Derivation of capture and reaction cross sections from quasi-elastic and elastic back-scattering

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1. Introduction

Capture cross section σ_{cap} is one important ingredient to predict the production cross sections of exotic and superheavy nuclei in cold, hot, sub-barrier astrophysical fusion reactions.

Direct measurement:

$$\sigma_{cap}(E_{c.m.})=\sigma_{qf}(E_{c.m.})+\sigma_{ff}(E_{c.m.})+\sigma_{ER}(E_{c.m.})$$

This requires different experimental setups and a large amount of beam time.

Direct measurement of reaction cross section:

$$\sigma_R(E_{c.m.})=\sigma_{in}(E_{c.m.})+\sigma_{tr}(E_{c.m.})+\sigma_{cap}(E_{c.m.})+\sigma_{BU}(E_{c.m.})+\sigma_{DIC}(E_{c.m.})$$

Conservation of total reaction flux:

Traditional method consists in deriving parameters of the complex optical potential which fits experimental elastic scattering angular distribution and then to derive the reaction cross sections predicted by this potential.

Using full angular distribution is necessary?

2. Quasi-elastic scattering and capture process

Conservation of reaction flux:

$$P_{qe}(E_{c.m.},J)+P_{cap}(E_{c.m.},J)=1$$

P_{cap} - **capture probability (transmission)**

P_{qe} - **quasielastic probability (reflection)**

$$P_{qe}=P_{el}+P_{in}+P_{tr}$$

P_{el} – *elastic probability*

P_{in} – *inelastic probability*

P_{tr} – *transfer probability*

Quasi-elastic probability for angular momentum J=0:

$$P_{qe}^{ex}(E_{c.m.}, J=0) = \sigma_{qe}(E_{c.m.}, \theta=180^{o}) / \sigma_{Ru}(E_{c.m.}, \theta=180^{o})$$

the ratio of quasi-elastic differential cross section and Rutherford differential cross section at 180 degrees . This ratio can be measured!

$$P_{cap}(E_{c.m.}, J=0) = 1 - P_{qe}^{ex}(E_{c.m.}, J=0)$$

Checking of equation

$$P^{ex}_{el,qe,BU}\left(E_{\mathrm{c.m.}}, J=0\right)=\frac{\sigma_{el,qe,BU}\left(E_{\mathrm{c.m.}},\theta=180^{o}\right)}{\sigma_{Ru}\left(E_{\mathrm{c.m.}},\theta=180^{o}\right)}$$

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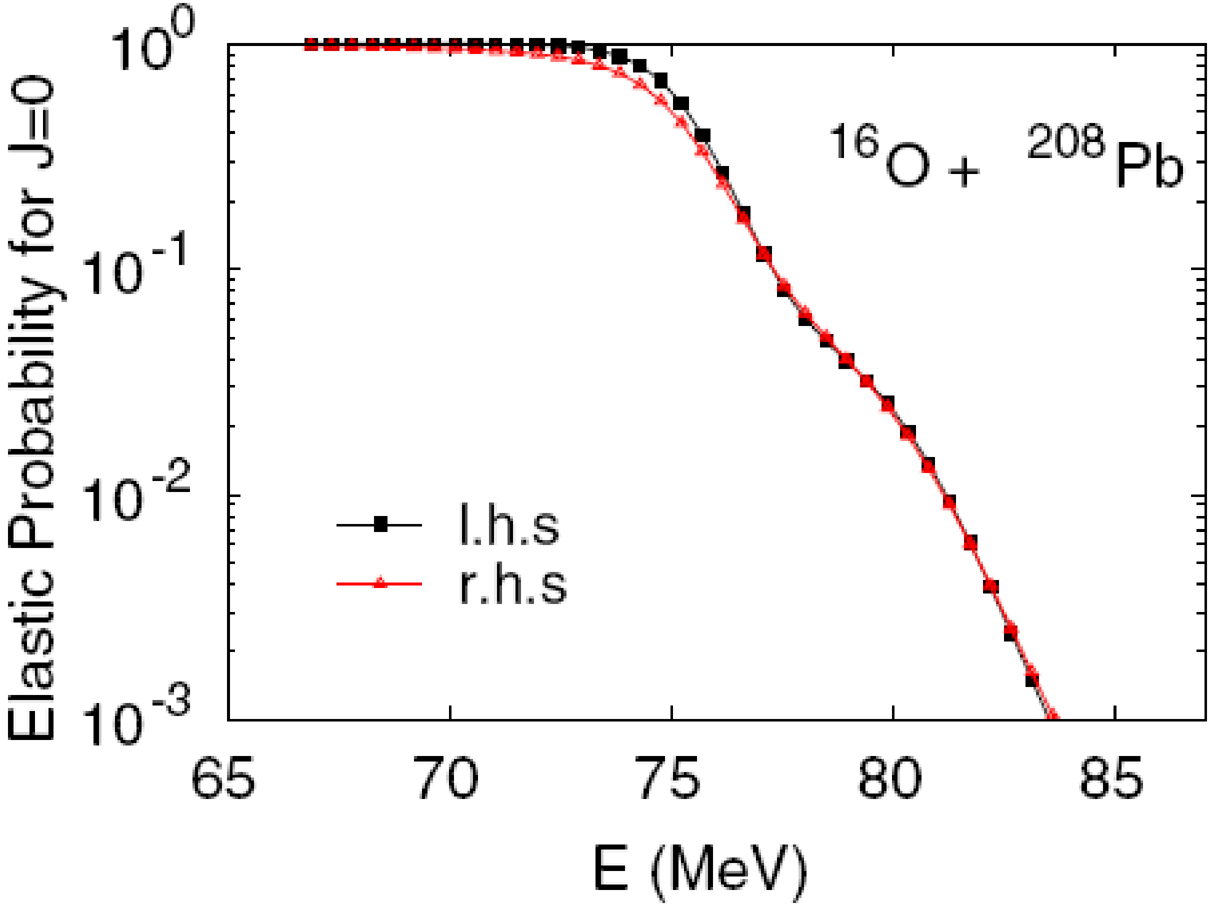


Capture cross section:

$$\sigma_{cap}(E_{c.m.}) = \pi \lambda\!\!\!^{-2} \sum_{J=0}^{J_{crit}} (2J+1) P_{cap}(E_{c.m.}, J)$$

$\lambda\!\!\!^{-2} = \hbar^2 / (2\mu E_{c.m.})$ **- reduced de Broglie wave-length,**
$E_{c.m.}$ **- energy in c.m. ,** μ **- reduced mass ,**
J **- angular momentum ,** J_{crit} **- critical ang. m.**

Height of Coulomb barrier:

$$V_b(J) = V_b(0) + \frac{\hbar^2 J(J+1)}{2\mu R_b^2} = V_b(J=0) + E_b^{rot}(J)$$

J-dependence of $P_{cap}(E_{c.m.}, J)$ at given energy one can approximate by shifting energy :

$$P_{cap}(E_{c.m.}, J) = P_{cap}(E_{c.m.} - E_b^{rot}, J=0)$$

Capture cross section :

$$\sigma_{cap}(E_{c.m.}) = \pi \lambda\!\!\!^{-}{}^2 \sum_{J=0}^{J_{crit}} (2J+1)\left[1 - P_{qe}^{ex}(E_{c.m.} - E_b^{rot}, J=0)\right]$$

$$\sum_J \ldots \to \int dE \ldots, \qquad J \to E = E_{c.m.} - E_b^{rot}$$

$$\sigma_{cap}(E_{c.m.}) = \frac{\pi R_b^2}{E_{c.m.}} \int_{E_{c.m.} - E_b^{rot}(J_{crit})}^{E_{c.m.}} dE\left[1 - P_{qe}^{ex}(E, J=0)\right]$$

Height of the Coulomb barrier [next expantion in J(J+1)]:

$$V_b(J) = V_b(J=0) + \frac{\hbar^2 J(J+1)}{2\mu R_b^2} + \frac{\hbar^4 [J(J+1)]^2}{2\mu^3 \omega_b^2 R_b^2}$$

Capture cross section :

$$\sigma_{cap}(E_{c.m.}) =$$

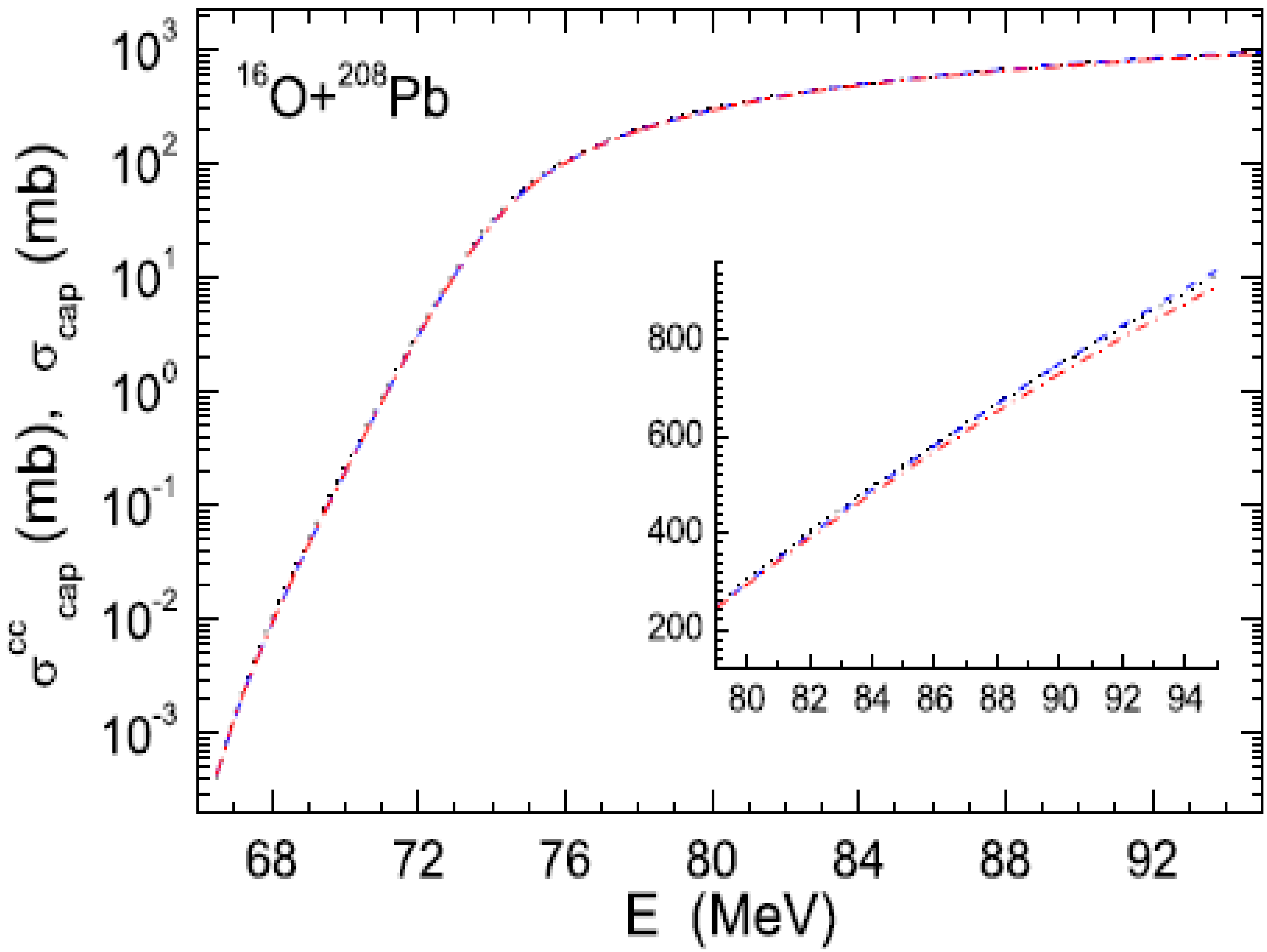
$$\frac{\pi R_b^2}{E_{c.m.}} \int_{E_{c.m.} - E_b^{rot}(J_{crit})}^{E_{c.m.}} dE \, [1 - P_{qe}^{ex}(E, J=0)][1 - \frac{4(E_{c.m.} - E)}{\mu \omega_b^2 R_b^2}]$$

$$E_b^{rot}(J_{crit}) = \frac{\hbar^2 J_{crit}(J_{crit}+1)}{2\mu R_b^2}$$

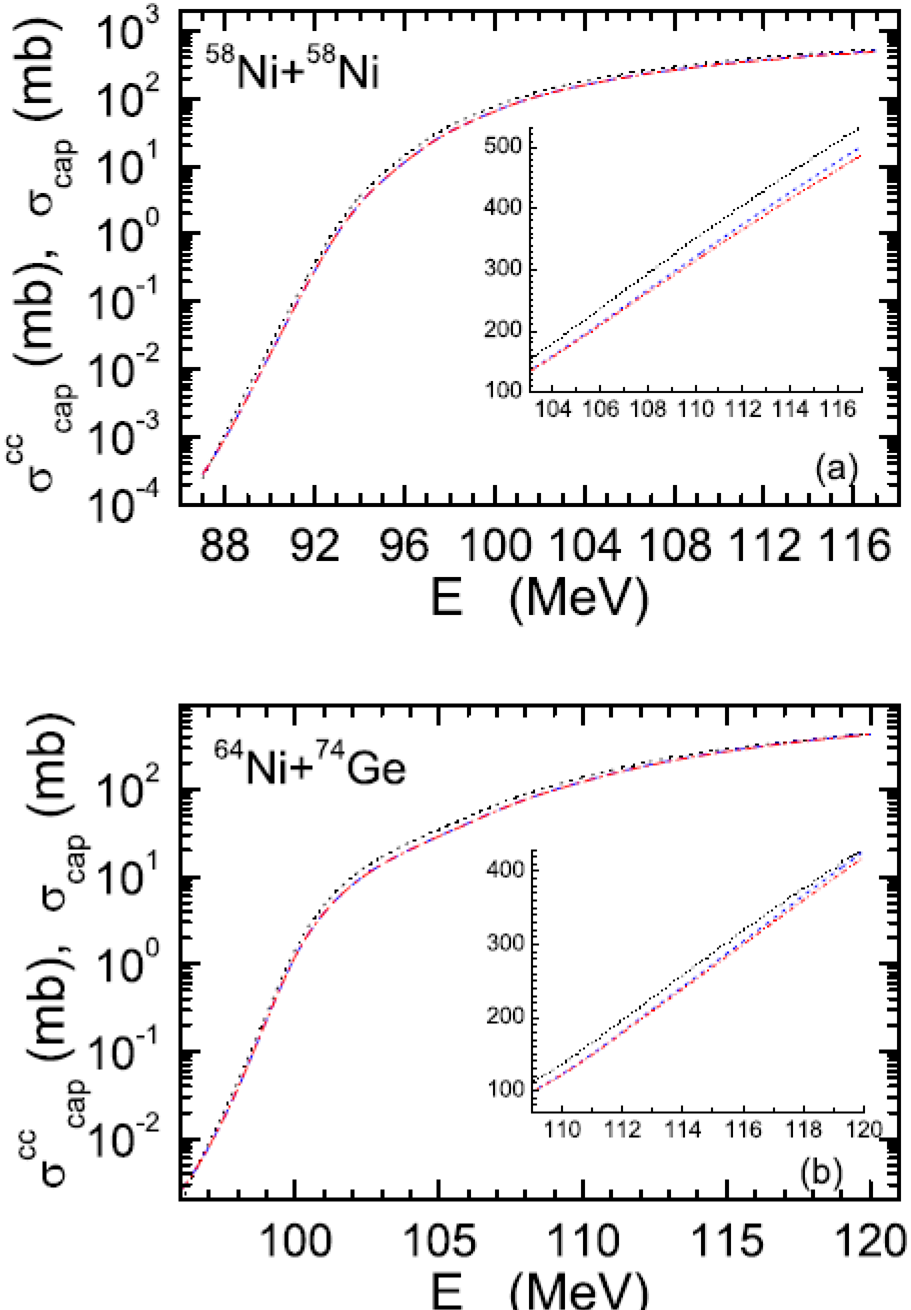
TESTING A FORMULA OF CAPTURE CROSS SECTION AGAINST COUPLED-CHANNELS CALCULATIONS

TABLE I. Parameters of both the bare Woods-Saxon potential between the colliding nuclei (second column) and the uncoupled Coulomb barriers. Energy is in MeV, while radius and diffuseness are in femtometers.

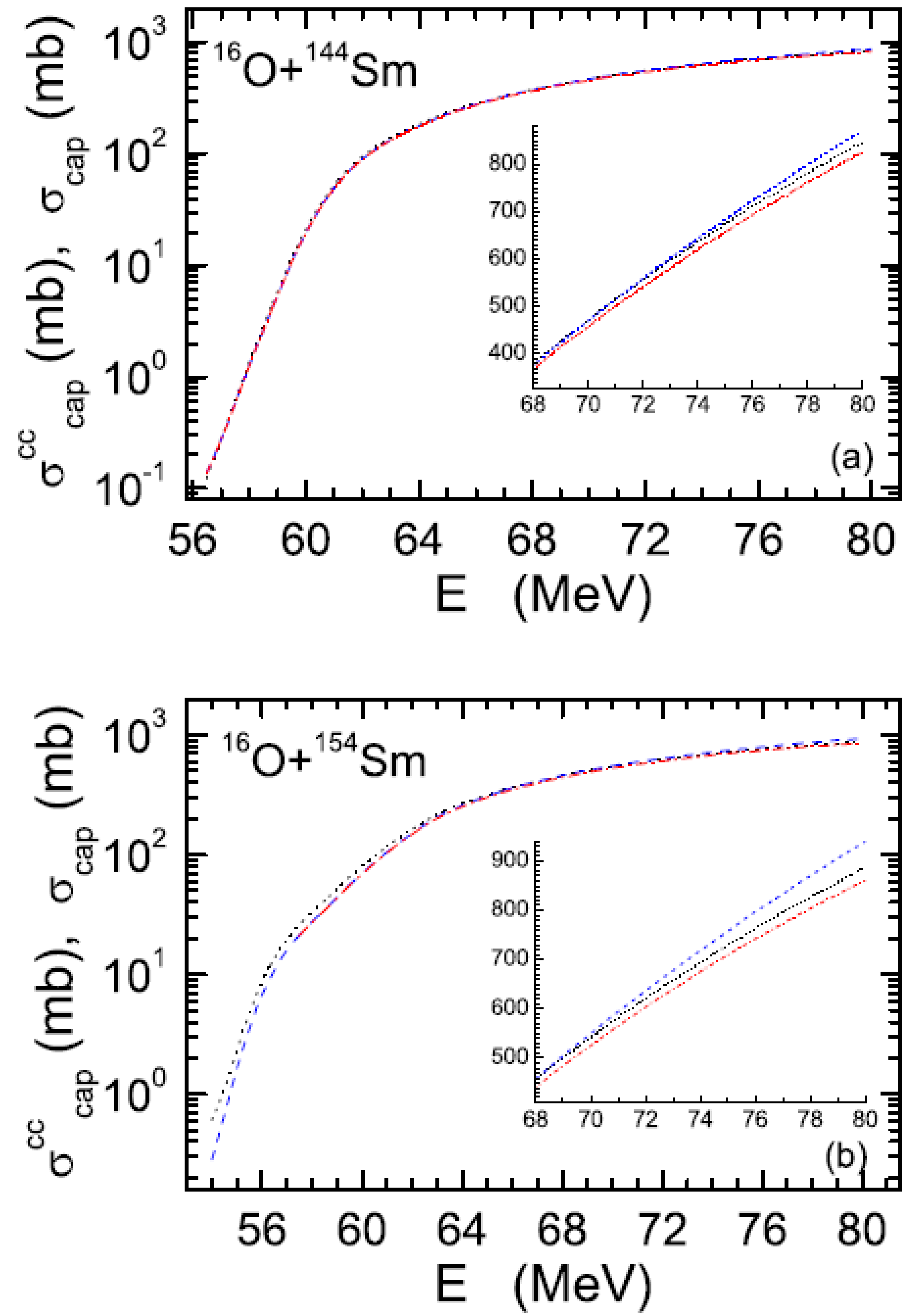
Reactions	(V_0, r_0, a_0)	V_B	R_B	$\hbar\omega_B$
$^{16}O + ^{144}Sm$	(−105.10,1.10,0.75)	61.25	10.82	4.25
$^{16}O + ^{154}Sm$	(−165,0.95,1.05)	59.41	10.81	3.48
$^{16}O + ^{208}Pb$	(−100,1.17,0.66)	75.04	11.86	4.76
$^{58}Ni + ^{58}Ni$	(−220,1.04,0.7)	101.16	10.38	4.13
$^{64}Ni + ^{74}Ge$	(−76.19,1.21,0.63)	106.57	11.37	3.68



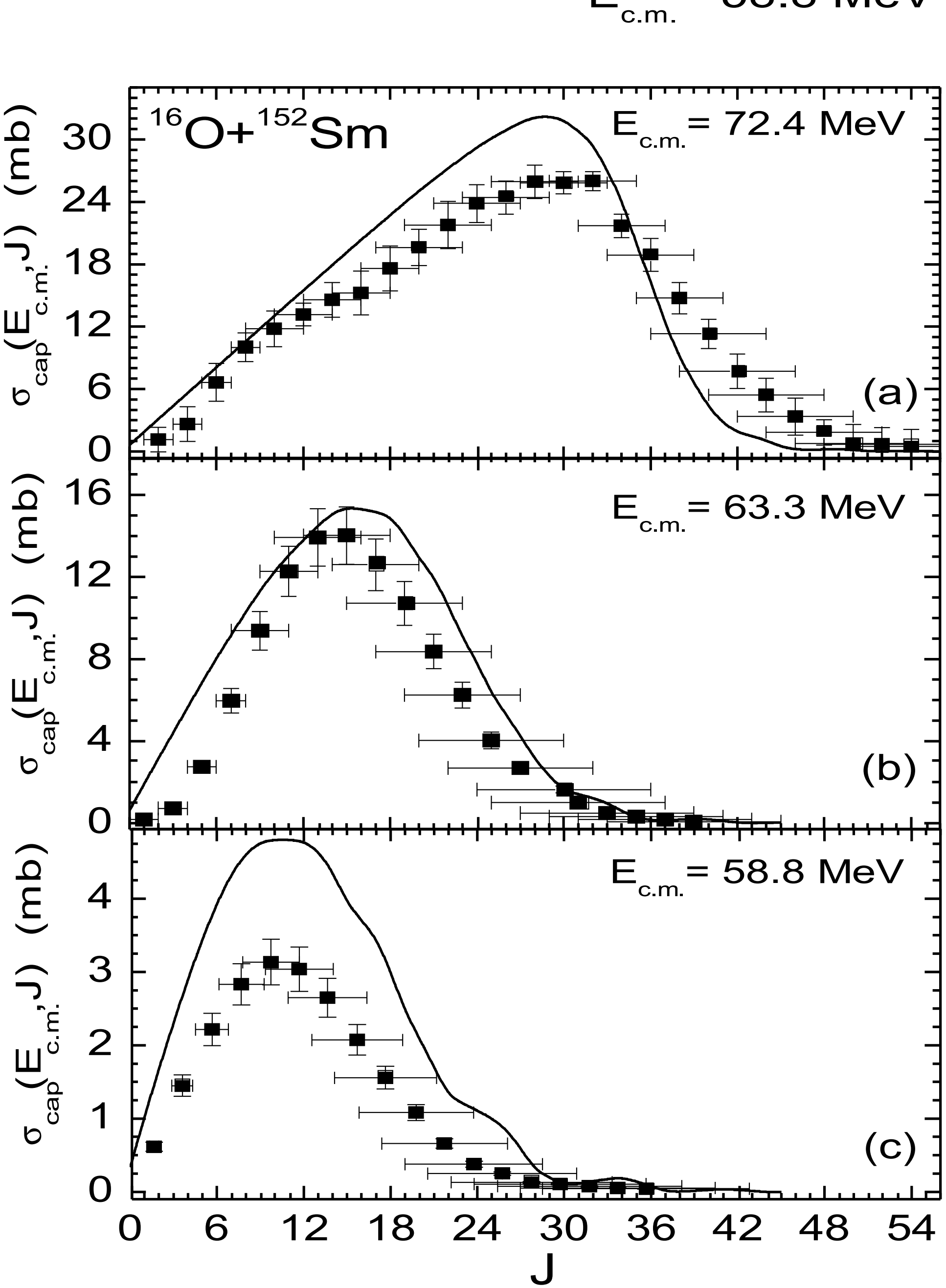
^{16}O+^{208}Pb
σ^{cc}_{cap} (mb), σ_{cap} (mb)
E (MeV)

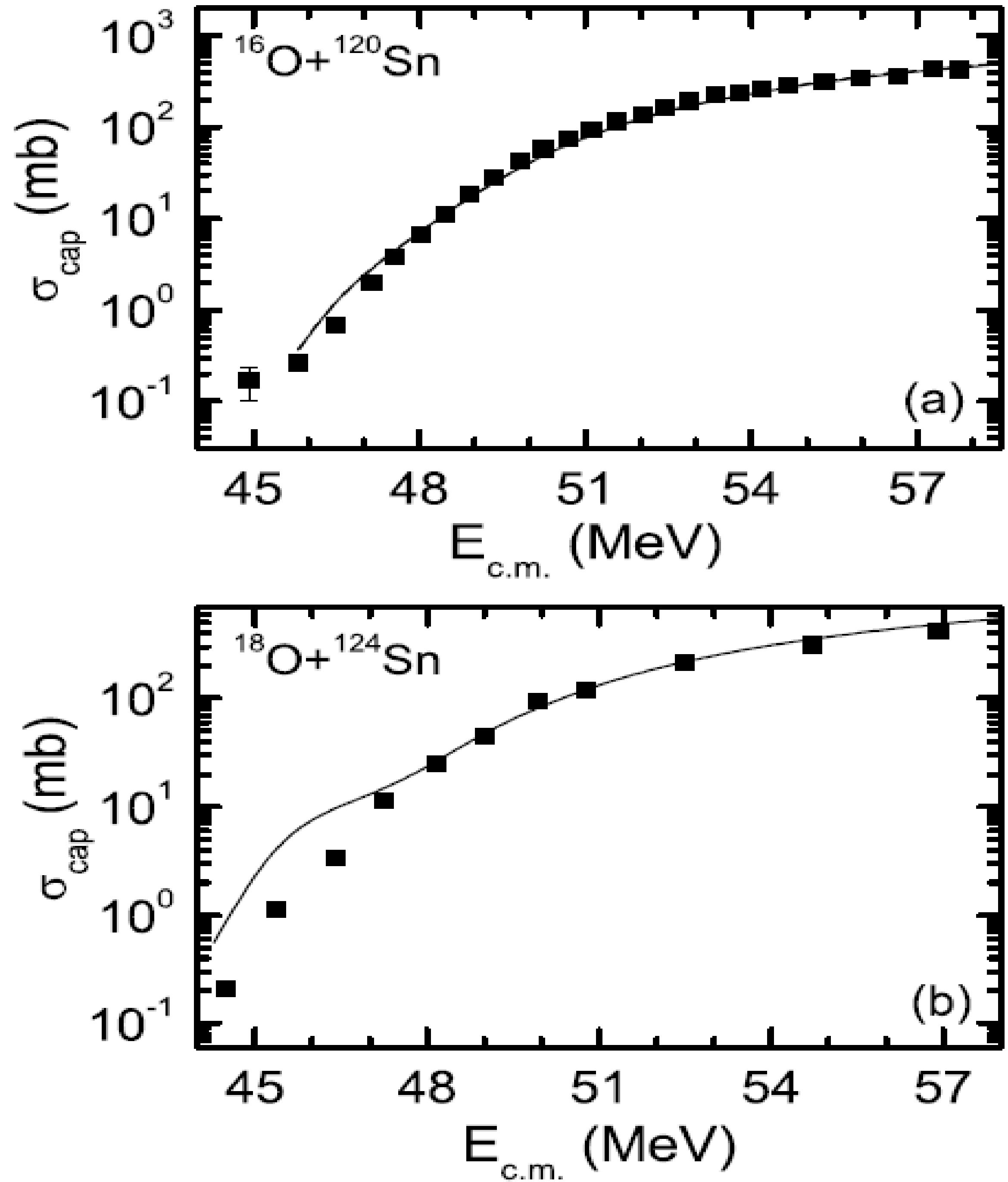


^{58}Ni+^{58}Ni
(a)
^{64}Ni+^{74}Ge
(b)
σ^{cc}_{cap} (mb), σ_{cap} (mb)
E (MeV)

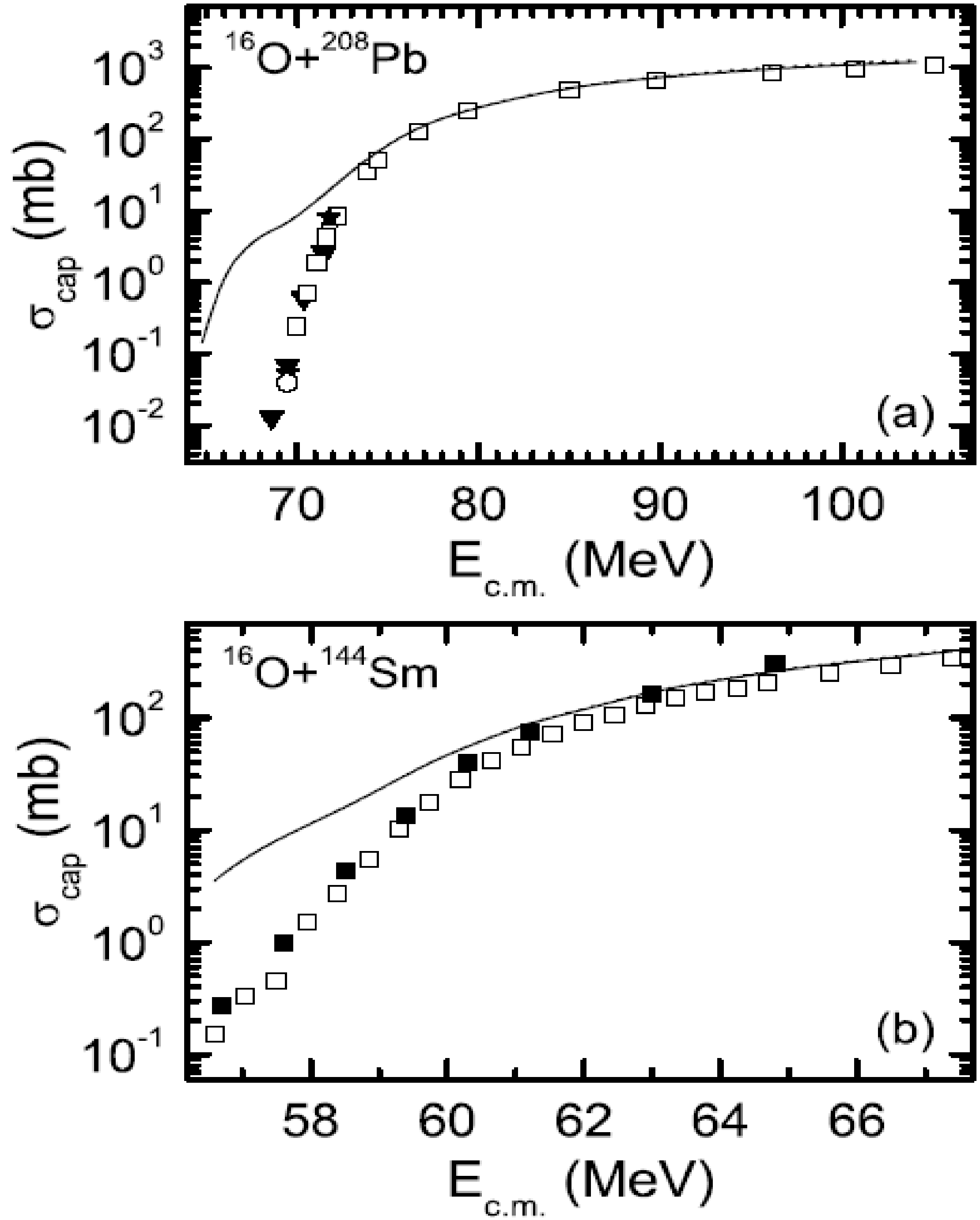


^{16}O+^{144}Sm
(a)
^{16}O+^{154}Sm
(b)
σ^{cc}_{cap} (mb), σ_{cap} (mb)
E (MeV)

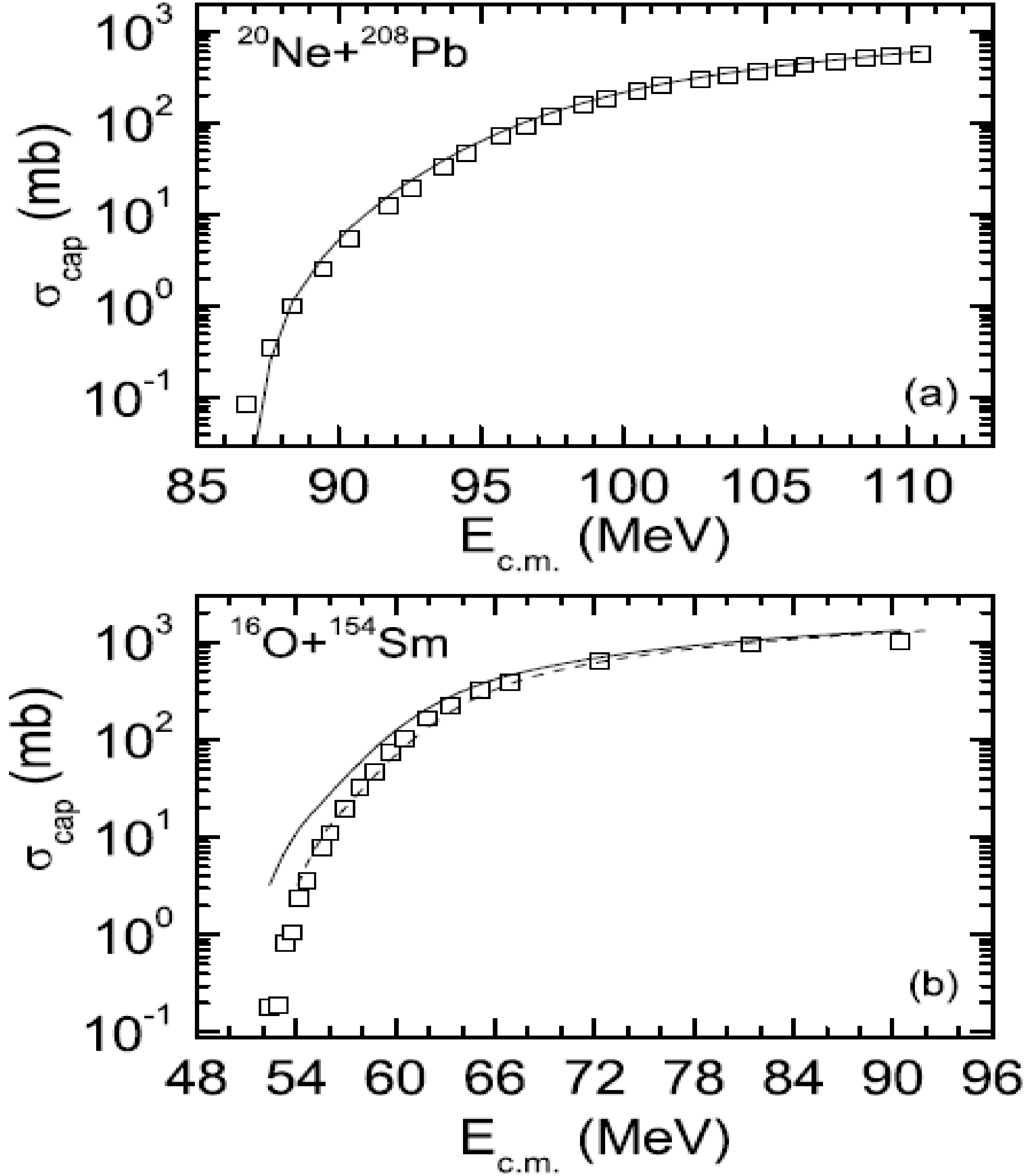

$E_{c.m.}$ = 58.8 MeV
$^{16}O+^{152}Sm$
$E_{c.m.}$ = 72.4 MeV
(a)
$E_{c.m.}$ = 63.3 MeV
(b)
$E_{c.m.}$ = 58.8 MeV
(c)
$\sigma_{cap}(E_{c.m.},J)$ (mb)
J
0
6
12
18
24
30
36
42
48
54
4
8
16
1
2
3



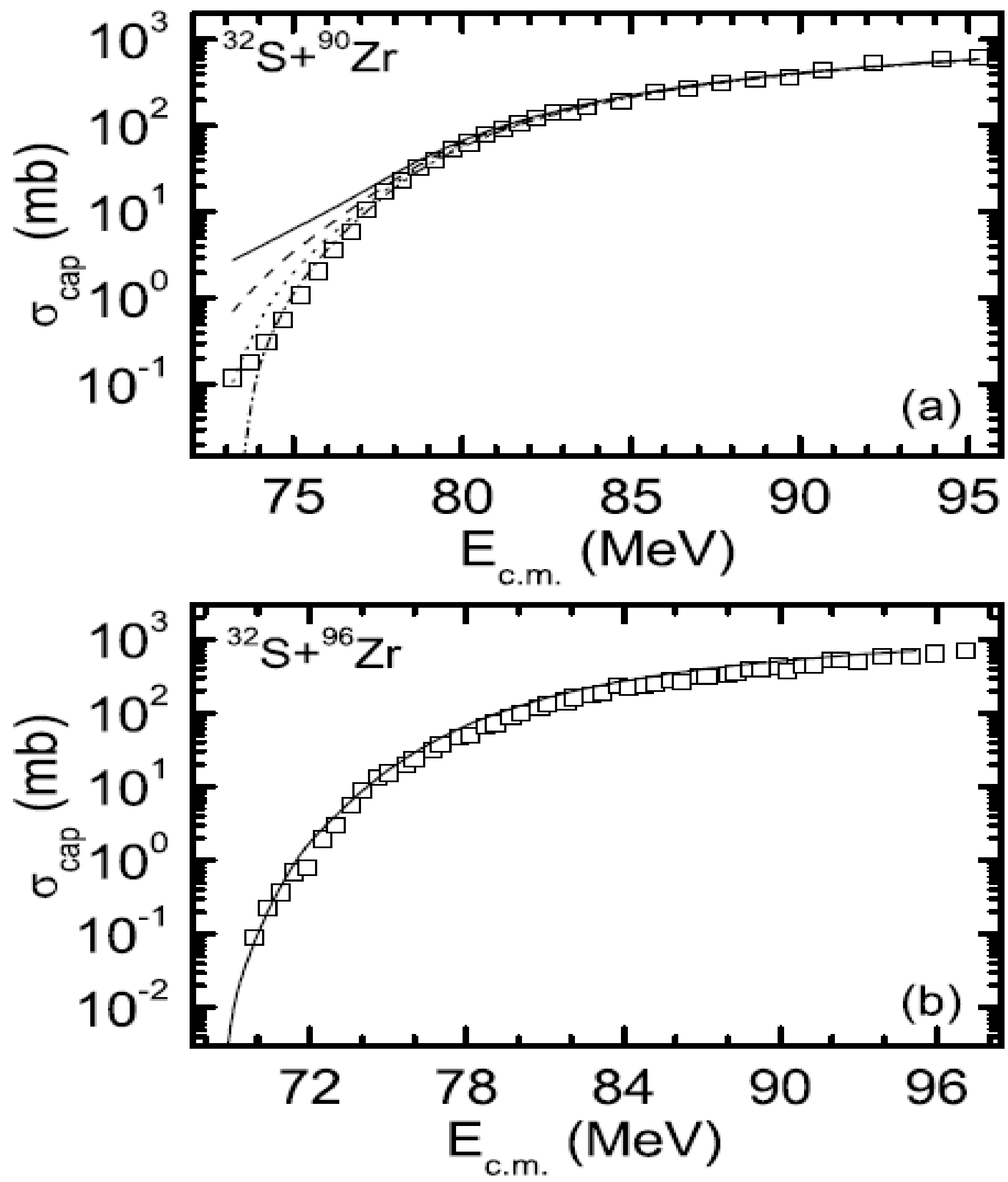
^{16}O+^{120}Sn
(a)
σ_{cap} (mb)
$E_{c.m.}$ (MeV)
^{18}O+^{124}Sn
(b)
σ_{cap} (mb)
$E_{c.m.}$ (MeV)



^{16}O+^{208}Pb
(a)
σ_{cap} (mb)
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
70
80
90
100
$E_{c.m.}$ (MeV)
^{16}O+^{144}Sm
(b)
σ_{cap} (mb)
10^{-1}
10^{0}
10^{1}
10^{2}
58
60
62
64
66
$E_{c.m.}$ (MeV)



²⁰Ne+²⁰⁸Pb
(a)
σcap (mb)
Ecm (MeV)
¹⁶O+¹⁵⁴Sm
(b)
σcap (mb)
Ecm (MeV)



^{32}S+^{90}Zr
(a)
σ_{cap} (mb)
$E_{c.m.}$ (MeV)
^{32}S+^{96}Zr
(b)
σ_{cap} (mb)
$E_{c.m.}$ (MeV)

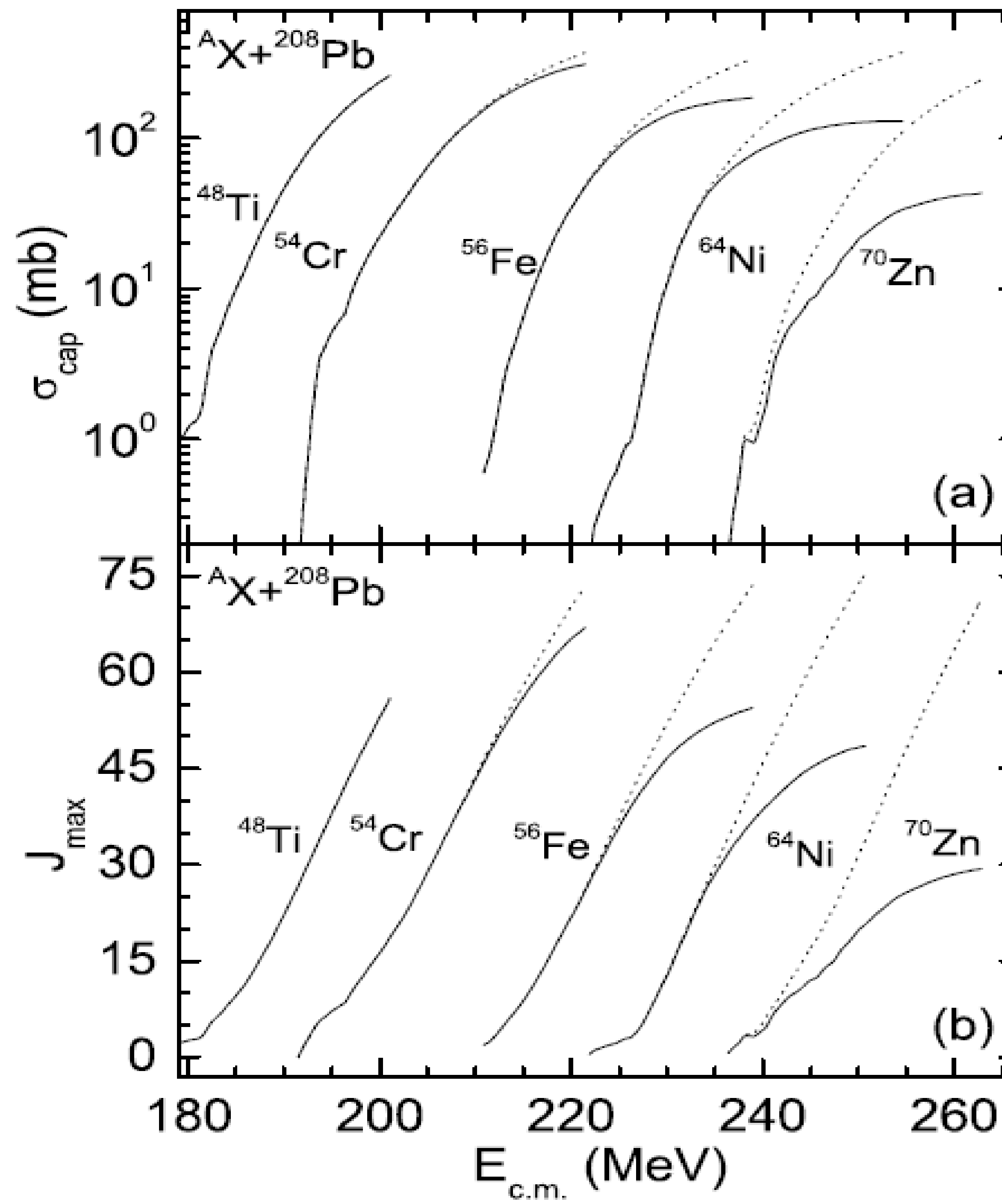
For systems with $Z_1*Z_2<2000$, factors

$$J_{crit} \quad \textbf{and} \quad [1-\frac{4(E_{c.m.}-E)}{\mu\omega_b^2 R_b^2}]$$

weakly influence the results of calculations.

Capture cross section is model independent :

$$\sigma_{cap}(E_{c.m.})=\frac{\pi R_b^2}{E_{c.m.}}\int_0^{E_{c.m.}} dE[1-P_{qe}^{ex}(E,J=0)]$$



$^{A}X+^{208}Pb$
σ_{cap} (mb)
10^2
10^1
10^0
^{48}Ti
^{54}Cr
^{56}Fe
^{64}Ni
^{70}Zn
(a)
$^{A}X+^{208}Pb$
J_{max}
75
60
45
30
15
0
^{48}Ti
^{54}Cr
^{56}Fe
^{64}Ni
^{70}Zn
(b)
180
200
220
240
260
$E_{c.m.}$ (MeV)

Average angular momentum of captured system:

$$< J > = \frac{\pi R_b^2}{E_{c.m.}\sigma_{cap}(E_{c.m.})} *$$

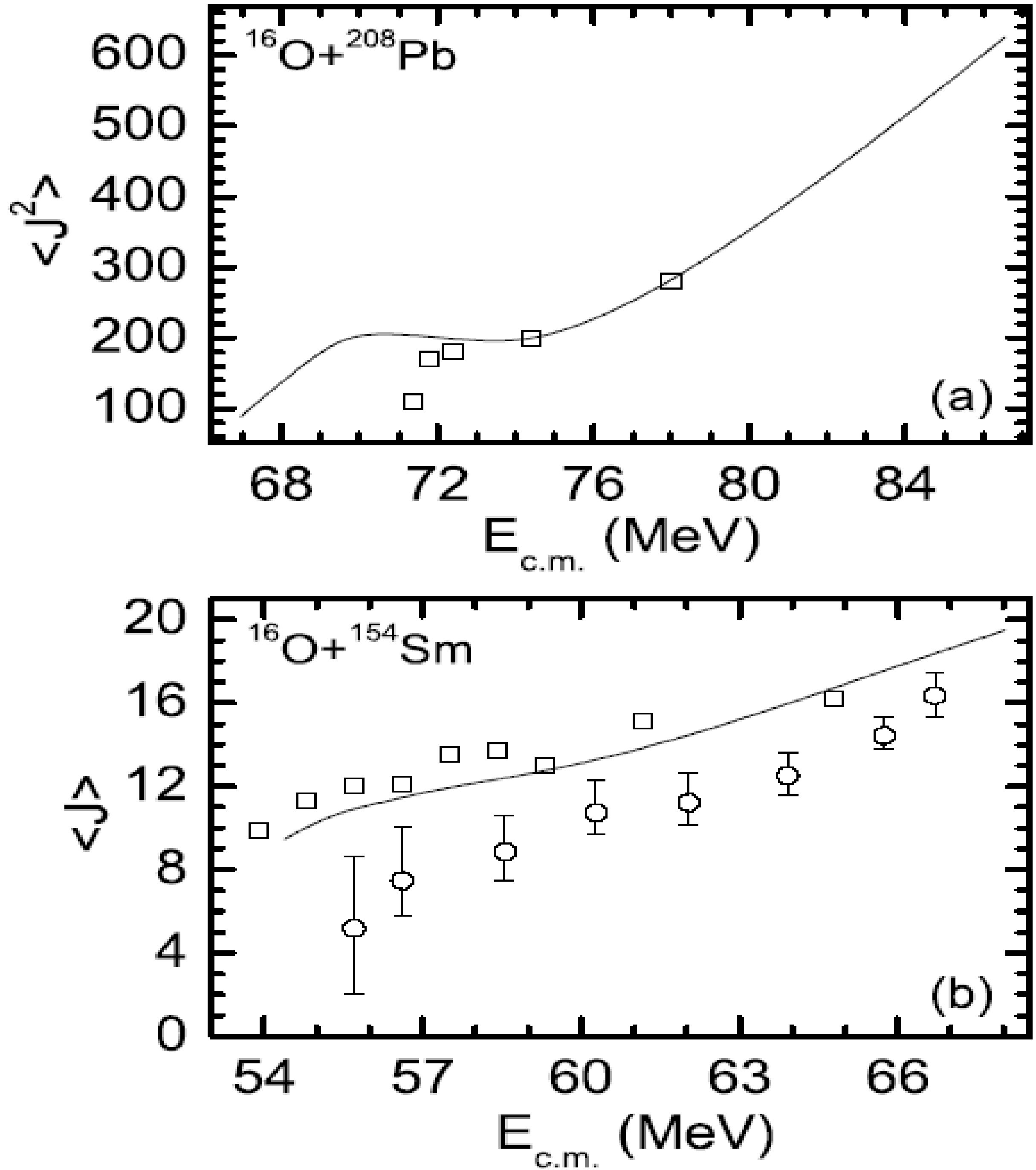
$$* \int_{E_{c.m.}-E_b^{rot}}^{E_{c.m.}} dE [1 - P_{qe}^{ex}(E, J=0)][1 - \frac{4(E_{c.m.} - E)}{\mu \omega_b^2 R_b^2}] *$$

$$* [(\frac{2\mu R_b^2}{\hbar^2}(E_{c.m.} - E) + \frac{1}{4})^{1/2} - \frac{1}{2}]$$

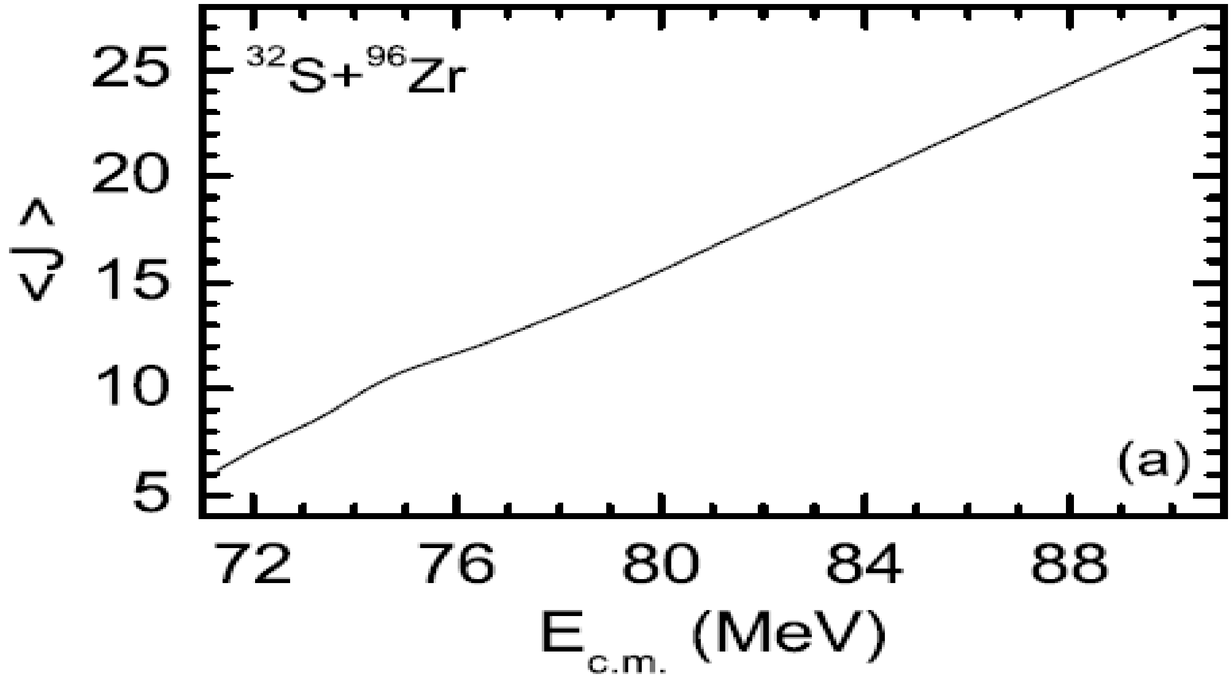
Second moment of angular momentum of captured system:

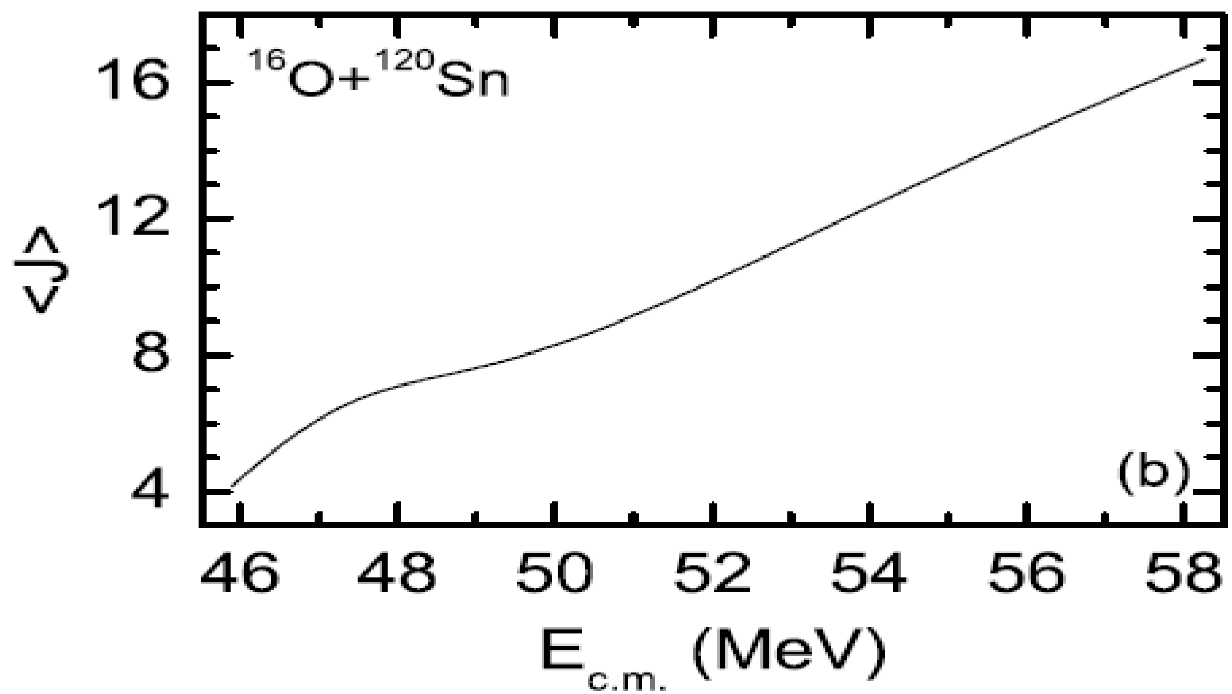
$$< J(J+1) > = \frac{2\pi\mu R_b^4}{\hbar^2 E_{c.m.} \sigma_{cap}(E_{c.m.})} *$$

$$\int_{E_{c.m.}-E_b^{rot}}^{E_{c.m.}} dE [1 - P_{qe}^{ex}(E, J=0)][1 - \frac{4(E_{c.m.} - E)}{\mu \omega_b^2 R_b^2}][E_{c.m.} - E]$$



16O+208Pb
(a)
<J2>
600
500
400
300
200
100
68
72
76
80
84
Ec.m. (MeV)
16O+154Sm
(b)
<J>
20
16
12
8
4
0
54
57
60
63
66
Ec.m. (MeV)


^{32}S+^{96}Zr
(a)
<J>
25
20
15
10
5
72
76
80
84
88
E$_{c.m.}$ (MeV)


^{16}O+^{120}Sn
(b)
<J>
16
12
8
4
46
48
50
52
54
56
58
E$_{c.m.}$ (MeV)

Weakly bound nuclei

Conservation of reaction flux:

$$P_{qe}(E_{c.m.}, J) + P_{cap}(E_{c.m.}, J) + P_{BU}(E_{c.m.}, J) = 1$$

P_{BU} **- breakup probability**

$$P_{cap}(E_{c.m.}, J=0) = 1 - P_{qe}^{ex}(E_{c.m.}, J=0) - P_{BU}^{ex}(E_{c.m.}, J=0)$$

$$P_{BU}^{ex}(E_{c.m.}, J=0) = \sigma_{BU}(E_{c.m.}, \theta=180^{o}) / \sigma_{Ru}(E_{c.m.}, \theta=180^{o})$$

the ratio of breakup differential cross section and Rutherford differential cross section at 180 degrees. This ratio can be measured!

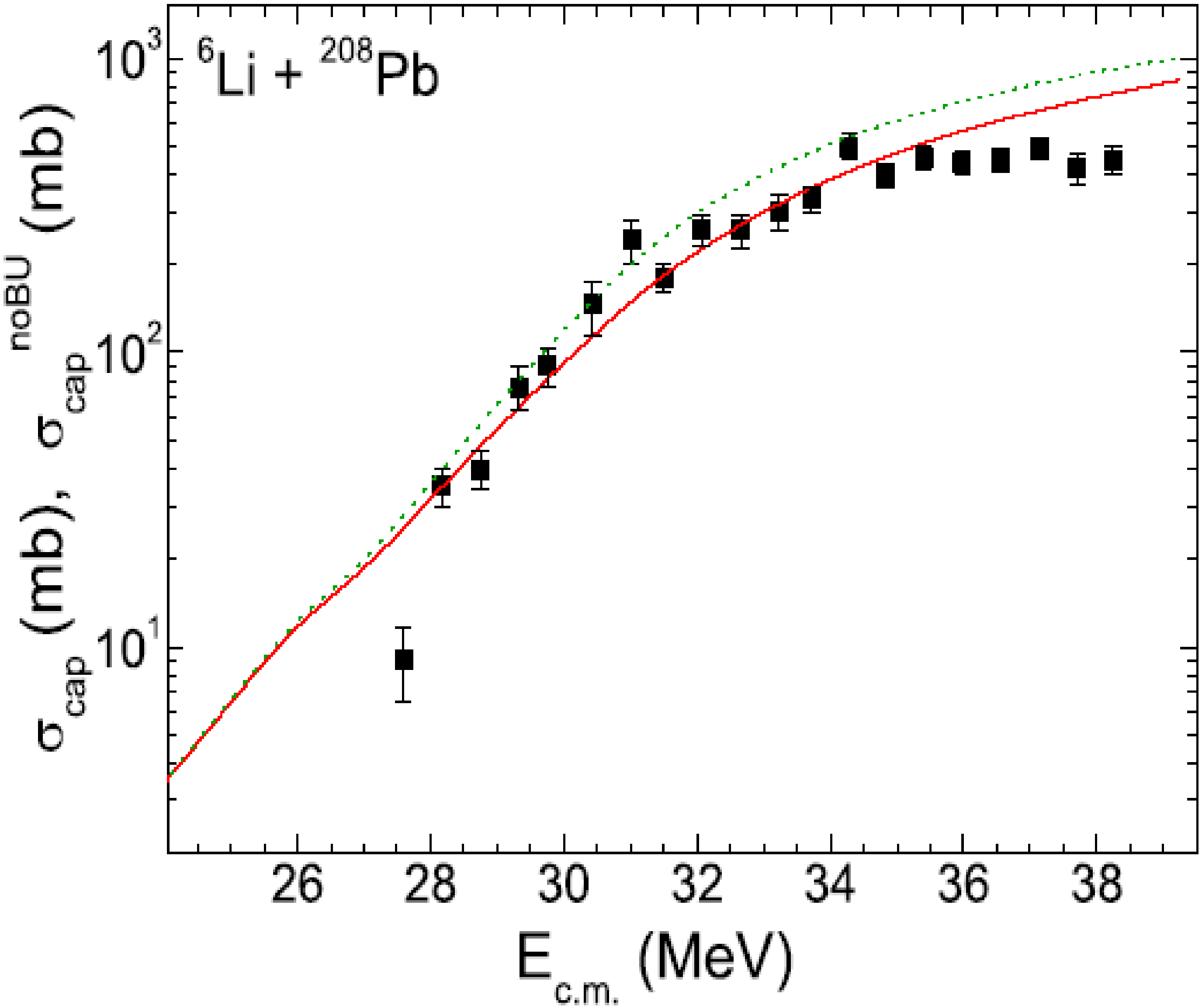
Capture cross section:

$$\sigma_{cap}(E_{c.m.})=$$
$$\frac{\pi R_b^2}{E_{c.m.}}\int_{E_{c.m.}-E_b^{rot}}^{E_{c.m.}} dE\, P_{cap}(E, J=0)[1-\frac{4(E_{c.m.}-E)}{\mu\omega_b^2 R_b^2}]$$

Capture probability in absence of breakup:

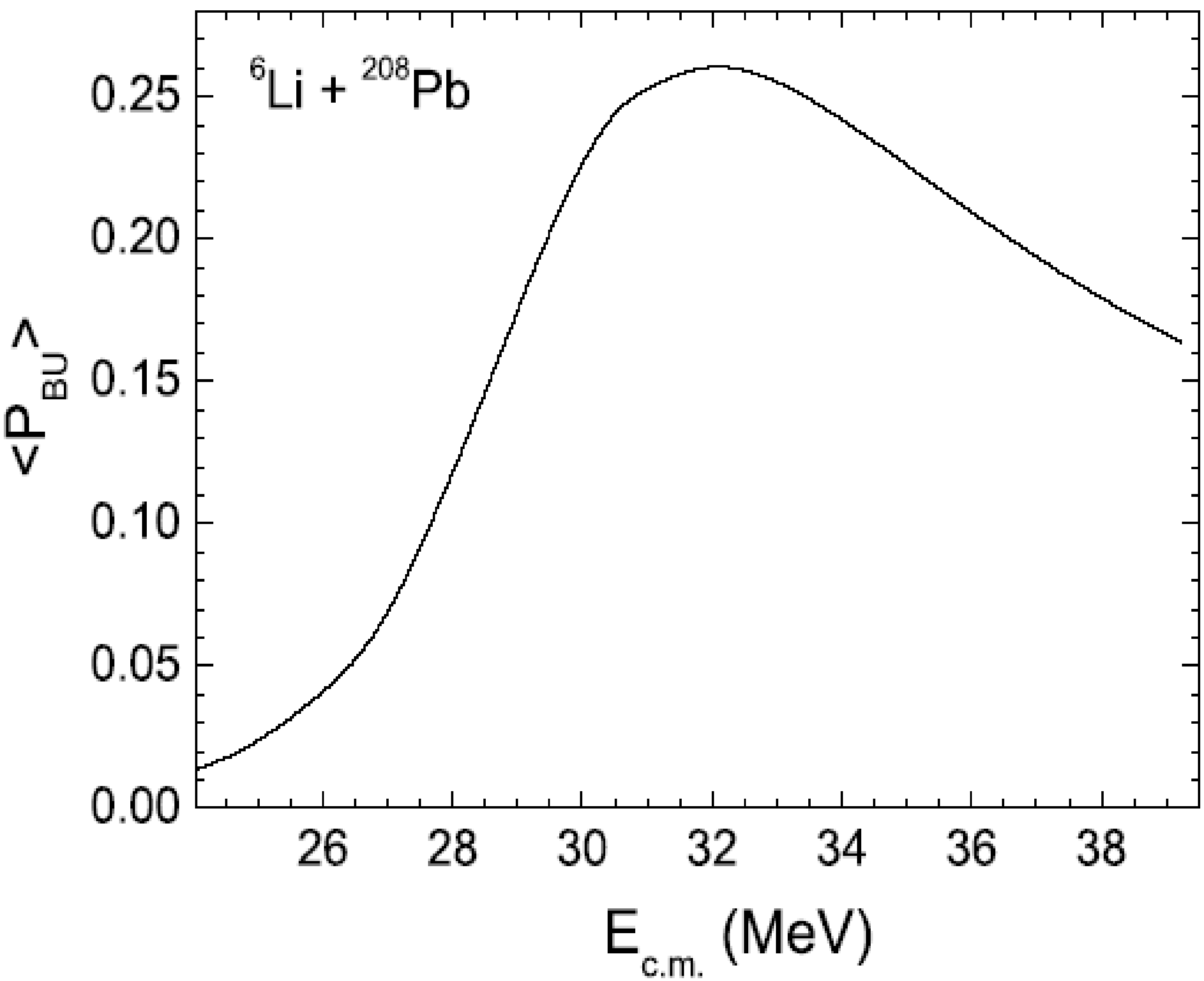
$$P_{cap}^{noBU}(E_{c.m.}, J=0)=1-\frac{P_{qe}^{ex}(E_{c.m.}, J=0)}{1-P_{BU}^{ex}(E_{c.m.}, J=0)}$$

Capture cross section in absence of breakup: $\sigma_{cap}^{noBU}(E_{c.m.})$



^{6}Li + ^{208}Pb
σ_{cap} (mb), σ_{cap}^{noBU} (mb)
$E_{c.m.}$ (MeV)
10^3
10^2
10^1
26
28
30
32
34
36
38

$$\langle P_{BU} \rangle (E_{c.m.}) = 1 - \frac{\sigma_{cap}(E_{c.m.})}{\sigma_{cap}^{noBU}(E_{c.m.})}$$



3. Elastic scattering and reaction cross section

Conservation of reaction flux:

$$P_{el}(E_{c.m.}, J=0)+P_R(E_{c.m.}, J=0)=1$$

$P_R = P_{cap} + P_{in} + P_{tr} + P_{BU}$ **- reaction probability**

P_{el} **- elastic probability measured at backward angle**

$P_{cap} - capture\ probability$

$P_{in} - inelastic\ probability$

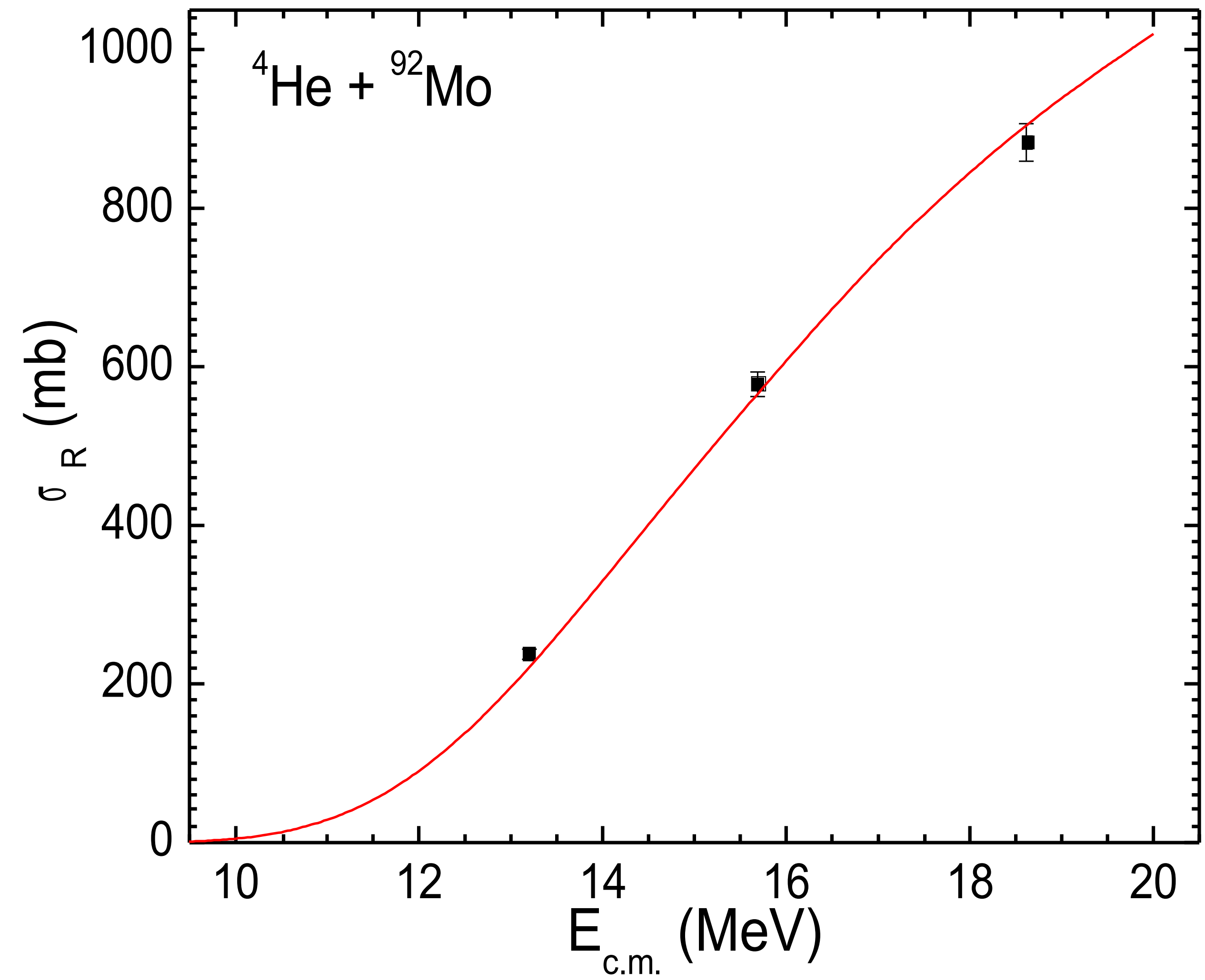
$P_{tr} - transfer\ probability$

$P_{BU} - breakup\ probability$

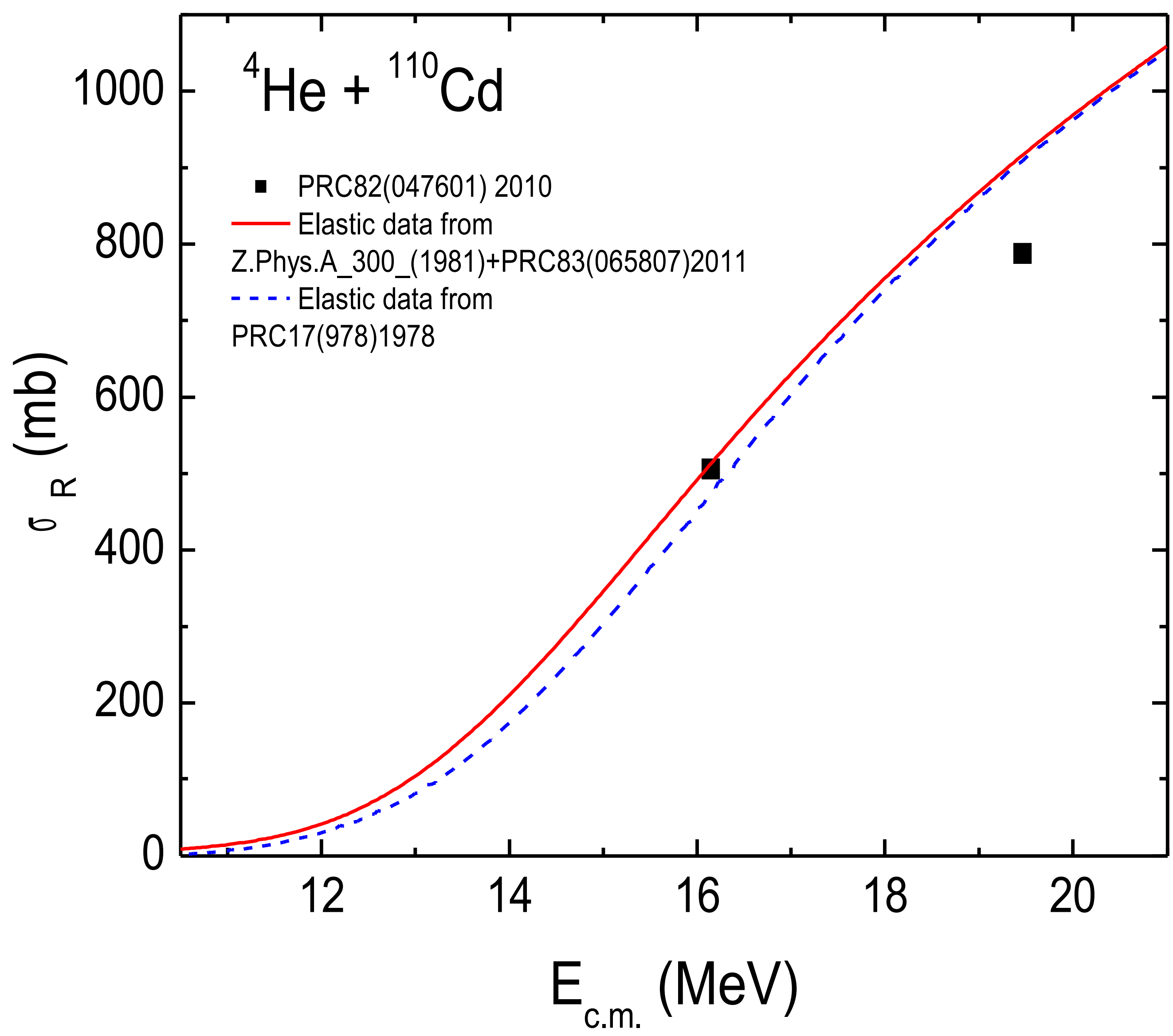
Reaction cross section :

$$\sigma_R(E_{c.m.}) =$$

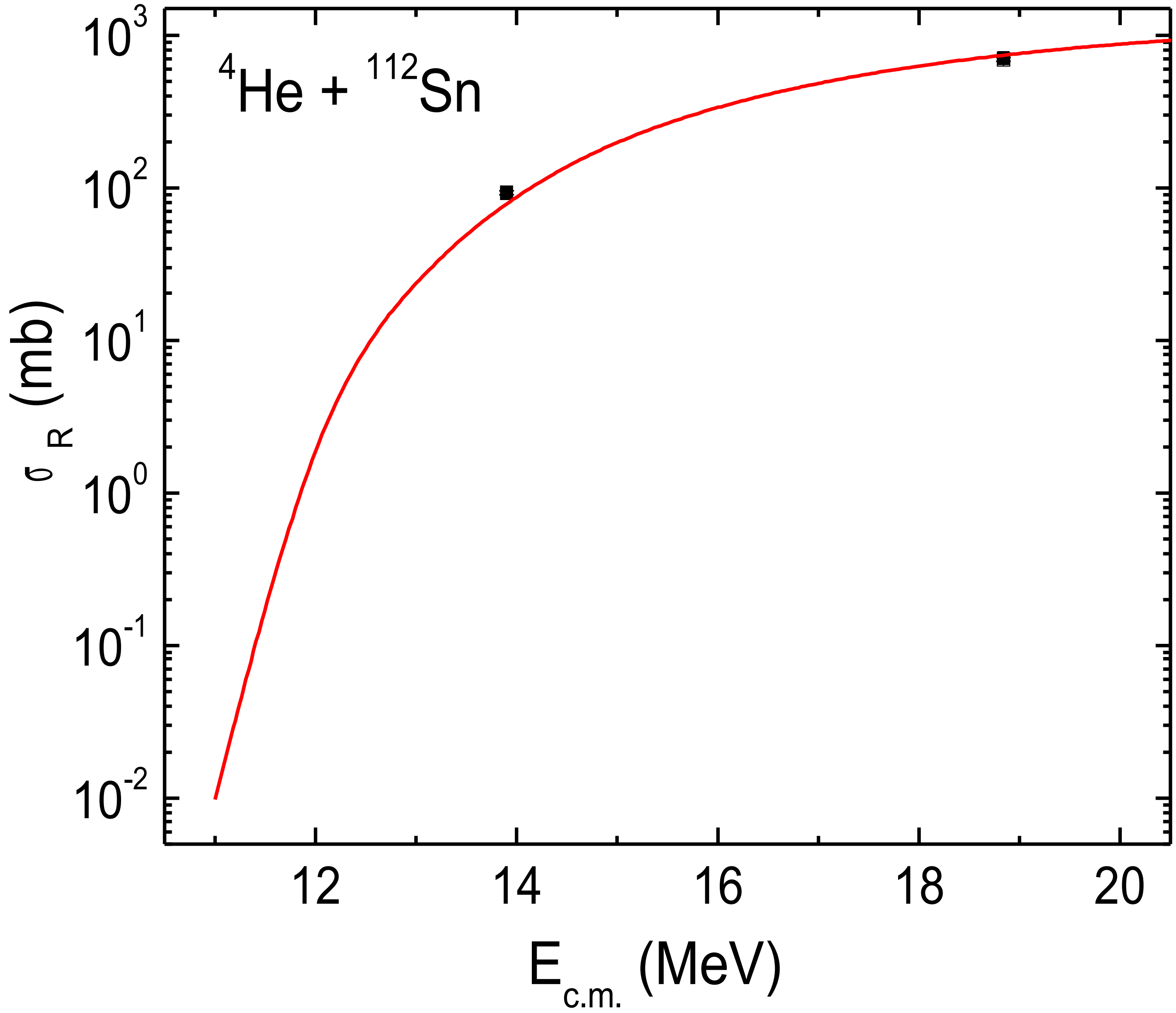
$$= \frac{\pi R_b^2}{E_{c.m.}} \int_0^{E_{c.m.}} dE \left[1 - P_{el}^{ex}(E, J=0)\right]\left[1 - \frac{4(E_{c.m.} - E)}{\mu \omega_b^2 R_b^2}\right]$$



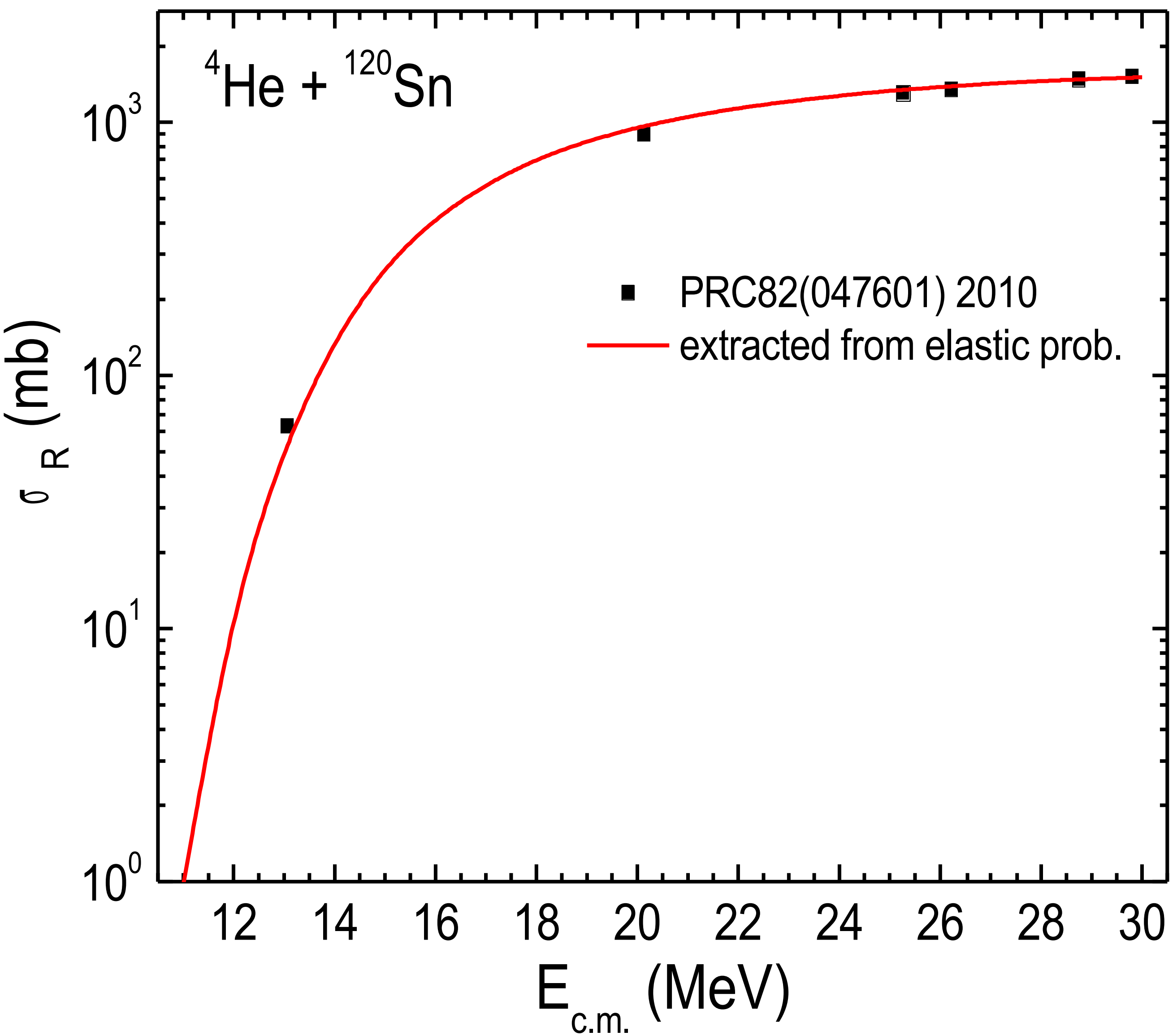
^{4}He + ^{92}Mo
σ_R (mb)
$E_{c.m.}$ (MeV)
0
200
400
600
800
1000
10
12
14
16
18
20



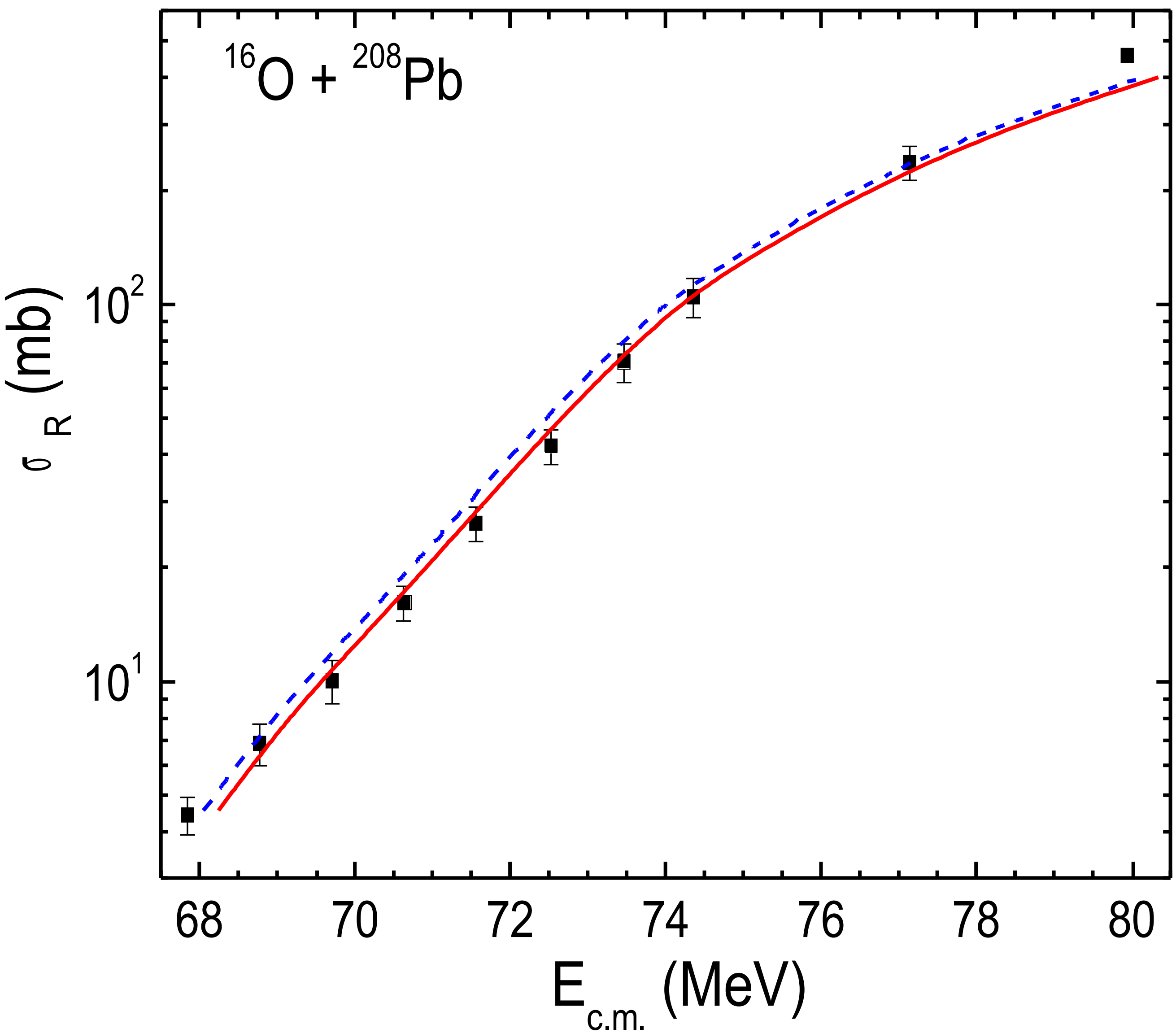
^{4}He + ^{110}Cd
PRC82(047601) 2010
Elastic data from
Z.Phys.A_300_(1981)+PRC83(065807)2011
Elastic data from
PRC17(978)1978
σ_R (mb)
0
200
400
600
800
1000
12
14
16
18
20
$E_{c.m.}$ (MeV)



^{4}He + ^{112}Sn
10^3
10^2
10^1
10^0
10^{-1}
10^{-2}
σ_R (mb)
12
14
16
18
20
$E_{c.m.}$ (MeV)

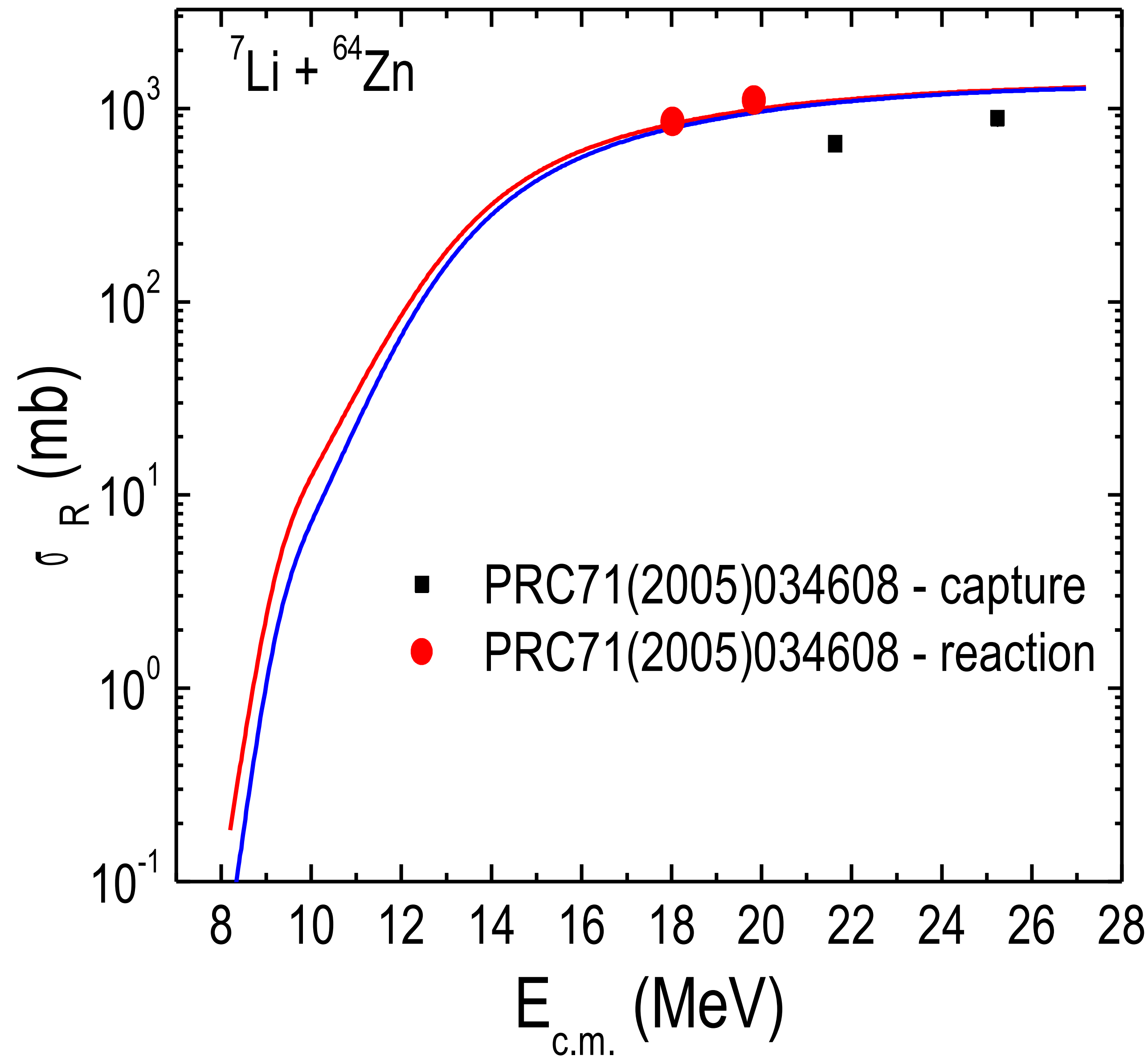


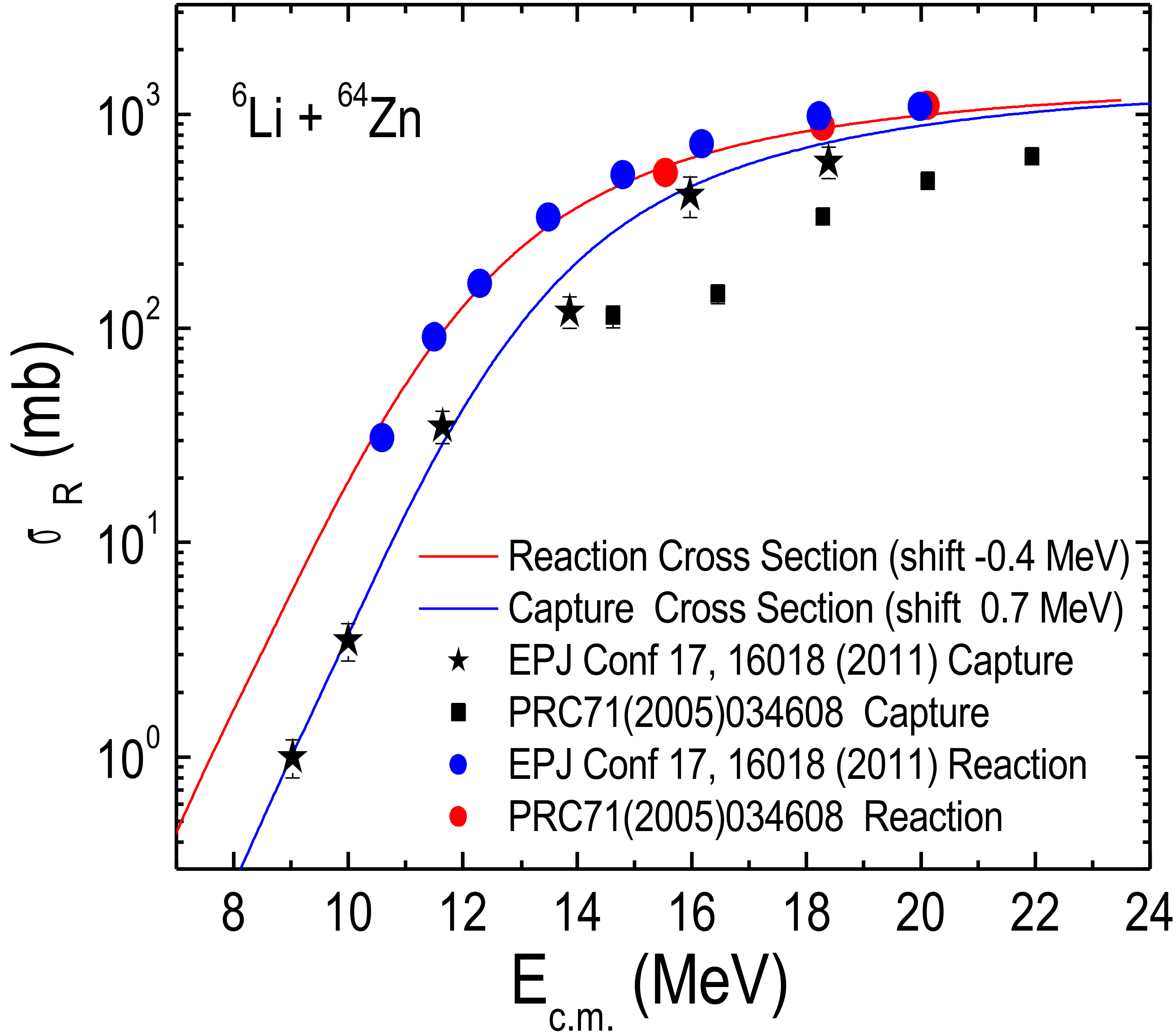
^{4}He + ^{120}Sn
PRC82(047601) 2010
extracted from elastic prob.
σ_R (mb)
$E_{c.m.}$ (MeV)
10^0
10^1
10^2
10^3
12
14
16
18
20
22
24
26
28
30



16O + 208Pb
σ R (mb)
10^2
10^1
68
70
72
74
76
78
80
E c.m. (MeV)

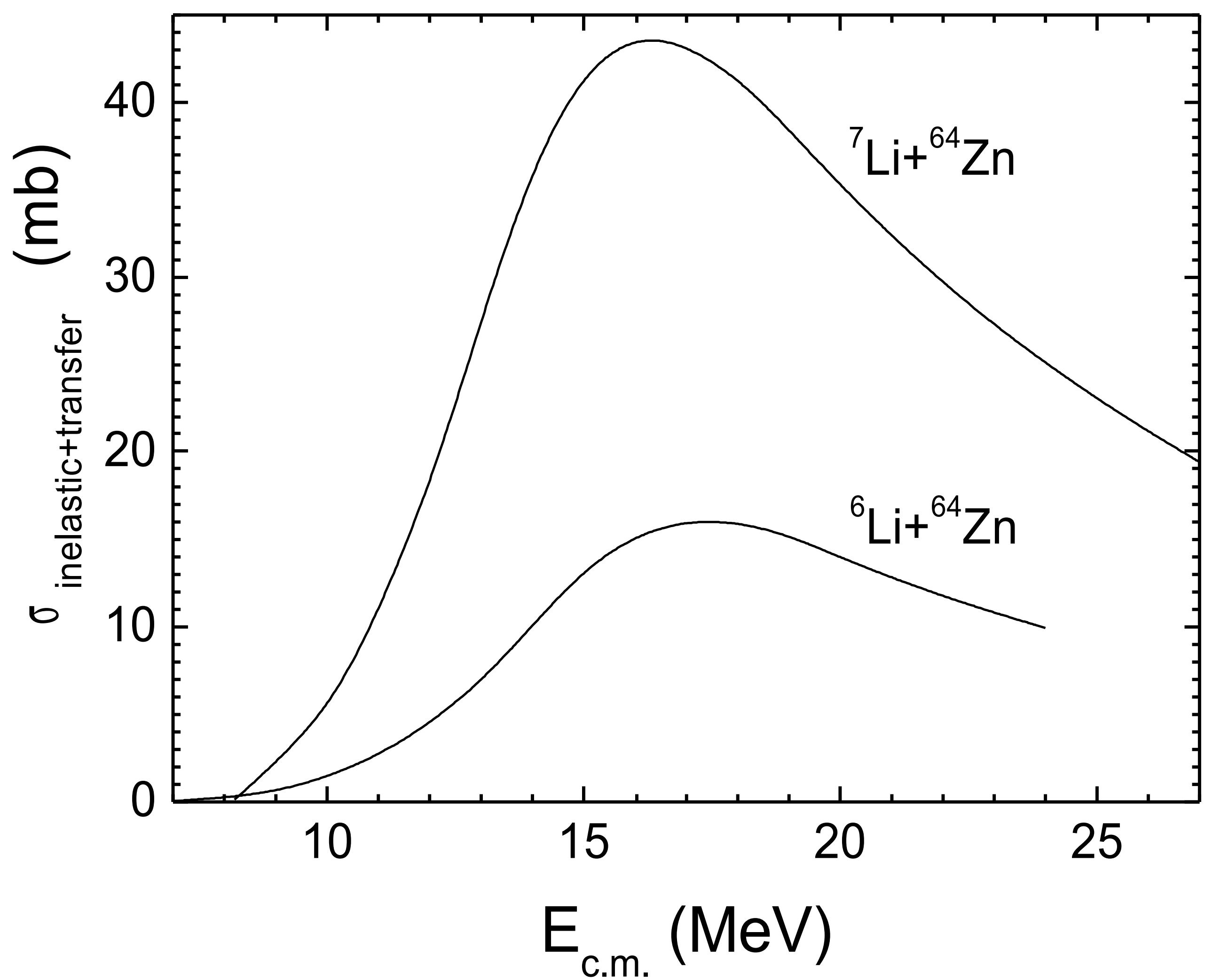
Weakly bound nuclei





6Li + 64Zn
σ R (mb)
E c.m. (MeV)
10^0
10^1
10^2
10^3
8
10
12
14
16
18
20
22
24
Reaction Cross Section (shift -0.4 MeV)
Capture Cross Section (shift 0.7 MeV)
EPJ Conf 17, 16018 (2011) Capture
PRC71(2005)034608 Capture
EPJ Conf 17, 16018 (2011) Reaction
PRC71(2005)034608 Reaction

$$\sigma_R(E_{c.m.}) - \sigma_{cap}(E_{c.m.}) \approx \sigma_{tr}(E_{c.m.}) + \sigma_{in}(E_{c.m.})$$



4. CONCLUSIONS:

Relashionship between quasielastic (elastic) excitation function at one backward angle and capture (reaction) cross sections is working well.

Extraction of capture (reaction) cross sections from quasielastic (elastic) at backscattering is possible with reasonable uncertainties as long as the deviation between quasielastic (elastic) cross section and Rutherford cross section exceeds the experimental uncertainties significantly.

Quasielastic (elastic) technique at one backward angle could be important and simple tool in study of capture (reaction) cross sections.

THANK YOU FOR ATTENTION !

Measured quasi-elastic probability

$$P_{qe}^{ex} = \sigma_{qe}(E_{c.m.}, \theta = 170^o) / \sigma_{Ru}(E_{c.m.}, \theta = 170^o)$$

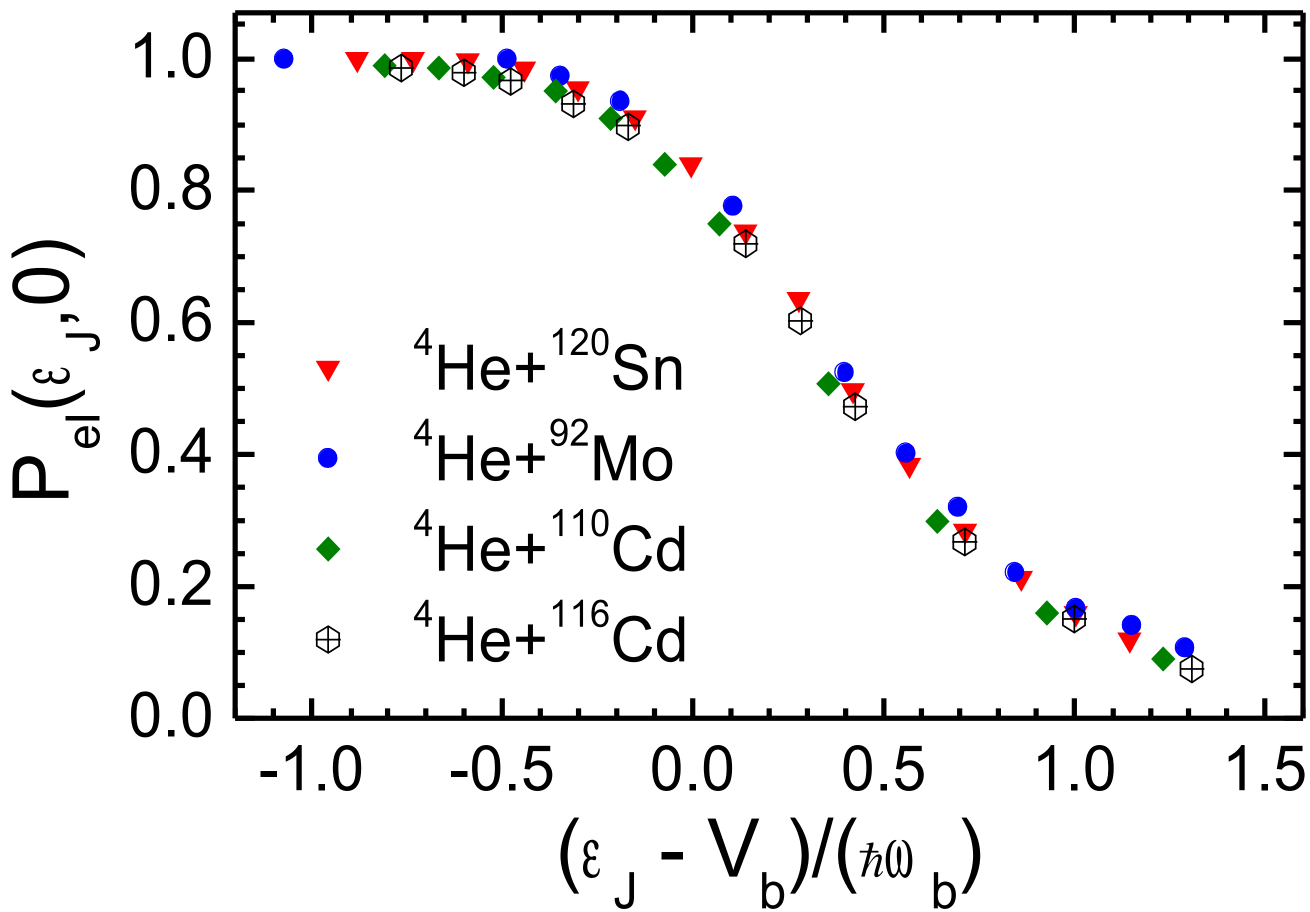
at $\theta = 170$ **degrees.**

At $\theta_0 = 180$ **degrees**

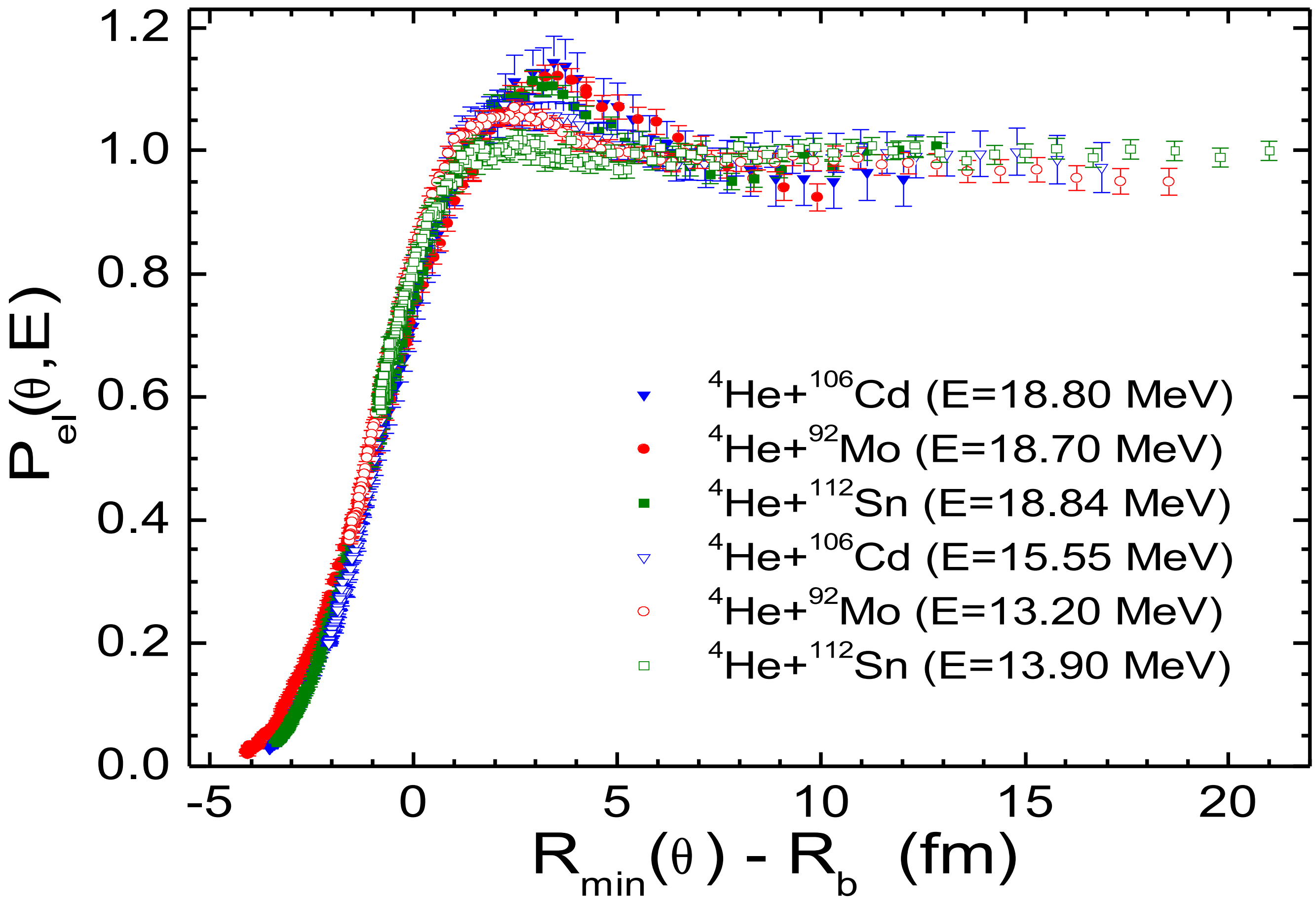
$$P_{qe}^{ex} = \sigma_{qe}(E_{c.m.}^{eff}, \theta = 170^o) / \sigma_{Ru}(E_{c.m.}^{eff}, \theta = 170^o)$$

with effective energy $E_{c.m.}^{eff} = 2 E_{c.m.} \frac{sin[\theta/2]}{1 + sin[\theta/2]},$

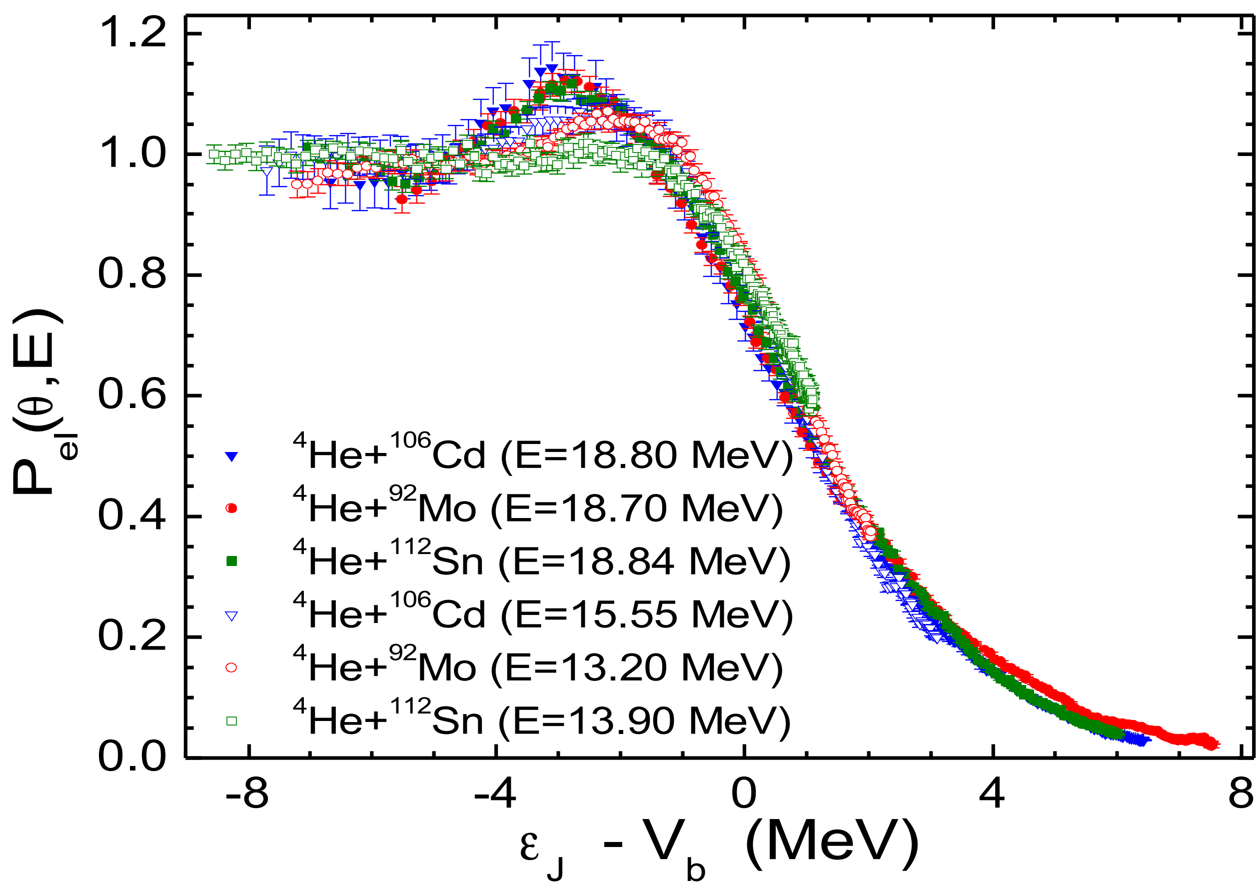
which takes into account the centrifugal energy for the Rutherford trajectory.



$$\epsilon_J = E_{c.m.} - E_R(J) = E_{c.m}[1 - sin(\theta_{c.m.}/2)]/[1 + sin(\theta_{c.m.}/2$$



1.2
1.0
0.8
0.6
0.4
0.2
0.0
-5
0
5
10
15
20
P_el(θ,E)
R_min(θ) - R_b (fm)
4He+106Cd (E=18.80 MeV)
4He+92Mo (E=18.70 MeV)
4He+112Sn (E=18.84 MeV)
4He+106Cd (E=15.55 MeV)
4He+92Mo (E=13.20 MeV)
4He+112Sn (E=13.90 MeV)



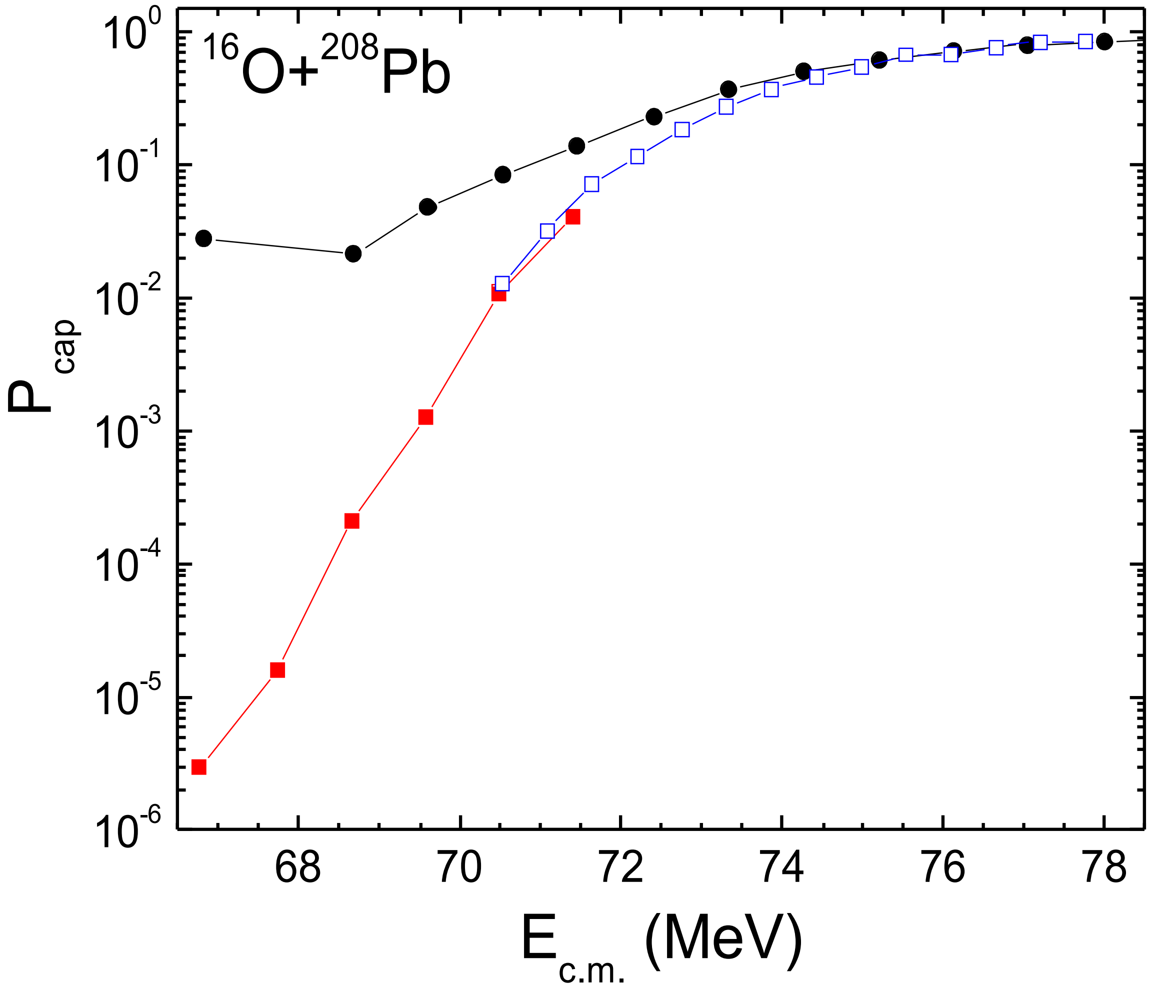
1.2
1.0
0.8
0.6
0.4
0.2
0.0
-8
-4
0
4
8
P_el(θ,E)
ε_J - V_b (MeV)
4He+106Cd (E=18.80 MeV)
4He+92Mo (E=18.70 MeV)
4He+112Sn (E=18.84 MeV)
4He+106Cd (E=15.55 MeV)
4He+92Mo (E=13.20 MeV)
4He+112Sn (E=13.90 MeV)

Fusion s-wave probabilities from experimental quasi-elastic back-scattering probabilities

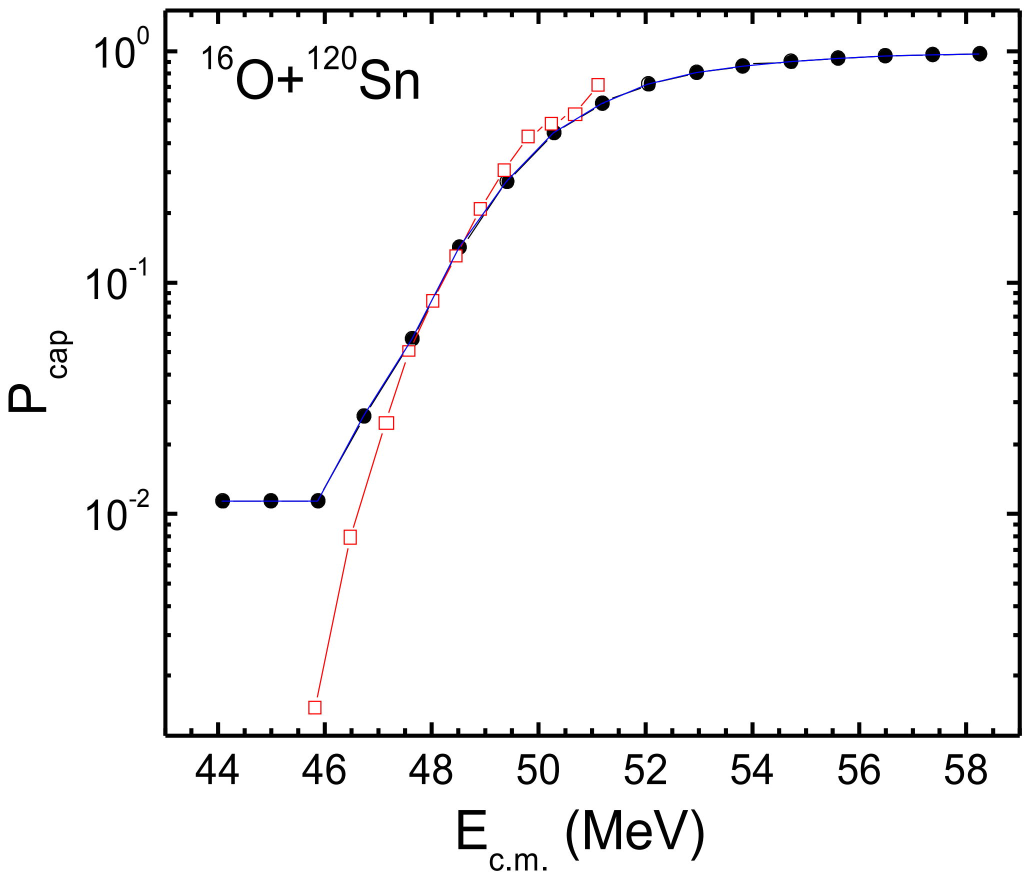
$$P_{fus}(E_{c.m.}, J=0) = \frac{1}{\pi R_b^2} \frac{d\left[E_{c.m.}\sigma_{fus}(E_{c.m.})\right]}{dE_{c.m.}}$$

Fusion s-wave probabilities from experimental fusion excitation function

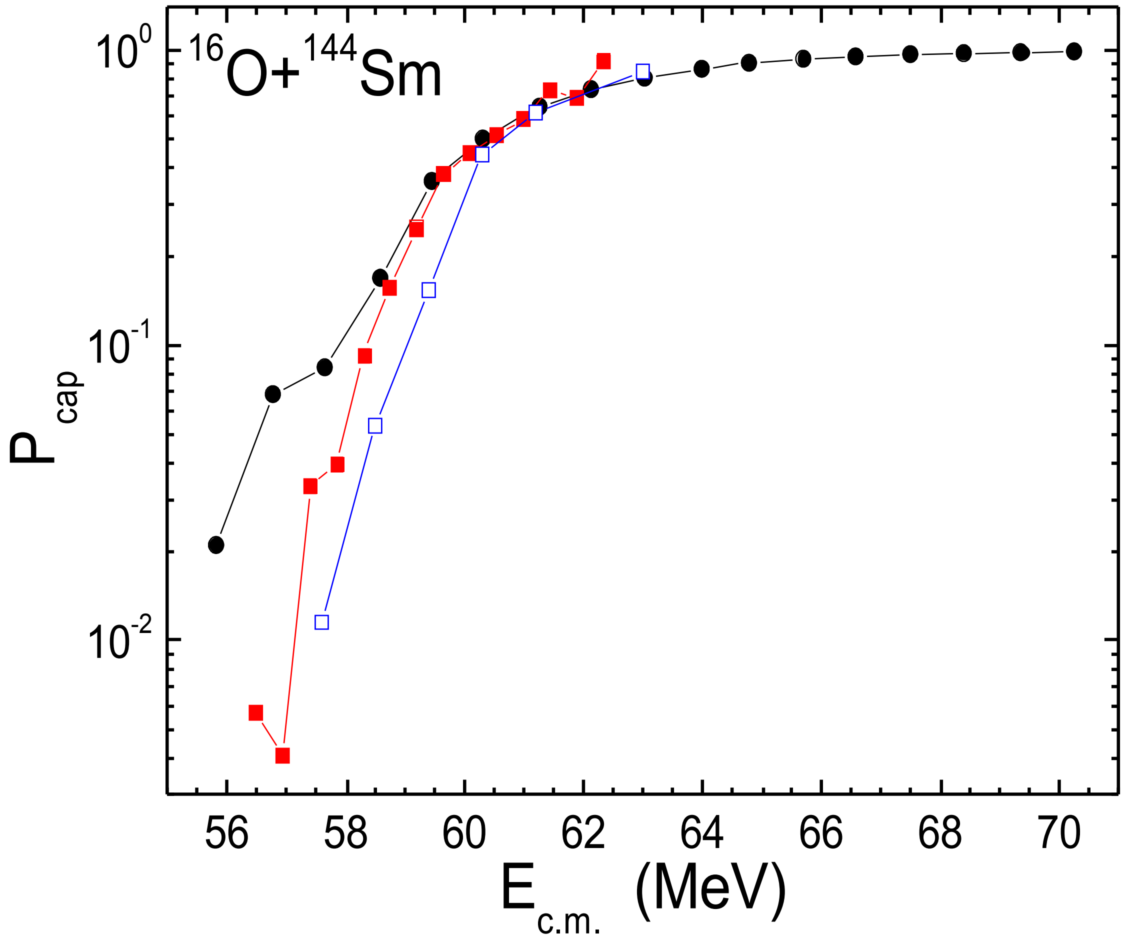
$$P_{fus}(E_{c.m.}, J=0) = 1 - P_{qe}^{ex}(E_{c.m.}, J=0)$$



$^{16}O+^{208}Pb$
P_{cap}
$E_{c.m.}$ (MeV)
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
68
70
72
74
76
78



^{16}O+^{120}Sn
10^0
10^{-1}
10^{-2}
P_{cap}
44
46
48
50
52
54
56
58
$E_{c.m.}$ (MeV)



^{16}O+^{144}Sm
10^{0}
10^{-1}
10^{-2}
P_{cap}
56
58
60
62
64
66
68
70
$E_{c.m.}$ (MeV)