy level required using the iteration procedure. This assumption is known as a gluon dominance which is used in description of multiplicity of pi-meson in hadron collisions (E. S. Kokoulina and V. A. Nikitin, (2005), arXiv:hep-ph/0502224).

We present approximate solution for D^a, G^a and demonstrate the application of this function to the problem of calculation of QCD radiative correction calculation in some particular processes.

A let of efforts was denoted to problem of calculation of sub - processes cross section in next-to-leading approximation. Main attention was paid to 2-loop level contributions (N. Kodonakis and R. Vogt, Phys. Rev. **D 68**, 14014 (2003); N. Kodonakis, Phys. Rev. **D 64**, 014009 (2001).). As a result the terms of order $(\alpha_s L^2)$, $(\alpha_s L^2)^2$ was taken into account.

Motivation: Conclusion

However the emission of real (soft and hard) gluons with the 1-loop radiative corrections was not considered. The role of radiative (virtual and real) corrections leads to the change of the $\left(\alpha L^2\right)^n$–regime to a single-logarithmical regime (i.e. only terms $\sim (\alpha L)^n$ remains) in inclusive experimental approach.
Single-logarithmical approach is determined by renormalization group evolution equation and thus allows us to use the Structure Function approach to obtain the cross section in leading (i.e. $(\alpha L)^n$) and next-to-leading (i.e. $\alpha (\alpha L)^n$) approximation.
Functions $W^a(x)$ describe the probability to find parton a inside a proton with off mass shell about one GeV squared. These functions was builded in (Martin A. D. et al. Eur.Phys.J. C(2009),63: 189-285.) as a self-consistent analysis of many sub-processes, and besides shown to satisfy the momentum and number sum rules. For experimental set-up with the product of subprocess detected which moves at large angles with invariant mass square exceeding some threshold value $s_{thr} = 4z_{th}E^2$ the role of "sea"-partons in the proton can be neglected.

Motivation: Appendix A

Let introduce three distributions $D^a = D^a_q(y,t)$ with $a = q, g, \bar{q}$ which describe the number of partons a with energy fraction y inside the parent quark. Similarly one must introduce three quantities $\bar{D}^a$ and three distributions G^a. Keeping in mind the absence of transition of a quark (antiquark) to antiquark (quark) $P_{q\bar{q}} = P_{\bar{q}q} = 0$ in lowest order of perturbation theory, the evolution equations of these 3 sets of distributions will have a form similar to ones for quark and gluon densities inside a proton given above. Similar consideration was used in frames QED in paper (Arbuzov A.,et al. Particles and Nuclei, v 41 N3,p722). For quark densities

$$\frac{d}{dt}D^q(x,t) = \frac{\alpha(t)}{2\pi}\int\limits_x^1 \frac{dy}{y}[D^q(y,t)P_{qq}(\frac{x}{y}) + D^g(y,t)P_{qg}(\frac{x}{y})];$$

$$\frac{d}{dt}D^{\bar{q}}(x,t) = \frac{\alpha(t)}{2\pi}\int\limits_x^1 \frac{dy}{y}[D^{\bar{q}}(y,t)P_{qq}(\frac{x}{y}) + D^g(y,t)P_{qg}(\frac{x}{y})];$$

$$\frac{d}{dt}D^g(x,t) = \frac{\alpha(t)}{2\pi}\int\limits_x^1 \frac{dy}{y}[D^g(y,t)P_{gg}(\frac{x}{y}) + D^{\bar{q}}(y,t)P_{gq}(\frac{x}{y}) + D^q(y,t)P_{gq}(\frac{x}{y})]. \quad (21)$$

Motivation: Appendix A

Similar set with the replacement $D^a \to \bar{D}^a$ take place for anti-quark densities. For gluon densities we have

$$\frac{d}{dt}G^{\bar{q}}(x,t) = \frac{d}{dt}G^q(x,t) = \frac{\alpha(t)}{2\pi}\int\limits_x^1 \frac{dy}{y}[G^q(y,t)P_{qq}(\frac{x}{y}) + G^g(y,t)P_{qg}(\frac{x}{y})];$$

$$\frac{d}{dt}G^g(x,t) = \frac{\alpha(t)}{2\pi}\int\limits_x^1 \frac{dy}{y}[G^g(y,t)P_{gg}(\frac{x}{y}) + G^{\bar{q}}(y,t)P_{gq}(\frac{x}{y}) + G^q(y,t)P_{gq}(\frac{x}{y})]. \quad (22)$$

It follows from these sets of equations

$$\frac{d}{dt}(D^g + G^g)(x,t) = \frac{\alpha(t)}{2\pi}\int\limits_x^1 \frac{dy}{y}[(D^g + G^g)(y,t)P_{gg}(\frac{x}{y}) +$$

$$(D^{\bar{q}} + G^{\bar{q}})(y,t)P_{gq} + (D^q + G^q)(y,t)P_{gq}(\frac{x}{y})]; \qquad (23)$$

Motivation: Appendix A

$$\frac{d}{dt}(D^q + G^q)(x,t) = \frac{\alpha(t)}{2\pi}\int\limits_x^1 \frac{dy}{y}[(D^q + G^q)(y,t)P_{qq}(\frac{x}{y}) + (D^g + G^g)(y,t)P_{qg}(\frac{x}{y})]. \quad (24)$$

When omitting the densities of anti-quarks $D^{\bar{q}}, G^{\bar{q}}$ inside quark and gluon,and identify $D^q + G^q = q, D^g + G^g = G$ we reproduce Altarelly-Parizi equations. Our statement about numerically small contribution of the intermediate quark (anti-quark) states in the evolution of gluon density follows from iteration procedure in solving the first equation of gluon set. So it can be taken into account by including as a relevant contribution to K-factor. Besides only light quarks must be considered, describing the experiments without quark jets production.

Motivation: Appendix B

A quark dominance consist in suggestion $D^q >> \bar{D}^q, G^q$. Gluon dominance imply $G^g >> D^g, \bar{D}^g$ and besides $D^g >> D^{\bar{q}}$. Set of equations in these approximations reads as

$$\frac{d}{dt}D^q(x,t) = \frac{\alpha(t)}{2\pi}\int\limits_x^1 \frac{dy}{y}D^q(y,t)P_{qq}(\frac{x}{y});$$

$$\frac{d}{dt}D^{\bar{q}}(x,t) = \frac{\alpha(t)}{2\pi}\int\limits_x^1 \frac{dy}{y}D^g(y,t)P_{qg}(\frac{x}{y});$$

$$\frac{d}{dt}D^g(x,t) = \frac{\alpha(t)}{2\pi}\int\limits_x^1 \frac{dy}{y}D^q(y,t)P_{qq}(\frac{x}{y});$$

$$\frac{d}{dt}G^q(x,t) = \frac{d}{dt}G^{\bar{q}}(x,t) = \frac{\alpha(t)}{2\pi}\int\limits_x^1 \frac{dy}{y}G^g(y,t)P_{qg}(\frac{x}{y});$$

(25)

Motivation: Appendix B

$$\frac{d}{dt}G^g(x,t) = \frac{\alpha(t)}{2\pi}\int_x^1 \frac{dy}{y}G^g(y,t)P_{gg}(\frac{x}{y}). \tag{26}$$

Note that the equation for D^q coincide with equation for non-singlet quark density $q_{NS} = q - \bar{q}$. The equations for D^q and G^g are given above. Keeping in mind the solution of evolution equations

$$\frac{\partial}{\partial t}A(x,\beta_{qt}) = C_F\frac{\alpha_{qt}}{2\pi}[a\beta_{qt} + b\beta_{qt}^2 + ...] = \frac{d\beta_{qt}}{dt}[a\beta_{qt} + b\beta_{qt}^2 + ...];$$
$$A(x,\beta_q) = a\frac{1}{2}\beta_q^2 + b\frac{1}{3}\beta_q^3 + ..., \tag{27}$$

we obtain

$$D^g(x,\beta_q) = G^q(x,\beta_q) = \frac{1}{2}[1 + (1-x)^2]\beta_q^2 + O(\beta_q^3);$$
$$D^{\bar{q}}(x,\beta_q) = D^{q'}(x,\beta_q) = D^{\bar{q}'}(x,\beta_q) = \frac{1}{2}\phi(x)\beta_q^2 + O(\beta_q^3),$$
$$\phi(x) = \frac{1}{3x}(1-x)(4 + 7x + 4x^2) + 2(1+x)\ln x. \tag{28}$$

Keeping in mind the problem of description of inelastic processes in high energy collision of protons it seems to be naturally consider proton as an objects with definite contents from quarks and gluons. It implied the presence of the preliminary evolution from mass shell to virtuality of order $1GeV^2$ of all the constituents of proton. Note that due to condition $x_1x_2 > z_{th}$ only valence quark and gluons inside proton take part in Drell-Yan process. We will choice the density of the valence quarks and gluons approximately as ones found in paper (Martin A. D. et al. Eur.Phys.J. C63 (2009)189-285):

$$\begin{aligned} xu_v(x,Q_0^2) &= A_u x^{\eta_1}(1-x)^{\eta_2}(1+\epsilon_u\sqrt{x}+\gamma_u x); \\ A_u &= 0,2; \eta_1 = -0,73; \eta_2 = 3,3; \\ xd_v(x,Q_0^2) &= A_d x^{\eta_3}(1-x)^{\eta_4}(1+\epsilon_d\sqrt{x}+\gamma_d x); \\ A_d &= 18; \eta_3 = 0,1; \eta 42 = 6; \\ xg(x,Q_0^2) = A_g x^{\delta_g}(1-x)^{\eta_g}(1+\epsilon_g\sqrt{x}+\gamma_g x) &+ A_{g'} x^{\delta'_g}(1-x)\eta'_g, \\ A_g &= 0.0012216. \end{aligned} \tag{29}$$

Motivation: Appendix C

Numerical constants must be chosen in such a way to satisfy the constrains from number sum rules

$$\int_0^1 dx W_u(x) = 2; \quad \int_0^1 dx W_d(x) = 1, \tag{30}$$

and, besides the momentum sum rule

$$\int_0^1 dx x [W_u(x) + W_d(x) + W_g(x) + S(x)] = 1, \tag{31}$$

with $S(x)$ is the sea contribution.
Where $\eta_u = 8.9924$; $\eta_d = 7.4730$; $\eta_g = 2.3882$; $\eta'_g = 0$; $A'_g = 0$; $\delta_g = -0.83657$; $\delta'_g = 0$; $\gamma_g = 1445.5$; $\epsilon_u = -2.3737$, $\epsilon_d = -4.3654$, $\epsilon_g = -38.997$.
More complicated expressions for densities which was extracted from description of fixed target, HERA and Tevatron experiments are presented in (Martin A. D. et al. Eur.Phys.J. C63 (2009)189-285).

Motivation: Acknowledgements

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