

## Inha University, Republic of Korea

# Nucleon Properties in Finite Nuclei

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#### Content

- Topological models and soliton
- Medium modifications
- Nucleon's structure changes in nuclear matter
- Nuclear matter
- Consistency (difference) with (from) other soliton approaches
- Nucleon in finite nuclei and few and many body systems
- Summary and Outlook

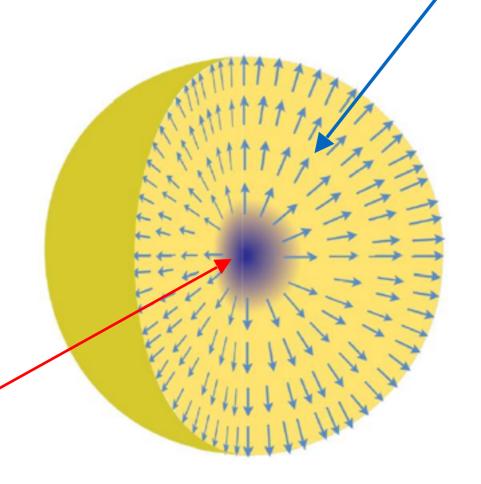
### Topological models and soliton

#### Why topological models?

- At fundamental level we may have
  - fermions -> bosons are made from fermions
  - bosons -> fermions are nontrivial topological structures

#### **Structure**

- What is a nucleon and, in particular, its core?
- The structure treatment depends on the energy scale
- At the limit of large number colours the core still has the mesonic content



Shell is made

meson cloud

from the

Core.. Made from what?

### Topological models and soliton

**Shrinks** 

**Swells** 

#### **Stabilisation**

- Soliton has the finite size and the finite energy
- One needs at least two terms in the effective (mesonic) Lagrangian

#### Prototype: Skyrme model

[T.H.R. Skyrme, Pros.Roy.Soc.Lond. A260 (1961)]

Nonlinear chiral effective meson (pionic) theory

$$\mathcal{L} = \frac{F_{\pi}^2}{16} \operatorname{Tr} \left( \partial_{\mu} U \partial^{\mu} U^{\dagger} \right) - \frac{1}{16e^2} \operatorname{Tr} \left[ U^{\dagger} \partial_{\mu} U, U^{\dagger} \partial_{\nu} U \right]^2$$
Shrinks Swells

Hedgehog solution (nontrivial mapping)

$$U = \exp\left\{\frac{i\overline{\tau} \overline{\pi}}{2F_{\pi}}\right\} = \exp\left\{i\overline{\tau} \overline{n}F(r)\right\}$$

### Topological models and soliton

#### The free space Lagrangian in use

[G.S.Adkins et al. Nucl.Phys. B228 (1983)]

$$\mathcal{L} = \frac{F_{\pi}^2}{16} \operatorname{Tr} \left( \partial_{\mu} U \partial^{\mu} U^{\dagger} \right) - \frac{1}{16e^2} \operatorname{Tr} \left[ U^{\dagger} \partial_{\mu} U, U^{\dagger} \partial_{\nu} U \right]^2 + \frac{F_{\pi}^2 m_{\pi}^2}{16} \operatorname{Tr} \left( U + U^{\dagger} - 2 \right)$$

- Nontrivial structure:
   topologically stable
   solitons with the
   corresponding conserved
   topological number
   (baryon number) A
- Nucleon is quantised state of the classical solitonskyrmion

$$U = \exp\{i\overline{\tau} \,\overline{\pi} / 2F_{\pi}\} = \exp\{i\overline{\tau} \,\overline{n}F(r)\}$$

$$B^{\mu} = \frac{1}{24\pi^{2}} \varepsilon^{\mu\nu\alpha\beta} Tr(L_{\nu}L_{\alpha}L_{\beta}) \qquad L_{\alpha} = U^{+}\partial_{\alpha}U$$

$$A = \int d^{3}rB^{0}$$

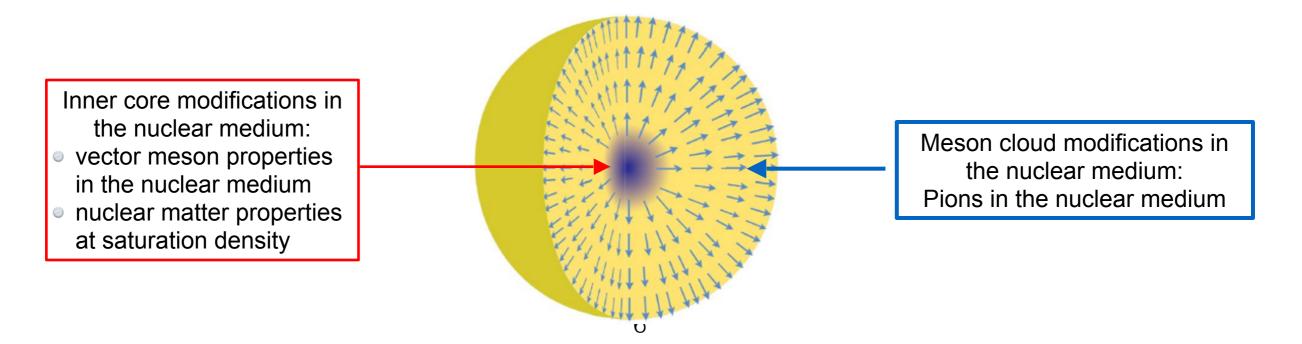
$$\begin{split} H &= M_{cl} + \frac{\overline{S}^2}{2I} = M_{cl} + \frac{\overline{T}^2}{2I}, \\ &|S &= T, s, t> = (-1)^{t+T} \sqrt{2T + 1} D_{-t,s}^{S=T}(A) \end{split}$$

#### What happens in the nuclear medium?

- The medium effects
  - Deformations (swelling or shrinking, multipole deformations) of nucleons
  - Possible characteristic changes: effective mass, charge distributions, form factors
  - NN interactions may change
  - etc.
- One should be able to describe all those phenomena

#### Soliton in the nuclear medium (phenomenological way)

- Outer shell modifications (informations from pionic atoms)
- Inner core modifications, in particular, at large densities (nuclear matter properties)



#### "Outer shell" modifications

- In free space three types of pions can be treated separately: isospin breakin
- In nuclear matter: three types of polarization operators
- Optic potential approach: parameters from the pionnucleon scattering, including isospin dependents

$$(\partial^{\mu}\partial_{\mu} + m_{\pi}^2)\vec{\pi}^{(\pm,0)} = 0$$

$$(\partial^{\mu}\partial_{\mu} + m_{\pi}^2 + \hat{\Pi}^{(\pm,0)})\vec{\pi}^{(\pm,0)} = 0$$

$$\hat{\Pi}^0 = 2\omega U_{\text{opt}} = \chi_s(\rho, b_0, c_0) + \vec{\nabla} \cdot \chi_p(\rho, b_0, c_0) \vec{\nabla}$$

	$\pi ext{-atom}$	$T_{\pi} = 50 \text{ MeV}$
$b_0 [m_{\pi}^{-1}]$	] - 0.03	- 0.04
$b_1 [m_{\pi}^{-1}]$	] - 0.09	- 0.09
$c_0 [m_{\pi}^{-3}]$	] 0.23	0.25
$c_1 [m_{\pi}^{-3}]$	] 0.15	0.16
g'	0.47	0.47

"Outer shell" modifications [U.Meissner et al., EPJ A36 (2008)]

$$\mathcal{L}_{2}^{*} = \frac{F_{\pi}^{2}}{16} \alpha_{\tau} \operatorname{Tr} \left( \partial_{0} U \partial_{0} U^{\dagger} \right) - \frac{F_{\pi}^{2}}{16} \alpha_{s} \operatorname{Tr} \left( \partial_{i} U \partial_{i} U^{\dagger} \right)$$

$$\mathcal{L}_m^* = -\frac{F_\pi^2 m_\pi^2}{16} \alpha_m \operatorname{Tr} \left(2 - U - U^+\right)$$

- Due to the nonlocality of optic potential the kinetic term is also modified
- Due to energy and momentum dependence of the optic potential parameters following parts of the kinetic term is modified in different form:
  - Temporal part
  - Space part

$$\hat{\Pi}^0 = 2\omega U_{\text{opt}} = \chi_s(\rho, b_0, c_0) + \vec{\nabla} \cdot \chi_p(\rho, b_0, c_0) \vec{\nabla}$$

	$\pi\text{-atom}$	$T_{\pi} = 50 \text{ MeV}$
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$c_1 \left[ m_{\pi}^{-3} \right]$	0.15	0.16
g'	0.47	0.47

#### "Inner core" modifications

[ UY & H.Ch. Kim, PRC83 (2011); UY, JKPS62 (2013); UY, PRC88 (2013) ]

$$\mathcal{L}_{4}^{*} = -\frac{1}{16e^{2}\zeta_{\tau}} \operatorname{Tr} \left[ U^{\dagger} \partial_{0} U, U^{\dagger} \partial_{i} U \right]^{2} + \frac{1}{32e^{2}\zeta_{s}} \operatorname{Tr} \left[ U^{\dagger} \partial_{i} U, U^{\dagger} \partial_{j} U \right]^{2}$$

- May be related to
  - Vector meson properties in nuclear matter
  - Nuclear matter properties

$$\zeta_{\tau,s} = \zeta_{\tau,s}(\rho, \delta\rho, \text{parameters})$$

#### **Final Lagrangian**

[UY, JKPS62 (2013); UY, PRC88 (2013)]

Separated into two parts

$$\mathcal{L}^* = \mathcal{L}^*_{\mathrm{sym}} + \mathcal{L}^*_{\mathrm{asym}}$$

Isoscalar part

$$\mathcal{L}_{\mathrm{sym}}^* = \mathcal{L}_2^* + \mathcal{L}_4^* + \mathcal{L}_m^*$$

Isovector part

$$\mathcal{L}_{\mathrm{asym}}^* = \mathcal{L}_{\delta m}^* + \mathcal{L}_{\delta \rho}^*$$

- Nuclear matter stabilisation
- Asymmetric matter properties

$$\mathcal{L}_{2}^{*} = \frac{F_{\pi}^{2}}{16} \alpha_{\tau} \operatorname{Tr} \left( \partial_{0} U \partial_{0} U^{\dagger} \right) - \frac{F_{\pi}^{2}}{16} \alpha_{s} \operatorname{Tr} \left( \partial_{i} U \partial_{i} U^{\dagger} \right)$$

$$\frac{1}{16e^{2}\zeta_{\tau}} \operatorname{Tr} \left[ U^{\dagger} \partial_{0} U, U^{\dagger} \partial_{i} U \right]^{2} + \frac{1}{32e^{2}\zeta_{s}} \operatorname{Tr} \left[ U^{\dagger} \partial_{i} U, U^{\dagger} \partial_{j} U \right]^{2}$$

$$\mathcal{L}_m^* = -\frac{F_\pi^2 m_\pi^2}{16} \, \alpha_m \text{Tr} \, \left( 2 - U - U^+ \right)$$

$$\mathcal{L}_{\delta m}^{*} = -\frac{F_{\pi}^{2}}{32} \sum_{a=1}^{2} (m_{\pi^{\pm}}^{2} - m_{\pi^{0}}^{2}) \operatorname{Tr} (\tau_{a} U) \operatorname{Tr} (\tau_{a} U^{\dagger})$$

$$\left( \mathcal{L}_{\delta\rho}^{*} \right) = -\frac{F_{\pi}^{2}}{16} m_{\pi} \alpha_{e} \, \varepsilon_{ab3} \text{Tr} \left( \tau_{a} U \right) \text{Tr} \left( \tau_{b} \partial_{0} U^{\dagger} \right)$$

#### Reparametrization

[UY, PRC88 (2013)]

- Five density dependent parameters
- Rearrangment (technical simplification)

$$1 + C_1 \frac{\rho}{\rho_0} = f_1 \left(\frac{\rho}{\rho_0}\right) \equiv \sqrt{\frac{\alpha_p^0}{\gamma_s}}$$

$$1 + C_2 \frac{\rho}{\rho_0} = f_2 \left(\frac{\rho}{\rho_0}\right) \equiv \frac{\alpha_s^{00}}{(\alpha_p^0)^2 \gamma_s}$$

$$1 + C_3 \frac{\rho}{\rho_0} = f_3 \left(\frac{\rho}{\rho_0}\right) \equiv \frac{(\alpha_p^0 \gamma_s)^{3/2}}{\alpha_s^{02}}$$

Shell modifications 
$$F_{\pi,\tau} \to F_{\pi,\tau}^*, \quad e_{\tau} \to e_{\tau}^*, \quad m_{\pi} \to m_{\pi}^*, \quad F_{\pi,s} \to F_{\pi,s}^*, \quad e_s \to e_s^*$$

$$1 + C_2 \frac{\rho}{\rho_0} = f_2 \left(\frac{\rho}{\rho_0}\right) = \frac{\alpha_s^{00}}{(\alpha_p^0)^2 \gamma_s} \qquad \frac{\alpha_e}{\gamma_s} = f_4 \left(\frac{\rho}{\rho_0}\right) \frac{\rho_n - \rho_p}{\rho_0} = \frac{C_4 \frac{\rho}{\rho_0}}{1 + C_5 \frac{\rho}{\rho_0}} \frac{\rho_n - \rho_p}{\rho_0}$$

#### Structure: Energy momentum tensor

- It allows to address the questions like:
  - How are the total angular momentum and angular momentum of the nucleon shared among its constituents?
  - How are the strong forces experienced by its constituents distributed inside the nucleon?
- EMT form factors studied in lattice QCD, ChPT and in different models (chiral quark soliton model, Skyrme model, etc.)
- We made further step studying EMT form factors in nuclear matter

Structure: Pressure distribution inside the nucleon in free space and in symmetric matter [H.C.Kim, P. Schweitzer, UY, Phys.Lett. B718 (2012)]

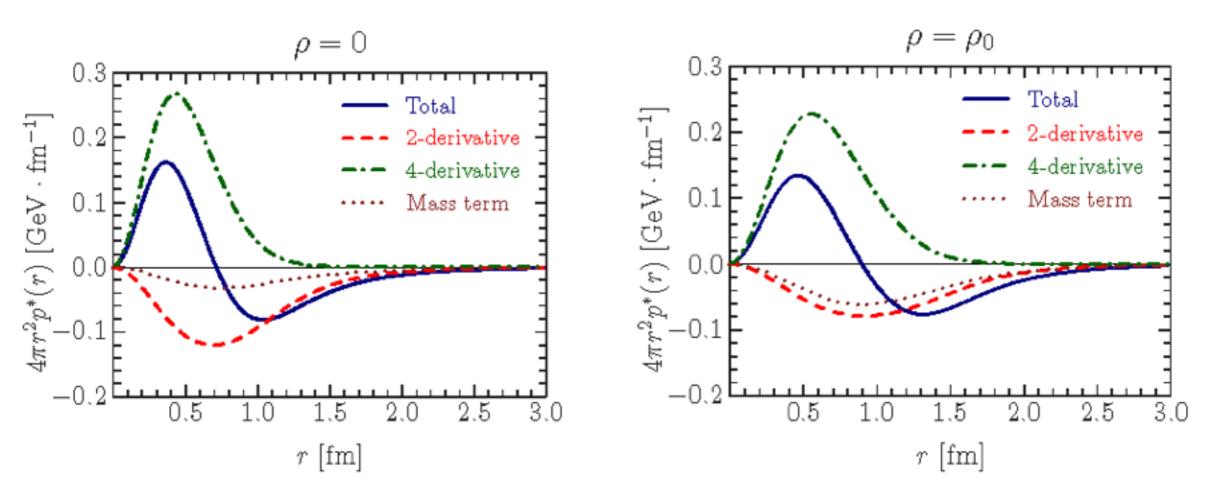


FIG. 3: (Color online) The decomposition of the pressure densities  $4\pi r^2 p^*(r)$  as functions of r, in free space ( $\rho = 0$ ) and at  $\rho = \rho_0$ , in the left and right panels, respectively. The solid curves denote the total pressure densities, the dashed ones represent the contributions of the 2-derivative (kinetic) term, the long-dashed ones are those of the 4-derivative (stabilizing) term, and the dotted ones stand for those of the pion mass term.

#### Stability and applicability [H.C.Kim, P. Schweitzer, UY, Phys.Lett. B718 (2012)]

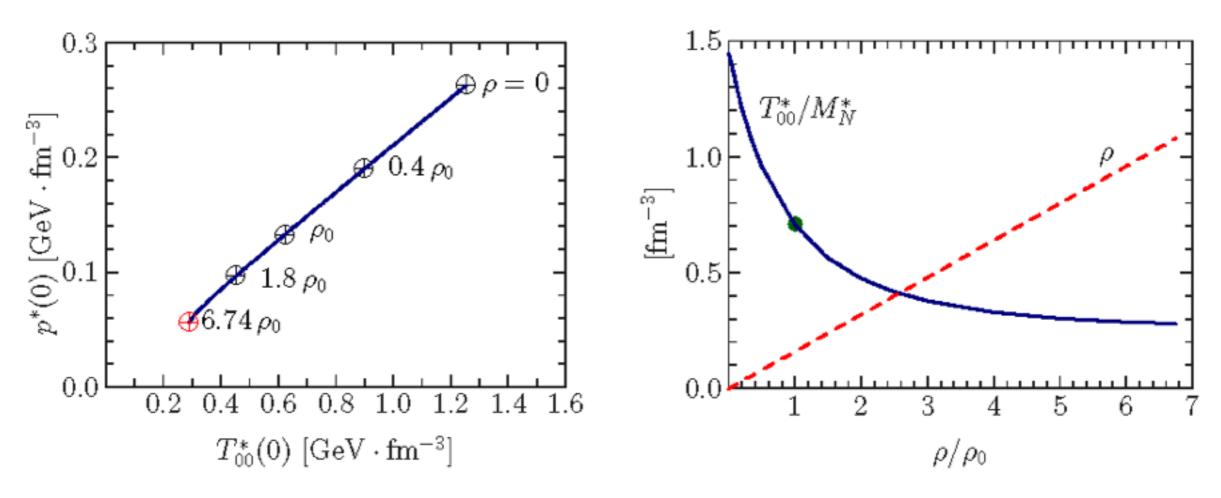


FIG. 5: (Color online) In the left panel, the correlated change of  $p^{\star}(0)$  and  $T_{00}^{\star}(0)$  drawn with  $\rho$  varied. In the right panel, the  $T_{00}^{\star}/M_N^{\star}$  and  $\rho$  depicted as a function of  $\rho/\rho_0$ . The maximal density is given as about 6.74  $\rho_0$ , above which the Skyrmion does not exist anymore. The filled circle on the solid curve represents the value of  $T_{00}^{\star}/M_N^{\star}$  at normal nuclear matter density.

#### **Nucleon in nuclear matter**

[UY, PRC88 (2013)]

Isovector mass

$$m_{N,s}^* = M_S^* + \frac{3}{8\Lambda^*} + \frac{\Lambda^*}{2} \left( a^{*2} + \frac{\Lambda_{\text{env}}^{*2}}{\Lambda^{*2}} \right)$$

Isovector mass

$$\Delta m_{np}^* = a^* + \frac{\Lambda_{\text{env}}^*}{\Lambda^*}$$

Mass of the nucleon

$$m_{n,p}^* = m_{N,s}^* - \Delta m_{np}^* T_3$$

#### Nuclear matter

The binding-energy-formula terms in the framework of present model

$$\varepsilon(A,Z) = -a_V + a_S \frac{(N-Z)^2}{A^2} + \mathbb{W}$$

We are ready to reproduce

- Volume term
- Infinite and asymmetric nuclear matter
   Asymmetry term
  - - Isospin asymmetric environment
  - Surface and Coulomb terms
    - Nucleons in a finite volume
  - Finite nuclei properties
    - Local density approximation

#### Nuclear matter

#### The volume term and Symmetry energy

 At infinite nuclear matter approximation the binding energy per nucleon takes the form

$$\varepsilon(\lambda, \delta) = \varepsilon_V(\lambda) + \varepsilon_S \delta^2 + O(\delta^4) \equiv \varepsilon_V(\lambda) + \varepsilon_A(\lambda, \delta)$$

- $\circ$   $\lambda$  is normalised nuclear matter density
- $\circ$   $\delta$  is asymmetry parameter
- $\epsilon_S$  is symmetry energy
- In our model
  - Symmetric matter
  - Asymmetric matter

$$\varepsilon_V(\lambda) = m_{N,s}^*(\lambda,0) - m_N^{\text{free}}$$

$$\varepsilon_{A}(\lambda,\delta) = \varepsilon(\lambda,\delta) - \varepsilon_{V}(\lambda)$$

$$= m_{N_{S}}^{*}(\lambda,\delta) - m_{N_{S}}^{*}(\lambda,0) + m_{N_{V}}^{*}(\lambda,\delta)\delta$$

#### Nuclear matter

#### **Nuclear matter properties**

Symmetric matter properties (pressure, compressibility and third derivative)

$$p = \rho_0 \lambda^2 \frac{\partial \varepsilon_V(\lambda)}{\partial \lambda} \bigg|_{\lambda=1}, \quad K_0 = 9 \rho^2 \frac{\partial^2 \varepsilon_V(\lambda)}{\partial \rho^2} \bigg|_{\rho=\rho_0} \qquad Q = 27 \lambda^3 \frac{\partial^3 \varepsilon_V(\lambda)}{\partial \lambda^3} \bigg|_{\lambda=1}$$

Symmetry energy properties (coefficient, slop and curvature)

$$\varepsilon_s(\lambda) = \varepsilon_s(1) + \frac{L_s}{3}(\lambda - 1) + \frac{K_s}{18}(\lambda - 1)^2 + \mathbb{W}$$

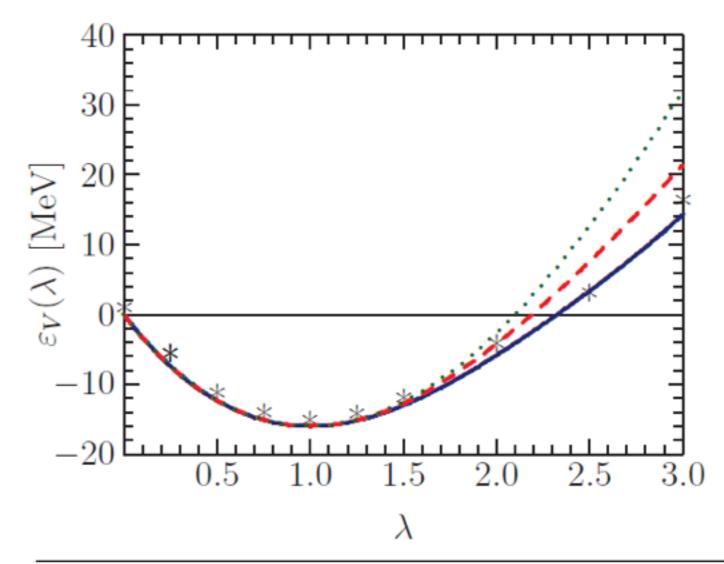
### Symmetric matter

#### **Volume energy**

[UY, PRC88 (2013)]

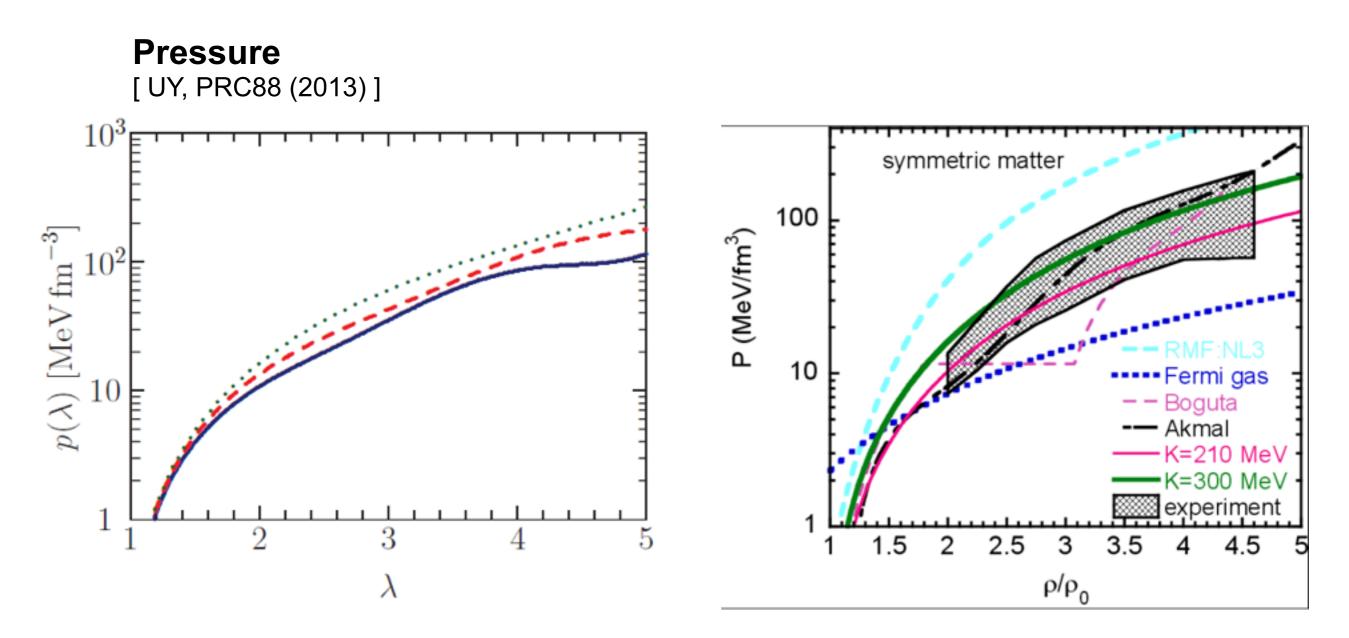
- Set I solid
- Set II dashed
- Set III dotted

For comparison: Akmal-Pandharipande-Ravenhall (APR) predictions
[PRC 58, 1804 (1998)] are given by stars.
(From Arigonna 2 body interactions + 3 body interactions)



Set	$C_1$	$C_2$	$C_3$	$\varepsilon_V(\rho_0)$ (MeV)	$K_0$ (MeV)	Q (MeV)
I	-0.279	0.737	1.782	-16	240	-410
II	-0.273	0.643	1.858	-16	250	-279
III	-0.277	0.486	2.124	-16	260	-178

### Symmetric matter



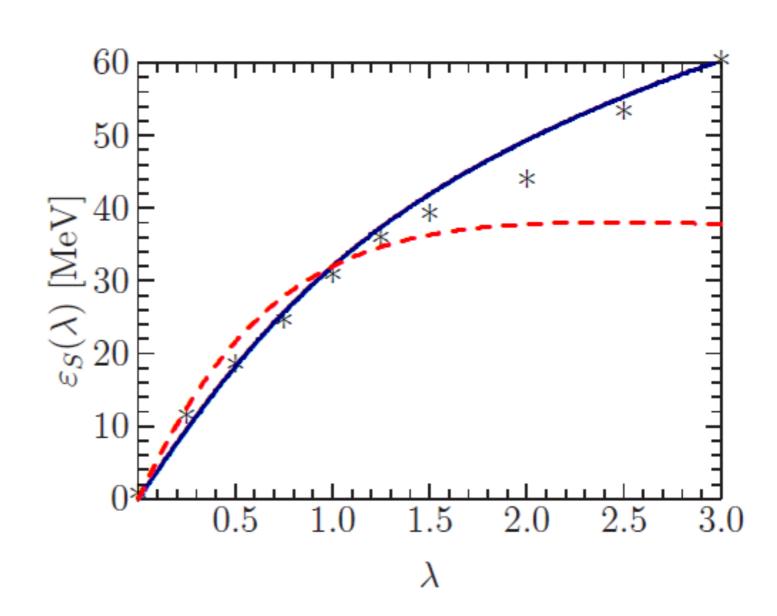
For comparison: Right figure from Danielewicz- Lacey-Lynch, Science 298, 1592 (2002). (Deduced from experimental flow data and simulations studies)

### Asymmetric matter

#### Symmetry energy

- Solid  $L_s = 70 \,\mathrm{MeV}$
- Dashed  $L_s = 40 \,\mathrm{MeV}$

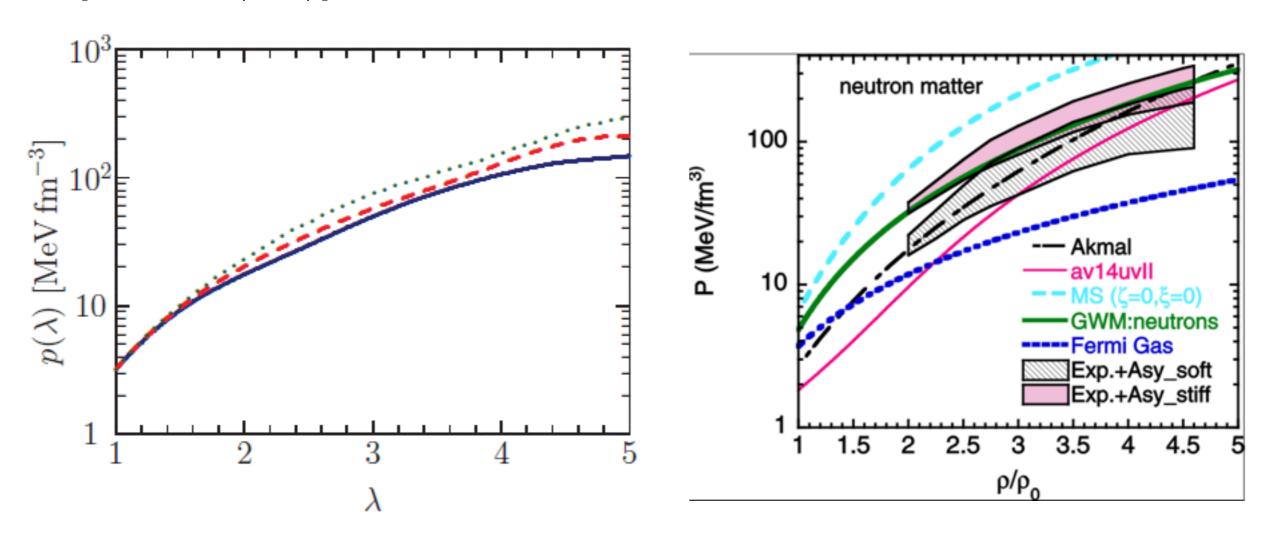
For comparison: Akmal-Pandharipande-Ravenhall (APR) predictions
[PRC 58, 1804 (1998)] are given by stars.
(From arigonna 2 body interactions + 3 body interactions)



### Asymmetric matter

#### **Pressure in neutron matter**

[UY, PRC88 (2013)]



For comparison: Right figure from

Danielewicz-Lacey-Lynch, Science 298, 1592 (2002).

(Deduced from experimental flow data and simulations studies)

### Asymmetric matter

#### Low density behaviour of symmetry energy

For comparison:

Trippa-Colo-Vigezzi [PRC 77, 061304 (2008)];

From analysis of GDR (208Pb).

Consequently one can predict in this model:

$$K_{\tau} = K_s - 6L_s$$

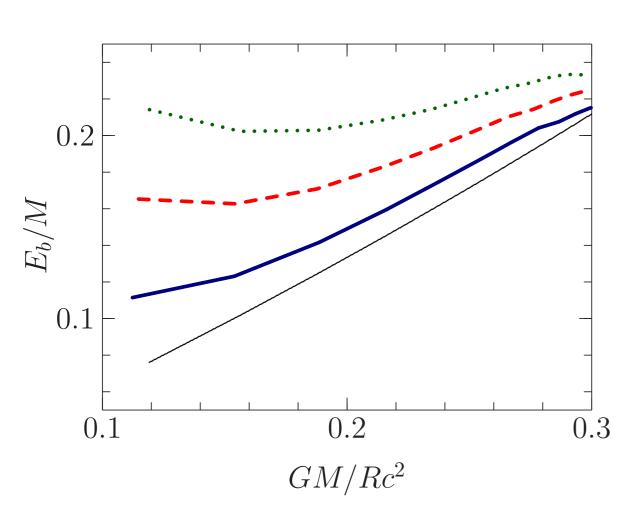
$$K_{0,2} = K_{\tau} - \frac{Q}{K_0} L_s$$

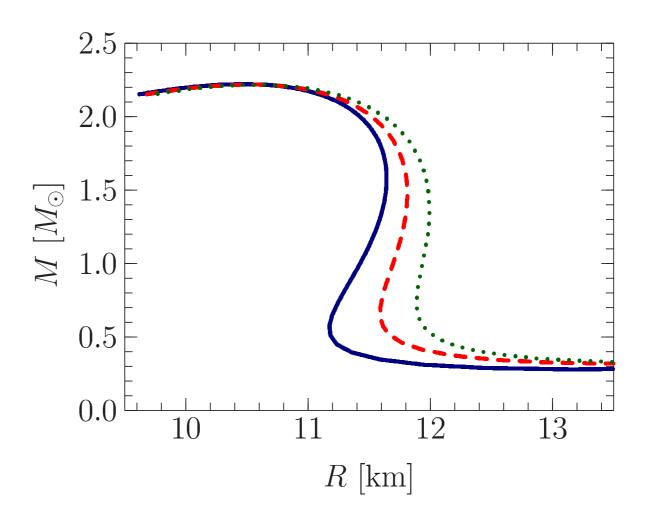
$$23.3 < \varepsilon_s (\rho = 0.1 \text{fm}^{-3}) < 24.9 \text{ MeV}$$

$\varepsilon_S( ho_0)$	$L_S$	$K_S$	$K_{ au}$	$K_{0,2}$	$\varepsilon_S(0.1 { m fm}^{-3})$
[MeV]	$[\mathrm{MeV}]$	[MeV]	$[\mathrm{MeV}]$	[MeV]	[MeV]
32	40	-181	-301	-257	25.15
32	50	-160	-310	-254	24.15
32	60	-126	-306	-239	23.22
32	70	-80	-290	-211	22.37
32	80	-21	-261	-172	21.57
32	90	50	-220	-119	20.82
32	100	134	-166	-55	20.13
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### Neutron stars

#### Neutron star properties [UY, arXiv:1506.06481[nucl-th]]





#### Neutron stars

#### Neutron star properties [UY, arXiv:1506.06481[nucl-th]]

TABLE III: Properties of the neutron stars from the different sets of parameters (see Tables I and II for the values of parameters):  $n_c$  is central number density,  $\rho_c$  is central energy-mass density, R is radius of the neutron star,  $M_{\text{max}}$  is possible maximal mass, A is number of baryons in the star,  $E_b$  is binding energy of the star. In the left panel we represent the neutron star properties corresponding to the maximal mass  $M_{\text{max}}$  and in right panel approximately 1.4 solar mass neutron star properties. The last two lines are results from the Ref. [21].

Set	$n_c$	$ ho_c$	R	$M_{ m max}$	A	$E_b$	$n_c$	$ ho_c$	R	M	A	$E_b$
	$[\mathrm{fm}^{-3}]$	$[10^{15}\mathrm{gr/cm}^3]$	[km]	$[M_{\odot}]$	$[10^{57}]$	$[10^{53}\mathrm{erg}]$	$[\mathrm{fm}^{-3}]$	$[10^{15}\mathrm{gr/cm}^3]$	[km]	$[M_{\odot}]$	$[10^{57}]$	$[10^{53}\mathrm{erg}]$
III-a	1.046	2.445	10.498	2.226	3.227	8.721	0.479	0.861	11.587	1.402	1.898	3.503
III-b	1.045	2.444	10.547	2.223	3.216	8.557	0.471	0.861	11.772	1.402	1.895	3.453
III-c	1.037	2.424	10.616	2.221	3.200	8.397	0.460	0.832	11.953	1.402	1.887	3.339
III-d	1.047	2.452	10.494	2.221	3.213	8.598	0.481	0.867	11.619	1.402	1.893	3.422
III-e	1.044	2.440	10.554	2.218	3.203	8.495	0.473	0.858	11.809	1.403	1.890	3.384
III-f	1.040	2.433	10.609	2.216	3.189	8.311	0.464	0.842	11.992	1.403	1.887	3.334
SLy230a [21]	1.15	2.69	10.25	2.10	2.99	7.07	0.508	0.925	11.8	1.4	1.85	2.60
SLy230b [21]	1.21	2.85	9.99	2.05	2.91	6.79	0.538	0.985	11.7	1.4	1.85	2.61

One can find density functionals from the reparametrization scheme

[UY, PRC88 (2013)]

Five density dependent parameters

Rearrangment (technical simplification)

$$1 + C_1 \frac{\rho}{\rho_0} = f_1 \left(\frac{\rho}{\rho_0}\right) \equiv \sqrt{\frac{\alpha_p^0}{\gamma_s}}$$

$$1 + C_2 \frac{\rho}{\rho_0} = f_2 \left(\frac{\rho}{\rho_0}\right) \equiv \frac{\alpha_s^{00}}{(\alpha_p^0)^2 \gamma_s}$$

$$1 + C_3 \frac{\rho}{\rho_0} = f_3 \left(\frac{\rho}{\rho_0}\right) \equiv \frac{(\alpha_p^0 \gamma_s)^{3/2}}{\alpha_s^{02}}$$

Shell modifications Core modifications 
$$F_{\pi,\tau} \to F_{\pi,\tau}^*, \quad e_{\tau} \to e_{\tau}^*, \quad m_{\pi} \to m_{\pi}^*, \\ F_{\pi,s} \to F_{\pi,s}^*, \quad e_{s} \to e_{s}^*$$

$$1 + C_2 \frac{\rho}{\rho_0} = f_2 \left(\frac{\rho}{\rho_0}\right) \equiv \frac{\alpha_s^{00}}{(\alpha_p^0)^2 \gamma_s} \qquad \frac{\alpha_e}{\gamma_s} = f_4 \left(\frac{\rho}{\rho_0}\right) \frac{\rho_n - \rho_p}{\rho_0} = \frac{C_4 \frac{\rho}{\rho_0}}{1 + C_5 \frac{\rho}{\rho_0}} \frac{\rho_n - \rho_p}{\rho_0}$$

$$1 + C_2 \frac{\rho}{\rho_0} = f_2 \left(\frac{\rho}{\rho_0}\right) \equiv \frac{(\alpha_p^0 \gamma_s)^{3/2}}{1 + C_5 \frac{\rho}{\rho_0}} = \frac{C_4 \frac{\rho}{\rho_0}}{1 + C_5 \frac{\rho}{\rho_0}} \frac{\rho_n - \rho_p}{\rho_0}$$

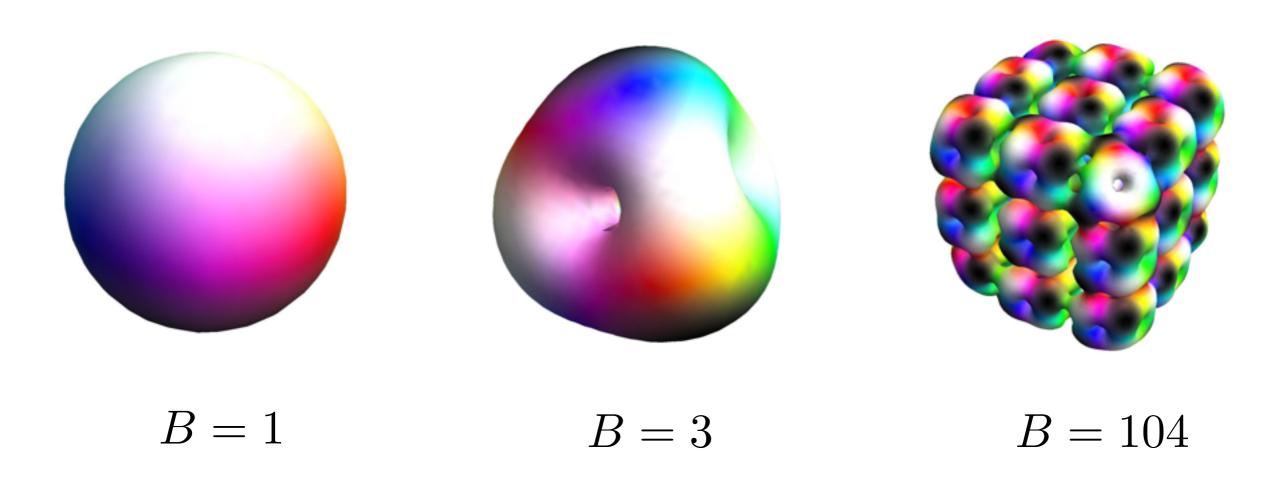
Low energy constants in nuclear at normal nuclear matter density  $\rho_0$ 

	Present model	ChPT [1]	QCD sum rules [2]		
$F_{\pi,t}^*$ / $F_{\pi}$	0.37	0.74	0.79		
$F_{\pi,s}^*$ / $F_{\pi}$	0.72	< 0	0.78		

<sup>[1]</sup> U. Meissner, J. Oller, A. Wirzba, Annals Phys. 297 (2002) 27.

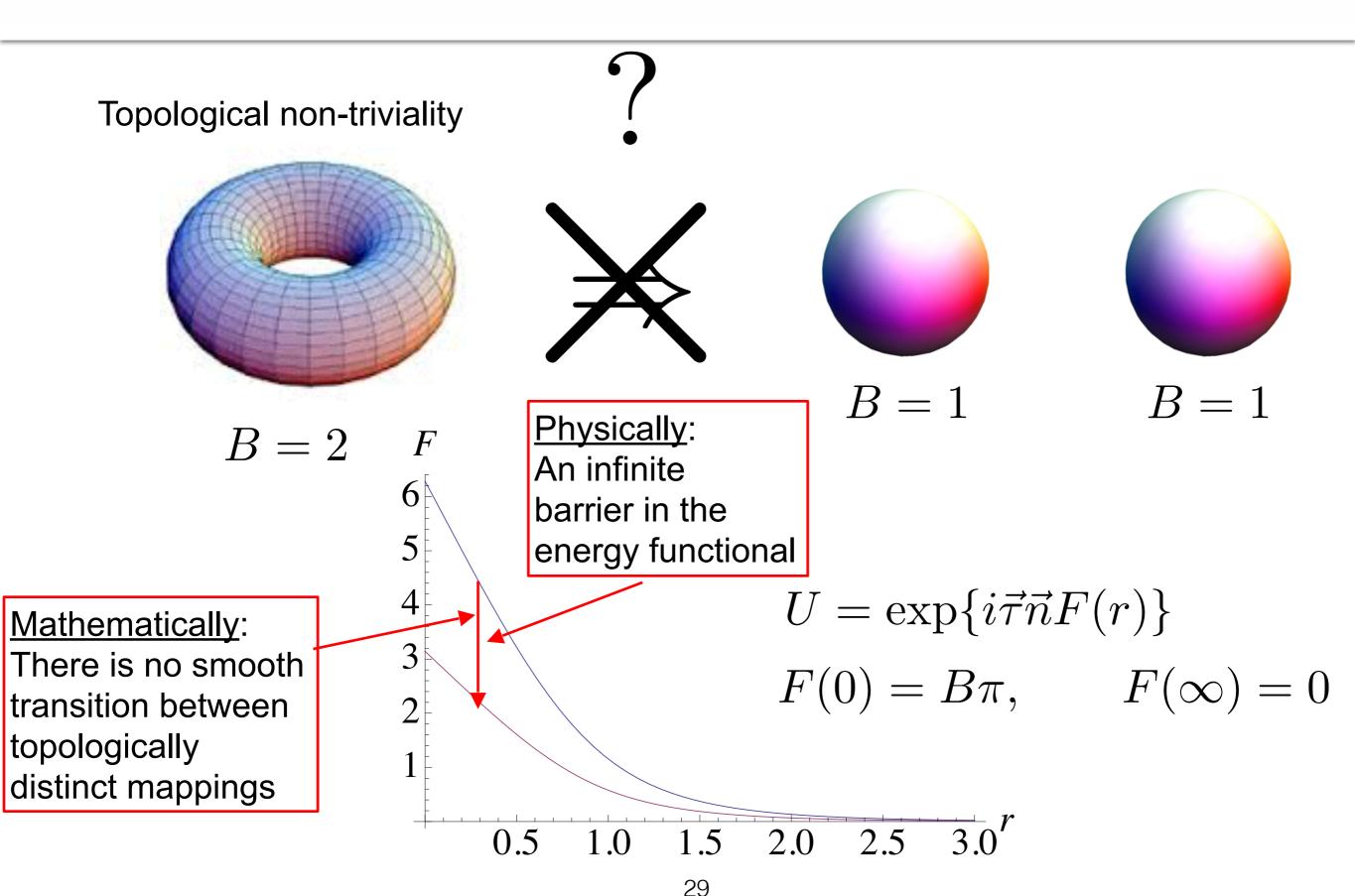
<sup>[2]</sup> H. Kim, M. Oka, NPA720 (2003) 368.

Surface of constant baryon density skyrmions [Feist, D.T.J. et al. Phys.Rev. D87 (2013)]

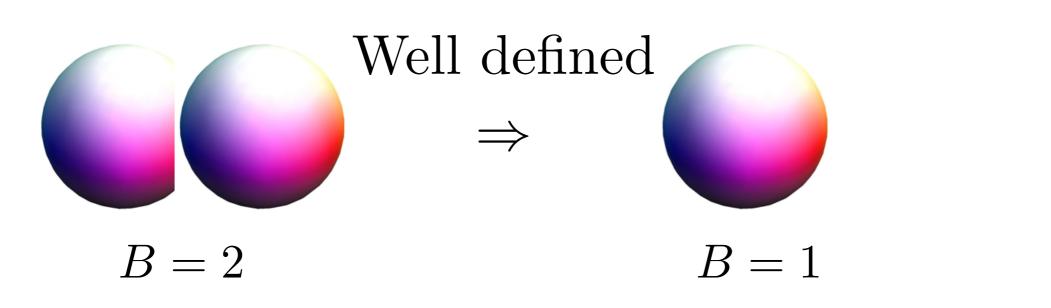


$$\mathcal{L} = \frac{F_{\pi}^{2}}{16} \operatorname{Tr} \left( \partial_{\mu} U \partial^{\mu} U^{\dagger} \right) - \frac{1}{16e^{2}} \operatorname{Tr} \left[ U^{\dagger} \partial_{\mu} U, U^{\dagger} \partial_{\nu} U \right]^{2}$$

Changes from a nucleus to a nucleus ("calibration")



Physically consistent picture (ansatz product)



Overlapping at small distances

One can reproduce SS potential and project it into NN potential.

$$U_{\text{system}} = U(\vec{r}_1)U(\vec{r}_2)$$

$$U = \exp\{i\vec{\tau}\vec{n}F(r)\}$$

$$F(0) = \pi, \qquad F(\infty) = 0$$

Well separated at large distances

B=1

#### Other approaches

- Classical crystalline structures
  - Cubic structure
    [M. Kutschera et al. Phys. Rev. Lett. 53 (1984)]
    [I. R. Klebanov, Nucl. Phys. B 262 (1985)]
  - Phase structure analysis using FCC crystal [H.-J. Lee et al. Nucl. Phys. A 723 (2003)]
- Skyrmions in hypersphere
  - System properties from the single skyrmion in hypersphere
     [N. S. Manton and P. J. Ruback, Phys. Lett. B 181 (1986)]

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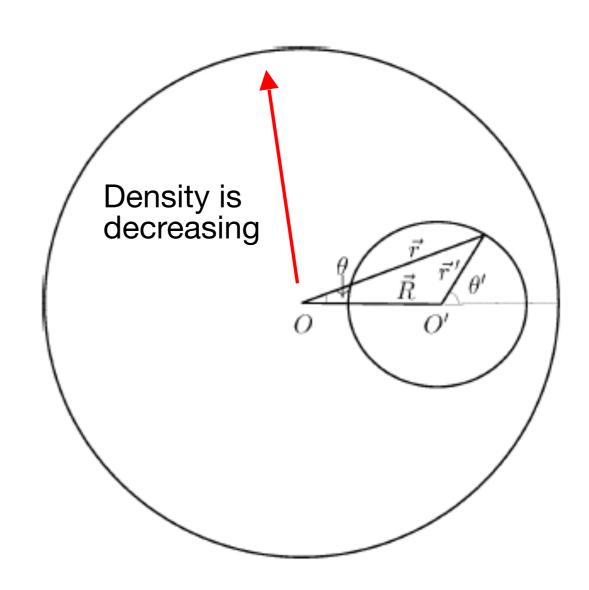
#### Finite nuclei

#### Nucleon in a nucleus (local density approximation)

The problem is coupled partial differential equations

$$f(F_{\tilde{r}\tilde{r}}, F_{\theta\theta}, F_{\tilde{r}}, F_{\theta}, \Theta_{\theta}, F, \Theta) = 0,$$
  
$$g(\Theta_{\theta\theta}, \Theta_{\theta}, F_{\tilde{r}}, F_{\theta}, \Theta, F) = 0,$$

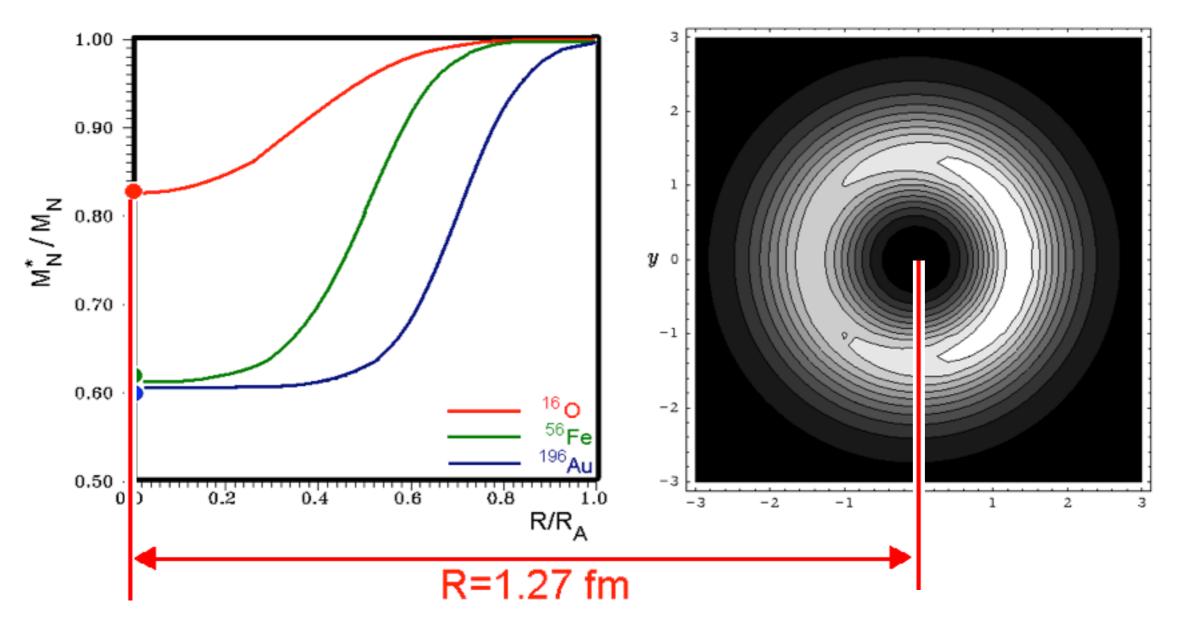
One can formulate the minimization method



#### Finite nuclei

# Nucleon may deform (core modification is not taken into account) [UY, et al., NPA200 (2002)]

The single baryon density distribution in finite nuclei. Ex: Skyrmion located at distance 1.25 fm from the centre of oxygen



### Summary and Outlook

- Within the applicability range, the model describes at same footing
  - the single hadrons properties
    - in separate state
    - in the community of their partners (EM and EMT form factors)
  - as well as the properties of that whole community
    - infinite nuclear matter properties (volume and symmetry energy properties)
    - matter under extreme conditions (neutron stars)
    - few and many nucleon systems (mirror nuclei, rare isotopes, halo nuclei,...)
    - nucleon knock-out reactions
    - Effective NN interactions

### Summary and Outlook

#### Applicability and extensions of the approach

- Nucleon tomography
  - [H.Ch. Kim, UY, PLB726 (2013)]
  - [J.H.Jung, UY, H.Ch.Kim, Jour. Phys. G41 (2014)]
  - [J.H.Jung, UY, H.Ch.Kim, P. Schweitzer. PRD89 (2014)]
- Nuclear matter [UY, UY, JKPS62 (2013); UY, PRC88 (2013)]
- Neutron stars [UY, arXiv:1506.06481[nucl-th]]
- Vector mesons in nuclear matter
  - [J.H.Jung, UY, H.Ch.Kim, PLB 723 (2013)]

Thank you very much for your attention!