Josephson junction detectors for Majorana and Dirac fermions

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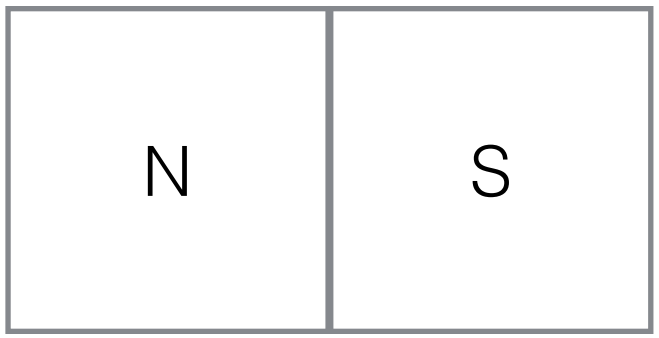
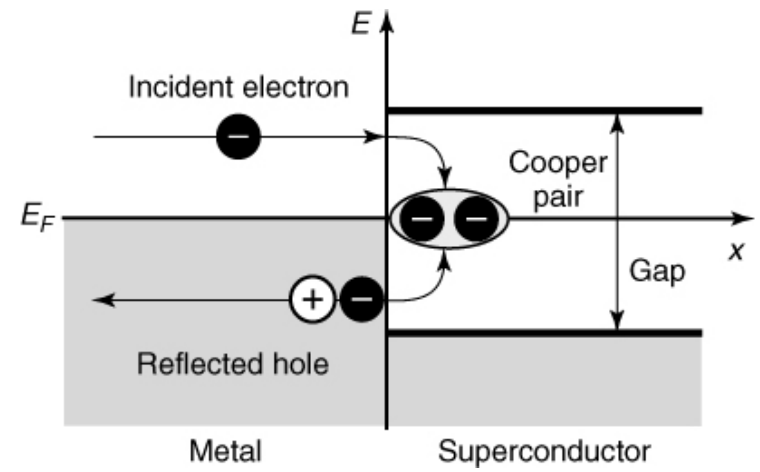
- *Pre-print in preparation*

Motivation

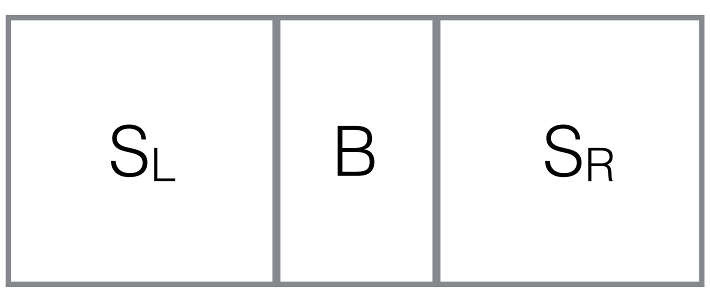
- Condensed matter systems whose low-energy quasiparticles obey Dirac-like equations -
- *Graphene*
  - ☆ *Novosolev et. al., Science (2004)*
  - ☆ *CWJ Beenakker RMP (2008), A Castro Neto et al., RMP (2009)*
- *Topological Insulators*
  - ☆ *Balents and Moore (2007), Fu and Kane (2007), Roy (2009)*
  - ☆ *MZ Hassan RMP (2010), Zhang et. al (2011)*
- Possibility of realisation of Majorana fermions in condensed matter systems -
- *1d chain of spineless fermions*
  - ☆ *Kitaev, Phys. Uspekhi (2001)*
- *1d nanowires*
  - ☆ *Lutchyn et. al PRL (2010)*
  - ☆ *Oreg et. al PRL (2010)*
- *Localised subgap states in superconductors ~ unconventional superconducting pairing symmetry ~ zero energy mid-gap states*
  - ☆ *Mourik et. al Science (2012)*
- Junctions of unconventional superconductors ~ *4 π periodic Josephson effect*
  - ☆ *Domnguez et. al. PRB (2012)*
  - ☆ *Houzet et. al PRL (2013)*

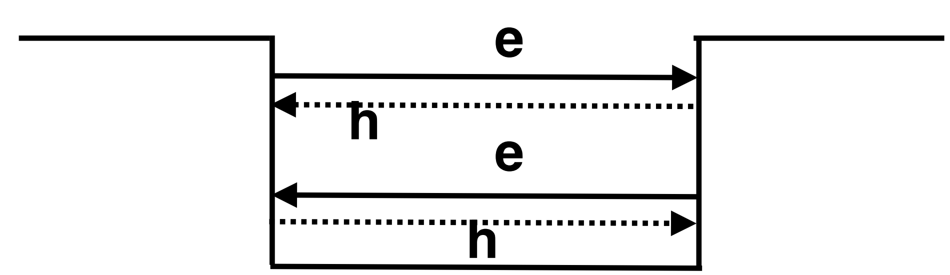
Andreev bound states in Josephson junctions





Andreev reflection at a normal-metal superconductor interface

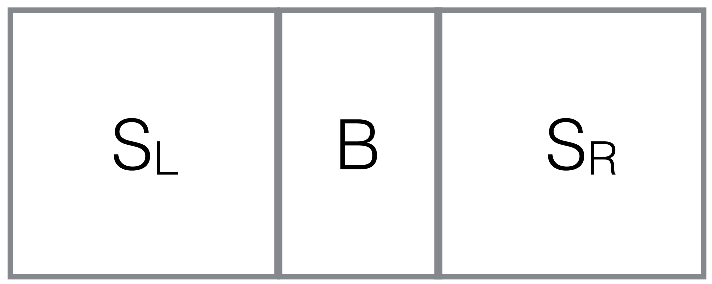


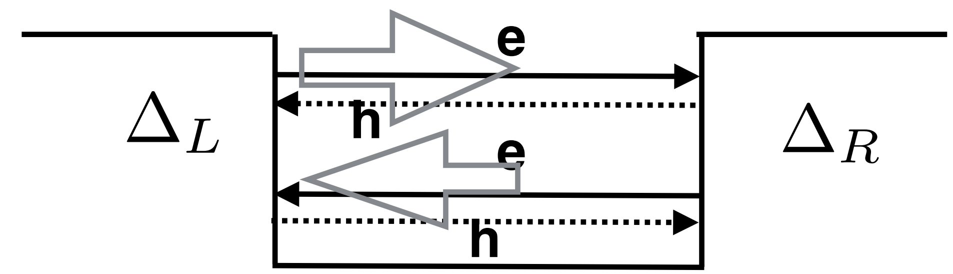


Andreev bound states

☆ *Zagoskin, Quantum theory of many body systems (1998)*

Andreev bound states in Josephson junctions





Andreev bound states

- *Solution of the Bogoliubov-de Gennes equation in the three different regions.*

$$((H_\beta + V(x))\tau_z + (\Delta_\beta(x)\tau_+ + h.c))\psi_\beta = E\psi_\beta$$

$$\psi_\beta = [\psi_{\beta\uparrow}, \psi^\dagger_{\beta\downarrow}]$$

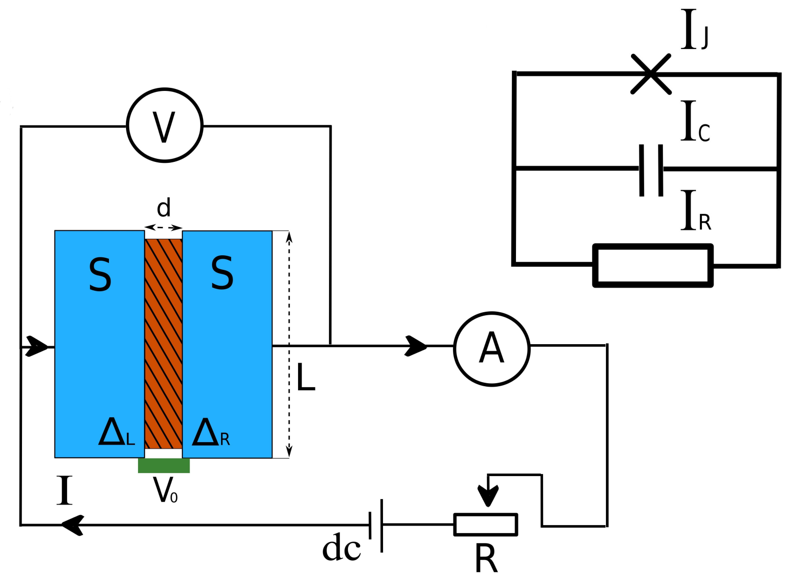
$$\beta = \mathrm{L, R}$$

- *Match the boundary conditions at the boundary between the regions S_L and B, B and S_R.*
- *Dispersion for the bound Andreev states, E(ϕ).*
- Superconducting current is given by:

$$I_J = \frac{2e}{\hbar}\sum_n \frac{\partial E_n(\phi)}{\partial \phi}$$

☆ *Zagoskin, Quantum theory of many body systems (1998)*

The RCSJ model



Schematic representation of the junction

Current-phase relation for the model in presence of an external radiation

$$\ddot{\phi} + \beta\dot{\phi} + I_J(\phi)/I_c = I/I_c + A/I_c \sin(\omega\tau)$$

$$\dot{\phi} = V \qquad \tau = \sqrt{\hbar C_0/(2eI_C)}$$

Dissipation parameter $\beta = \sqrt{\dfrac{\hbar}{2eR^2C_0}}$

$\beta \lessgtr 1 \Rightarrow$ *under-damped* (*over-damped*)

Junctions hosting Majorana quasi-particles

$$I_J = \frac{e\Delta_0}{\hbar}\sqrt{D}\sin(\phi/2)$$

dimensionless barrier strength

$$D = 1/(1 + (2V_0/\hbar v_F k_F)^2)$$

☆ *H-J K et al, EPJB, 37, 349 (2004)*

Junctions hosting Dirac quasi-particles

Thin barrier limit $V_0 \to \infty\ ,\ \ d \to 0$

$$\chi = V_0 d/\hbar v_F$$

$$I_J = g_0 \int_{-\pi/2}^{\pi/2} d\gamma \frac{\cos(\gamma)\sin(\phi)T(\gamma,\chi)}{\sqrt{1 - T(\gamma,\chi)\sin^2(\phi/2)}}$$

☆ *KS et. al PRL, 97, 217001 (2006),*

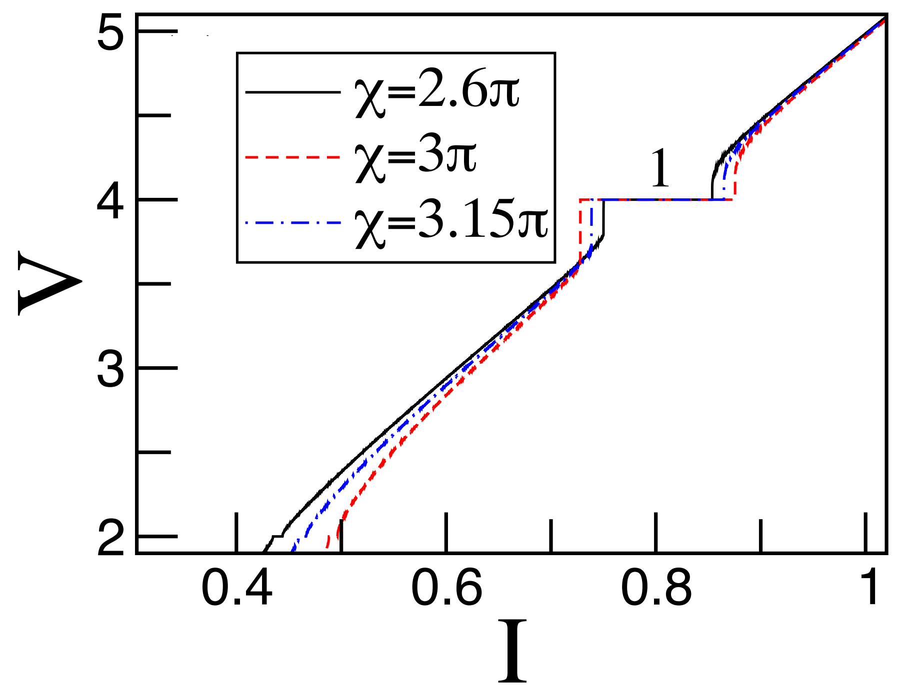
☆ *MM, KS PRB 76, 054513 (2007)*

Josephson junctions hosting Dirac particles

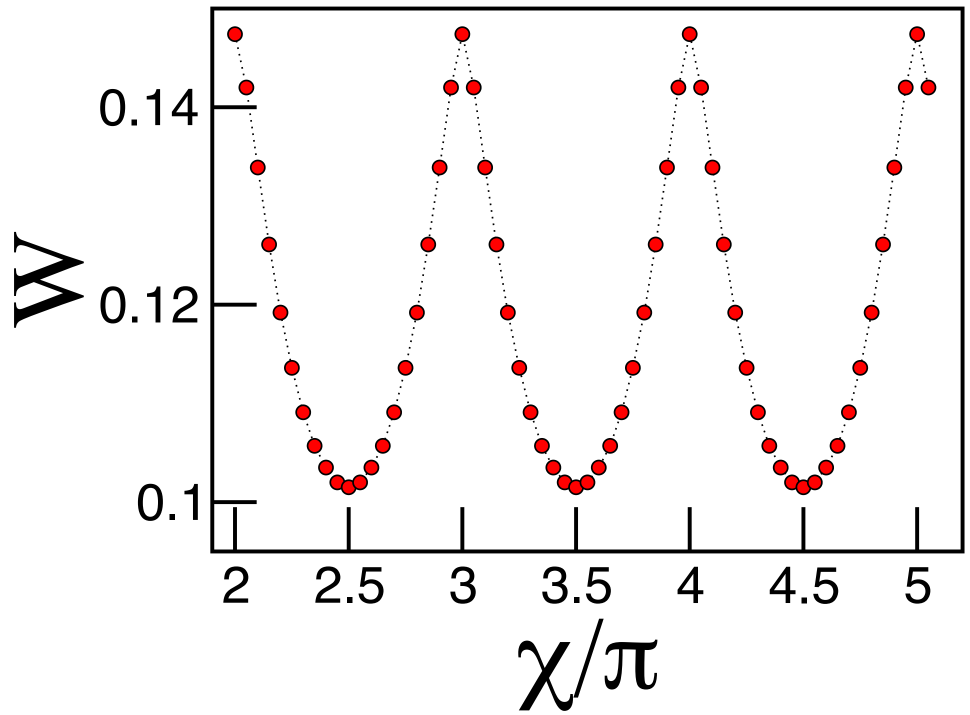
Thin barrier limit $V_0 \to \infty\ ,\ \ d \to 0$

$$\chi = V_0 d/\hbar v_F$$

Current-voltage characteristics of the Josephson junctions hosting Dirac particles:



Variation of the width of the Shapiro steps with the barrier strength:



$$W \sim I_J \sim T(\gamma,\chi) = \frac{\cos^2(\gamma)}{1-\cos^2(\chi)\sin^2(\gamma)}$$

☆ *Katnelson et al. Nat. Phys. 2, 620 (2006),*
☆ *KS et al PRB 76, 184514 (2007)*

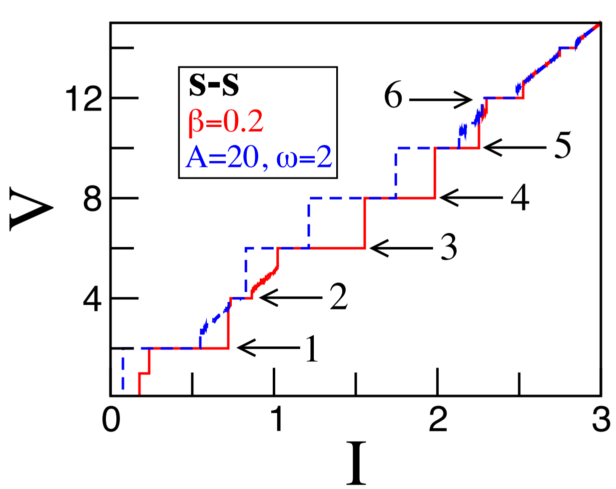
Conventional junctions

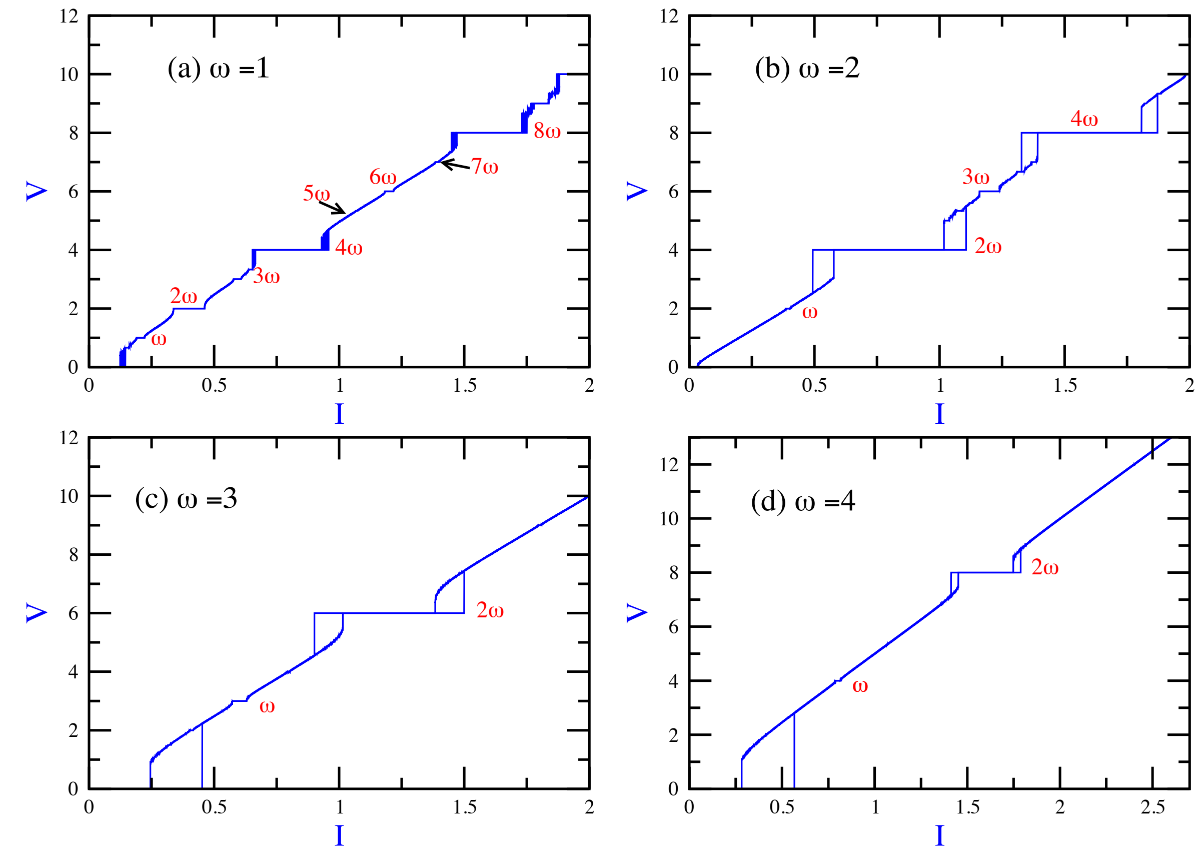
$$W \sim I_J \sim (1+Z^2/4)^{-1/2}$$

☆ *BTK, PRB, 25, 4515 (1982)*

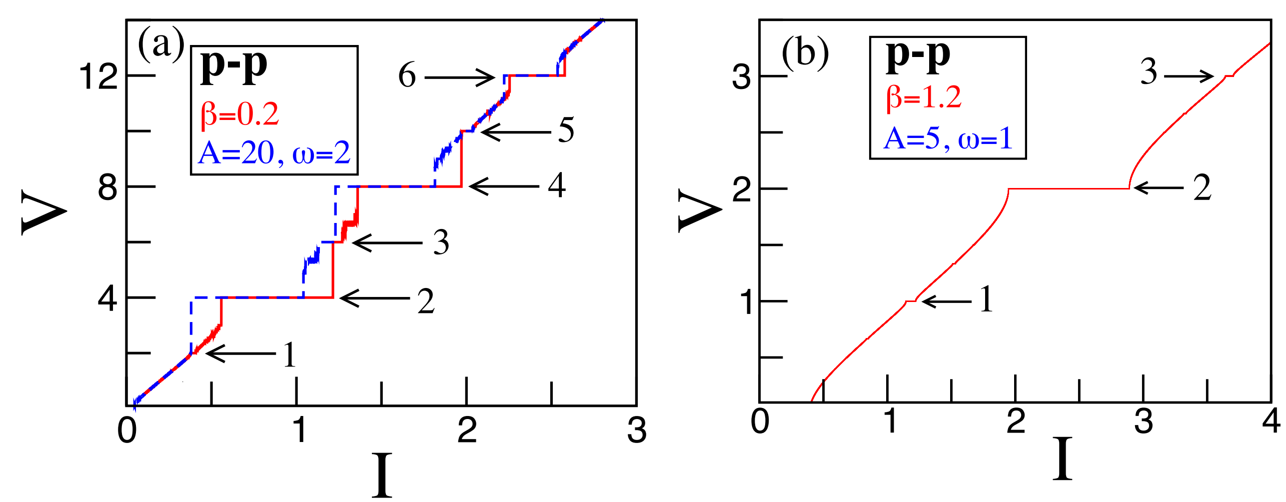
Josephson junctions hosting Majorana quasi-particles

- Appearance of ***both*** odd and even steps in the current-voltage characteristics.
- Even steps are ***enhanced*** compared to the odd steps for a significant range of external coupling.
- Similar qualitative feature is obtained both in the under-damped and over-damped region.
- In contrast, for a *s-wave* junction, the width of the Shapiro steps for the odd and even harmonics are comparable for the range of β varying from the under-damped to over-damped region.





Variation with external frequency



$\beta < 1$ $\qquad$ $\beta > 1$

$$\eta = W_{even}(2\omega)/W_{odd}(\omega)$$

- In the regime *β, ω, A* >> 1, perturbative analysis of the non-linear term.

$$\phi = \sum_n \epsilon^n \phi_n, \quad I = \sum_n \epsilon^n I_n$$

☆ *Kornev et. al, J Phys. Conf. Ser., 43, 1105 (2006)*

- I_0 ~ applied current, $I_{n>0}$ ~ determined from $\langle \dot{\phi}_{n>0} \rangle = 0$

- For n < 2

$$\ddot{\phi}_n + \beta\dot{\phi}_n = f_n(t) + I_n$$

where

$$f_0 = A\sin(\omega t)$$

$$f_1 = -\sin(\phi_0/2)$$

$$\Delta I_s^{even} = 2J_n\left(\frac{A}{2\omega\sqrt{\beta^2+\omega^2}}\right)$$

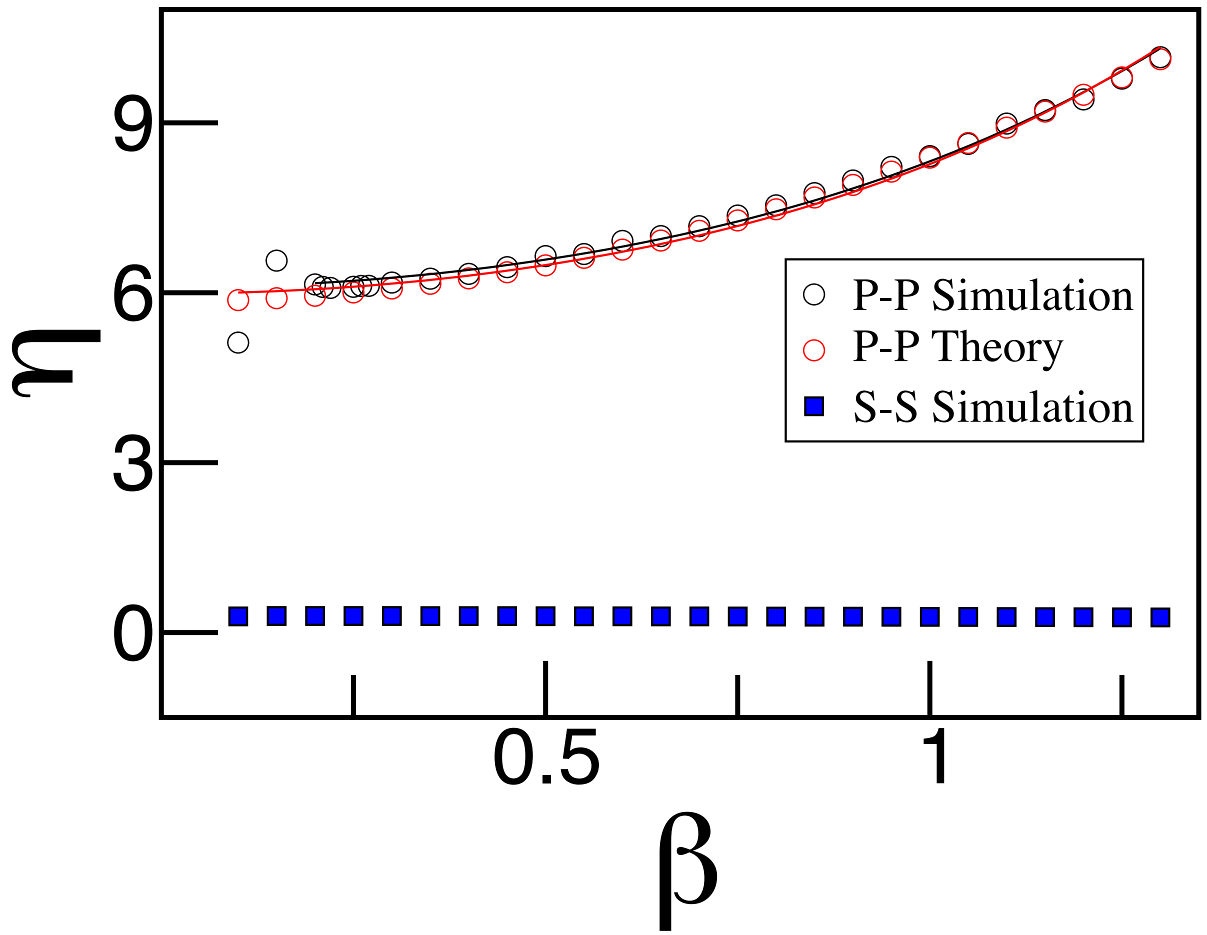
~ contribution from the harmonics

$$\Delta I_s^{odd} = \sum_{n>m} \frac{J_n\left(\frac{A}{2\omega\sqrt{\beta^2+\omega^2}}\right) J_{2m+1-n}\left(\frac{A}{2\omega\sqrt{\beta^2+\omega^2}}\right)}{2(\beta^2 + (2m+1-2n)^2\omega^2/4)}$$

~ contribution from the sub-harmonics

$$\boxed{\eta = \frac{\Delta I_s^{even}}{\Delta I_s^{odd}}}$$

Plot of the ratio of the step width η as a function of dissipation parameter β



$$\eta = \alpha_0 \exp(\alpha_1 \beta^2)$$

Simulation:

$$\alpha_0 = 6.09, \alpha_1 = 0.31$$

Theory:

$$\alpha_0 = 5.98, \alpha_1 = 0.32$$

- For p-wave junction η has exponential dependence on the junction capacitance C_0
- This provides a universal *phase sensitive signature* for the presence of Majorana fermions.

Summary and outlook

- Josephson junctions hosting Dirac and Majorana quasiparticles subjected to external radiation.

- The current-voltage characteristics shows novel oscillatory nature for junctions with graphene. This is a manifestation of the transmission resonance condition of the Dirac quasiparticles.

- The current-voltage characteristics of junctions with p-wave pairing symmetry shows presence of both odd and even steps in the Shapiro step structures. This is contrast with the existing studies in 1d nanowires where the dominance of the even harmonics is found and is associated with the presence of Majorana fermions.