Peripheral processes with vector exchanges.

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Introduction

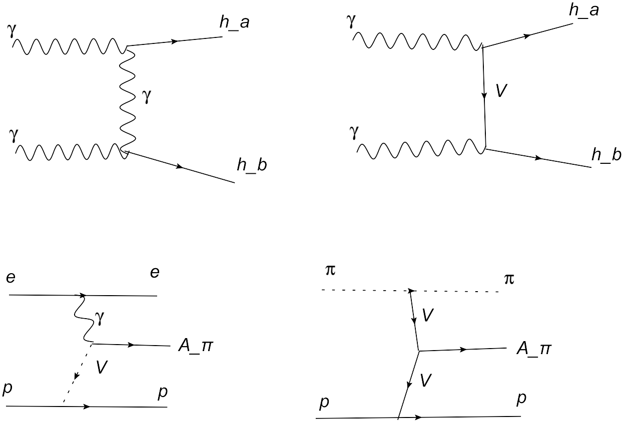
We investigate the production of light mesons pairs and elementary atoms (positronium atoms Ps and pionium atoms A_π) in high energy $\gamma\gamma, \pi p, ep$ collisions:

$$\gamma(k_1) + \gamma(k_2) \to h_a(p_1) + h_b(p_2); \quad h_a, h_b = \pi, \eta, \eta\prime, \sigma, Ps, A_\pi \quad (1)$$

$$\begin{aligned} e(p_1) + p(p_2) &\to e(p_1') + p(p_2') + A_\pi \\ \pi(p_1) + p(p_2) &\to \pi(p_1') + p(p_2') + A_\pi \end{aligned} \quad (2)$$

Due to peripheral kinematics $s = (k_1 + k_2)^2, t = q^2 = (p_1 - k_1)^2; s >> |q^2|$ the created objects h_a, h_b have energies approximately equal to the energies of colliding particles and move along the directions of initial particles motion (center of mass of initial particles implied).

The remarkable property of the relevant cross sections-they become independent from center of mass energy s of colliding particles starting from some threshold energy $\sqrt{s} \sim 2 - 3GeV$. The nondecreasing feature of pairs yield is a result of vector nature of the interaction (photon or vector meson exchanges in the t-channel).In peripheral kinematics one can use the perturbation models of hadrons like Chiral Perturbation Theory (ChPT) or Nambu–Jona-Lasinio (NJL) model to describe the sub-processes at the relevant vertexes.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Meson pairs production in photon-photon collisions $\gamma(k_1) + \gamma(k_2) \rightarrow h_a(p_1) + h_b(p_2)$

One can expresses the matrix elements of this reactions through so called impact factors, which are nothing else than the matrix elements of a sub-processes with virtual photon : $\gamma(k_1) + \gamma^*(q) \rightarrow h_a$ and $\gamma(k_2) + \gamma^*(q) \rightarrow h_b$ or vector meson : $\gamma(k_1) + V(q) \rightarrow h_a$ and $\gamma(k_2) + V(q) \rightarrow h_b$.
Making use the Sudakov variables one obtains the connection of matrix element of process (1) with impact factors (with power accuracy):

$$M = \frac{2s}{q^2 - m_V^2} M^a M^b; \quad M^a = \frac{J^a_\mu k_2^\mu}{s}; \quad M^b = \frac{J^b_\nu k_1^\nu}{s}. \qquad (3)$$

The impact factors M^a, M^b don't decrease with energy and can be described in terms of perturbation strong interaction models like Nambu-Jona-Lasinio model or Chiral Perturbation Theory.
The differential cross section of the processes (1) reads:

$$d\sigma^{ab\to h_a h_b} = \frac{d^2q}{(4\pi)^2(\vec{q}^2 + m_V^2)^2} \sum_{spins} |M^a|^2 \sum_{spins} |M^b|^2 \qquad (4)$$

Thus the knowledge of relevant impact factors allows one to calculate the cross sections of processes (1).

Mesons production through photon exchange.

Consider the production of $\pi^0\pi^0$ pair in $\gamma\gamma$ collisions with photon exchange in the t-channel. The current algebra gives for the matrix element of neutral pion decay to two photons $\pi^0(p) \to \gamma(k_1, e_1) + \gamma(k_2, e_2)$:

$$M(\pi^0 \to \gamma\gamma) = \frac{\alpha}{\pi F_\pi}(k_1 e_1 k_2 e_2), \tag{5}$$

where $(abcd) = a^\alpha b^\beta c^\gamma d^\delta \epsilon_{\alpha\beta\gamma\delta}$ and $k_i, e_i(k_i)$ are the momenta and polarization vectors of real photons, $\alpha = \frac{e^2}{4\pi} = 1/137$ is the fine structure constant and $F_\pi = 92.2MeV$ is the pion decay constant measured in the $\pi^+ \to \mu^+\nu_\mu$ decay rate. The pion radiative decay width is given by the textbook formula:

$$\Gamma(\pi^0 \to 2\gamma) = (\frac{m_\pi}{4\pi})^3(\frac{\alpha}{F_\pi})^2 = 7.76eV \tag{6}$$

The decay amplitude (5) can be used as impact factor in $\pi^0\pi^0$ production.

More elaborated impact factors considering the photon virtuality can be obtained if one calculates the triangle fermion loop with the light u and d quarks as a fermions. Quarks charges and number of colors result in a factor $3((2/3)^2 - (1/3)^2) = 1$. After standard procedure of denominators joining, calculating the relevant trace in the fermions spin indices and integration over the loop momenta we obtain:

$$M(\pi^0 \to \gamma\gamma^*) = \frac{\alpha}{2\pi F_\pi} |\,[\vec{e}, \vec{q}]\,| N_\pi J_\pi(z), \quad N_\pi J_\pi(0) = 1, z = \frac{\vec{q}^2}{m_q^2}$$

$$J_\pi(z) = \int_0^1 dx \int_0^1 \frac{ydy}{1 - \rho^2 y^2 x(1-x) + zy(1-y)x}, \quad \rho = \frac{m_\pi}{m_q} \quad (7)$$

Here m_q is the constituent quark mass, which we put $m_q = m_u = m_d \approx 280 MeV$, whereas N_π is the normalization constant.

The similar expression for the sub-process of the scalar meson decay $\sigma \to \gamma\gamma^*$ reads:

$$M(\sigma \to \gamma\gamma^*) = \frac{5\alpha}{6\pi F_\sigma}|(\vec{e},\vec{q})|N_\sigma J_\sigma(z); N_\sigma J_\sigma(0) = 1, F_\sigma \approx F_\pi$$

$$J_\sigma(z) = \int_0^1 dx \int_0^1 \frac{y(1-4y^2x(1-x))dy}{1-\rho^2y^2x(1-x)+zy(1-y)x}, \quad \rho = \frac{m_\sigma}{m_q}. \quad (8)$$

The combination of quarks charges and color factor give a coefficient $3((2/3)^2 + (1/3)^2) = 5/3$. The nontrivial difference in numerators is a result of scalar nature of σ meson. The amplitudes $M(\pi^0 \to \gamma\gamma^*), \quad M(\sigma \to \gamma\gamma^*)$ are nothing else than impact factors, one needs to calculate the cross sections of neutral mesons pairs production.

For cross section we get:

$$\frac{d\sigma}{dz} = (\frac{\alpha m_q}{4\pi F_\pi})^4 \frac{(zN_\pi J_\pi(z))^4}{\pi m_q^2 (1+z^2)^2}(1+2cos^2\phi_0), \qquad (9)$$

where ϕ_0 is the azimuthal angle between the initial photons polarization vectors. In the case of pions production one can safely neglect the small term $y^2x(1-x)m_\pi^2/m_q^2 < 0.05$ in the denominator ($N_\pi = 1/2$) with the result:

$$J_\pi(z) = \int\limits_0^1 dx \int\limits_0^1 \frac{ydy}{1+zy(1-y)x} = \frac{4}{z}\ln^2\left(\sqrt{1+\frac{z}{4}}+\sqrt{\frac{z}{4}}\right).(10)$$

The total cross section:

$$\begin{aligned}
\sigma^{\gamma\gamma\to\pi_0\pi_0} &= \sigma_0(1+2\cos^2\phi_0)I, \\
\sigma_0 &= \frac{\alpha^4 m_q^2}{2^7\pi^5 f_\pi^4} \approx 2,6\times 10^{-2} pb, \\
I &= \frac{1}{4}\int\limits_0^\infty \frac{dz}{z^4}\ln^8(\sqrt{1+z}+\sqrt{z}) = 0.3557. \quad (11)
\end{aligned}$$

Above expressions allow one to calculate any combination of light meson pairs production in $\gamma\gamma$ collisions.

Bound states production

The considered approach is especially efficient in investigation of bound states formation in $\gamma\gamma$ collisions. As a typical examples we examine the production of simplest atoms being the bound state of two charged pions (pionium atom A_π) and atom constructed from two leptons (positronium atom Ps). To determine the pionium impact factor $\gamma\gamma* \to A_\pi$ we take advantage of well known QED amplitude for the process $\gamma(k_1, e_1) + \gamma(q) \to \pi^-(q_-) + \pi_+(q_+)$:

$$M^{\gamma\gamma\to\pi\pi} = \frac{4\pi\alpha}{s}[\frac{(2q_-e_1)((-2q_+ + q)k_2)}{2q_-k_1} + \frac{(-2q_+e_1)((2q_- - q)k_2)}{-2q_+k_1}$$

Accounting that in atom the pions have the same velocity $q_+ = q_- = p/2; 2(pk_1) = 4m^2 + \vec{q}^2$ expressing p, ϵ through the Sudakov variables:

$$p = \alpha_p k_2 + \beta_p k_1 + q_\perp; \quad e = \beta_e k_1 + e_\perp, \tag{13}$$

the expression in curly brackets simplified:

$$(pe_1)((p-q)k_2 - (pk_1)(e_1 k_2) = -2s(\vec{q}\vec{e}_1) \tag{14}$$

Finally the amplitude of two pions production at same velocities takes the form:

$$M^{\gamma\gamma\to\pi\pi} = \frac{32\pi\alpha(\vec{e}\vec{q})}{\vec{q}^2 + 4m_\pi^2} \tag{15}$$

In order to obtain the amplitude for pionium production we use the well known receipt allowing to connect the amplitude of the two free particles production with the production amplitude of bound state A_π :

$$M^{\gamma\gamma\to A_\pi} = M^{\gamma\gamma\to\pi\pi}\frac{i\Psi(\vec{r}=0)}{\sqrt{m}} \tag{16}$$

Finally for the amplitude of pionium production in $\gamma\gamma$ collisions we get:

$$M^{\gamma\gamma\to A_\pi} = \frac{4\pi\alpha(2\vec{e}_1\vec{q})}{4m_\pi^2+\vec{q}^2}\frac{\Psi(0)}{\sqrt{m_\pi}}, \tag{17}$$

The amplitude of positronium creation $\gamma(k_1, e_1) + \gamma^*(q) \to Ps(p)$ can be obtained in the same way. The difference with above consideration is in the receipt of passage from free e^+e^- pair in a final state of the QED process $\gamma(k_1, e_1) + \gamma^*(q) \to e^+(q_+) + e^-(q_-)$ to the bound state, which requires a substitution:

$$\bar{u}...v \to \frac{im_l\alpha^{3/2}}{\pi} \tag{18}$$

The amplitude of free electron-positron e^+e^- pair production with the same momenta $q_+ = q_- = p/2$ can be obtain using the standard expression for QED amplitude:

$$M^{\gamma\gamma\to e^+e^-} = \frac{4\pi\alpha}{s}\frac{1}{pk_1}\left[(pe_1)((p-q)k_2 - (pk_1)(e_1k_2)\right] \tag{19}$$

$$M^{\gamma\gamma\to Ps} = \frac{2m_l\sqrt{\pi\alpha^5}}{4m_l^2 + \vec{q}^2}|[\vec{e}_1, \vec{q}]|, \tag{20}$$

With the help of obtained equations one can calculates the differential cross section of any of the processes

$$\gamma + \gamma \to S_1 + S_2; \qquad S_1, S_2 = A_\pi, Ps \tag{21}$$

For total cross sections of bound state production by photon exchange mechanism (Fig.1a) we get:

$$\sigma^{\gamma\gamma \to PsPs} = \frac{\pi\alpha^8}{96} r_e^2 (1 + 2\cos^2\phi_0);$$

$$\sigma^{\gamma\gamma \to A_\pi A_\pi} = (\frac{r_e}{4r_\pi})^2 \sigma^{\gamma\gamma \to PsPs}; \sigma^{\gamma\gamma \to PsA_\pi} = \frac{\pi\alpha^8}{64} r_\pi^2 (3 - 2\cos^2\phi_0);$$

$$\sigma^{\gamma\gamma \to Ps\pi_0} = \frac{\alpha^7}{32\pi^2 F_\pi^2}(1 + 2\cos^2\phi_0); r_e = \frac{\alpha}{m_e}, r_\pi = \frac{\alpha}{m_\pi}, \tag{22}$$

Rough estimates of these cross sections shows that they are really very small the quantities of order $10^{-8} nb$.

Vector meson exchanges

Up to now we considered production processes provided by photon exchange (Fig.1a). From the other hand the exchange by vector meson (Fig. 1b) also gives nondecreasing with energy contribution in the processes (1). The problem with such type exchanges is connected with the fact that Born approximation depicted on Fig. 1b badly violated for strong interactions.

To take into account the higher order contributions of strong interaction one would replaces the exchanged vector meson propagator by its reggeized one:

$$\frac{1}{t-m_V^2} \to \alpha' \frac{1-e^{-i\pi\alpha(t)}}{2}\Gamma(1-\alpha(t))(\frac{s}{s_0})^{\alpha(t)}, \qquad (23)$$

$\alpha(t)$ is the Regge trajectory of vector meson

$$\alpha(t) = \alpha(0) + \alpha' t \tag{24}$$

Γ function contains the pole propagator $1/sin(\pi\alpha(t))$ and in the limit $t \to m_V^2$ reduced to the standard propagator. In calculation of vector meson exchanges contribution in cross section the simplified expression will be used:

$$R(s,t) = (\frac{s}{s_0})^{2(\alpha(t)-1)} \approx \frac{s_o}{s}, \quad s_0 \approx 1GeV. \tag{25}$$

The impact factors corresponding to vector mesons exchanges depend on the considered process and would be obtained as it has been done above in consideration of photon exchange. Let us begin from process of two charged pions production $\gamma\gamma \to \pi^+\pi^-$, for which the photon exchange is absent. The main contribution to this reaction exchange gives the ρ exchange. The matrix element of radiative decay of charged ρ-meson $\rho^+(p,\epsilon) \to \pi^+(p_\pi) + \gamma(k,e)$ is known $M = g_+(p\epsilon k e)$. The relevant decay width is

$$\Gamma^{\rho^+ \to \pi^+\gamma} = \frac{g_+^2}{96\pi}\left(\frac{M_\rho^2 - m_\pi^2}{M_\rho}\right)^3. \tag{26}$$

Comparing with experimental value of $\rho^+ \to \pi^+\gamma$ branching ratio $B = 4,5 \times 10^{-4}$, $\Gamma_{exp} = 67 KeV$, we find $g_+ \approx 0.21 GeV^{-1}$.

For the case of virtual photon one can modifies these relations by replacement $g_+ \to g_+ F(z)$ with

$$F(z) = \frac{4}{z}\ln^2(\sqrt{1+\frac{z}{4}} + \sqrt{\frac{z}{4}}), z = \frac{\vec{q}^2}{m_q^2}. \tag{27}$$

For the differential cross section of the process $\gamma\gamma \to \pi_+\pi_-$ in peripheral kinematic we get

$$d\sigma = \frac{d\vec{q}^2 d\phi}{32\pi^2}\frac{|M^{(1)}|^2|M^{(2)}|^2}{(\vec{q}^2 + M_\rho^2)^2}, \tag{28}$$

$$M^{(1)} = \frac{g_+}{2}[\vec{q}\vec{e}_1]F(z); M^{(2)} = \frac{g_+}{2}[\vec{q}\vec{e}_2]F(z). \tag{29}$$

The averaging in azimuthal angles leads to
$\int\limits_0^{2\pi} \frac{d\phi}{2\pi}([\vec{q}\vec{e}_1]^2([\vec{q}\vec{e}_2])^2 = \frac{1}{8}(1 + 2\cos^2\phi_0)$.

For the total cross section we obtain

$$\sigma = \frac{g_+^4 M_q^2}{32\pi}(1 + 2\cos^2\phi_0) I, \tag{30}$$

$$I = \int\limits_0^\infty \frac{dz}{z^2(z + (\frac{M_\rho}{2M_q})^2)^2} \ln^8(\sqrt{1 + \frac{z}{4}} + \sqrt{\frac{z}{4}}) \approx 0,372, \tag{31}$$

$$\sigma_{periph}^{\gamma\gamma\to\pi_+\pi_-} \approx 60 nb. \tag{32}$$

We see that this mechanism dominate starting from $\sqrt{s} = 1 GeV$.

Production of pionium atom in high energy charged pion-proton and electron-proton collisions

A lot of attention was paid for the problem of measurement of the lifetime τ of the bound state of two oppositely charged pions. The main aim is to check the predictions of low-energy hadron theories such as CHPT, NJL for the difference $a_0^0 - a_0^2$ of $\pi\pi$ s-wave scattering lengths (isospin I=0,2)

$$\Gamma = \frac{1}{\tau} = \frac{2}{9}\sqrt{\frac{2(m_{\pi^+} - m_{\pi^0})}{m_\pi}}(a_0^0 - a_0^2)^2 m_\pi^3 \alpha^3. \qquad (33)$$

The experiment Dirac at PS CERN measured the lifetime of pionium atoms created in collisions of high energy proton with the dense target.

Let us now consider the peripheral mechanism of creation of two charged pions in collision of high energy electron or pion with proton:

$$\begin{aligned} \pi(p_1) + p(p_2) &\to \pi(p_1') + A(p) + p(p_2'); \\ e(p_1) + p(p_2) &\to e(p_1') + A(p) + p(p_2') \end{aligned} \tag{34}$$

For the case of electron-proton collision the pion pair is created by virtual photon emitted by electron and virtual ρ (ω) meson emitted by proton. For the case of π-meson proton collisions it is produced by two virtual ρ mesons.

Matrix element of considered processes have the form:

$$\frac{G}{(q_1^2 - M_1^2)(q_2^2 - M_2^2)} J_1(p_1)_{\mu_1} T_{\mu\nu} J_p(p_2)_{\nu_1} G^{\mu\mu_1} G^{\nu\nu_1}, \qquad (35)$$

with G is the product of the relevant coupling constants, $M_{1,2}$-masses of the exchanged vector particles; J_1, J_p -are the currents,connected with the initial particles; tensor $T_{\mu\nu}$ describes the conversion of two vector mesons to pion pair. Main contribution in peripheral kinematics (non-vanishing in limit $s \to \infty$) arises from such form of denominators Of nominators of Green functions

$$G^{\mu\mu_1} = \frac{2}{s} p_2^{\mu} p_1^{\mu_1}; G^{\nu\nu_1} = \frac{2}{s} p_2^{\nu} p_1^{\nu_1}. \qquad (36)$$

The standard procedure of forming the bound state consist in choice of kinematics when the 4- momenta of π-meson are equal and take into account the probability for them to be in the same point, which is described by factor $\psi(r)_{r=0} = \psi = m\alpha^{3/2}/\sqrt{4\pi n}$.

Matrix element of the sub-process of creation of pion pair with equal 4-momenta by two virtual vector particles

$$V_\mu(q_1) + V_\nu(q_2) \to \pi_-(q) + \pi_+(q), 2q = q_1 + q_2, \qquad (37)$$

described by tensor

$$T_{\mu\nu} = \frac{2}{D}[q_{2\mu}q_{1\nu} + Dg_{\mu\nu}], D = -\frac{1}{2}[4m^2 + \vec{q}_1^{\,2} + \vec{q}_2^{\,2}], \qquad (38)$$

where we use Sudakov parametrization of the transferred 4-vectors q_1, q_2 as

$$\begin{aligned} q_1 &= \beta_1\tilde{p}_1 + \alpha_1\tilde{p}_2 + q_{1\perp} \approx \beta\tilde{p}_1 + q_{1\perp}, \\ q_2 &= \beta_2\tilde{p}_1 + \alpha_2\tilde{p}_2 + q_{2\perp} \approx \alpha_2\tilde{p}_2 + q_{2\perp}, \\ q_\perp\tilde{p}_{1,2} &= 0, q_\perp^2 = -\vec{q}^{\,2}, \tilde{p}_i^2 = 0; \tilde{p}_1\tilde{p}_2 = s. \end{aligned} \qquad (39)$$

As a result we have for matrix element of process $e + p \rightarrow e' + p' + A$

$$M^{ep\rightarrow epA} = \frac{4s}{\vec{q}_1^2 + m_e^2\beta_1^2}\frac{G_e}{\vec{q}_2^2 + M_V^2}\Phi_e\Phi_A\Phi_p\psi, \qquad (40)$$

with $G_e = 4\pi\alpha g_\pi g_p$, g_π, g_p-coupling constant of interaction of ρ-meson with pion and proton,

$$\Phi_e = \frac{1}{s}\bar{u}(p_1')\hat{p}_2 u(p_1); \qquad (41)$$

$$\Phi_A = \frac{1}{s}p_1^\mu p_2^\nu T_{\mu\nu} = -\frac{\vec{q}_1\vec{q}_2}{D}; \qquad (42)$$

$$\Phi_p = \frac{1}{s}\bar{u}(p_2')\Gamma_\mu u(p_1)p_2^\mu, \Gamma_\mu = \gamma_\mu F_1 + \frac{1}{4M_p}(\hat{q}_2\gamma_\mu - \gamma_\mu\hat{q}_2)F_2, \qquad (43)$$

$F_1 = F_1(q_2^2), F_2 = F_2(q_2^2)$-Dirac and Pauli form-factors of proton.

Phase volume of the final state

$$d\Gamma = \frac{(2\pi)^4}{(2\pi)^9} \frac{d^3 p_1'}{2E_1'} \frac{d^3 p_2'}{2E_2'} \frac{d^3 p_A}{2E_A} \delta^4(p_1 + p_2 - p_1' - p_2' - p_A), \quad (44)$$

Introducing the integrations on two exchange momentum factor

$$d^4 q_1 \delta^4(p_1 - q_1 - p_1') d^4 q_2 \delta^4(p_2 - q_2 - p_2'), \quad (45)$$

with Sudakov parametrization $d^4 q = (s/2) d\alpha d\beta d^2 \vec{q}$ and

$$\frac{d^3 p_1'}{2E_1'} = d^4 p_1' \delta((p_1')^2 - m_1^2); \frac{d^3 p_2'}{2E_2'} = d^4 p_2' \delta((p_1')^2 - m_1^2);$$
$$\frac{d^3 p_A}{2E_A} = d^4 p_A \delta(p_A^2 - 4m^2), \quad (46)$$

we obtain

$$d\Gamma = \frac{1}{(2\pi)^5} \frac{1}{4s} \frac{d\beta_1}{\beta_1} d^2 \vec{q}_1 d\vec{q}_2. \quad (47)$$

Using the summed on spin states of squares of matrix elements of the sub-processes

$$\sum |\Phi_e|^2 = 2; \sum |\Phi_p|^2 = 2[F_1^2 + \frac{\vec{q}_2^2}{4M_p^2} F_2^2];$$
$$|\Phi_A|^2 = \frac{4(\vec{q}_1 \vec{q}_2)^2}{(4m^2 + \vec{q}_1^2 + \vec{q}_2^2)^2}, \qquad (48)$$

we obtain for the cross section

$$d\sigma^{ep \to epA} = \frac{\alpha^5 g_\pi^2 g_p^2}{2\pi^2} \frac{m^2 \vec{q}_1^2 d\vec{q}_1^2 \vec{q}_2^2 d\vec{q}_2^2}{(4m^2 + \vec{q}_1^2 + \vec{q}_2^2)^2 (\vec{q}_1^2 + m_e^2 \beta_1^2)^2 (\vec{q}_2^2 + M_\rho^2)^2}$$
$$\times [F_1^2 + \frac{\vec{q}_2^2}{4M_p^2} F_2^2] \frac{d\beta_1 (1 - \beta_1)}{\beta_1}; \frac{4m^2}{s} < \beta_1 < 1. (49)$$

Similar expression for the cross section with initial π meson instead electron:

$$d\sigma^{\pi p\to\pi pA} = \frac{\alpha^3 g_\pi^6 g_p^2}{64\pi^4}\frac{m^2\vec{q}_1^2 d\vec{q}_1^2\vec{q}_2^2 d\vec{q}_2^2}{(4m^2+\vec{q}_1^2+\vec{q}_2^2)^2(\vec{q}_1^2+M_\rho^2)^2(\vec{q}_2^2+M_\rho^2)^2}$$
$$\times[F_1^2+\frac{\vec{q}_2^2}{4M_p^2}F_2^2]\frac{d\beta_1(1-\beta_1)}{\beta_1}; \frac{4m^2}{s}<\beta_1<1. (50)$$

Further estimation consists in calculation of the total cross sections σ^e, σ^π by means of integration on $\beta_1, \vec{q}_1^2, \vec{q}_2^2$. The dependence on invariant mass squared s_A of two photons arising from decay of pionium can be approximated as

$$\frac{d\sigma}{\sigma ds_A} = \frac{1}{\pi}\frac{2m\Gamma}{(s_A - 4m^2)^2 + 4m^2\Gamma^2}, \qquad (51)$$

with Γ is the width of the pionium resonance. The modern experiments give $\gamma_A = 33eV$.

Our estimation for the total cross sections give:

$$\sigma^e = \sigma_0^e D^e, \sigma_0^e = \frac{\alpha^5 g_\pi^2 g_p^2 m^2}{2\pi^2 M_\rho^4} \approx 0.3pb; \quad (5$$

$$D^e = J_N[l_m^2 + l_\pi(l_m - 1) - 2], J_N = \int_0^\infty \frac{xN^2 dx}{(x+4)^2(x+N)^2} \approx 0.845; \quad (5$$

$$l_m = \ln \frac{s}{4m^2}, l_\pi = \ln \frac{m^2}{m_e^2}. (5$$

For $s = 100 GeV^2$, $N = (M_\rho/m)^2 = 30$ and $g_\pi = g_p = 6$ we obtain $D^e \approx 100$. Such a value of cross section $\sigma^e \approx 30pb$ is too small to be measured at DESY facility .

$$\sigma^\pi = \sigma_0^\pi D^\pi, \sigma_0^\pi = \frac{\alpha^3 g^8 m^2}{64\pi^2 M_\rho^4} \approx 217 nb; \qquad (55)$$

$$D^\pi = (l_m - 1)I, I = \int\int_0^\infty \frac{x_1 x_2 dx_1 dx_2}{(x_1+x_2)^2(x_1+1)^2(x_2+1)^2} \approx 0.133. \qquad (56)$$

estimation give $D^\pi \approx 0.82$. The total cross section turns out to be of order $\sigma^\pi \approx 178 nb$ for $s = 80 GeV^2$. In conclusion we note that the contribution of the channels with both virtual photons exchanged is of order

$$\sigma^e = \sigma_0 D^e_{\gamma\gamma}; \sigma^\pi = \sigma_0 D^\pi_{\gamma\gamma}, \sigma_0 = \frac{8\alpha^7}{m^2 \approx} 1,8 \times 10^{-3} pb. \qquad (57)$$

In spite of a rather large enhancement factors $D^e_{\gamma\gamma} \sim 10 D^\pi_{\gamma\gamma} \sim 10^2$ the relevant contributions seems to be negligible.

Effect of vector meson reggeization

Consideration of hadronic processes in peripheral kinematics in Born approximation is non-adequate. The effect of converting the ordinary vector mesons to the relevant Regge poles must be taken into account. It results in an additional factor in the total cross section

$$R = [\frac{s_1 s_2}{s_0^2}]^{2(\alpha(0)-1)}, \tag{58}$$

with $s_1 = (p_1 + q_2)^2 \approx s\alpha_2, s_2 = (p_2 + q_1)^2 \approx s\beta_1$-are the partial invariant mass squared, $s_0 \approx 1 GeV^2$- the usual scale Regge theory parameter; $\alpha(0) \approx 0.5$ is the intercept of ρ-meson trajectory.

Keeping in mind the kinematical relation $s_1 s_2 \approx 4m^2 s$, we obtain:

$$R \approx \frac{s_0^2}{4sm^2}. \quad (59)$$

For $s = 80 GeV^2$ it results in factor

$$R \approx 0.16. \quad (60)$$

So the realistic cross section for Protvino facility is expected $\sigma^\pi \approx 28 nb$.