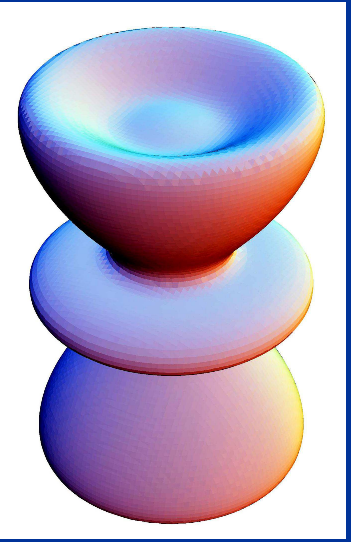
BLTP, JINR

Non Abelian Composite Configurations in AdS_4 Spacetime



Ya Shnir

Thanks to my collaborators:
O.Kichakova, J.Kunz and E.Radu

Brazil-JINR Forum, 18 June 2015

Outline

- SU(2) solitons in AdS spacetime
  - Monopoles and Bartnik-McKinnon solutions
  - Thermodynamics of AdS Black Holes
  - Hairy AdS Black Holes
  - Self-gravitating SU(2) monopoles
  - SU(2) monopoles and dyons in AdS spacetime
  - Dyons in AdS Holography

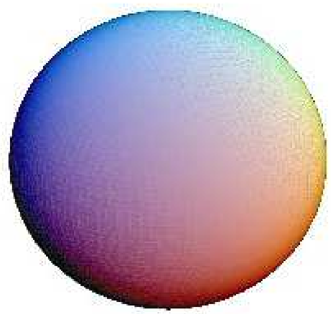
Yang-Mills-Higgs Theory

$$S = \frac{1}{2}\int d^4x\,\{\mathrm{Tr}\ \mathrm{F}_{\mu\nu}\mathrm{F}^{\mu\nu} + \mathrm{Tr}\ (\mathrm{D}_\mu\Phi)(\mathrm{D}^\mu\Phi) - \mathrm{V}(\Phi)\}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ie[A_\mu, A_\nu]$$

$$D_\mu\Phi = \partial_\mu\Phi + ie[A_\mu, \Phi]$$

$$V(\Phi) = \lambda\ (\Phi^2 - \eta^2)^2$$



´t Hooft-Polyakov static spherically symmetric solution: monopole

$$\phi^a = \frac{r^a}{er^2}H(e\eta r)$$

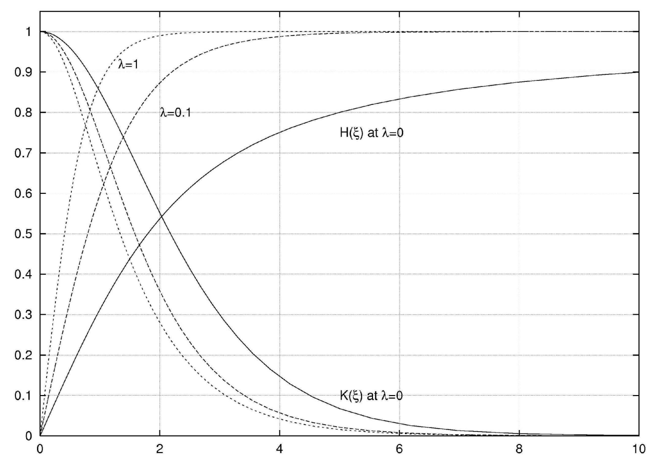
$$A_n^a = \varepsilon_{amn}\frac{r^m}{er^2}(1 - K(e\eta r))$$

$$M = -\int d^3x\sqrt{-g}T_0^0;$$

$$g = \int d^3x\sqrt{-g}\ \mathrm{Tr}({}^*F^{0n}D_n\Phi);$$

$$Q = \int d^3x\sqrt{-g}\ \mathrm{Tr}(F^{0n}D_n\Phi);$$

$$J = 2\int d^3x\sqrt{-g}\ \mathrm{Tr}(F_{r\varphi}F^{r0} + F_{\theta\varphi}F^{\theta0} + D_\varphi\Phi D^0\Phi);$$



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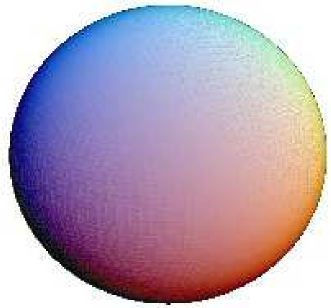
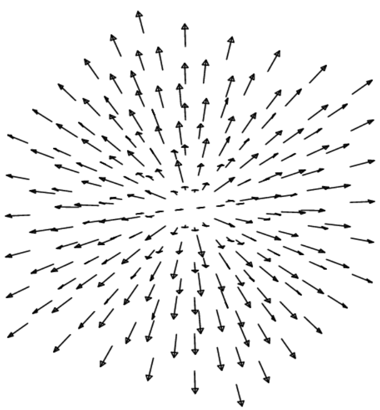
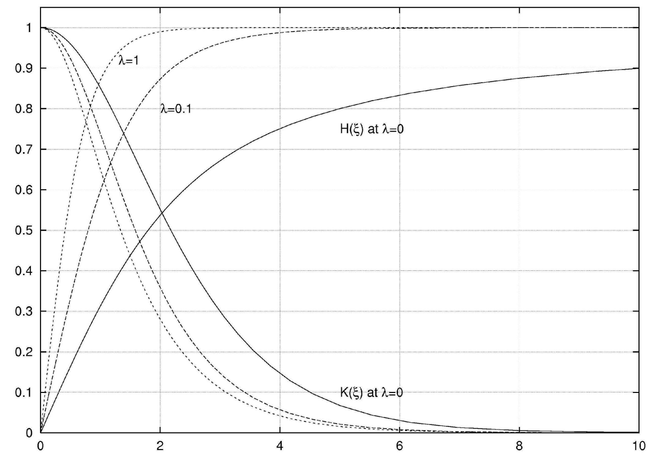


´t Hooft-Polyakov static spherically symmetric solution: monopole

Hedgehog ansatz





Monopole core: $R_c \sim m_V^{-1}$

Localised solitons: Gravity vs Yang-Mills

Pure gravity (attraction)

$$L = -\frac{R}{16\pi G}$$

Lichenrowitz: there are black holes but there are no gravitational solitons, the only globally regular, asymptotically flat, static vacuum solution to the Einstein eqs with finite energy is Minkowski space.

Pure Yang-Mills (attraction/repulsion)

$$L = \frac{1}{2}\mathrm{Tr}\ F_{\mu\nu}^2$$

Deser, Coleman: Classical Yang-Mills theory in 3+1 dim is scale invariant - there is no soliton solutions

Israel's theorem: Static Einstein-Maxwell black holes are spherically symmetric

'No-hair' theorem: Stationary black holes are completely characterized by their mass **M**, charge **Q** and angular momentum **J**

Einstein-Yang-Mills model

$$S = \frac{1}{2}\int d^4x\sqrt{-g}\,\{(R-2\Lambda) - \mathrm{Tr}\ \mathrm{F}_{\mu\nu}\mathrm{F}^{\mu\nu}\}$$

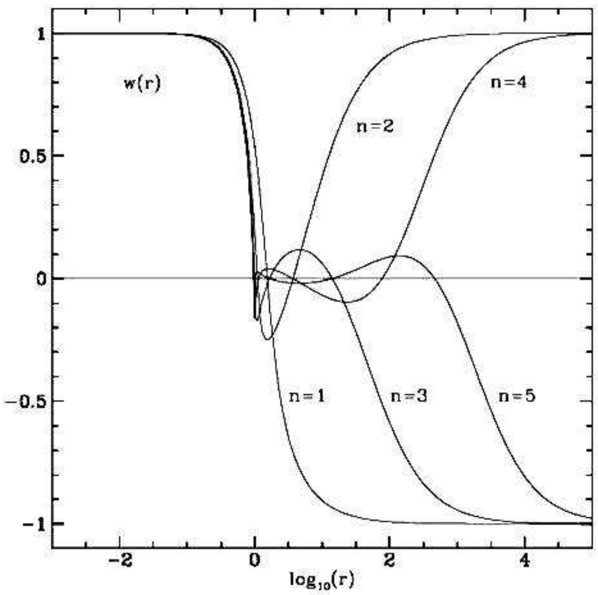
$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = T_{\mu\nu}; \qquad D_\mu F^\mu_\nu = \nabla_\mu F^\mu_\nu + [A_\mu, F^\mu_\nu] = 0$$

Spherical symmetry:

$$ds^2 = -\sigma^2(r)N(r)dt^2 + \frac{1}{N(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

Static asymptotically flat solution

$$A^a_k = \varepsilon_{iak}\frac{x^k}{r^2}(w(r) - 1)$$



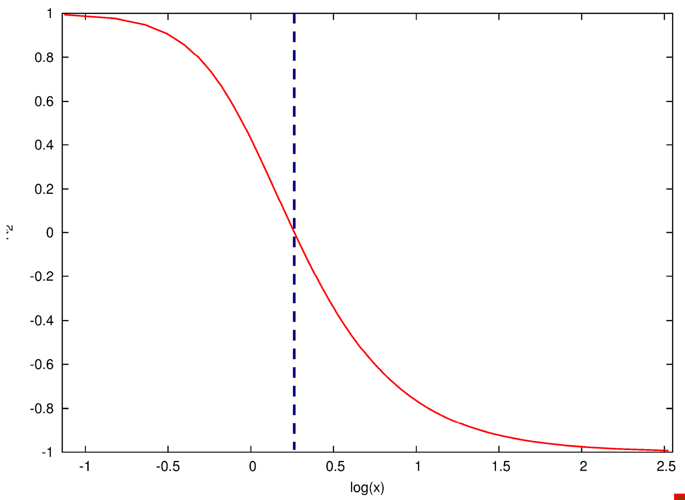
The Bartnik-McKinnon solitons (1988)

- Found numerically by the shooting method;
- The solution is globally regular;
- Analytic proof of existence of solutions of the differential equation;
- Gauge function $\boldsymbol{\omega}(\mathbf{r})$ has at least one zero, the solutions are characterized by the number of nodes of the $\boldsymbol{\omega}(\mathbf{r})$

Properties of the solutions

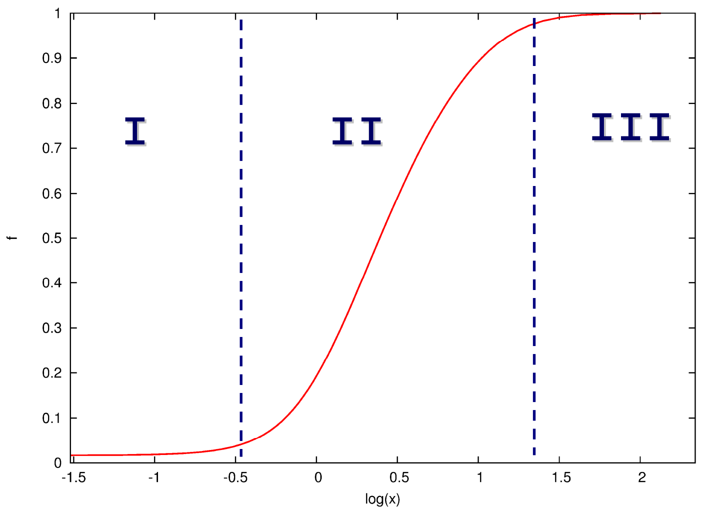
Dimensionless variables: $x = \frac{e}{\sqrt{4\pi G}} r \sim \frac{r}{l_{Pl}}; \quad \tilde{M} = eM\sqrt{\frac{G}{4\pi}} \sim \frac{M}{M_{Pl}}$

$$M_{Pl} \sim 1/\sqrt{G}; \quad l_{Pl} \sim \sqrt{G}$$



- **Region I:** **Yang-Mills field is almost trivial, the metric is close to Schwarzchild**
- **Region II:** **Yang-Mills field corresponds to monopole the metric is almost Reissner–Nordström**
- **Region III:** **Yang-Mills field is almost trivial, the metric is asymptotically Schwarzchild**

All Bartnik-McKinnon configurations are sphalerons



Galtsov, Volkov: **There are EYM black hole solutions with long-range non-abelian fields (hairy black holes)**

BM solutions are static asymptotically flat gravitationally bound EYM sphaleron solutions; the exterior of the limiting solution approaches RN black hole

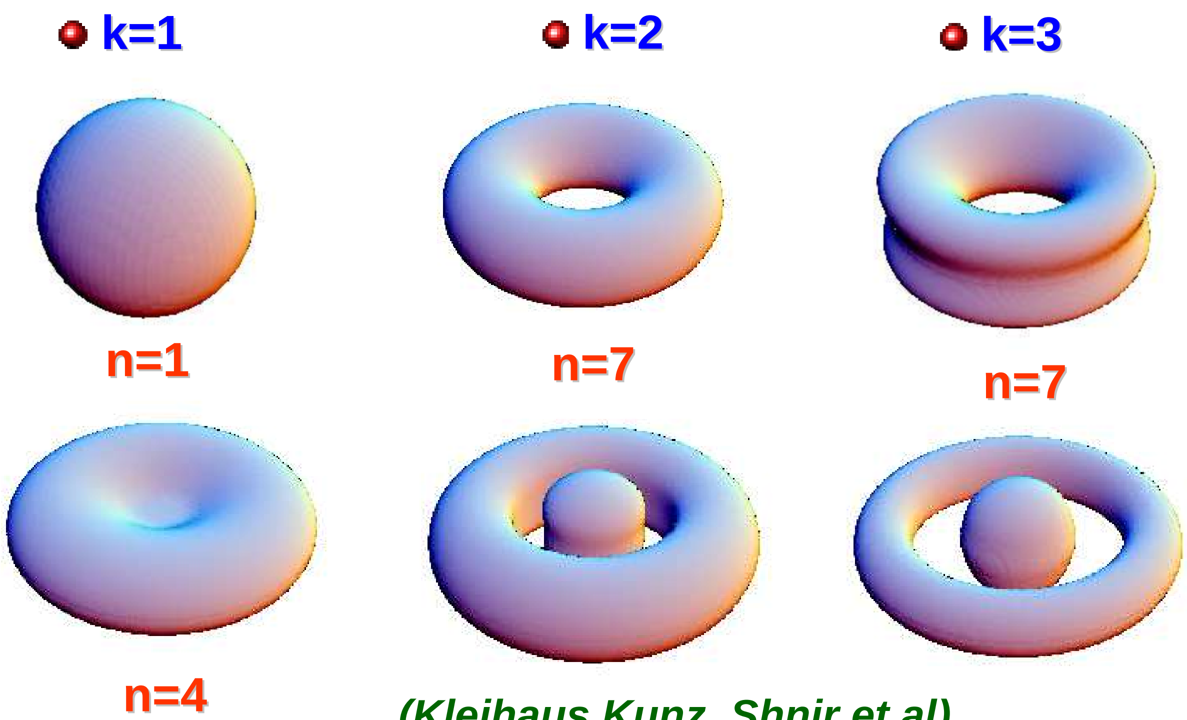
Generalised Bartnik-McKinnon solitons

Axial symmetry:

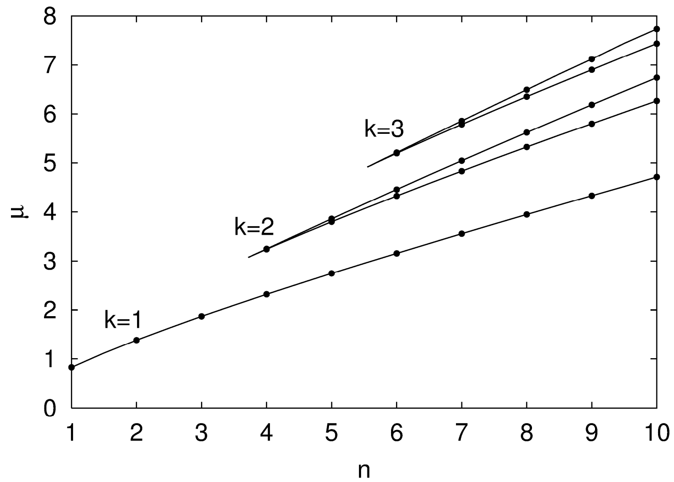
$$ds^2 = -f\,dt^2 + \frac{m}{f}\left(dr^2 + r^2 d\theta^2\right) + \frac{l}{f} r^2 \sin^2\theta\, d\varphi^2$$

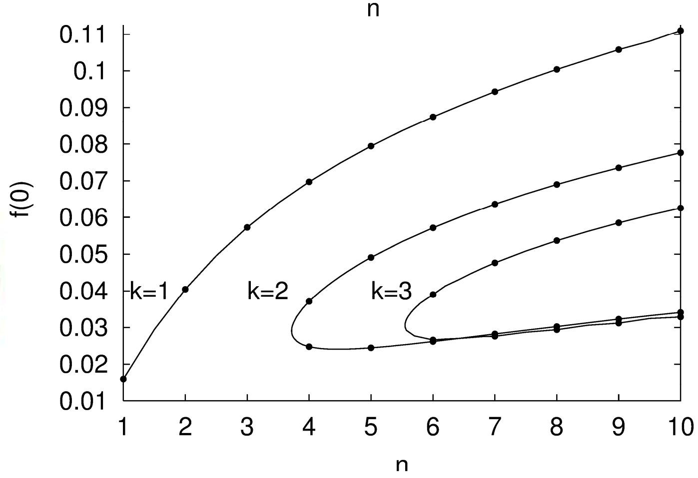
$$A_\mu dx^\mu = \left(\frac{K_1}{r}dr + (1-K_2)d\theta\right)\frac{\tau_\varphi^{(n)}}{2e} - n\sin\theta\left(K_3\frac{\tau_r^{(n,k)}}{2e} + (1-K_4)\frac{\tau_\theta^{(n,k)}}{2e}\right)d\varphi$$

Bartnik-McKinnon solutions are composite states



(Kleihaus,Kunz, Shnir et al)

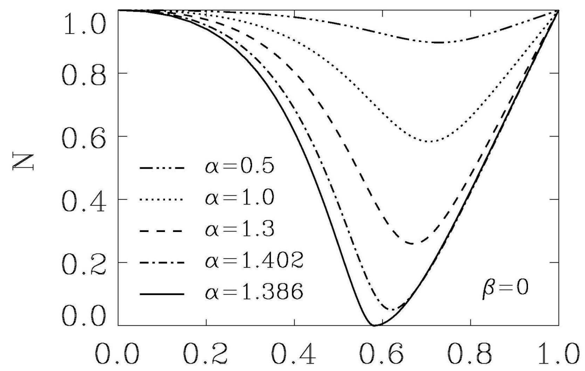


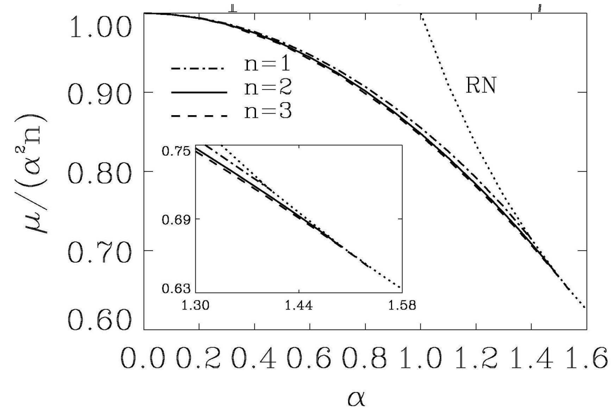


Self-gravitating Dyons

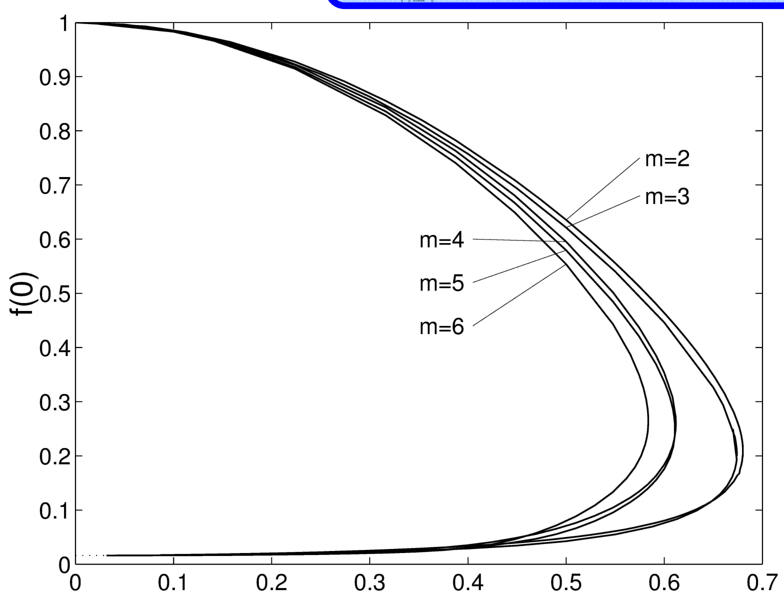
- **n=1, m=1** — Branch of gravitating solutions links the monopole to the RN black hole

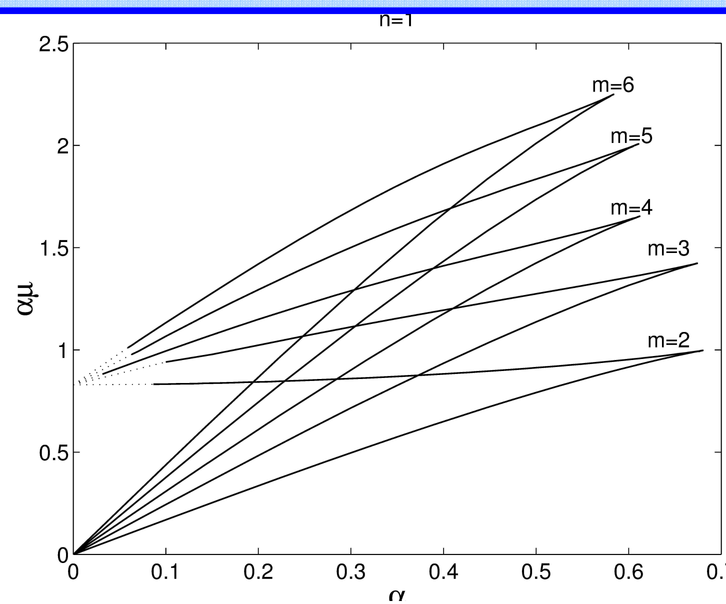
Dimensionless parameters of the model: $\alpha^2 = 4\pi^2 G\eta^2$, $\beta^2 = e^2/\eta$





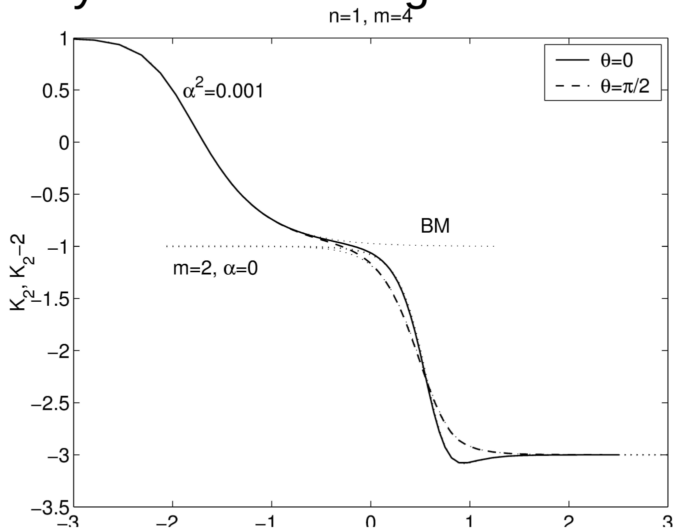
- **n=1, m=2,3..** — Branch of gravitating MA-chains is linked to the BM solutions

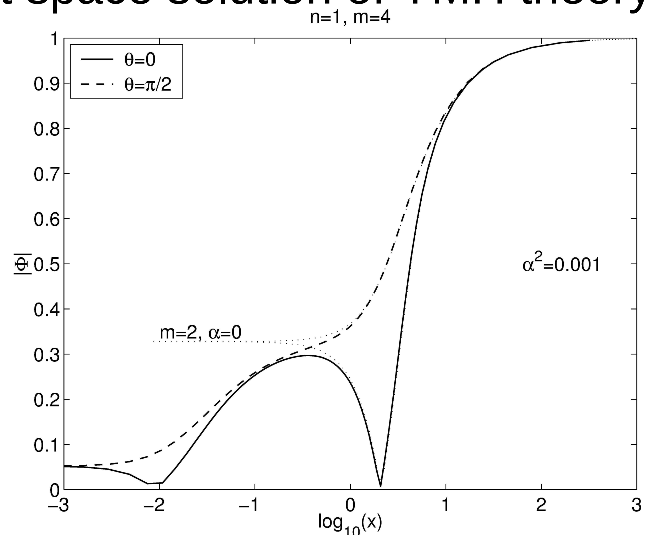


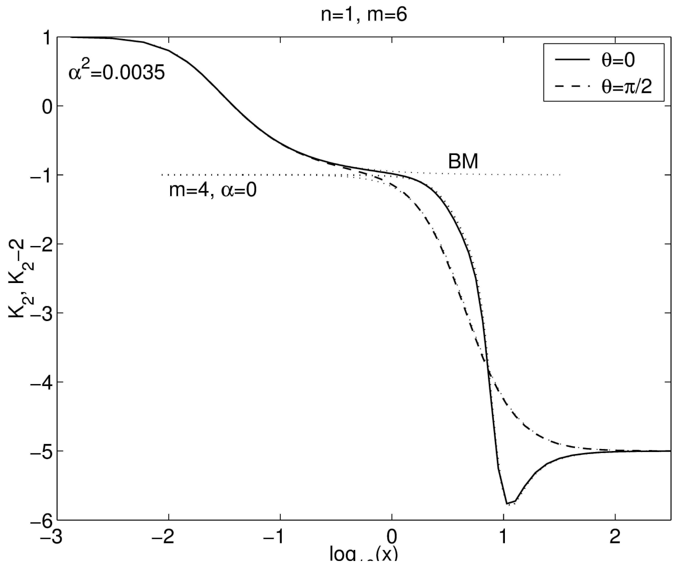


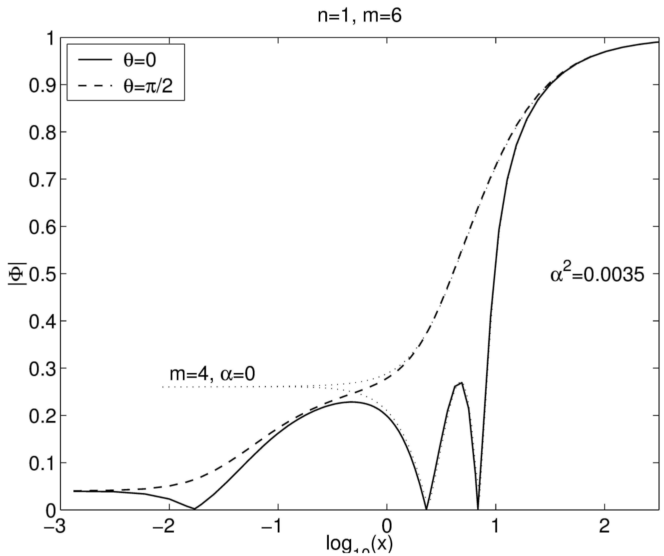
From gravitating Dyons to Bartnik-MacKinnon solutions

As gravity increases, the second branch of graviting axially symmetric **n**-MA chain evolve toward composite system of a Bartnik-McKinnon solution of EYM theory in the inner region and an outer **n – 2** flat space solution of YMH theory.





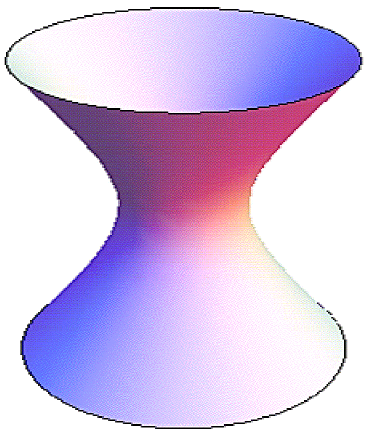




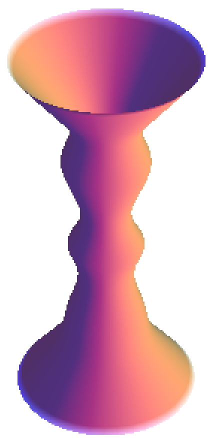
AdS Bartnik-McKinnon solitons

(Maison, Winstanley, Radu, Bjoraker, Hosotani et al)

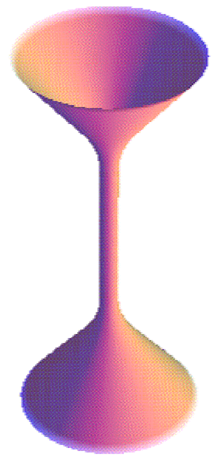
- Found numerically: There are continous families of solutions;
- Boundary conditions on the gauge function $\boldsymbol{\omega}(\mathbf{r})$ can be relaxed: $\boldsymbol{\omega}(\mathbf{r}) = 1 - \boldsymbol{\omega}_0$
- Gauge function $\boldsymbol{\omega}(\mathbf{r})$ may have no zero, the solutions possess a non-integer non-topological magnetic charge $Q_g = n(1 - \boldsymbol{\omega}_0^2)$
- There are rotating and electrically charged BM solitons
- There are stable configurations, both colored black holes and self-gravitating lumps



Fixed AdS space



Asymptotically AdS space



Limiting AdS

AdS SU(2) EYM theory

$$A_\mu dx^\mu = \left(\frac{K_1}{r}dr + (1-K_2)d\theta\right)\frac{\tau_\varphi^{(n)}}{2e} - n\sin\theta\left(K_3\frac{\tau_r^{(n,k)}}{2e} + (1-K_4)\frac{\tau_\theta^{(n,k)}}{2e}\right)d\varphi$$

Boundary conditions:

- **Odd *m*:**

$$H_1 = 0, \quad H_2 = 1 - m + w_0$$

$$H_3 = \frac{\cos\theta}{\sin\theta}\left(\cos((m-1)\theta) - 1\right) + w_0\sin((m-1)\theta)$$

$$H_4 = -\frac{\cos\theta}{\sin\theta}\sin((m-1)\theta) + w_0\cos((m-1)\theta)$$

Magnetic charge: $Q_M = n|1 - w_0^2|$

Nonquantized Monopoles

- **Even *m*:**

$$H_1 = 0, \quad H_2 = 1 - mw_0$$

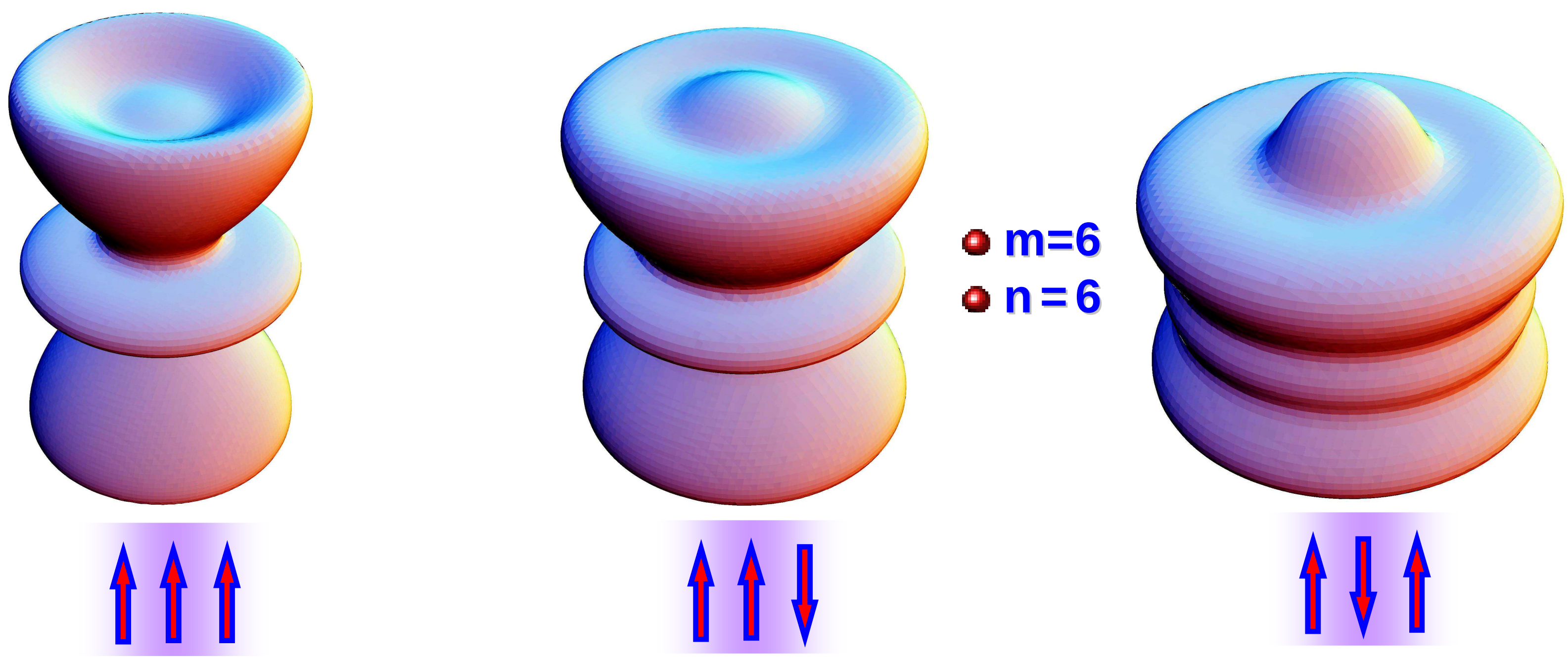
$$H_3 = w_0\frac{\cos(m-1)\theta - \cos\theta}{\sin\theta}; \quad H_4 = 1 - w_0 - w_0\frac{\sin(m-1)\theta}{\sin\theta}$$

Magnetic charge: $Q_M = \frac{mn}{2}|(1-w_0)w_0|$

AdS - YM composite solitons

Probe limit: $ds^2 = -N(r)dt^2 + \frac{dr^2}{N(r)} + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)\,; \quad N(r) = 1 + \frac{r^2}{l^2}$

- n = 4 and n = 6 solution consists of 2 and 3 constituents, respectively, each of them representing a n = 2 soliton.
- Each of the components of the composite configuration possesses a magnetic dipole moment, whose magnitude increases with n
- Dipole-dipole interaction energy becomes a significant part of the total energy



Bartnik-McKinnon solitons in asympotically AdS space

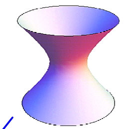
Axial symmetry in the bulk:

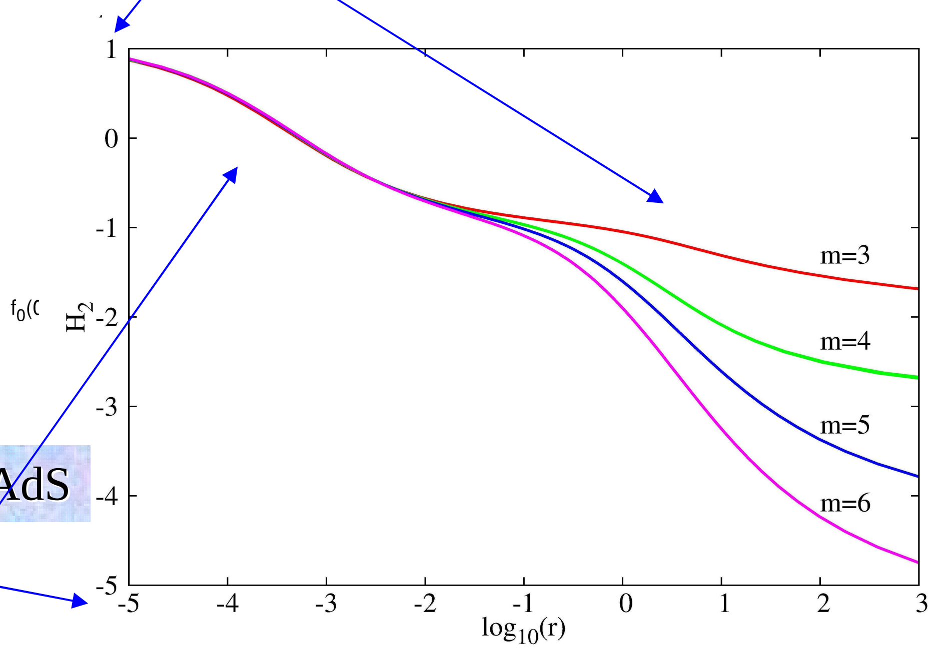
$$ds^2 = -f\left(1 - \frac{\Lambda}{3}r^2\right)dt^2 + \frac{m}{f}\left(\frac{dr^2}{1-\frac{\Lambda}{3}r^2} + r^2 d\theta^2\right) + \frac{l}{f}r^2\sin^2\theta d\varphi^2$$

Gravitational coupling: $\alpha = \frac{\sqrt{4\pi G}}{el} \to 0$

Two branches of the solutions

Fixed AdS





Throat + AdS

$l \to \infty \ (\Lambda \to 0)$

strongly coupled gravitating configurations in the asymptotically flat spacetime

$G \to 0$

YM solutions in a fixed globally AdS background

Introducing temperature: AdS black holes

- *Hawking temperature is dual to the temperature of the system on the boundary in d=3*
- *Temperature of the black hole is proportional to the surface gravity, T=κ/2π*
- *Entropy of a black hole is proportional to surface area of event horizon*
- *Dynamics in the bulk yields the boundary thermal field theory including non-equilibrium processes (dissipation)*

AdS Schwarzschild:

$$ds^2 = \frac{dr^2}{N(r)} + r^2(d\theta^2 + \sin^2\theta d\phi^2) - N(r)\sigma(r)^2 dt^2; \qquad N(r) = 1 - \frac{2m(r)}{r} - \frac{\Lambda}{3}r^2$$

Spherical symmetry: $A_k^a = \varepsilon_{iak}\frac{x^k}{r^2}(w(r) - 1)$

ADM mass: $\lim_{r\to\infty} m(r) = M$

AdS Reissner-Nordström: $N(r) = 1 - \frac{2m(r)}{r} + \frac{Q^2}{r^2} - \frac{\Lambda}{3}r^2$

Magnetic charge: $Q = \frac{1}{4\pi}\int_{S^2} d\theta d\phi \ \mathrm{Tr}\ (F_{\theta\phi}\cdot\tau_r)$

Thermodynamics of AdS Black Holes

- **Temperature:** $$T = \frac{1}{4\pi r_h}\left(1 - \frac{Q^2}{r^2} + \frac{3r_h^2}{l^2}\right)$$
- **Entropy:** $$S = 4\pi r_h^2$$
- **Free energy:** $$F = M - TS$$

Canonical ansemble - $\boldsymbol{Q}_{\mathbf{m}}$ is fixed

Two exact solutions:

ω=1

$$N(r) = \left(1 - \frac{r_h}{r}\right)\left(1 + \frac{1}{l^2}[r^2 + r r_h + r_h^2]\right); \qquad \sigma(r) = 1$$

SAdS

$$Q = 0, \quad M = \frac{r_h}{2}\left(1 + \frac{r_h^2}{l^2}\right)$$

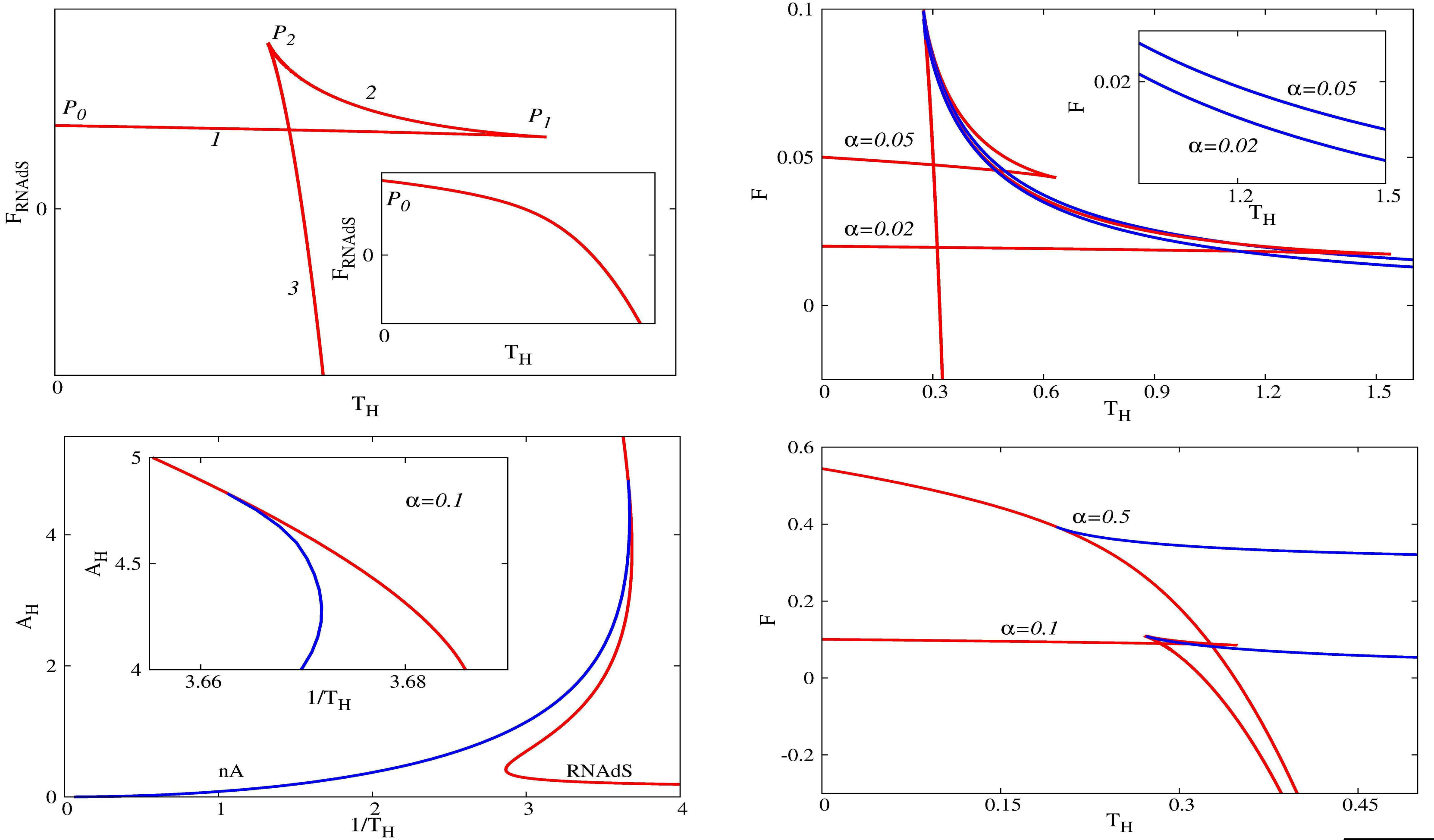
ω=0

$$N(r) = \left(1 - \frac{r_h}{r}\right)\left(1 + \frac{1}{l^2}[r^2 + r r_h + r_h^2] - \frac{\alpha^2}{r r_h}\right); \quad \sigma(r) = 1$$

RNAdS

$$Q = 1, \quad M = \frac{1}{2r_h}\left(\alpha^2 + r_h^2(1 + \frac{r_h^2}{l^2})\right)$$

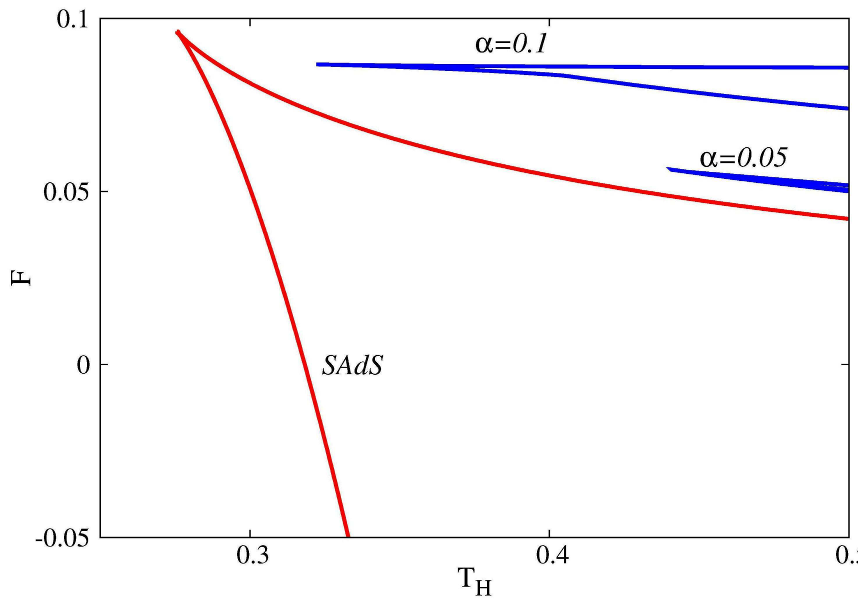
Thermodynamics of RNAdS Black Holes

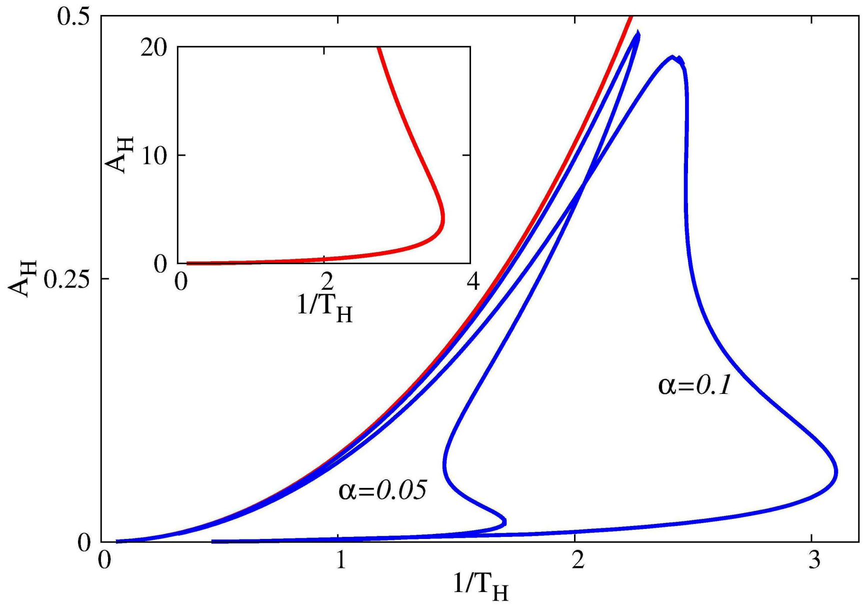


- *There is soliton limit of the nA RNAdS solutions: as $r_h \rightarrow 0$* $\omega(r) = 1/\sqrt{1+\frac{r^2}{l^2}}$

- *Free energy of all hairy solutions is minimized by the RNAdS solution*

Thermodynamics of SAdS Black Holes





- *Hairy solitons with Q=0 possess non-vanishing magnetic field in the bulk*
- *Hairy BH do not appear as perturbation of the SAdS solutions*
- *The branch structure of the solutions:* $\alpha = \sqrt{4\pi G}/el \to 0$
- *First law of thermodynamics is satisfied by the generic hairy solutions:*

$$dM = TdS + \Phi dQ$$

Einstein-deTurck Equations

(M. Headrick, S. Kitchen and T. Wiseman, Class. Quant. Grav. 27 (2010) 035002)

Harmonic Einstein equations:

$$R_{\mu\nu} - \nabla_{(\mu}\xi_{\nu)} - \Lambda g_{\mu\nu} = 2\alpha^2 T_{\mu\nu}$$

DeTurck choice of ξ :

$$\xi^\mu = g^{\nu\rho}\left(\Gamma^\mu_{\nu\rho} - \bar{\Gamma}^\mu_{\nu\rho}\right)$$

Spacetime metric:

$$ds^2 = f_1(r,\theta)\frac{dr^2}{N(r)} + S_1(r,\theta)(r d\theta + S_2(r,\theta)dr)^2$$
$$f_2(r,\theta) r^2 \sin^2\theta d\phi^2 - f_0(r,\theta)N(r)dt^2$$

Reference metric:

$$ds^2 = \frac{dr^2}{N(r)} + r^2(d\theta^2 + \sin^2\theta d\phi^2) - N(r)dt^2$$

$$N(r) = 1 + \frac{r^2}{l^2}$$

Advantage: better quality of the numerical results

AdS/gauge theory correspondence

- The dual gravitational system has at least one extra dimension z; the field theory properties can be extracted by working on the boundary.

- The extra dimension z should be interpreted as an energy scale.

- It represents the renormalisation group flow of the quantum field theory defined on the boundary.

- The AdS/CFT correspondence "geometrises" the field theory energy scale.

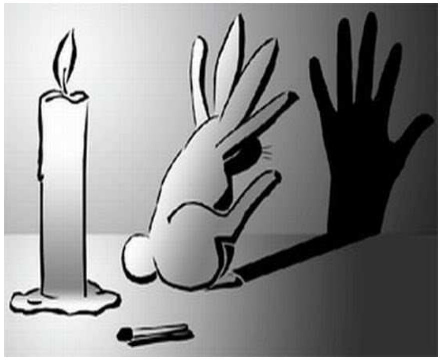
- Geometrisation: in the dual bulk gravitational description the energy scale is treated geometrically on an equal footing to the spatial directions of the boundary field theory

Large N gauge theory Strong coupled field theory at finite timperature d-dim space-time	⟷	Classical Einstein gravity d+1 - dim space-time

AdS SU(2) EYMH theory

$$S = \frac{1}{2}\int d^4x\sqrt{-g}\,\{(R-2\Lambda) - \mathrm{Tr}\,\mathrm{F}_{\mu\nu}\mathrm{F}^{\mu\nu} - \mathrm{Tr}\,(\mathrm{D}_\mu\Phi)(\mathrm{D}^\mu\Phi) - \mathrm{V}(\Phi)\}$$

(Maison, Breitenlocher, Shaposhnik, Moreno, Tong, Bolognesi, Kunz, Radu, Shnir..)



Boundary CFT

V = 0: Gauge field A_μ $\Leftrightarrow$ triplet of conserved currents J_μ^a

Scalar field Φ^a $\Leftrightarrow$ scalar operators $\mathcal{Q}^a$

BPS limit:

$$\Phi^a(z) \to \eta^a + \frac{C^a}{z^3} + \dots$$

SU(2) global symmetry on the boundary is broken:

$$\partial^\mu J_\mu^a = \varepsilon^{abc}\eta^b\mathcal{Q}^c \qquad \eta^b = \mathrm{const} \Rightarrow U(1)$$

- **V < 0:** Abelian symmetry in the bulk $\Leftrightarrow$ U(1) conserved boundary current J_μ, massive gauge boson $\Leftrightarrow$ charged spin-1 operator

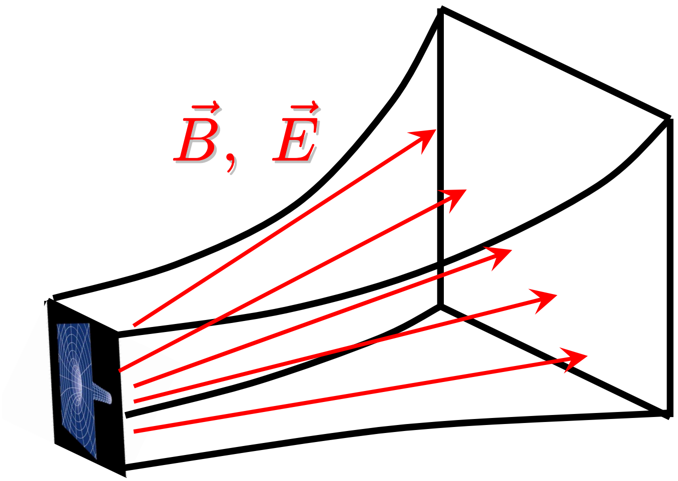
  Scalar field Φ $\Leftrightarrow$ relevant scalar operator

$$\Phi^a(z) \to n^a\left(\eta + \frac{C_0}{z^{\Delta_-}} + \frac{C_1}{z^{\Delta_+}} + \dots\right)$$

- **V > 0:** Scalar field is irrelevant

Holographic AdS dyon

BPS limit: V = 0



$$A_k = B\varepsilon_{ki}x_i + \ldots \Longrightarrow L_{QFT}(A) = A_i J_i$$

$$\Phi^a = \eta^a + \frac{C^a}{r^3} + \ldots \Longrightarrow L_{QFT}(\Phi) = \Phi \cdot \mathcal{O}(x)$$

$$A_0^a = e\eta\hat{n}^a\left(\mu + \frac{Q}{r} + \ldots\right)$$

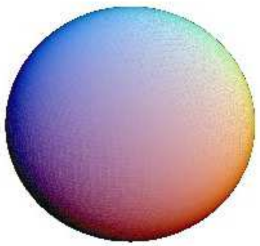
On the boundary: d=2+1 Abelian Quantum Field Theory which undergoes a phase transition exhibiting condensation below a critical temperature.

Abelian Higgs model at finite temperature

In the bulk we have:

- d=3+1 Yang-Mills-Higgs theory;
- Schwarzschild-AdS black hole

Holographic p-wave superconductors



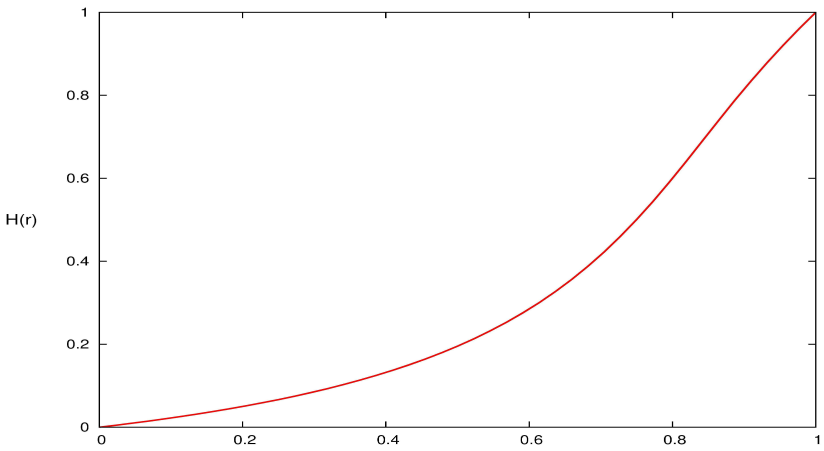
$$ds^2 = \frac{dr^2}{N(r)} + r^2(d\theta^2 + \sin^2\theta d\varphi^2) - N(r)dt^2$$

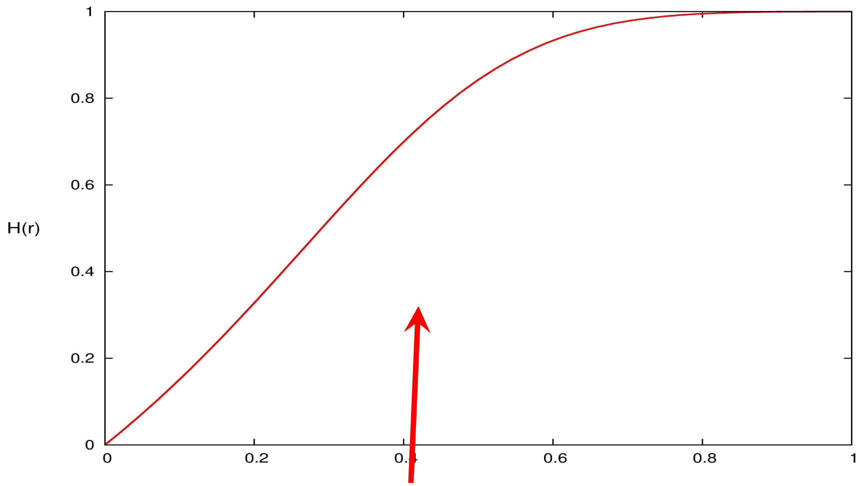
$$N(r) = 1 - \frac{\Lambda}{3}r^2 - \frac{2M}{r}$$

$$N(r) = (1 - \frac{r_h}{r})\left[1 - \frac{\Lambda}{3}\left(r^2 + rr_h + r_h^2\right)\right]$$

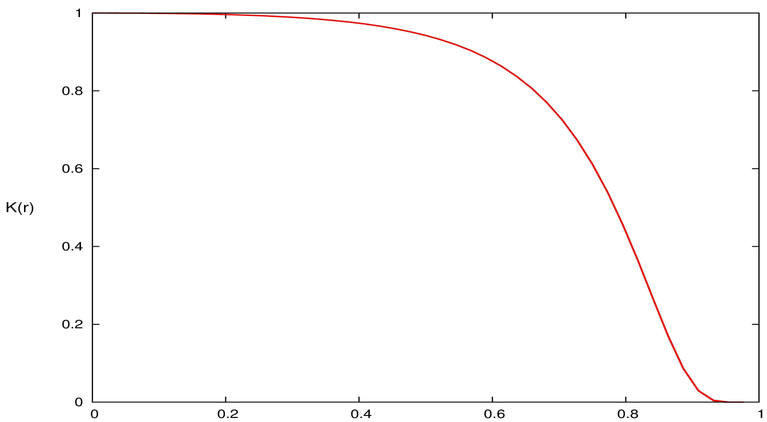
Dyons in AdS space

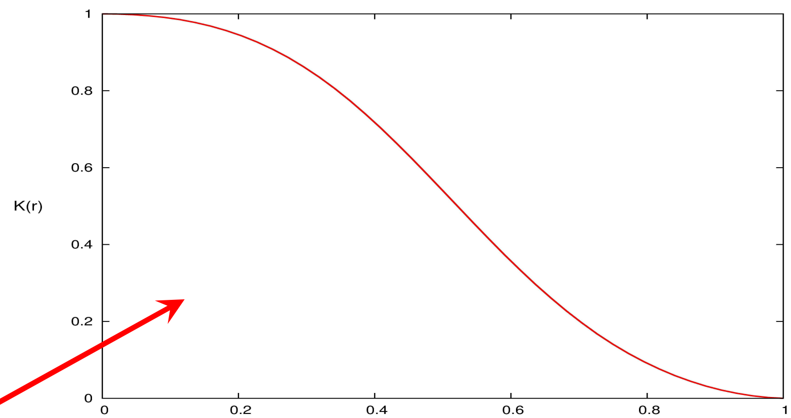
- **Λ=0**
- **Λ=-3**





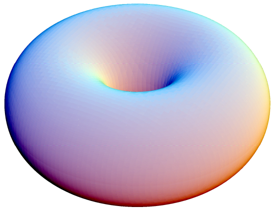
The Higgs field on the boundary becomes a constant
It does not qualify as a proper order parameter

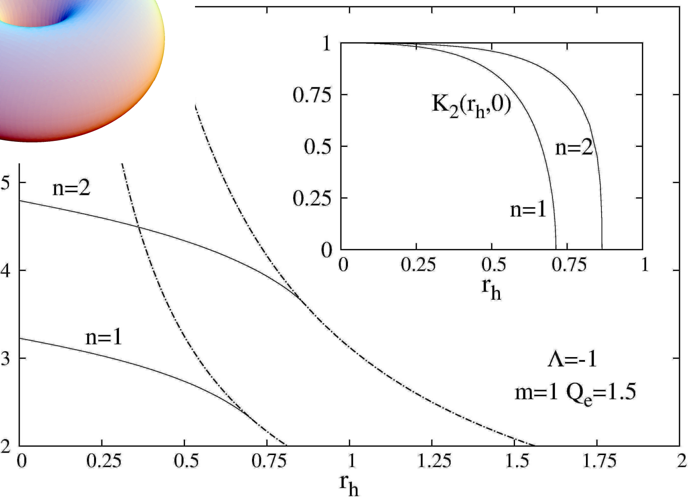


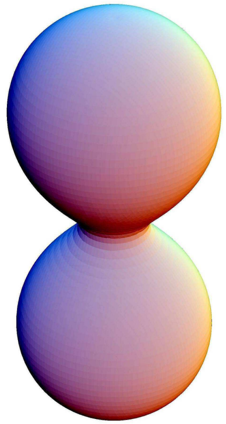


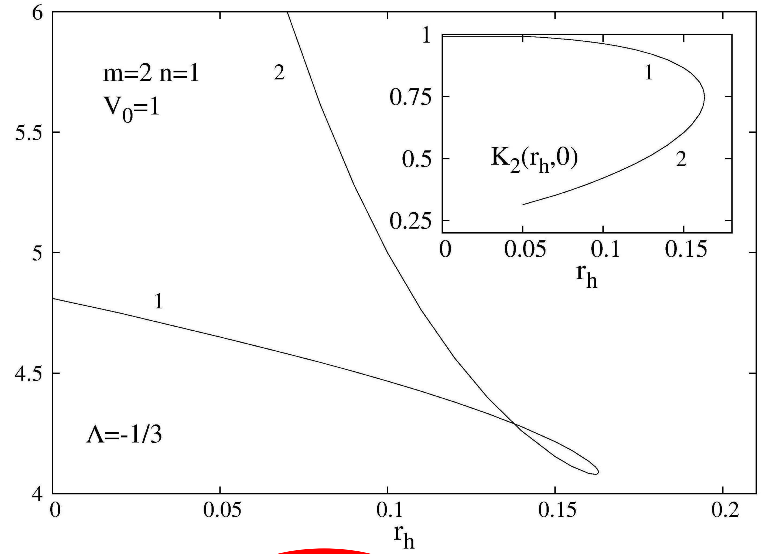
The order parameter is the gauge function K

Some more numerics..



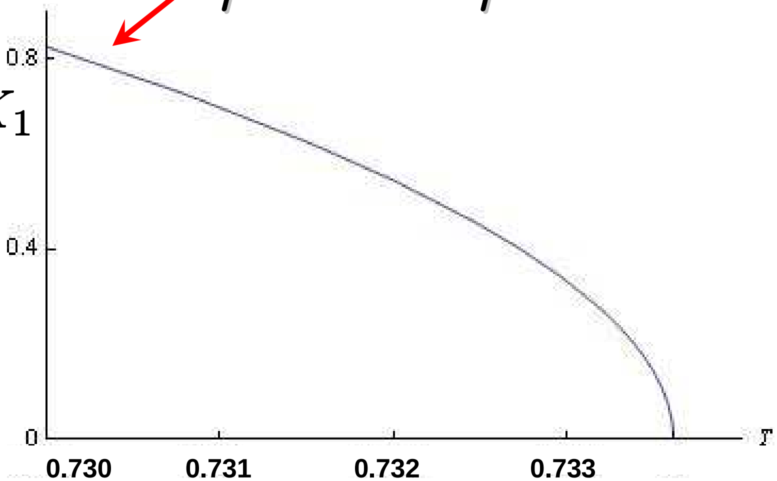


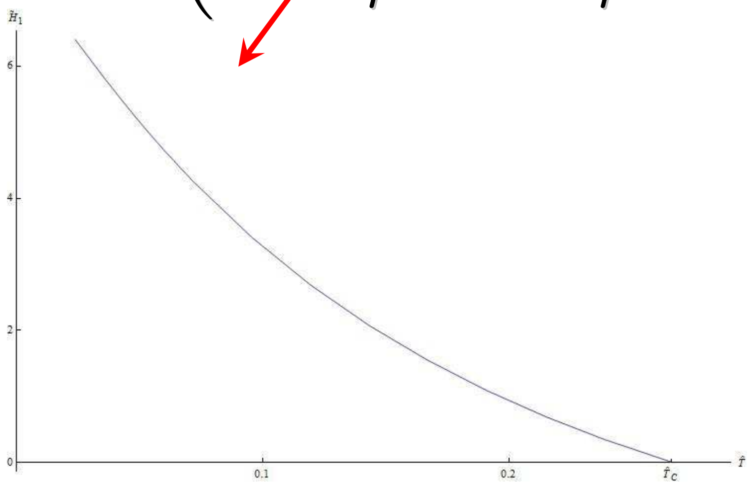




$$K(r) = \frac{K_1(T)}{r^{\nu+1}} + \frac{K_2(T)}{r^{\nu+3}} + \dots$$

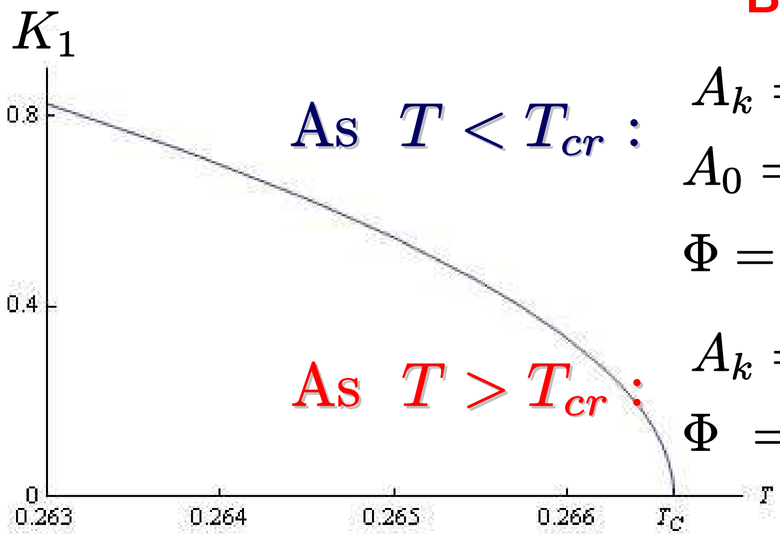
$$H(r) = \eta\left(1 - \frac{H_1(T)}{r^3} + \frac{H_2(T)}{r^5} + \dots\right)$$





Physics in the bulk/boundary

Interpretation: Phase transition in the bulk at T=T_{cr}



	Bulk:	**Boundary:**
As $T < T_{cr}$:	$A_k = B\varepsilon_{ki}x_i + \dots$	$L_{QFT}(A) = A_i J_i$
	$A_0 = C_0 + \frac{C_1}{r^\circ} + \dots$	$L_{QFT}(\Phi) = \Phi \cdot \mathcal{O}(x)$
	$\Phi = v + \frac{C}{r^\alpha} + \dots$	
As $T > T_{cr}$:	$A_k = \; const$	$J_k = 0; B = 0$
	$\Phi \; = \; const$	$\mathcal{O} = 0$

- To the right of T_{cr} the conguration becomes trivial, SU(2) global symmetry is restored.
- To the left of T_{cr} the congurations, which correspond to v.e.v.'s in the dual field theory are non-trivial.
- There is a finite temperature continuous symmetry breaking transition.
- The system condenses below a critical temperature T.
- Fitting the curves one confirms that this is a second order phase transition:

$$K_1 \propto (T_{cr} - T)^{1/2}; \qquad H_1 \propto (T_{cr} - T)$$

Summary and Outlook

- We constructed generalized BM AdS solutions
- Using de Turck approach we obtained static axially symmetric dyonic solutions in AdS spacetime
- Dyonic black hole in AdS yields phase transition on the boundary at critical temperature
- Vortex condensation?

Thank you for your attention!