Study of statistical properties of scattered photons from a driven three-level emitter embedded in 1D open space waveguide

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Introduction
- Statistical property of scattered photons
- Realization of strong interaction between matter and photon field in 1D
- Open space formalism for photon transport

Description of the Model
- Hamiltonian of the system

Techniques:
- How to find single, two or multiple photons scattering state in the full system

Results
- Strongly correlated photons: Electromagnetically Induced Transparency (EIT)
- Behaviour of second order correlation with system parameters

Conclusion
1. Photons are neutral particles, do not interact with each other. Photons should arrive independently of one another from a source (an ideal laser or a single frequency).

2. Non-classical correlations e.g. bunching (spatial attraction) and antibunching (spatial repulsion).


\[ g^2(\tau) = \frac{\langle I(t)I(t+\tau) \rangle}{\langle I(t) \rangle^2} \]

For light from an ideal coherent laser, \( g^2(\tau) = 1 \), for bunched light \( g^2(\tau) \leq g^2(\tau = 0) \), and for anti-bunched light \( g^2(\tau) \geq g^2(\tau = 0) = 0 \).

4. To study statistics in few photon level, one needs strong light-matter interaction.
Efficient strong coupling between matter and photon field in 1D open space:

- Highly confined propagating microwave photon modes in a 1D open superconducting transmission line and a large dipole moment of an artificial atom such as a superconducting qubit,
- Line defects in photonic crystals coupled to quantum dots and surface plasmons of a metallic nanowire coupled to quantum dots or nanocrystals.

**Figure:** Transmon qubits acting as artificial atoms (in Green) coupled to a 1D superconducting transmission line (in Blue).

**Figure:** Line defects in photonic crystal coupled to quantum dots.
Scattering of probe photons by three level emitter (3LE) ⇒ Study of one and two photon transport incident on a single three-level emitter (atom), when the photons are restricted to a one-dimensional system.

Exact theoretical approach, based upon real-space equations of motion and the Bethe ansatz. (Ref. Shen and Fan. PRL 98, 153003 (2007))
The full Hamiltonian describing the scattering of photons from a driven 3LE embedded in a 1D photonic waveguide:

\[ \mathcal{H} = \mathcal{H}_{wg} + \mathcal{H}_{3LE} + \mathcal{H}_c \]

1. Hamiltonian for Free probe photons in wave-guide: \( \mathcal{H}_{wg} \)
2. Hamiltonian for a driven 3LE: \( \mathcal{H}_{3LE} \)
3. Hamiltonian describing the interaction between probe photons and 3LE: \( \mathcal{H}_c \)
We consider a linear energy-momentum dispersion \( E_k = v_g k \) for the free probe photons in waveguide → Time evolution ≡ Space evolution.

Divide the positive and negative momentum photons as right-moving modes and left-moving modes. \( a_R(x) \) \( [a_L(x)] \) is the annihilation operator of a right-(left-) moving photon at position \( x \).

\[ |2> \]
\[ v \]
\[ |1> \]
\[ \Omega \]
\[ |3> \]
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Wire Hamiltonian

Hamiltonian for Free probe photons in waveguide (Linear dispersion $E_k = v_g k$)

$$\mathcal{H}_{wg} = -i v_g \int dx \left[ a_R^\dagger(x) \partial_x a_R(x) - a_L^\dagger(x) \partial_x a_L(x) \right],$$

where $v_g$ is the group velocity of the photons.
The Hamiltonian of a driven 3LE embedded in a 1D open waveguide:

$$\mathcal{H}_{3LE} = (E_2 - i\gamma_2/2)|2\rangle\langle 2| + (E_2 - \Delta - i\gamma_3/2)|3\rangle\langle 3| + (\Omega_c/2)(|3\rangle\langle 2| + |2\rangle\langle 3|),$$

where spontaneous emission loss: $-i\gamma_2/2$ and $-i\gamma_3/2$ to the energy of the respective states $|2\rangle$ and $|3\rangle$.

The excited state $|2\rangle$ of the emitter is connected to the state $|3\rangle$ by a classical laser beam (Control beam) with Rabi frequency $\Omega_c$ (Proportional to the amplitude of control beam and dipole moment of 3LE),
1. Transitions $|1\rangle - |2\rangle$ and $|2\rangle - |3\rangle$ would couple to different polarizations of light by selection rule.

2. A probe beam in the waveguide is sent near resonant to the transition $|1\rangle - |2\rangle$.

3. We also consider that there is no direct transition between the states $|1\rangle$ and $|3\rangle$ by dipole selection rule.
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Within Rotating-wave approximation (RWA) the Hamiltonian for the 3LE side-coupled to the propagating light fields locally at $x = 0$:

$$\mathcal{H}_c = V |2\rangle \langle 1| (a_R(0) + a_L(0)) + h.c.,$$

where $V$ is coupling strength between emitter and probe photons.
Introduce a new basis, even-odd basis of probe photons, defined by

\[ a_e(x) = (a_R(x) + a_L(-x))/\sqrt{2}, \quad a_o(x) = (a_R(x) - a_L(-x))/\sqrt{2} \]

In even-odd mode, photons at \( x < 0 \) represent an event before scattering, and photons at \( x > 0 \) after scattering.

In the even-odd basis the Hamiltonian can be decoupled as \( \mathcal{H} = \mathcal{H}_e + \mathcal{H}_o \), where

\[
\begin{align*}
\mathcal{H}_e &= -iv_g \int dx \ a_e^\dagger(x) \partial_x a_e(x) + \mathcal{H}_{3LE} \\
&\quad + \bar{V} \langle a_e^\dagger(0)|1\rangle\langle 2 | + | 2 \rangle\langle 1 | a_e(0) \rangle, \quad \text{and} \\
\mathcal{H}_o &= -iv_g \int dx \ a_o^\dagger(x) \partial_x a_o(x),
\end{align*}
\]

where \( \bar{V} = \sqrt{2}V \).
Single photon state

\[ |k\rangle = \int dx \{ A_1 [g_k(x) a_e^\dagger(x) |0, 1\rangle + e_k |0, 2\rangle + f_k |0, 3\rangle] + B_1 h_k(x) a_o^\dagger(x) |0, 1\rangle \}, \]

where the constants \( A_1 = B_1 = 1/\sqrt{2} \) for a right-moving photon with the incoming state \( |k\rangle_{in} = (1/\sqrt{2\pi}) \int dx e^{ikx} a_R^\dagger(x) |0, 1\rangle \).

- Here \( g_k(x) \) and \( h_k(x) \) are the amplitude of a single photon in the even and odd field modes when the emitter in the ground state.
- For an incident photon coming from the left, \( g_k(x < 0) = h_k(x < 0) = e^{ikx}/\sqrt{2\pi} \).
- The amplitude of the excited states \( |2\rangle \) and \( |3\rangle \) are respectively given by \( e_k \) and \( f_k \).
- The basis state \( |0, i\rangle \) denotes zero photon in the waveguide and the emitter in the \( i^{th} \) state.
Stationary Schrödinger equations, $\mathcal{H}|k\rangle = E_k|k\rangle$ with $E_k = v_g k$ gives

$$-i v_g \partial_x g_k(x) - E_k g_k(x) + \tilde{V} e_k \delta(x) = 0 \Rightarrow g_k(0^+) = g_k(0^-) - i \frac{\tilde{V}}{v_g} e_k,$$

$$(E_2 - i \gamma_2/2 - E_k)e_k + \tilde{V} g_k(x) \delta(x) + \frac{\Omega_c}{2} f_k = 0,$$

$$(E_2 - \Delta - i \gamma_3/2 - E_k)f_k + \frac{\Omega_c}{2} e_k = 0,$$

$$-i v_g \partial_x h_k(x) - E_k h_k(x) = 0.$$ 

Regularization of amplitudes across the emitter position,

$$g_k(0) = \frac{[g_k(0^-) + g_k(0^+) ]}{2}$$

and the initial boundary conditions to solve the above differential equations of the amplitudes.
Single photon state

\[ g_k(x) = h_k(x) \left[ \theta(-x) + t_k \theta(x) \right], \quad h_k(x) = \frac{e^{ikx}}{\sqrt{2\pi}}, \]
\[ t_k = \frac{\chi - i\Gamma/2}{\chi + i\Gamma/2} \quad \text{(Single photon transmission amplitude)}, \]
\[ e_k = \frac{\bar{V}}{\sqrt{2\pi}(\chi + i\Gamma/2)}, \quad f_k = 0.5\Omega_c e_k/(E_k - E_2 + \Delta + i\gamma_3/2) \]

Here \( \theta(x) \) is the step function, \( \Gamma = \bar{V}^2/v_g = 2V^2/v_g \) and

\[ \chi = E_k - E_2 + i\gamma_2/2 - \frac{\Omega_c^2}{4(E_k - E_2 + \Delta + i\gamma_3/2)}. \]

Now onwards we set \( v_g = 1 \).

For an incident photon from the left,

- Single-photon transmission amplitude of right moving photon : 
  \[ \tilde{t}_k = (1 + t_k)/2 = \chi/(\chi + i\Gamma/2) \]
- Single photon reflection amplitude left moving photon: 
  \[ \tilde{r}_k = (t_k - 1)/2 = -0.5i\Gamma/(\chi + i\Gamma/2) \]
Two-photon state

Two-photon Initial state

\[ |k_1, k_2\rangle_{in} = \int dx_1 dx_2 \phi_k(x_1, x_2) \frac{1}{\sqrt{2}} a_R^\dagger(x_1) a_R^\dagger(x_2)|0, 1\rangle, \]

where \( \phi_k(x_1, x_2) = (e^{ik_1 x_1 + ik_2 x_2} + e^{ik_1 x_2 + ik_2 x_1})/(2\sqrt{2\pi}) \) with \( k = (k_1, k_2) \).

Two-photon Scattering state:

\[ |k_1, k_2\rangle = \int dx_1 dx_2 \left[ A_2 \{ g(x_1, x_2) \frac{1}{\sqrt{2}} a_e^\dagger(x_1) a_e^\dagger(x_2)|0, 1\rangle + (e(x_1) a_e^\dagger(x_1)|0, 2\rangle + f(x_1) a_e^\dagger(x_1)|0, 3\rangle \delta(x_2) \} \right. \]

\[ + B_2 \{ j(x_1; x_2) a_e^\dagger(x_1) a_o^\dagger(x_2)|0, 1\rangle + (v(x_1) a_o^\dagger(x_1)|0, 2\rangle + w(x_1) a_o^\dagger(x_1)|0, 3\rangle \delta(x_2) \} \]

\[ + C_2 h(x_1, x_2) \frac{1}{\sqrt{2}} a_o^\dagger(x_1) a_o^\dagger(x_2)|0, 1\rangle \right], \]

\[ g(x_1, x_2) = \frac{1}{\sqrt{2}} \left[ \sum_P g_{kP_1}(x_1) g_{kP_2}(x_2) + \sum_{PQ} B_{kP_1, kP_2}^{(2)}(x_{Q_1}, x_{Q_2}) \theta(x_{Q_2}) \right], \]

\[ j(x_1; x_2) = \sum_P g_{kP_1}(x_1) h_{kP_2}(x_2), \quad h(x_1, x_2) = \frac{1}{\sqrt{2}} \sum_P h_{kP_1}(x_1) h_{kP_2}(x_2), \]

\[ \Gamma_k(x - y) = (d_- \epsilon_k e^{i(s-t)|x-y|} + d_+ \epsilon_k e^{i(s+t)|x-y|}), \quad d_\pm = \left( \frac{1}{2\beta} \pm \frac{\epsilon}{2\Omega_c} \right), \]

\[ s = -E_2 + \Delta/2 + i(\gamma_2 + \gamma_3)/4, \quad t = \sqrt{\epsilon^2 + 4\Omega_c^2}/4 \]

\[ B_{kP_1, kP_2}^{(2)}(x_{Q_1}, x_{Q_2}) = -i\tilde{V}_\beta (1 - t_{kP_1}) \Gamma_k P_2(x_{Q_12}) h_{kP_1}(x_{Q_1}) h_{kP_2}(x_{Q_1}) \theta(x_{Q_12}) \]

← Two-photon bound state

Here \( P = (P_1, P_2) \) and \( Q = (Q_1, Q_2) \) are permutation of \((1, 2)\), and \( x_{Q12} = x_{Q1} - x_{Q2} \).
Electromagnetically Induced Transparency

Transmission coefficient for right moving photon: $T_k = |\tilde{t}_k|^2$, when $g(x_1 > 0, x_2 < 0)$.

\[ \frac{(E_k - E_2)}{\Gamma} \]

\[ \Omega_c/\Gamma = 0 \quad \Omega_c/\Gamma = 1/2 \quad \Omega_c/\Gamma = 1 \quad \Omega_c/\Gamma = 5/2 \]

Figure: $T_k$ vs detuning $(E_k - E_2)$; Parameters: $\Delta/\Gamma = \gamma_2/\Gamma = 1/4, \gamma_3/\Gamma = 1/40$

- Appearance of EIT at two-photon resonance, $E_k - E_2 = -\Delta$ at weak control beam ($\Omega_c < \Gamma$).
- Destructive quantum interference between the two paths that lead to a transition to $|2>$ $\Rightarrow$ Cancellation of the population of the state $|2>$ ("dark state").
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Study of photon statistics by measuring second-order spatial coherence of the scattered photons

\[ g^2(x_2 - x_1) = \frac{\langle \psi | a_m^\dagger(x_1) a_m^\dagger(x_2) a_m(x_2) a_m(x_1) | \psi \rangle}{\langle \psi | a_m(x_1) a_m(x_1) | \psi \rangle \langle \psi | a_m^\dagger(x_2) a_m(x_2) | \psi \rangle}, \]

where \( m = R(L) \) for the transmitted (reflected) photons for an incident probe beam from the left.

\(|\psi\rangle\) is a \( N \)-photon scattering Fock state with incident momenta \( k_1, k_2..k_N \).
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A single emitter becomes saturated by a single photon as one emitter can absorb only one photon at a time \( \rightarrow \) a strong photon-photon nonlinearity is created by an emitter for two incident photons.
2nd order correlation (Intensity-Intensity correlation)

- Study of photon statistics by measuring second-order spatial coherence of the scattered photons

$$g^2(x_2 - x_1) = \frac{\langle \psi | a_m^\dagger(x_1) a_m^\dagger(x_2) a_m(x_2) a_m(x_1) | \psi \rangle}{\langle \psi | a_m^\dagger(x_1) a_m(x_1) | \psi \rangle \langle \psi | a_m^\dagger(x_2) a_m(x_2) | \psi \rangle},$$

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- $|\psi\rangle$ is a $N$-photon scattering Fock state with incident momenta $k_1, k_2..k_N$.

- A single emitter becomes saturated by a single photon as one emitter can absorb only one photon at a time $\rightarrow$ a strong photon-photon nonlinearity is created by an emitter for two incident photons.
Keeping higher order contributions in the numerator and denominator: 2nd order correlation for two photon state

\[
g^2(x_2 - x_1) = \frac{1}{|(t_{k_1} \pm 1)(t_{k_2} \pm 1)|^2} \left( |\sum_P(t_{kP_1} \pm 1)(t_{kP_2} \pm 1)\tilde{h}_{kP_1}(x_1)\tilde{h}_{kP_2}(x_2)|^2 + 2i \sum_{PQ} V \beta (t_{kP_1} - 1)\Xi_{kP_2}(x_{Q12})\tilde{h}_{kP_1}(x_{Q_1})\tilde{h}_{kP_2}(x_{Q_1})\theta(x_{Q12})|^2 \right). \]

\[ +(-) \text{ sign for the transmitted (reflected) probe beam.} \]

\[
\tilde{h}_k(x) = e^{ikx} \theta(x)/\sqrt{2}, \quad \Xi_k(x_1 - x_2) = \sum_{j=\pm} d_j \varepsilon_j(k)e^{i(s+j\Omega_c/4\beta)|x_1-x_2|}. \]

\[
\varepsilon_{\pm}(k) = V/(E_k + s \pm \Omega_c/4\beta), \quad s = -(E_2 - \Delta/2) + i(\gamma_2 + \gamma_3 + \Gamma)/4
\]

\[
\beta = \Omega_c/\sqrt{\epsilon^2 + 4\Omega^2}, \quad d_{\pm} = (1/(2\beta) \pm \epsilon/(2\Omega_c)), \quad \epsilon = -2\Delta + i(\gamma_2 + \Gamma - \gamma_3). \]

We use \(P = (P_1, P_2)\) and \(Q = (Q_1, Q_2)\) for permutation of \((1, 2)\) and \(x_{Q12} = x_{Q_1} - x_{Q_2}\).
Two photon resonance:
\[ E_{k_1} = E_{k_2} = E_2 - \Delta. \]

First Row: \( g^2 \) vs \( \Omega_c \):
- two-photon resonance \( \delta = (E_k - (E_2 - \Delta)) = 0 \)
- and \( \gamma_3/\Gamma = 1/40 \)

Second Row: \( g^2 \) vs \( \delta \):
- \( \Omega_c/\Gamma = 3/10 \) and \( \gamma_3/\Gamma = 1/8 \).
- The other parameters are \( \Delta = 0 \), \( \gamma_2/\Gamma = 0.31 \).

\( g^2 \) shows antibunching of the transmitted probe photons ⇒ Two probe photons cannot transmit through the emitter simultaneously.

This happens for a Rabi frequency when a complete dark state is not yet formed.

Further increase \( \Omega_c \) a dark state is formed ⇒ \( g^2(x_2 - x_1) = 1 \).
We calculate the exact one photon and two-photon wave functions for this model.

Appearance of EIT at two-photon resonance, $E_k - E_2 = -\Delta$ when a weak control beam is switched on.

Second-order coherence of the scattered probe photons from $\Lambda$ or ladder-type 3LE can be tuned by changing Rabi frequency of the control beam. It can be measured experimentally using a Hanbury Brown and Twiss measurement setup.

Generalization: To derive multiphoton scattering states and second-order coherence of the scattered probe photons in case of multiple interacting multilevel emitters.
THANK YOU !