# Description of $e^+e^- \to \pi^+\pi^-(\pi')$ processes in the extended NJL model

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March 16, 2012

# Outline

- Motivation to study $e^+e^- \to \pi^+\pi^-(\pi')$
- Remind the Nambu-Jona-Lasinio model
- NJL model with radially excited mesons
- Process $e^+e^- \to \pi^+\pi^-$
- Process $e^+e^- \to \pi\pi'$
- $\tau$ decays
- Conclusions

# Motivation

Studies of $e^+e^-$ annihilation into $\pi\pi(\pi')$ at colliding electron-positron beams provide interesting information about meson interactions at low energies
[CMD-2: R.R. Akhmetshin et al., Phys. Lett. B 2003]

The same interactions can be also found in the tau lepton decay $\tau \to \pi\pi\nu_\tau$ studied at number of experiments [CLEO, ALEPH, Belle, BaBar]

It can serve as a test of models for the pion transition form factor

# Motivation

- A. B. Arbuzov, E. A. Kuraev and M. K. Volkov, "Processes $e^+e^- \to \pi^0(\pi^{0\prime})\gamma$ in the NJL model," European Physics Journal. A **26**, 3337 (2011)
- A. B. Arbuzov, E. A. Kuraev and M. K. Volkov, "Production of $\omega\pi^0$ pair in electron-positron annihilation," Phys. Rev. C **83**, 048201 (2011)
- A. I. Ahmadov, E. A. Kuraev and M. K. Volkov, "Generalized polarizability of neutral pions of the process $e^-e^+ \to \pi^0\pi^0\gamma$ in NJL model," Int. J. Mod. Phys. A **26**, 3337 (2011)
- A. B. Arbuzov and M. K. Volkov "Two-photon decays and photoproduction on electrons of $\eta(550)$, $\eta'(958)$, $\eta(1295)$ and $\eta(1475)$ mesons, Phys. Rev. C **84**, 058201 (2011)"
- A. I. Ahmadov, E. A. Kuraev and M. K. Volkov, "Production of $\pi^0\rho^0$ pair in electron-positron annihilation in the Nambu-Jona-Lasinio model," arXiv:1111.2124 (accepted to PEPAN letters).

For the ground meson states we use the standard NJL Lagrangian:

$$\Delta\mathcal{L}_1 = \bar{q}\left[i\hat{\partial} - m + eQ\hat{A} + ig_\pi\gamma_5\tau_3\pi^0 + \frac{g_\rho}{2}\gamma_\mu\left(I\omega_\mu + \tau_3\rho^0_\mu\right)\right]q$$

$$Q = diag(2/3, -1/3), \qquad I = diag(1,1), \qquad m = diag(m_u, m_d)$$

$m_u = 280$ MeV $\quad g_\pi = m_u/f_\pi, \quad f_\pi = 93$ MeV, $\quad g_\rho \approx 6.14 \quad (g_\rho^2/(4\pi) \approx 3)$

[M.K. Volkov, Phys. Part. Nucl. 1986]

# Extended NJL model

For the first radially exited meson states we can use the extended NJL:

$$\begin{aligned}
\Delta\mathcal{L}_2 &= \bar{q}\Big\{ i\hat{\partial} - m + eQA \\
&+ \left[ g_{\pi_1}\frac{\sin(\alpha+\alpha_0)}{\sin(2\alpha_0)} + g_{\pi_2} f(k^{\perp 2})\frac{\sin(\alpha-\alpha_0)}{\sin(2\alpha_0)} \right]\tau^3\gamma_5\pi(p) \\
&- \left[ g_{\pi_1}\frac{\cos(\alpha+\alpha_0)}{\cos(2\alpha_0)} + g_{\pi_2} f(k^{\perp 2})\frac{\cos(\alpha-\alpha_0)}{\cos(2\alpha_0)} \right]\tau^3\gamma_5\pi'(p) \\
&+ \left[ g_{\rho_1}\frac{\sin(\beta+\beta_0)}{\sin(2\beta_0)} + g_{\rho_2} f(k^{\perp 2})\frac{\sin(\beta-\beta_0)}{\sin(2\beta_0)} \right]\omega, \rho_\mu(p) \\
&- \left[ g_{\rho_1}\frac{\cos(\beta+\beta_0)}{\sin(2\beta_0)} + g_{\rho_2} f(k^{\perp 2})\frac{\cos(\beta-\beta_0)}{\sin(2\beta_0)} \right]\tau^3\rho^{0\prime}_{\mu}(p)\Big\} q
\end{aligned}$$


M.K. Volkov & C. Weiss, PRD 1997;
M.K. Volkov, Yad. Fiz. 1997;
M.K. Volkov, D. Ebert and M. Nagy, IJMPA 1998;
A.B. Arbuzov, E.A. Kuraev and M.K. Volkov, Yad. Fiz. 2011

# Extended NJL model

$$g_{\pi_1} = g_\pi , \qquad g_{\rho_1} = g_\rho$$

$$g_{\pi_2} = \left[4 I_2^{f^2}\right]^{-1/2}, \qquad g_{\rho_2} = \left[\frac{2}{3} I_2^{f^2}\right]^{-1/2} = \sqrt{6} g_{\pi_2}$$

$$I_m^{f^n} = -iN_c \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{\left(f({k^\perp}^2)\right)^n}{(m^2 - k^2)^m}, \quad n = 1, 2, \quad m = 1, 2$$

The form factor is taken in a simple polynomial form:

$$f({k^\perp}^2) = (1 - d|{k^\perp}^2|)\Theta(\Lambda^2 - |{k^\perp}^2|),$$

$$k^\perp = k - \frac{(kp)p}{p^2}, \qquad d = 1.78 \ \mathrm{GeV}^{-2},$$

$k$ and $p$ are the quark and meson momenta, $\Lambda = 1.03$ GeV

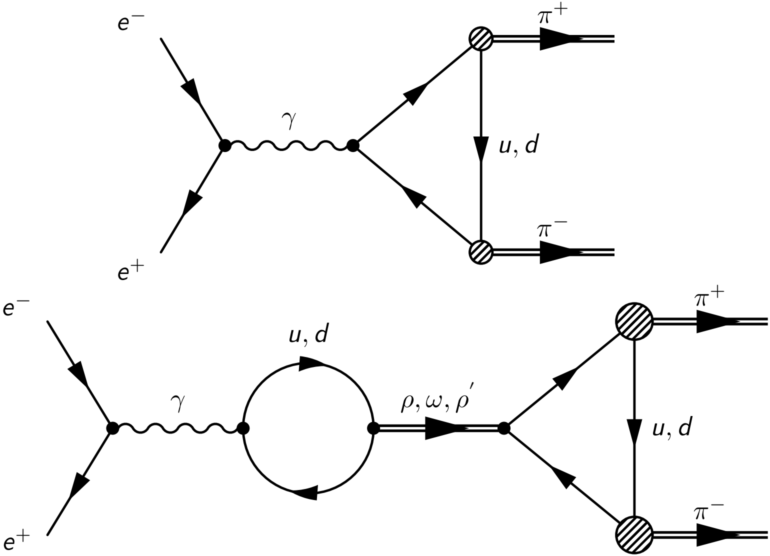
Angles $\alpha_0 = 59.06^\circ$, $\alpha = 59.38^\circ$, $\beta_0 = 61.53^\circ$ and $\beta = 76.78^\circ$ describe mixing of the ground and excited states for pions and vector mesons.

$\gamma \to \rho'$ transition:

$$C_{\gamma\rho'} \frac{e}{g_\rho} (g^{\nu\nu'} q^2 - q^\nu q^{\nu'})$$

$$C_{\gamma\rho'} = \frac{\sin(\beta + \beta_0)}{\sin(2\beta_0)} + \Gamma \frac{\sin(\beta - \beta_0)}{\sin(2\beta_0)}$$

$$\Gamma = \frac{I_2^f}{\sqrt{I_2 I_2^{f^2}}} \approx 0.47$$



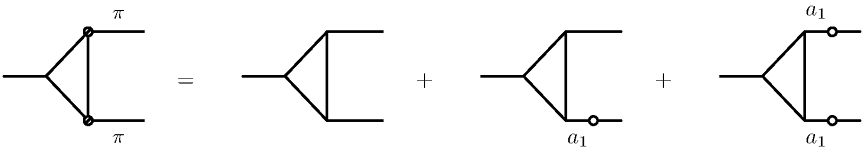
e⁻
e⁺
γ
u, d
π⁺
π⁻
e⁻
e⁺
γ
u, d
ρ, ω, ρ′
u, d
π⁺
π⁻

The amplitude of $e^+e^- \to \pi^+\pi^-$ takes the form

$$T = \frac{4\pi\alpha}{s} f_{a_1}(s)\,(B_{\rho\gamma} + B_\omega + B_{\rho'})\,\bar{e}\gamma_\mu e(p^\mu_{\pi^+} - p^\mu_{\pi^-})\pi^+\pi^- \,,$$

$$s = (p_{e^+} + p_{e^-})^2$$

where $B_{\rho\gamma}$ is the contribution of photon and $\rho(770)$ meson, $B_\omega$ is the contribution of $\omega(780)$ meson and $B_{\rho'}$ is the contribution of $\rho'(1450)$ meson.




For description $\gamma\pi\pi$ and $\rho\pi\pi$ vertexes we can use amplitude for $\rho \to \pi\pi$ with $\pi - a_1$ transitions.

$$g_\rho \left(Z + (1 - Z) + (f_{a_1}(p^2) - 1)\right) \rho^-_\mu (p^\mu_{\pi^+} - p^\mu_{\pi^-})\pi^+\pi^- \,,$$

$$f_{a_1}(p^2) = 1 + \left(\frac{p^2 - m^2_\pi}{(g_\rho F_\pi)^2}\right)\left(1 - \frac{1}{Z}\right) \,,$$

where $Z = (1 - 6m_u^2/m_{a_1}^2)^{-1}$ is additional renormalizing factor after accounting of $\pi - a_1$ transitions.

# $\rho(770)$ MESON CONTRIBUTION

Transition $\gamma - \rho$ takes the form

$$\frac{e}{g_\rho}(g^{\nu\nu'}q^2 - q^\nu q^{\nu'}) \, .$$

The $\gamma - \omega$ transition differs from the above just by factor 1/3.
Thus, contribution of $\gamma$ and $\rho(770)$ reads

$$B_{\rho\gamma} = 1 + \frac{s}{m_\rho^2 - s - i\sqrt{s}\Gamma_\rho(s)} = \frac{1 - i\sqrt{s}\Gamma_\rho(s)/m_\rho^2}{m_\rho^2 - s - i\sqrt{s}\Gamma_\rho(s)} m_\rho^2 \, .$$

This expression coincides with VMD model.

# $\omega$ MESON CONTRIBUTION

We can describe $\omega\pi\pi$ vertex using the amplitude for $\omega \to \pi\pi$ decay

$$C(m_\rho^2)\omega_\mu(p^\mu_{\pi^+} - p^\mu_{\pi^-})\pi^+\pi^- ,$$

where $C(s) = C_1(s) + C_2(s)$.

$C_1$ describes amplitude $\omega \to \rho \to \pi\pi$ through the quark loop with accounting of difference between quark masses ($m_d - m_u \approx 3.66$ MeV)

$$C_1(s) = \frac{8(\pi\alpha_\rho)^{3/2} m_\omega^2}{3(m_\omega^2 - s - i\sqrt{s}\Gamma_\rho(s))} \frac{3}{(4\pi)^2} \log\left(\frac{m_d}{m_u}\right)^2 .$$

$C_2$ describes amplitude $\omega \to \gamma \to \rho \to \pi\pi$.

$$C_2(s) = -\sqrt{\frac{\pi}{\alpha_\rho}} \frac{2\alpha s}{3(m_\omega^2 - s - i\sqrt{s}\Gamma_\rho(s))} .$$

Thus, $\omega$ contribution reads

$$B_\omega = \frac{C(s)}{3g_\rho} \frac{s}{m_\omega^2 - s - i\sqrt{s}\Gamma_\omega(s)} .$$

Far from the resonance the width should be modified. Our crude approximation:

$$\Gamma_{\rho'}(s) = \begin{cases} \Gamma_{\rho'\to 2\pi}, & \sqrt{s} \le 2M_\pi, \\ \Gamma_{\rho'\to 2\pi} + \Gamma_{\rho'\to\omega\pi}\frac{\sqrt{s}-2M_\pi}{M_\omega - M_\pi}, & 2M_\pi < \sqrt{s} \le M_\omega + M_\pi \\ \Gamma_{\rho'\to 2\pi} + \Gamma_{\rho'\to\omega\pi} + (\Gamma_{\rho'} - \Gamma_{\rho'\to 2\pi} - \Gamma_{\rho'\to\omega\pi})\frac{\sqrt{s}-M_\omega-M_\pi}{M_{\rho'}-M_\omega-M_\pi}, & \\ \qquad M_\omega + M_\pi < \sqrt{s} \le M_{\rho'}, & \\ \Gamma_{\rho'}, & M_{\rho'} < \sqrt{s} \end{cases}$$

# $\rho'(1450)$ MESON CONTRIBUTION

The vertex $\rho'\pi\pi$ is proportional to

$$C_{\rho'\pi\pi} = -\left( \frac{\cos(\beta+\beta_0)}{\sin(2\beta_0)} g_{\rho_1} + \frac{\cos(\beta-\beta_0)}{\sin(2\beta_0)} \frac{I_2^f}{I_2} g_{\rho_2} \right)$$

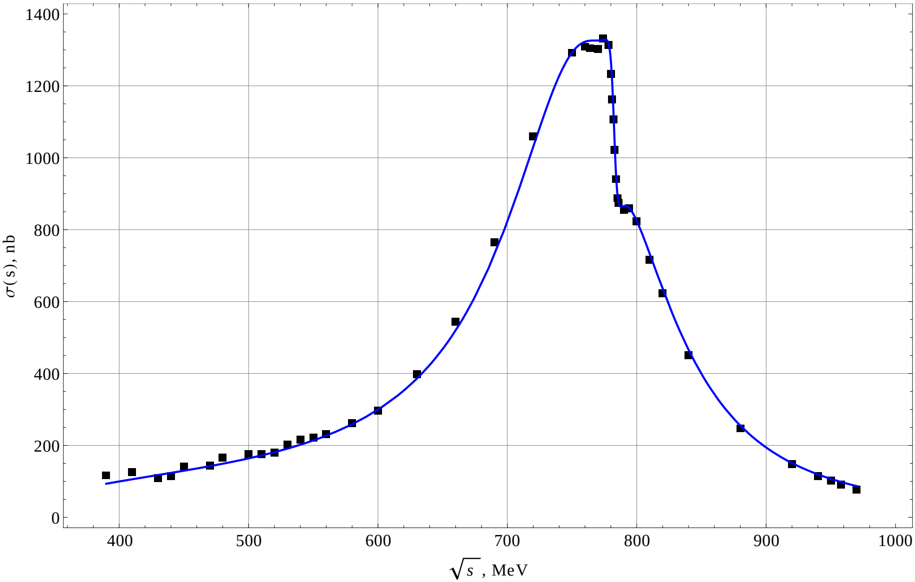
Thus, contribution of $\rho'(1450)$ takes the form

$$B_{\rho'} = \frac{C_{\gamma\rho'} C_{\rho'\pi\pi}}{g_\rho} \frac{s}{m_{\rho'}^2 - s - i\sqrt{s}\Gamma_{\rho'}(s)}$$

For total cross-section we get

$$\sigma(s) = \frac{\alpha^2\pi}{12s} f_{a_1}^2(s)(1-4m_\pi^2/s)^{3/2} \left|B_{\rho\gamma} + B_\omega + B_{\rho'}\right|^2$$

# Cross-section $e^+e^- \to \pi^+\pi^-$



# Cross-section $e^+e^- \to \pi\pi'$

The main contribution to $e^+e^- \to \pi\pi'$ is given by $\rho'(1450)$.

$$\sigma(s) = \frac{\alpha^2\pi}{12s}(1 - 4m_\pi^2/s)^{3/2}\left|B_{\rho\gamma}^{\pi\pi'} + B_{\rho'}^{\pi\pi'}\right|^2,$$

where

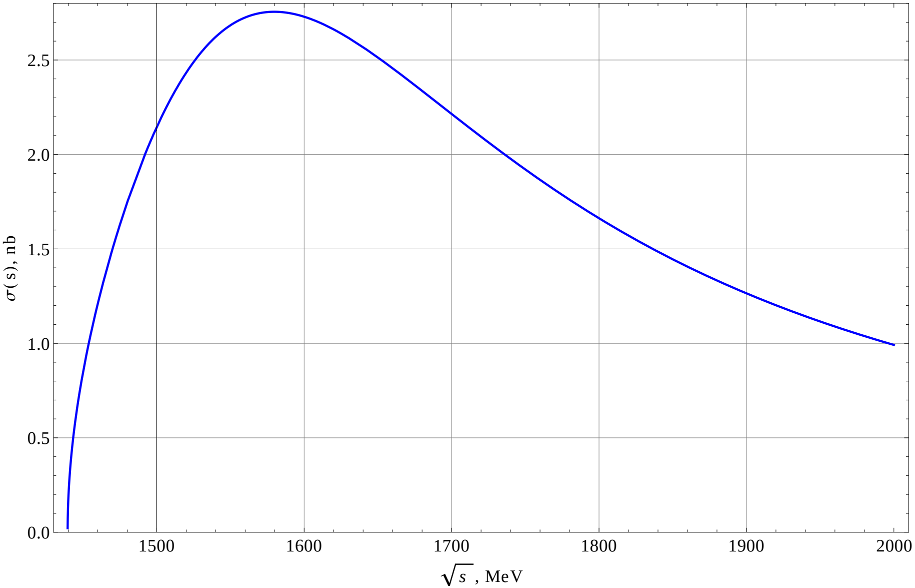
$$B_{\rho\gamma}^{\pi\pi'} = \frac{C_{\rho\pi\pi'}}{g_\rho}\left(1 + \frac{s}{m_\rho^2 - s - im_\rho\Gamma_\rho}\right) = \frac{C_{\rho\pi\pi'}}{g_\rho}\frac{1 - i\Gamma_\rho/m_\rho}{m_\rho^2 - s - im_\rho\Gamma_\rho}m_\rho^2$$

and

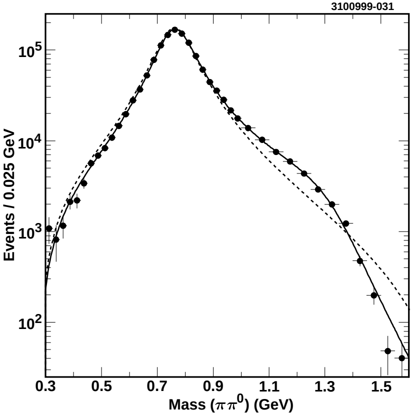
$$B_{\rho'}^{\pi\pi'} = \frac{C_{\gamma\rho'}C_{\rho'\pi\pi'}}{g_\rho}\frac{s}{m_{\rho'}^2 - s - im_{\rho'}\Gamma_{\rho'}}$$

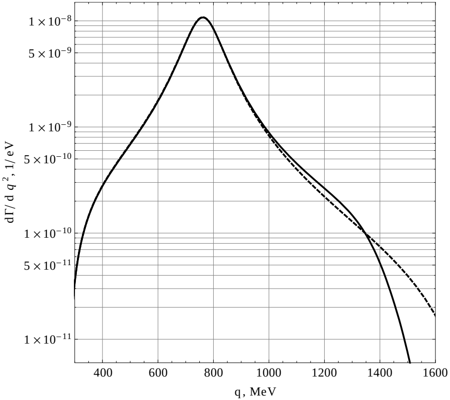
$C_{\rho\pi\pi'}$ and $C_{\rho'\pi\pi'}$ was defined similary $C_{\rho'\pi\pi}$.

# Cross-section $e^+e^- \to \pi\pi'$



Using the same Feynman amplitudes we can get values for decays $\tau \to \pi\pi(\pi')\nu$
$\mathcal{B}(\tau \to \pi\pi\nu) = 24.86$ % (PDG $25.51 \pm 0.09$ %) [arXiv:1202.0506]
$\beta \approx C_{W\rho'} C_{\rho'\pi\pi}/g_\rho = -0.092$
$\mathcal{B}(\tau \to \pi\pi'\nu) = 0.26$ % (PDG ?)



# Outlook

- Processes $e^+e^- \to \pi\pi(\pi')$ were considered in framework of NJL
- A satisfactory agreement with experimental data is observed
- A qualitative agreement with fitted parameters is obtained
- No any additional parameter was introduced

Thanks for your attention