Spontaneous breaking of conformal invariance in the Standard Model

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- Energy scales in fundamental interactions
- Naturalness problem of SM
- Quark condensate
- Radiatively induced symmetry breaking
- Spontaneous conformal symmetry breaking in SM
- Conclusions

Motivation (I)

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- We have to look for alternatives ...
- Symmetry principles to be exploited
- Correspondence to SM should be preserved

Motivation (II)

Higgs boson with $m_H \approx 126$ GeV makes the SM stable up to the Planck energy scale, i.e. 10^{19} GeV.

New physics is not required?

F. Bezrukov, M.Y. Kalmykov, B.A. Kniehl, M. Shaposhnikov, JHEP'2012

S. Alekhin, A. Djouadi and S. Moch, Phys. Lett. B'2012

A.V. Bednyakov, A.F. Pikelner and V.N. Velizhanin, Nucl. Phys. B'2013, 2014; Phys. Lett. B'2014.

...

Motivation (III)

At the EW scale we have a remarkable empirical relation

$$v = \sqrt{M_H^2 + M_W^2 + M_Z^2 + m_t^2}$$

for today PDG values we have a perfect agreement within experimental errors

$$246.22 = 246 \pm 1 \text{ GeV}$$

Obviously, there should be some tight clear relation between the top quark mass and the Higgs boson one (or the EW scale)
Note also

$$2\frac{m_h^2}{m_t^2} = 1.05 \approx 1 \approx 2\frac{m_t^2}{m_h^2} = 0.99$$

Motivation (IV)

We will try to apply the mechanism of the chiral symmetry breaking to the SM

The Nobel Prize in Physics 2008 (one half) was awarded to Yoichiro Nambu "for the discovery of the mechanism of spontaneous broken symmetry in subatomic physics".

The prize in 2013 was awarded to Fran cois Englert and Peter Higgs "for the theoretical discovery of a mechanism that contributes to our understanding of the origin of mass of subatomic particles ..."

Mechanisms of Spontaneous Symmetry Breaking (SSB) in SM and QCD are similar but of different types

Fundamental Interactions

At present to the best of our knowledge we can distinguish

6 types of fundamental interactions:

1) U(1) gauge int.
2) SU(2) gauge int.
3) SU(3) gauge int.
4) Higgs Yukawa int.
5) Higgs self coupling
6) Gravity

Scale invariance breaking

The observed world is obviously not Scale Invariant (SI)

But many physical laws are SI, see e.g. Newtonian mechanics (w/o gravity) and Maxwell equations

There is only one term (the Higgs tachyon mass) in the SM Lagrangian, which explicitly breaks SI

then we have dimensional transmutation in QCD

and an explicitly dimensionful coupling constant in Gravity

All those make real troubles for the fundamental theory

Examples of SI breaking

1. In the Newtonian classical mechanics (w/o gravity), the laws are SI but solutions are not. The breaking happens due to the initial conditions. This is a case of **soft** symmetry breaking.

N.B. Dynamical symmetry breaking is a soft one (Y. Nambu)

2. In QED the SI is broken by the electron mass which enters the Lagrangian. This is an **explicit** symmetry breaking.

Due to quantum effects we have in QED also the Landau pole:

$$\alpha(Q^2) \approx \frac{\alpha(0)}{1 - \frac{\alpha(0)}{3\pi} \ln \frac{Q^2}{m_e^2}}, \qquad \alpha(0) \approx \frac{1}{137}, \qquad \alpha(Q_0^2) \to \infty$$

This problem is not resolved in QED, it is related to explicit SI breaking.

Generation of masses

Does the Higgs boson really give masses to everything that we see?

not really

Λ-term and dark matter in Cosmology?

the proton mass?

neutrino masses?

the Higgs mass itself?

We still do not understand the origin of masses

and of fundamental physical energy scales in general

Higgs boson in SM (I)

Remind the Standard Model mechanism:

$$V_{\text{Higgs}}(\phi) = \lambda(\Phi^\dagger\Phi)^2 + \mu^2\Phi^\dagger\Phi$$

Due to spontaneous symmetry breaking (SSB) of $O(4)$ symmetry

if $\mu^2 < 0$, one component of the complex scalar doublet field $\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$ acquires a non-zero vacuum expectation value

$$\langle\phi^0\rangle = v/\sqrt{2}$$

The vacuum stability condition $\lambda > 0$ is always assumed

Higgs boson in SM (II)

The $O(4)$ symmetry of the Higgs field is broken spontaneously but that does not protect the Higgs mass from huge renormalizations:

$$\Delta m_H^2 \sim \Lambda^2$$

contrary to the cases of m_W and m_Z which have typical

$$\Delta m_{W,Z}^2 \sim m_{W,Z}^2 \ln \frac{\Lambda^2}{m_{W,Z}^2}$$

That is known as the naturalness or fine tuning or hierarchy problem of SM

That is because m_W and m_Z have the pure SSB origin, while m_H is related to the tachyon mass term ($\mu^2 < 0$) which breaks the conformal symmetry of SM **explicitly**

Naturalness problem (I)

There are two general ways to solve the naturalness problem:

I. Cancel out the huge radiative corrections

— either due to some (super)symmetry

— or due to fine tuning (anthropic principle)

II. Make Λ small, i.e. $\Lambda \lesssim 1$ TeV with some new physics motivation

— but LHC and others do not see anything new at this scale

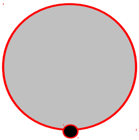
Naturalness problem (II)

Let us look at some details of the problem.

In the SM, the Λ^2 divergent terms cancel out everywhere except the corrections to the Higgs mass

They appear as scalar Passarino-Veltman integrals

$$A_0(m^2,\Lambda^2) = \int_\Lambda \frac{d^4k}{i\pi^2}\frac{1}{k^2-m^2+i\varepsilon} = \Lambda^2 - m^2 \ln\frac{\Lambda^2}{m^2} + \mathcal{O}\left(\Lambda^{-2}\right)$$



N.B. That is the so-called tadpole Feynman diagram

Naturalness problem (III)

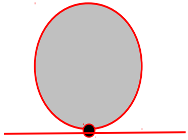
Two types of diagrams contribute:

— Higgs boson loop

— EW boson loop

— top-quark loop

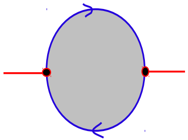
EW and Higgs boson loops



N.B. Actually longitudinal components of EW bosons, i.e. goldstones, are relevant

Naturalness problem (IV)

Top quark loop



$$\int_{\Lambda_t} \frac{d^4k}{i\pi^2} \frac{\mathrm{Tr}(\hat{k}+m_t)((\hat{p}-\hat{k})+m_t)}{(k^2-m_t^2)((p-k)^2-m_t^2} \to 4\int_{\Lambda_t} \frac{d^4k}{i\pi^2} \frac{1}{k^2-m_t^2} + \mathcal{O}(m_t^2)$$

$$= 4A_0(m_t^2, \Lambda_t^2) + \mathcal{O}(m_t^2)$$

Naturalness problem (V)

Combined in the lowest approximation (if Λ_i are the same)

$$M_H^2 = (M_H^0)^2 + \frac{3\Lambda^2}{8\pi^2 v^2}\left[M_H^2 + 2M_W^2 + M_Z^2 - 4m_t^2\right]$$

It is unnatural to have $\Lambda \gg M_H$.

The most natural option would be $\Lambda \sim M_H$, e.g. everything is defined by the EW scale. But this is not the case of the SM.

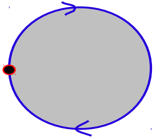
Obviously, the problem is caused by the explicit breaking of the conformal symmetry in the SM

Quark condensate (I)

By definition formally

$$\langle \bar{q}\, q \rangle \equiv -N_C \int_{\Lambda_q} \frac{d^4 k}{i(2\pi)^4} \frac{\mathrm{Tr}(\hat{k} + m_q)}{k^2 - m^2 + i\varepsilon} \sim -4 N_C m_q A_0(m_q^2, \Lambda_q^2)$$

In particular the top quark condensate gives $\langle \bar{t}\, t \rangle / m_t$ contribution to ΔM_H. This statement concerns as formal definitions as well as observables.



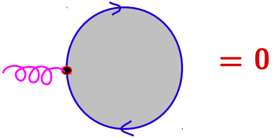
N.B. $\langle \bar{q}\, q \rangle \equiv 0 \qquad \text{if} \quad m_q = 0$

Quark condensate (II)

Light quark condensate is “measured”: $\sqrt[3]{\langle \bar{q}\, q \rangle} \simeq -250$ MeV

The “measurement” itself is possible due to nonperturbative effects at low energies

In perturbative QCD the condensate can not be accessed just due to the Furry theorem:



I.e. the condensate can be finite, but its contribution is exactly zero

Quark condensate (III)

We do not know exactly how does appear the low-energy QCD scale, but we see

$-\sqrt[3]{\langle \bar{q}\, q \rangle} \;\sim\; M_q \;\sim\; \Lambda_{\mathrm{QCD}}$
where M_q is the constituent light quark mass

Or $\langle \bar{q}\, q \rangle \sim -M_q \times \Lambda^2_{\mathrm{QCD}}$

Very likely that the Λ_{QCD} scale comes from outside QCD. The QCD dynamics just helps it to propagate into M_q and $\langle \bar{q}\, q \rangle$.

It is very likely that radiatively induced dimensional transmutation is realized in QCD. It means a SOFT breaking of conformal symmetry there.

Coleman-Weinberg mechanism (I)

S. Coleman & E. Weinberg 1973

Semi-classical conformal-invariant $V = \lambda\phi_c^4/4!$ is transformed by quantum loop corrections into

$$V_{\mathrm{eff}} = \frac{\lambda}{4!}\phi_c^4 + \frac{\lambda^2\phi_c^4}{256\pi^2}\left(\ln\frac{\phi_c^2}{M^2} - \frac{25}{6}\right)$$

where M is a scale, which should be introduced to avoid infrared divergences.

Minimization of the effective potential leads to $\langle\phi\rangle \neq 0$ and consequently to $m_\phi \neq 0$

Coleman-Weinberg mechanism (II)

Let us apply the C-W procedure for the case of scalar+fermion:

$$V_{\rm cl} = \lambda\phi_c^4/4! + y\phi_c\bar{f}f$$

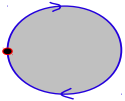
Scalar and fermion loops give:

$$\Delta V_{\rm sc} = \frac{1}{2}\int\frac{d^4k}{(2\pi^4)}\ln\left(1+\frac{\lambda\phi_c^2}{2k^2}\right) \to \frac{\lambda\Lambda^2}{256\pi^2}\phi_c^2 + \frac{\lambda^2\phi_c^4}{256\pi^2}\left(\ln\frac{\lambda\phi_c^2}{2\Lambda^2}-\frac{1}{2}\right)$$

$$\Delta V_f = -4N_C\int\frac{d^4k}{(2\pi^4)}\ln\left(1+\frac{ym_f\phi_c}{k^2-m_f^2}\right) \to -4N_C\frac{ym_f\Lambda_f^2}{16\pi^2}\phi_c$$

$$-4N_C\frac{y^2m_f^2\phi_c^2}{32\pi^2}\left(\ln\frac{ym_f\phi_c}{\Lambda_f^2}-\frac{1}{2}\right)$$

N.B. The first term in ΔV_f is the fermion tadpole



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Scalar and fermion loops give: **Renormalize** $\to 0!$

$$\Delta V_{\rm sc} = \frac{1}{2}\int \frac{d^4k}{(2\pi^4)} \ln\left(1 + \frac{\lambda\phi_c^2}{2k^2}\right) \to \frac{\lambda\mathbf{\Lambda^2}}{\mathbf{256}\pi^2}\phi_c^2 + \frac{\lambda^2\phi_c^4}{256\pi^2}\left(\ln\frac{\lambda\phi_c^2}{2\Lambda^2} - \frac{1}{2}\right)$$

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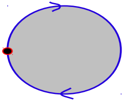

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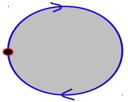
Renormalize $\to$?

N.B. The first term in ΔV_f is the fermion tadpole



Coleman-Weinberg mechanism (III)

Conformal-invariant unbroken phase (classical only):

$$m_\phi = m_f \equiv 0, \qquad \langle \phi \rangle \equiv 0, \qquad \langle \bar{f}\, f \rangle \equiv 0$$

In the softly broken phase for $\lambda \sim 1$ and $y \sim 1$:

$$m_\phi \sim m_f \sim M, \qquad \langle \phi \rangle \sim M, \qquad \langle \bar{f}\, f \rangle \sim -M^3$$

like in QCD, but non-perturbativity is not required

Let us look for a stable solution in the broken phase

SCSB for Higgs (I)

The dominant terms of Higgs interactions (for $\mu \equiv 0$) are

$$L_{\rm int} = -\frac{\lambda}{4}\phi^4 - \frac{y_t}{\sqrt{2}}\phi\,\bar{t}t$$

C.-W. mechanism gives the leading effective potential in the form

$$V_{\rm cond}(\phi) = \frac{\lambda}{4}\phi^4 + \frac{y_t}{\sqrt{2}}\langle\bar{t}\,t\rangle\phi$$

The extremum condition

$$\left.\frac{dV_{\rm cond}}{d\phi}\right|_{\phi=v} = 0 \quad \longrightarrow \quad v^3 = -\frac{y_t}{\sqrt{2}}\langle\bar{t}\,t\rangle$$

The Yukawa coupling $y_t \approx 0.99$ is known from $m_t = v\ y_t/\sqrt{2}$
The potential takes the form

$$V_{\rm cond}(\phi)|_{\phi=v+H} = V_{\rm cond}(v) + \frac{3\lambda v^2}{2}H^2 + \lambda v H^3 + \frac{\lambda}{4}H^4$$

SCSB for Higgs (II)

So the Higgs mass is

$$M_H^2 = 3\lambda v^2 = -\frac{3y_t\langle\bar{t}\,t\rangle}{\sqrt{2}\,v} = -\frac{3m_t\langle\bar{t}\,t\rangle}{v^2}$$

N.B. The difference from the SM is in the value of λ:

$$\lambda = \frac{2}{3}\lambda_{\mathrm{SM}}$$

Top quark condensate

To get $M_H = 126$ GeV we need $\langle \bar{t}\, t \rangle = (-123\ \mathrm{GeV})^3$

It is just a natural value according to the naturalness problem

There are no any (other) phenomenological restrictions on $\langle \bar{t}\, t \rangle$

Having non-zero top quark condensate does NOT lead to top quark bound states in our case

Naturalness (once more)

W. Bardeen (1995): radiative stability of the Higgs boson mass, i.e. resolution of the naturalness problem, can be ensured by the classical scale invariance

The constructed semi-classical solution is stable at least around the EW scale

For $\lambda \sim 1$ and $y_t \approx 1$ it is natural to have

$$m_t \sim M_H \sim v \sim \sqrt[3]{-\langle \bar{t}\, t \rangle}$$

Coleman-Weinberg: we have to introduce a finite scale but not into the Lagrangian. It can be a property of the quantum physical vacuum. How does the scale defines the observables depends on the model.

Conclusions

1. We proposed a simple modification of the SM based on the Nambu condensate mechanism. The difference from SM is only in 1.5 times lower value of the Higgs self-coupling λ

2. Here m_H and m_t are mutually related and define together EW scale

3. Our estimate of the top quark condensate value looks natural

4. The suggested mechanism automatically protects m_H from running away, since renormalization happens at the EW scale

5. The picture resembles the EW bootstrap suggested by Nambu and Bardeen at al. (1989). But their approaches were not based on the conformal symmetry. They just tried to cancel out the quadratic divergences.

6. Similar relations are used also in modern technicolor models, but the Higgs boson is composite there