Brane mechanism of spontaneously generated gravity

A. A. Zheltukhin

[a] Kharkov Inst. of Physics and Technology, Kharkov, 61108, Ukraine
[b] Nordita, KTH Royal Inst. of Technology and Stockholm University, SE 106 91 Stockholm, Sweden

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Dirac branes as minimal h-ws, Cartan multiplets and R squared gravity

The Dirac action for p-branes in $\mathbf{R}^{1,D-1}$

$$S = T_p \int d^{p+1}\xi \, \sqrt{|g|}. \tag{1}$$

The induced metric on hyper-ws Σ_{p+1} swept by brane world vector $\mathbf{x}(\xi^{\mu})$

$$g_{\mu\nu} := \partial_{\mu}\mathbf{x}\partial_{\nu}\mathbf{x},$$

The nonlinear wave EOM: $\quad \Box^{(p+1)}\mathbf{x} := \nabla^{\mu}\nabla_{\mu}\mathbf{x} = \frac{1}{\sqrt{|g|}}\partial_{\mu}(\sqrt{|g|}g^{\mu\nu}\partial_{\nu}\mathbf{x}).$

The moving frame $\mathbf{n}_A(\mathbf{x})$: $\quad \mathbf{n}_A(\mathbf{x})\mathbf{n}_B(\mathbf{x}) = \eta_{AB}, \ \ (A = 0, 1, .., D-1).$

The Cartan invariant differential forms ω^A and $\omega_A{}^B$:

$$d\mathbf{x} = \omega^A(d\xi)\mathbf{n}_A, \quad d\mathbf{n}_A = -\omega_A{}^B(d\xi)\mathbf{n}_B. \tag{2}$$

Projection of EOM on orts $\mathbf{n}_a(\xi) \perp$ to Σ_{p+1} results in the minimality conds.

$$Sp(l^a) := g^{\mu\nu}l^a_{\mu\nu} = 0, \quad (a = p+1, .., D-p-1) \tag{3}$$

where $l_{\mu\nu}{}^a$ is the second fundam. form of h-ws Σ_{p+1}

$$l_{\mu\nu}{}^a := \mathbf{n}^a\partial_{\mu\nu}\mathbf{x} \equiv \mathbf{n}^a\nabla_{\mu}\partial_{\nu}\mathbf{x}. \tag{4}$$

We use $(\omega^A, {\omega_A}^B)$ as new dynamical variables instead of the D-hedron $(\mathbf{x}, \mathbf{n}_A)$.
The Maurer-Cartan eqs.

$$d \wedge \omega_A + {\omega_A}^B \wedge \omega_B = 0, \tag{5}$$

$$d \wedge {\omega_A}^B + {\omega_A}^C \wedge {\omega_C}^B = 0 \quad \rightarrow \quad {F_A}^B = 0 \tag{6}$$

are integrab. conds. of Eqs. (2). Hyper-ws Σ_{p+1} spontaneously breaks Poincare symmetry of $\mathbf{R}^{1,D-1}$: $ISO(1, D-1) \rightarrow ISO(1, p-1) \times SO(D-p-1)$.
The frame $\mathbf{n}_A$ splits into two subsets: $\mathbf{n}_A = (\mathbf{n}_i, \mathbf{n}_a)$,
where $\mathbf{n}_i$, $(i, k = 0, 1, ..., p)$ are tangent and $\mathbf{n}_a \perp$ to Σ_{p+1}.
The rest symmetry formed by tangent Lorentz rotations in Σ_{p+1} and rotations in the $(D-p-1)$-dim. subspace $\perp$ to Σ_{p+1}.
The Nambu-Goldstone bosons are effectively described by the Cartan multiplets of the right gauge group $SO(1, D-1)$

$${\omega_A}^B(d\xi) = \begin{pmatrix} {A_i}^k(d\xi) & {W_i}^b(d\xi) \\ {W_a}^k(d\xi) & {B_a}^b(d\xi) \end{pmatrix}. \tag{7}$$

The diag. submatrices $A_{\mu i}{}^{k} d\xi^{\mu}$ and $B_{\mu a}{}^{b} d\xi^{\mu}$ form gauge fields in the fund. reps. of $SO(1,p)$ and $SO(D-p-1)$ subgroups. $W_{\mu i}{}^{b} d\xi^{\mu}$ form a charged vector multiplet in the bi-fund. rep. of $SO(1,p) \times SO(D-p-1)$ with the covar. derivative

$$(D_{\mu} W_{\nu})_{i}{}^{a} = \partial_{\mu} W_{\nu i}{}^{a} + A_{\mu i}{}^{k} W_{\nu k}{}^{a} + B_{\mu}{}^{a}{}_{b} W_{\nu i}{}^{b}. \tag{8}$$

The forms ω^{A} for the global translations of $\mathbf{R}^{1,D-1}$ are $\delta^{A}_{m} dx^{m}$.
Forms referred to a moving frame on Σ_{p+1} are projections of $d\mathbf{x}$ on $\mathbf{n}_{A}(\xi)$

$$\omega^{A} = d\mathbf{x}(\xi)\mathbf{n}^{A}(\xi) \equiv dx^{m} n_{m}{}^{A}(\xi). \tag{9}$$

PDE's (9) represent N-G translation modes $x^{m}(\xi)$ through $\omega^{A}_{m}(\xi)$ In view of orhogonality $\mathbf{n}_{a}(\xi) d\mathbf{x}(\xi) = 0$ we have

$$\omega^{a}(d\xi) = 0 \quad \rightarrow \quad d\mathbf{x}(\xi) = \omega^{i}(d)\mathbf{n}_{i}(\xi). \tag{10}$$

Then $ds^2 = d\mathbf{x}^2$ on Σ_{p+1} takes the form

$$ds^2 = \omega_{i}\omega^{i} = \omega^{i}_{\mu}\omega_{i\nu} d\xi^{\mu} d\xi^{\nu} \equiv g_{\mu\nu}(\xi) d\xi^{\mu} d\xi^{\nu}. \tag{11}$$

This shows that $\omega^{i}_{\lambda}(\xi)$ is the vielbein of Σ_{p+1}

$$g_{\mu\nu} := \omega^{i}_{\mu}\eta_{ik}\omega^{k}_{\nu}, \;\; \omega^{i}_{\mu}\omega^{\mu}_{k} = \delta^{i}_{k}. \tag{12}$$

Solution of M-C Eqs. (5) yields the tetrade postulate

$$D^{\parallel}_{[\mu}\omega^{i}_{\nu]} \equiv \partial_{[\mu}\omega^{i}_{\nu]} + A_{[\mu}{}^{i}{}_{k}\omega_{\nu]}{}^{k} = 0, \tag{13}$$

which expresses $A_{\mu}{}^{i}{}_{k}$ through ω^{i}_{μ} together with the constraints

$$\omega^{i}_{[\mu}W_{\nu]ia} = 0 \quad \rightarrow \quad W_{\mu i}{}^{a} = -l_{\mu\nu}{}^{a}\omega^{\gamma}_{i}, \tag{14}$$

where $l^{a}_{\mu\nu} = l^{a}_{\nu\mu}$ is the second fundamental form of Σ_{p+1}. As a result, $A_{\mu}{}^{i}{}_{k}$ and its strength $F_{\mu\nu i}{}^{k}$ are expressed through $\Gamma^{\rho}_{\nu\lambda}$ and the Riemann tensor $R_{\mu\nu}{}^{\gamma}{}_{\lambda}$

$$A^{ik}_{\mu} = \omega^{i}_{\rho}\Gamma^{\rho}_{\mu\lambda}\omega^{\lambda k} + \omega^{i}_{\lambda}\partial_{\mu}\omega^{\lambda k}, \tag{15}$$

$$F_{\mu\nu}{}^{i}{}_{k} = \omega^{i}_{\gamma}R_{\mu\nu}{}^{\gamma}{}_{\lambda}\omega^{\lambda}_{k}. \tag{16}$$

Then M-C Eqs. (6) are transformed into the Gauss-Ricci-Peterson-Codazzi eqs.

$$R_{\mu\nu}{}^{\gamma}{}_{\lambda} = l_{[\mu}{}^{\gamma a}l_{\nu]\lambda a}, \tag{17}$$

$$H_{\mu\nu a}{}^{b} := (\partial_{[\mu}B_{\nu]} + [B_{\mu}, B_{\nu}])_{a}{}^{b}, \quad H_{\mu\nu}{}^{ab} = l_{[\mu}{}^{\gamma a}l_{\nu]\gamma}{}^{b}, \tag{18}$$

$$\nabla^{\perp}_{[\rho}l_{\mu]\nu}{}^{a} = 0. \tag{19}$$

These eqs. and Eqs. (3) yield a complete set of data describing fundamental branes in terms of the Cartan multiplets of the gauge group $SO(D-p-1)$.

The $SO(D-p-1)$ and diff invariant action of p-branes sweeping a minimal hyper ws Σ^{min}_{p+1} and consistent with Eqs. (17-19) is given by

$$S_{Dir} = \frac{1}{k_p^2}\int d^{p+1}\xi\,\sqrt{|g|}\,\{-\frac{1}{4}Sp(H_{\mu\nu}H^{\nu\mu}) + \frac{1}{2}\nabla^{\perp}_{\mu}l_{\nu\rho a}\nabla^{\perp\{\mu}l^{\nu\}\rho a} - \nabla^{\perp}_{\mu}l^{\mu}_{\rho a}\nabla^{\perp}_{\nu}l^{\nu\rho a} + V_{Dir}(l)\}. \tag{20}$$

The diff invariant potential $V_{Dir}(l)$ encoding self-interaction of the N-G multiplet $l^a_{\mu\nu}$ in the gravitational *background* $g_{\mu\nu}(\xi^\rho)$ is

$$V_{Dir} = -\frac{1}{2}Sp(l_a l_b)Sp(l^a l^b) + Sp(l_a l_b l^a l^b) - Sp(l_a l^a l_b l^b) + \mathrm{c_p}, \tag{21}$$

where $\mathrm{c_p}$ is an integration constant.
To derive V_{Dir} we used the Bianchi identities

$$[\nabla^{\perp}_{\gamma},\,\nabla^{\perp}_{\nu}]l^{\mu\rho a} = R_{\gamma\nu}{}^{\mu}{}_{\lambda}l^{\lambda\rho a} + R_{\gamma\nu}{}^{\rho}{}_{\lambda}l^{\mu\lambda a} + H_{\gamma\nu}{}^{a}{}_{b}l^{\mu\rho b} \tag{22}$$

for the metric and Y-M covariant derivative

$$\nabla^{\perp}_{\mu}l_{\nu\rho}{}^{a} := \partial_{\mu}l_{\nu\rho}{}^{a} - \Gamma^{\lambda}_{\mu\nu}l_{\lambda\rho}{}^{a} - \Gamma^{\lambda}_{\mu\rho}l_{\nu\lambda}{}^{a} + B^{ab}_{\mu}l_{\nu\rho b}. \tag{23}$$

The Euler-Lagrange PDEs have a unique solution describing p-branes provided that the Ricci-Codazzi eqs.(18-19) were chosen as the *Cauchy initial data*.

The latter turned out to be invariants of the evolution prescribed by S_{Dir}.

The Gauss eqs. (17) are treated as the evolution PDEs for $g_{\mu\nu}$. They are consistent with the used variational principle since they have selected V_{Dir}.

Then the EOM become equivalent to the identities

$$\nabla^{\perp\mu}\mathcal{H}^{ab}_{\mu\nu} = 0, \quad \nabla^{\perp\mu}\nabla^{\perp}_{[\mu}l^{a}_{\nu]\rho} = 0 \tag{24}$$

produced by the covariant differentiation of the Ricci-Codazzi eqs.
They can be equivalently written in the form of the generalized Maxwell-Y-M and Newton eqs. in the gravit. field defined by Gauss eqs. (17)

$$\nabla^{\perp}_{\nu}H^{\nu\mu}_{ab} = j^{\mu}_{ab}, \quad j^{\mu}_{ab} = Sp(l_{[a}\nabla^{\perp\mu}l_{b]}), \quad \nabla^{\perp}_{\mu}j^{\mu}_{ab} = 0, \tag{25}$$

$$\nabla^{\perp}_{\mu}\nabla^{\perp\mu}l^{\nu\rho a} = \frac{1}{2}\frac{\partial V_{Dir}}{\partial l_{\nu\rho a}} \equiv (2l_b l^a l^b - l^a l_b l^b - l_b l^b l^a)^{\nu\rho} - l^{\nu\rho}_{b} Sp(l^b l^a). \tag{26}$$

We conclude that S_{Dir} (20) with the chosen potential V_{Dir} (21) reformulates the Dirac p-brane dynamics it terms of the Cartan multiplets.

The potential term V_{Dir} can be represented in the form

$$V_{Dir} = -\frac{1}{4}R_{\mu\nu\gamma\lambda}R^{\mu\nu\gamma\lambda} - \frac{1}{2}R_{\mu\nu}R^{\mu\nu} + \frac{1}{4}H_{\mu\nu ab}H^{\nu\mu ab} + \mathrm{c_p}. \tag{27}$$

Eq. (27) was derived using Eqs. (17-18) and (3). They yield the relaions

$$\frac{1}{2}R_{\mu\nu\gamma\lambda}R^{\mu\nu\gamma\lambda} = Sp(l_a l_b)Sp(l^a l^b) - Sp(l_a l_b l^a l^b), \tag{28}$$

$$\frac{1}{2}H^{ab}_{\mu\nu}H^{\mu\nu}_{ab} = Sp(l_a l_b l^a l^b) - Sp(l_a l^a l_b l^b), \quad Spl^a = 0. \tag{29}$$

These relations were combined with the quadratic reps of the Ricci tensor $R_{\mu\nu}$ and the scalar curvature R of the *minimal* hyper w-s Σ^{min}_{p+1}

$$R_{\mu\nu} = -(l^a l_a)_{\mu\nu}, \quad R = -Sp(l^a l_a). \tag{30}$$

The potential (27) contains the curvature squared terms considered in f(R) gravity. In the codimention 1, i.e. when $D = p + 2$, $B^{ab}_{\mu} \equiv 0$ since $a = b = p + 1$ and

S_{Dir} (20) is reduced to the action

$$S_{D=p+2} = -\frac{1}{k_p^2}\int d^{p+1}\xi\,\sqrt{|g|}\big(\frac{1}{2}\nabla_\mu l_{\nu\rho}\nabla^\mu l^{\nu\rho} - \nabla_\mu l^\mu_\rho \nabla_\nu l^{\nu\rho} + \frac{1}{2}(Sp(l^2)^2 - \mathrm{c_p}), \qquad (31)$$

where $l_{\lambda\rho} \equiv l_{\lambda\rho(p+1)} = -l^{(p+1)}_{\lambda\rho}$ and the metric covariant derivative ∇_μ is

$$\nabla_\mu l_{\nu\rho} := \partial_\mu l_{\nu\rho} - \Gamma^\lambda_{\mu\nu} l_{\lambda\rho} - \Gamma^\lambda_{\mu\rho} l_{\nu\lambda} \qquad (32)$$

Eqs. (27-30) shows that V_{Dir} in $S_{D=p+2}$ can be rewritten as

$$\frac{1}{2}(Sp(l^2))^2 = \frac{1}{2}R^2 \quad \rightarrow \quad V_{Dir} = -\frac{1}{2}(Sp(l^2))^2 + \mathrm{c_p} = -\frac{1}{2}R^2 + \mathrm{c_p}. \qquad (33)$$

Eqs. (25-26) are reduced to the eqs.

$$\Box l_{\nu\rho} = l_{\nu\rho} Sp(l^2) \equiv R\, l_{\nu\rho}, \quad Spl = 0, \qquad (34)$$

where $\Box \equiv \nabla_\mu \nabla^\mu$ is the D'Alembert-Beltrami operator for tensor fields on Σ^{min}_{p+1}.
This correspondence between Dirac p-branes and R^2 models does not generate the Hilbert-Einsten gravity.
For 3-branes the H-E term is forbidden in view of the scale symmetry of R^2 action (31) with a cosmological constant $c_3 = 0$.
Indeed, for $p = 3$ the coupling k_p is dimensionless because $[k_p] = [T_p]^{\frac{3-p}{2(p+1)}}$.

Spontaneously generated gravity and non-minimal hyper-worldsheets

The dilatation symmetry of 3-brane action is realized by the transf-s:

$$\xi'^{\mu} = e^{-\lambda}\xi^{\mu}. \quad g'_{\mu\nu}(\xi') = g_{\mu\nu}(\xi), \quad l'_{\mu\nu}(\xi') = e^{\lambda} l_{\mu\nu}(\xi) \tag{35}$$

This action is also invariant under global Weyl transf-s:

$$\xi'^{\mu} = \xi^{\mu}, \quad g'_{\mu\nu}(\xi') = e^{2\alpha} g_{\mu\nu}(\xi), \quad l'_{\mu\nu}(\xi') = e^{\alpha} l_{\mu\nu}(\xi) \tag{36}$$

These laws show that an abelian subgroup U_+ of $U(1) \times U(1)$ formed by $\alpha = \lambda$:

$$\xi'^{\mu} = e^{-\lambda}\xi^{\mu}, \quad g'_{\mu\nu}(\xi') = e^{2\lambda} g_{\mu\nu}(\xi), \quad l'_{\mu\nu}(\xi') = e^{2\lambda} l_{\mu\nu}(\xi) \tag{37}$$

yields a diff trans-on of 3-brane h-ws. So, diff-s protect U_+ symmetry.
The diff. invariant Spl creates a 1-dim. repres-n. of the Weyl and dilat. symm-s

$$Spl'(\xi') = e^{\lambda} Spl(\xi), \quad Spl'(\xi') = e^{-\alpha} Spl(\xi) \tag{38}$$

Then the condition $Spl = 0$ does not break the scale symmetry.
To create the H-E term this symmetry should be broken, e.g. as: $Spl = constant$.
Thus, we arrive at the idea of spontaneously generated gravity studied by Adler and Zee which is explained by the example of 4-dim. scale-invariant action

$$A = \int d^4x \sqrt{|g|}\,[\frac{\alpha}{2}\varphi^2 R + \frac{1}{2}\nabla_{\mu}\varphi\nabla^{\mu}\varphi - V(\varphi)] \tag{39}$$

including a scalar field φ and a dimensionless constant α.
$V(\varphi, g)$ is assumed to have a deep minimum at $\varphi_o = v$ which provides vev v for φ.
The expansion around the minimum generates the H-E term with the Newton constant $G_N \approx \frac{1}{\alpha v^2}$.
So, the scale symmetry of (39) is spontaneously broken that results in a 4-dim. gravity in the low energy limit.
On the contrary, in the early universe, v as a function of the temperature, is expected to vanish resulting in a scale-invariant R^2 action. However, this model prevents appearance of a cosmol. const. arising in such cases. Replacement of φ by a scalar $\bar{\psi}\psi$ proposed by Adler does not improve the situation.
3-brane brane action (31) quadratic in curvarture includes a cosmol. const. c_p. It encodes an R^2 action with zero vev for the field Spl

$$\phi := Spl \quad \rightarrow \quad <\phi>_0 = 0. \tag{40}$$

Thus, the restoration of the H-E term in brane action makes us search for a deformation of V_{Dir} to a new potential U which has its extremal at $l_{o\mu\nu} \neq 0$:

$$Spl_o \equiv <\phi>_0 = \mu, \tag{41}$$

where the constant μ has the dimension $[\mu] = [L^{-1}]$.
This generates a fundamental mass scale similarly to the Higgs effect in QFT.

To find a deformed diff invariant quartic potential U we explore the action

$$S = \frac{1}{k_p^2}\int d^{p+1}\xi\,\sqrt{|g|}(\frac{1}{2}\nabla_\mu l_{\nu\rho}\nabla^{\{\mu}l^{\nu\}\rho} - \nabla_\mu l^\mu_\rho\nabla_\nu l^{\nu\rho} - U(l)) \quad (42)$$

defined on a h-ws with codim 1, and obtain the following EOM

$$\frac{1}{2}\nabla_\mu\nabla^{[\mu}l^{\{\nu]\rho\}} = -[\nabla^\mu, \nabla^{\{\nu}]l_\mu{}^{\rho\}} - \frac{\partial U}{\partial l_{\nu\rho}}. \quad (43)$$

The h-ws metric $g_{\mu\nu}(\xi)$ in (42) is treated as a background field since its evolution is encoded by the embedding conditions, given by the Gauss's Theorema Egregium (17). For hypersurfaces Σ_{p+1} of codim 1 it takes the form

$$R_{\mu\nu\gamma\lambda} = -l_{\mu\gamma}l_{\nu\lambda} + l_{\nu\gamma}l_{\mu\lambda}. \quad (44)$$

The Gauss eqn. (44) combined with the Bianchi identities

$$[\nabla_\mu, \nabla_\nu]l^{\gamma\rho} = R_{\mu\nu}{}^\gamma{}_\lambda l^{\lambda\rho} + R_{\mu\nu}{}^\rho{}_\lambda l^{\gamma\lambda}. \quad (45)$$

permits to write the commutator in the r.h.s. of (43) as

$$-\frac{1}{2}[\nabla^\mu, \nabla^{\{\nu}]l_\mu^{\rho\}} = (l^2)^{\nu\rho}Spl - l^{\nu\rho}Sp(l^2), \quad (46)$$

where $Sp(l^2) := l_{\mu\rho}l^\rho_\nu g^{\mu\nu}$. Then EOM (43) is transformed to the PDE

$$\frac{1}{4}\nabla_\mu\nabla^{[\mu}l^{\{\nu]\rho\}} = (l^2)^{\nu\rho}Spl - l^{\nu\rho}Sp(l^2) - \frac{1}{2}\frac{\partial U}{\partial l_{\nu\rho}}. \quad (47)$$

A general homogenious quartic polynomial invariant under diffeomorphisms is

$$U = \frac{2}{3}SplSp(l^3) - \frac{1}{2}(Sp(l^2)^2 + b_2Sp(l^2)(Spl)^2 + b_4(Spl)^4 + b_4'Sp(l^4). \quad (48)$$

It contains arbitrary dimensionless parameters b_2, b_4, b_4', and its l-derivative is

$$\frac{1}{2}\frac{\partial U}{\partial l_{\nu\rho}} = Spl(l^2)^{\nu\rho} - [Sp(l^2) - b_2(Spl)^2]l^{\nu\rho} + 2b_4'(l^3)^{\nu\rho} \quad (49)$$
$$+[\frac{1}{3}Sp(l^3) + b_2Sp(l^2)Spl + 2b_4(Spl)^3)]\frac{\partial Spl}{\partial l_{\nu\rho}}.$$

For simplicity we choose an extension of V_{Dir} with $b_2 = b_4 = b_4' = 0$:

$$U \to V := \frac{2}{3}SplSp(l^3) - \frac{1}{2}(Sp(l^2)^2 + c_p. \quad (50)$$

Then EOM (47) reduces to the eqn.

$$\frac{1}{2}\nabla_\mu\nabla^{[\mu}l^{\{\nu]\rho\}} = -\frac{2}{3}Sp(l^3)\frac{\partial Spl}{\partial l_{\nu\rho}}. \quad (51)$$

Eq. (51) is the Euler-Lagrange eqn. given by S (42) with $V(l)$ substituted for $U(l)$

$$S = \frac{1}{k_p^2}\int d^{p+1}\xi\,\sqrt{|g|}\Big(\frac{1}{2}\nabla_\mu l_{\nu\rho}\nabla^{\{\mu}l^{\nu\}\rho} - \nabla_\mu l^\mu_\rho \nabla_\nu l^{\nu\rho} \tag{52}$$
$$-\frac{2}{3}Spl Sp(l^3) + \frac{1}{2}Sp(l^2)Sp(l^2) - \mathrm{c_p}\Big).$$

Extremals $l_o^{\nu\rho}$ are defined by Eq. (49) with zero b. They are roots of the eqn.

$$(l_o^2)^{\nu\rho}Spl_o - l_o^{\nu\rho}Sp(l_o^2) = 0 \tag{53}$$

which can equivalently be represented as

$$l^\nu_{o\alpha}(l_o^{\alpha\rho}Spl_o - g^{\alpha\rho}Sp(l_o^2)) = 0. \tag{54}$$

Supposing that the matrice $l^\nu_{o\alpha}$ is non-degenerate we obtain solution of (54):

$$l_{o\mu\nu} = \frac{Spl_o}{p+1}g_{\mu\nu}, \quad det\, l^\mu_{o\nu} \neq 0, \tag{55}$$

where $l^\mu_{o\nu} \equiv g^{\mu\gamma}l_{o\gamma\nu}$. Eq. (55) generates the recurrent relations:

$$(l_o^n)_{\mu\nu} = (\frac{Spl_o}{p+1})^n g_{\mu\nu}, \;\; \rightarrow \;\; Sp(l_o^n) = (p+1)(\frac{Spl_o}{p+1})^n. \tag{56}$$

We find that extremal (55) breaks neither the Weyl nor the dilatation symmetries.

However, we have not yet taken into account that extremals must obey the P-C embedding conds. (19) encoding the brane sector of sol-s of (47).

For h-ws of codim 1 the P-C eqs. are given by

$$\nabla_{[\mu} l_{\nu]\rho} = 0 \quad \longrightarrow \quad \nabla^{\rho} l_{\rho\nu} = \nabla_{\nu} Sp\, l \equiv \partial_{\nu} Spl. \tag{57}$$

The substitution of extremal solution (55) in the second of Eqs. (57) gives

$$\nabla^{\rho} l_{o\rho\nu} = \partial_{\nu} Spl_o \quad \rightarrow \quad \frac{1}{p+1} \partial_{\nu} Spl_o = \partial_{\nu} Spl_o \quad \rightarrow \quad Spl_o = \mu, \tag{58}$$

where μ is a constant. So, we obtain the desired extremal (41)

$$l_{o\mu\nu} = \frac{\mu}{p+1} g_{\mu\nu} \quad \rightarrow \quad Spl_o \equiv <\phi>_0 = \mu. \tag{59}$$

It is evident that this extremal gives a particular solution of EOM (51), because it vanishes its l-h and r-h sides

$$\frac{1}{2}\nabla_\mu \nabla^{[\mu} l_o^{\{\nu]\rho\}} = -\frac{2}{3} Sp(^3)\frac{\partial Spl}{\partial l_{\nu\rho}}|_{Spl=\mu} = 0. \tag{60}$$

The Gauss map (44) of the Riemannian tensor of the vacuum h-ws Σ^o_{p+1}

$$R_{o\mu\nu\gamma\lambda} = -l_{o\mu\gamma} l_{o\nu\lambda} + l_{o\nu\gamma} l_{o\mu\lambda}. \tag{61}$$

yields its explicit expression

$$R_{o\mu\nu\gamma\lambda} = -(\frac{\mu}{p+1})^2 (g_{\mu\gamma} g_{\nu\lambda} - g_{\nu\gamma} g_{\mu\lambda}). \tag{62}$$

It shows that the h-ws Σ^o_{p+1} has the negative constant curvature $R_o = g^{\mu\nu} R_{o\mu\nu}$

$$R_{o\mu\nu} = -\frac{p}{(p+1)^2}\mu^2 g_{\mu\nu}, \quad R_o = -\frac{p}{p+1}\mu^2. \tag{63}$$

Resume: the Weyl and scale invariant 3-brane action (52) with the potential

$$V_3 = \frac{2}{3} Spl Sp(l^3) - \frac{1}{2}(Sp(l^2))^2, \quad c_3 = 0 \tag{64}$$

has the classical vacuum solution breaking the above rigid symmetries.

Models of R squared gravity from p-branes

To discuss gravity models encoded by p-branes we use the compact notations

$$\phi := Spl, \quad \theta_n := Sp(l^n), \quad (n = 2, 3, 4) \tag{65}$$

in which the iscussed potential takes the form

$$V_p = \frac{2}{3}\phi\theta_3 - \frac{1}{2}(\theta_2)^2 - c_p. \tag{66}$$

The Gauss map (44) permits to express curvature invariants through homogenious polynomials constructe from traces of the tensor $l_{\mu\nu}$

$$\frac{1}{2}R_{\mu\nu\gamma\lambda}R^{\mu\nu\gamma\lambda} = -\theta_4 + (\theta_2)^2, \tag{67}$$

$$R_{\mu\nu} = (l^2)_{\mu\nu} - \phi l_{\mu\nu} \quad \rightarrow \quad R_{\mu\nu}R^{\mu\nu} = \theta_4 - 2\phi\theta_3 + \theta_2\phi^2,$$

$$R = \theta_2 - \phi^2 \;\; \rightarrow \;\; R^2 = (\theta_2)^2 - 2\theta_2\phi^2 + \phi^4.$$

The additional relation

$$R^{\mu\nu}l_{\mu\nu}\phi = \phi\theta_3 - \theta_2\phi^2 \tag{68}$$

represents the first term in V as

$$\phi\theta_3 = R^{\mu\nu}l_{\mu\nu}\phi + (R + \phi^2)\phi^2. \tag{69}$$

The latter expression combined with the reps: $(\theta_2)^2 = (R + \phi^2)^2$ for the second term in V_p yields the R^2 gravity interaction lagrangian

$$- V_p = \frac{1}{2}R^2 + \frac{1}{3}R\phi^2 - \frac{2}{3}R_{\nu\lambda}l^{\lambda\nu}\phi - \frac{1}{6}\phi^4 + c_p. \tag{70}$$

For $p = 3, c_3 = 0$ Eq. (70) gives the Weyl and scale invariant lagrangian realizing the Adler-Zee mechanism of spontaneously induced gravity due to the presence of the critical point $< \phi >_0 = \mu$.

The model (70) generalizes the known models describing inflation and reheating in the presence of scalar field φ similar to the Brans-Dicke one.

The latter scalar is changed by the massless tensor field $l_{\mu\nu}$, and its trace $\phi \equiv Spl$ has non-zero vev $< Spl >_0 = \mu$.

So, implementation of the massless tensor perturbations $l_{\mu\nu}$, associated with brane matter, supplies new tensor-tensor models of R^2 gravity which can be used for analyzing the current experiments. Note that scale-invariant models fit the experimental data from Planck (see P. A. R. A. et.al (Planck Collaboration), Planck 2015 results. XX. Constraint on inflation"'. arXiv: 1502.02114 [astro-ph].)

There is alternative way to express the first term in V as

$$\frac{2}{3}\phi\theta_3 = -\frac{1}{3}(\frac{1}{2}R_{\mu\nu\gamma\lambda}R^{\mu\nu\gamma\lambda} + R_{\mu\nu}R^{\mu\nu}) + \frac{1}{3}((\theta_2)^2 + \theta_2\phi^2). \tag{71}$$

This reps yields the following R^2 gravity lagrangian accompanied with scalar ϕ

$$V_p = -\frac{1}{6}L_{GB} - R_{\mu\nu}R^{\mu\nu} + \frac{1}{6}\phi^4 - c_p, \tag{72}$$

where L_{GB} is the Gauss-Bonnet term in $(p+1)$-dim. space-time associated with the h-ws Σ_{p+1}

$$L_{GB} := R_{\mu\nu\gamma\lambda}R^{\mu\nu\gamma\lambda} - 4R_{\mu\nu}R^{\mu\nu} + R^2 \tag{73}$$

which is known topological invariant for $p = 3$, but is a dynamical term for $p > 3$. Thus, for $p = 3, c_3 = 0$ expression (72) gives the Weyl and scale invariant lagrangian of R^2 gravity

$$-V_3 = R_{\mu\nu}R^{\mu\nu} - \frac{1}{6}\phi^4, \tag{74}$$

where the tensor field $l_{\mu\nu}$ is presented by only its invariant trace $\phi \equiv l_{\mu\nu}g^{\mu\nu}$.

Summary

1. The tensor dynamical variables $g_{\mu\nu}$ and $l_{\mu\nu}$, originating from the Gauss-Cartan geometric approach to embedded hypersurfaces, are used to reformulate p-brane description. This reveals the geometric structure of their non-linearities.

2. It is shown that the interaction potential of the hyper-ws multiplet $l_{\mu\nu}$ encodes scale-invariant models of R squared gravity.
This potential has the extremal $l_{o\mu\nu} = \frac{\mu}{p+1} g_{\mu\nu}$ spontaneously breaking the Weyl and scale global symmetries.

3. On this extremal the trace $Spl \equiv g^{\mu\nu} l_{\mu\nu}$ has the vev $Spl_o = \mu$.
The extremal h-ws has the constant curvature $R_o = -\frac{p}{p+1}\mu^2$.

4. These resuts yield brane realization of the Adler-Zee mechanism of spontaneously generated gravity arising from breaking of the scale symmetry. This proposes new tensor-tensor models of R^2 gravity alternative to the well-known scalar-tensor models.

THANK YOU FOR YOUR ATTENTION!