



**BLTP Bogoliubov Laboratory of
Theoretical Physics**

Joint Institute for Nuclear Research, Dubna, Russia



Institute of Nuclear Physics, Tashkent, Uzbekistan

FUSION MECHANISM OF MASSIVE NUCLEI

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Content

- Motivation of theoretical study of the fusion mechanism.
- Advance and of the cold and hot fusion reactions used in the synthesis of superheavy elements.
- Comparison of the 4 reactions leading to formation ^{220}Th to study the role of the nuclear shell effects and impact parameter of collision in formation of the observed evaporation residues.
- Conclusions

Motivation of study

An unambiguous estimation of the fusion cross section is difficult task for the experimental and theoretical point view.

- 1) overlap of the characteristics of the reaction products formed in different channels causes ambiguity in reconstruction of the realistic mechanism of the given reaction channel;
- 2) Theoretical model to calculate the cross section of processes in heavy ion collisions can be developed on the base of the realistic concept about reaction mechanism.



Mendeleev periodic table of the elements (2015)



1																		18																
IA																		VIII A																
IIA																		VIII A																
1	2																3																	
H 1,00794 Hydrogen	Be 9,01218 Beryllium																He 4,0026 Helium																	
3	4																10																	
Li 6,941 Lithium	Mg 24,3050 Magnesium																Ne 20,1797 Neon																	
11	12																	18																
Na 22,989768 Sodium	Ca 40,078 Calcium		III B			IV B			V B			VI B			VII B			VIII B																
19	20		21		22		23		24		25		26		27		28		29		30		31		32		33		34		35		36	
K 39,0983 Potassium	Zn 65,38 Zinc		Sc 44,95591 Scandium		Ti 47,88 Titanium		V 50,9415 Vanadium		Cr 51,9961 Chromium		Mn 54,93805 Manganese		Fe 55,847 Iron		Co 58,93320 Cobalt		Ni 58,6934 Nickel		Cu 63,546 Copper		Zn 65,38 Zinc		Ga 69,723 Gallium		Ge 72,61 Germanium		As 74,92159 Arsenic		Se 78,96 Selenium		Br 79,904 Bromine		Kr 83,80 Krypton	
37	38		39		40		41		42		43		44		45		46		47		48		49		50		51		52		53		54	
Rb 85,4678 Rubidium	Sr 87,62 Strontium		Y 88,90585 Yttrium		Zr 91,224 Zirconium		Nb 92,90638 Niobium		Mo 95,94 Molybdenum		Tc [98] Technetium		Ru 101,07 Ruthenium		Rh 102,90550 Rhodium		Pd 106,42 Palladium		Ag 107,8682 Silver		Cd 112,411 Cadmium		In 114,818 Indium		Sn 118,710 Tin		Sb 121,757 Antimony		Te 127,60 Tellurium		I 126,90447 Iodine		Xe 131,29 Xenon	
55	56		57		72		73		74		75		76		77		78		79		80		81		82		83		84		85		86	
Cs 132,90543 Cesium	Ba 137,327 Barium		La 138,9055 Lanthanum		Hf 178,49 Hafnium		Ta 180,9479 Tantalum		W 183,84 Tungsten		Re 186,207 Rhenium		Os 190,23 Osmium		Ir 192,22 Iridium		Pt 195,084 Platinum		Au 196,96654 Gold		Hg 200,59 Mercury		Tl 204,3833 Thallium		Pb 207,2 Lead		Bi 208,98037 Bismuth		Po [209] Polonium		At [210] Astatine		Rn [222] Radon	
87	88		89		104		105		106		107		108		109		110		111		112		113		114		115		116		117		118	
Fr [223] Francium	Ra 226,025 Radium		Ac [227] Actinium		Rf [261] Rutherfordium		Db [262] Dubnium		Sg [266] Seaborgium		Bh [262] Bohrium		Hs [265] Hassium		Mt [268] Meitnerium		Ds [271] Darmstadtium		Rg [272] Roentgenium		Cn [285] Copernicium		Nh [286] Nihonium		Fl 114 Flerovium		Lv 116 Livermorium		Ts [294] Tennessine		Og [294] Oganesson			

Лантаноиды Lanthanides

58	59	60	61		62	63	64	65	66	67	68	69	70	71
Ce 140,115 Cerium	Pr 140,90765 Praseodymium	Nd 144,24 Neodymium	Pm [145] Promethium		Sm 150,36 Samarium	Eu 151,965 Europium	Gd 157,25 Gadolinium	Tb 158,92534 Terbium	Dy 162,50 Dysprosium	Ho 164,93032 Holmium	Er 167,26 Erbium	Tm 168,93421 Thulium	Yb 173,04 Ytterbium	Lu 174,967 Lutetium

1
H 1,00794 Hydrogen

Актиноиды Actinides

90	91	92	93	94	95	96	97	98	99	100	101	102	103
Th 232,0381 Thorium	Pa 231,03588 Protactinium	U 238,0289 Uranium	Np [237] Neptunium	Pu [244] Plutonium	Am [243] Americium	Cm [247] Curium	Bk [247] Berkelium	Cf [251] Californium	Es [252] Einsteinium	Fm [257] Fermium	Md [258] Mendelevium	No [259] Nobelium	Lr [262] Lawrencium

0 - символ
 1 - порядковый номер
 18 - порядковый номер группы
 13 - порядковый номер периода
 0,0001 - температура плавления °C
 0,0001 - температура кипения °C
 0,0001 - температура плавления °C
 0,0001 - температура кипения °C

- металлы
- неметаллы
- металлоиды
- благородные газы



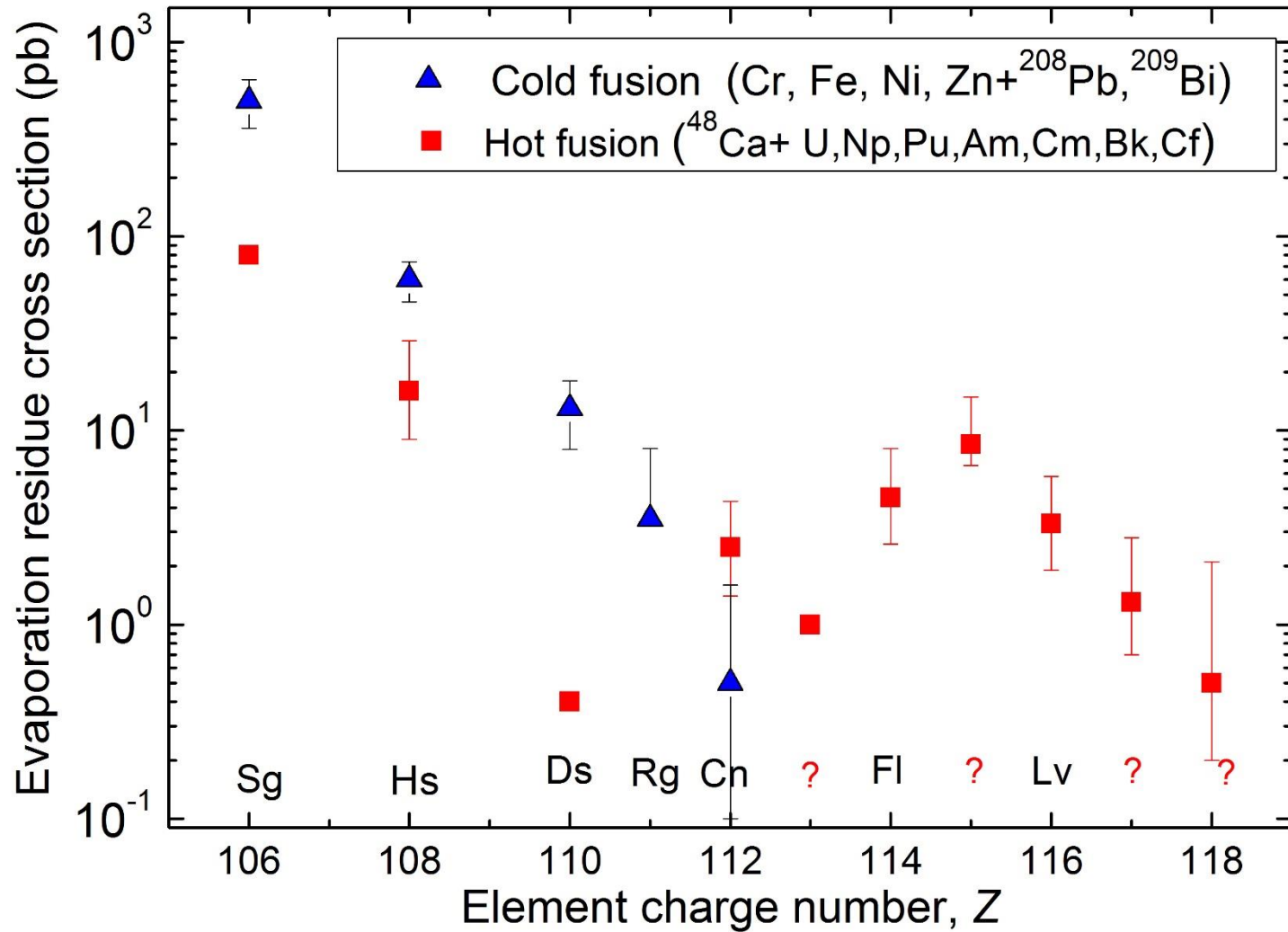
IB		IIB		IIIB		IIIV		IIIV		IIIV		IIIV		IIIV					
29	Цинк Zn 65,39 Zinc	30	Галлий Ga 69,723 Gallium	31	Германий Ge 72,61 Germanium	32	Мышьяк As 74,92159 Arsenic	33	Селен Se 78,96 Selenium	34	Бром Br 79,904 Bromine	35	Криpton	36	Аргон	37	Криpton		
45	Палладий Pd 106,42 Palladium	46	Серебро Ag 107,8682 Silver	47	Кадмий Cd 112,411 Cadmium	48	Индий In 114,818 Indium	49	Олово Sn 118,710 Tin	50	Сурьма Sb 121,757 Antimony	51	Теллур Te 127,60 Tellurium	52	Иод I 126,90447 Iodine	53	Ксенон Xe 131,29 Xenon	54	Криpton
77	Платина Pt 195,08 Platinum	78	Золото Au 196,96654 Gold	79	Ртуть Hg 200,59 Mercury	80	Таллий Tl 204,3833 Thallium	81	Свинец Pb 207,2 Lead	82	Висмут Bi 208,98037 Bismuth	83	Полоний Po [209] Polonium	84	Астат At [210] Astatine	85	Радон Rn [222] Radon	86	Радон
119	Дармштадтий Ds [269] Darmstadtium	110	Рентгений Rg [272] Roentgenium	111	Коперниций Cn [277] Copernicium	112	Нихоний	113	Флеровий Fl 288	114	Ливерморий Lv 292	115	Теннессин	116	Оганесон	117	Углерод	118	Оганесон



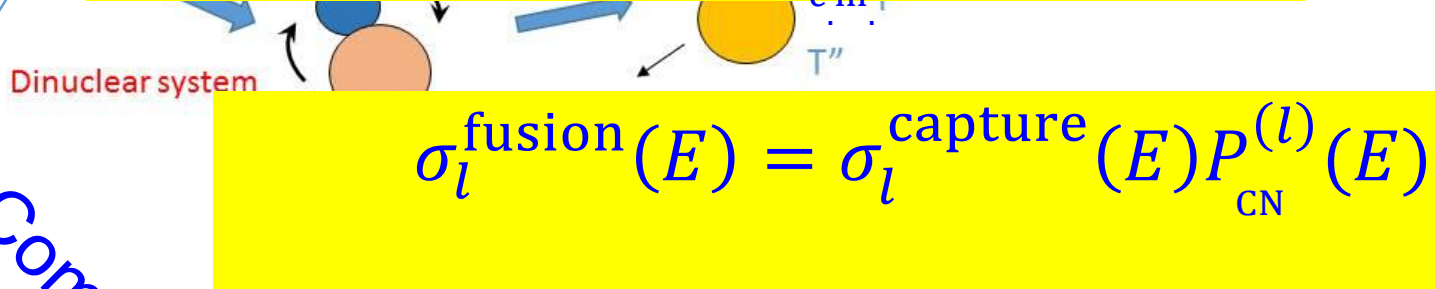
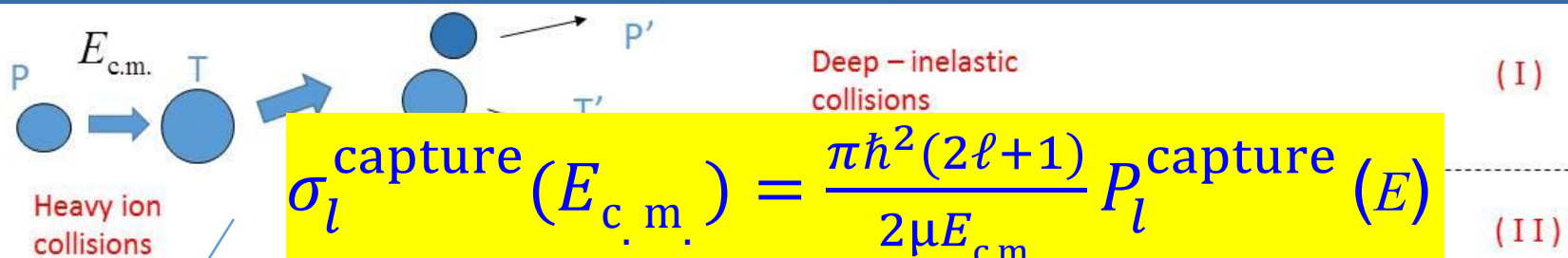
66	Гольмий Ho 164,93032 Holmium	67	Эрбий Er 167,26 Erbium	68	Тулий Tm 168,93421 Thulium	69	Иттербий Yb 173,04 Ytterbium	70	Лютеций Lu 174,967 Lutetium	71	Лютеций
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1	Водород H 1,00794 Hydrogen
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Synthesis of superheavy elements in the cold and hot fusion reactions.

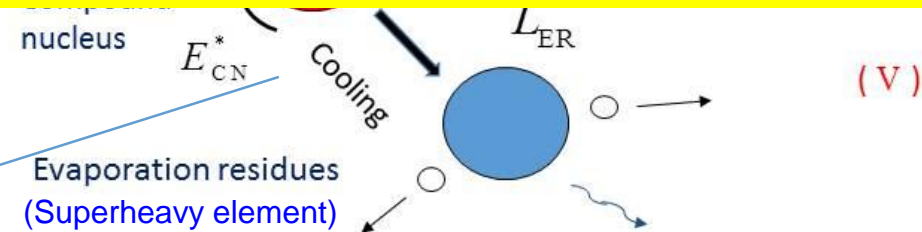


Reaction channels in heavy ion collisions at low energies

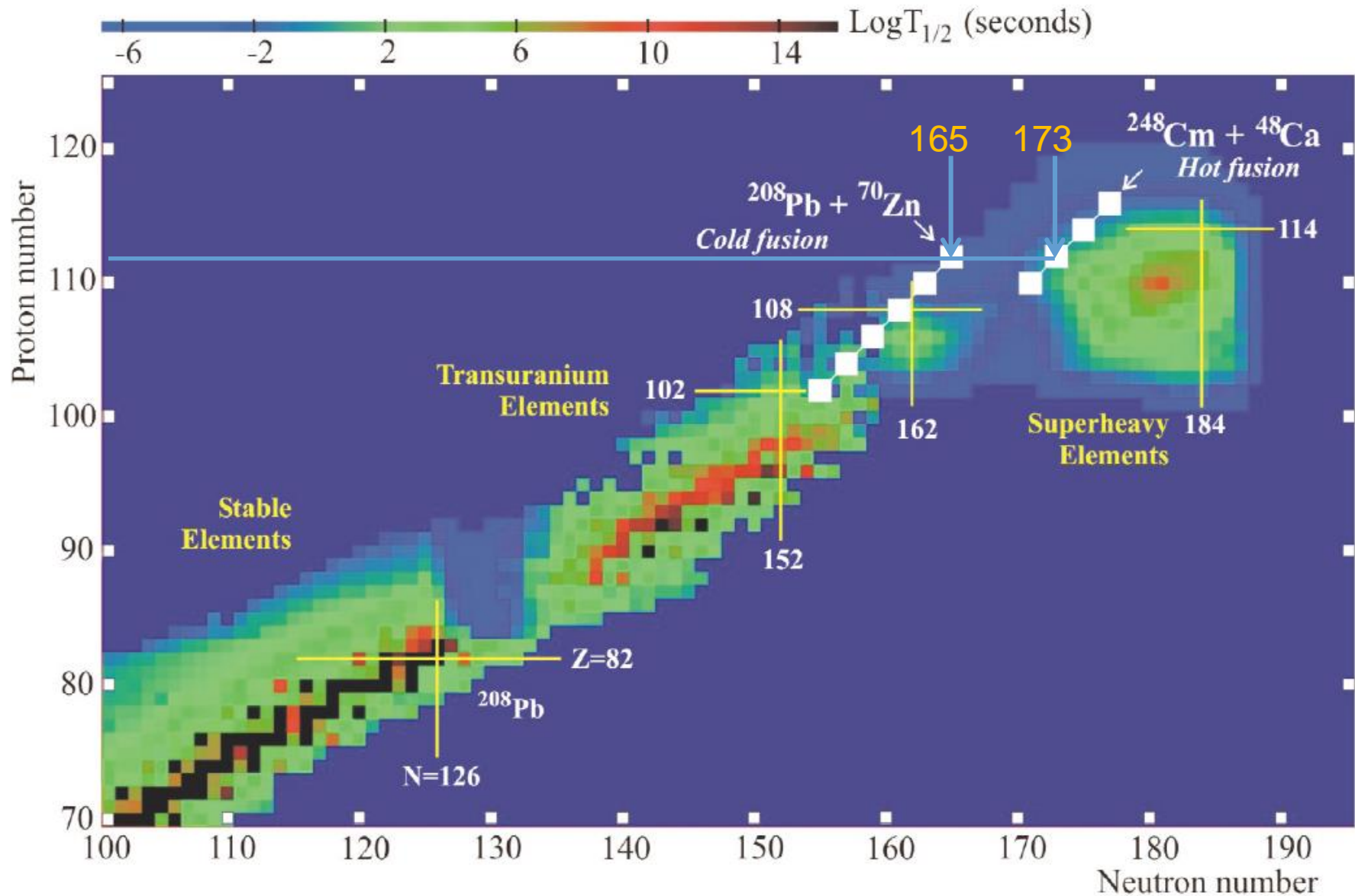


Complete fusion

$$\sigma_l^{\text{ER}}(E) = \sigma_l^{\text{capture}}(E) P_{\text{CN}}^{(l)}(E) W_{\text{survive}}^{(l)}(E)$$



Map of superheavy elements region



Transition of the advantage from the “cold” fusion reaction to the hot fusion reactions.

Cold fusion	Z _{CN}	N	B _f (MeV)	σ _{ER} (pb)	Hot fusion	N	B _f (MeV)	σ _{ER} (pb)
⁵⁴ Cr + ²⁰⁸ Pb [1]	106	156	6.05	500±140	²² Ne+ ²⁴⁸ Cm [3]	160	5.37	≈80
⁵⁸ Fe+ ²⁰⁸ Pb [1]	108	156	5.47	60±14	²⁶ Mg+ ²⁴⁸ Cm [4]	161	6.15	7 ⁺³ ₋₃
-	108	-	-	-	⁴⁸ Ca+ ²²⁶ Ra [5]	162	6.42	16 ⁺¹³ ₋₇
⁶⁴ Ni+ ²⁰⁸ Pb [1]	110	162	5.83	13±5	³⁴ S+ ²⁴⁴ Pu [6]	163	5.52	≈0.4
⁶⁴ Ni+ ²⁰⁹ Bi [1]	111	162	5.52	3.5 ^{+4.6} _{-3.5}	-	-	-	-
⁷⁰ Zn+ ²⁰⁸ Pb [1]	112	165	4.29	0.5 ^{+1.1} _{-0.4}	⁴⁸ Ca+ ²³⁸ U [7]	171	4.01	2.5 ^{+1.8} _{-1.1}
⁷⁰ Zn+ ²⁰⁹ Bi [2]	113	165	4.53	0.022 ^{+0.020} _{-0.013}	⁴⁸ Ca+ ²³⁷ Np [8]	169	3.93	1.0
	114	-	-	-	⁴⁸ Ca+ ²⁴⁴ Pu [7]	174	5.53	4.5 ^{+3.6} _{-1.9}
	115	-	-	-	⁴⁸ Ca+ ²⁴³ Am [9]	173	5.40	8.5 ^{+6.4} _{-3.7}
	116	-	-	-	⁴⁸ Ca+ ²⁴⁸ Cm [7]	176	6.22	3.3 ^{+2.5} _{-1.4}
	117	-	-	-	⁴⁸ Ca+ ²⁴⁹ Bk [10]	176	6.11	3.6 ^{+6.1} _{-2.5}
	118	-	-	-	⁴⁸ Ca+ ²⁴⁹ Cf [10]	176	5.99	0.5 ^{+1.6} _{-0.3}

1. S. Hofmann, Rev. of Mod. Phys. 2000. V.72. P.733.
2. K. Morita et al., J. Phys. Soc. of Jap, 81, (2012) 103201 .
3. Yu. A. Lazarev, Phys. Rev. Lett. 1994. V.73. P. 624.
4. J. Dvorak Phys. Rev. Lett. 2008. V.100. P.132503.
5. Yu. Ts. Oganessian Phys. Rev. C. 2013. V.87. P.034605.

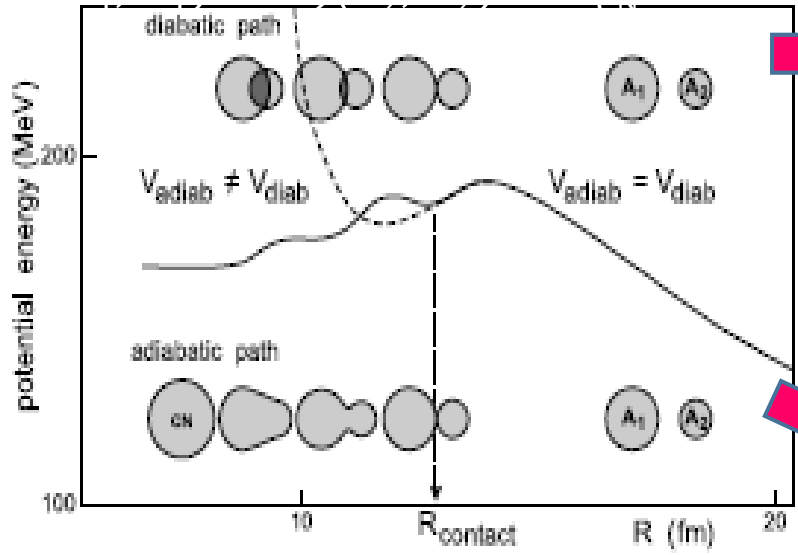
6. Yu. A. Lazarev Phys. Rev. .C. 1996. V.54. P.620.
7. Yu. Ts. Oganessian PRC 70, 2004. P.064609.
8. Yu. Ts. Oganessian, PRC 76. 2007, P.011601.
9. Yu. Ts. Oganessian PRC 69, 2004, P.021601(R) , PRC 87, 2013, P.014302.
10. Yu. Ts. Oganessian PRL 109, 2012, P.162501.



Two main concepts for complete fusion of massive nuclei



$$U(Z_1, A_1, Z_2, A_2, R, Q, g, g) = V_{\text{int}}(Z_1, A_1, Z_2, A_2, R) + V_{\text{ext}}(Z_1, A_1, Z_2, A_2, R) + V_{\text{ext}}(Z_2, A_2, Z_1, A_1, R)$$

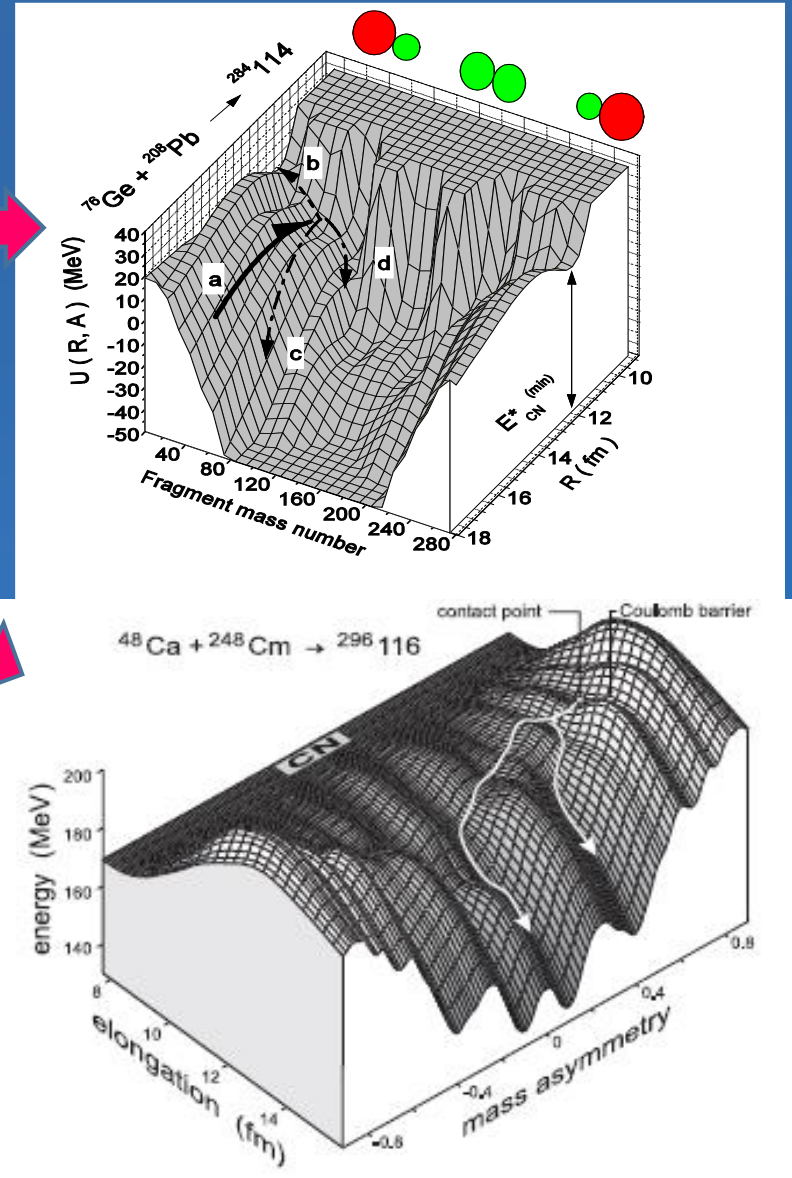


$$V = V_{\text{diab}}[1 - f(t)] + V_{\text{adiab}}f(t)$$

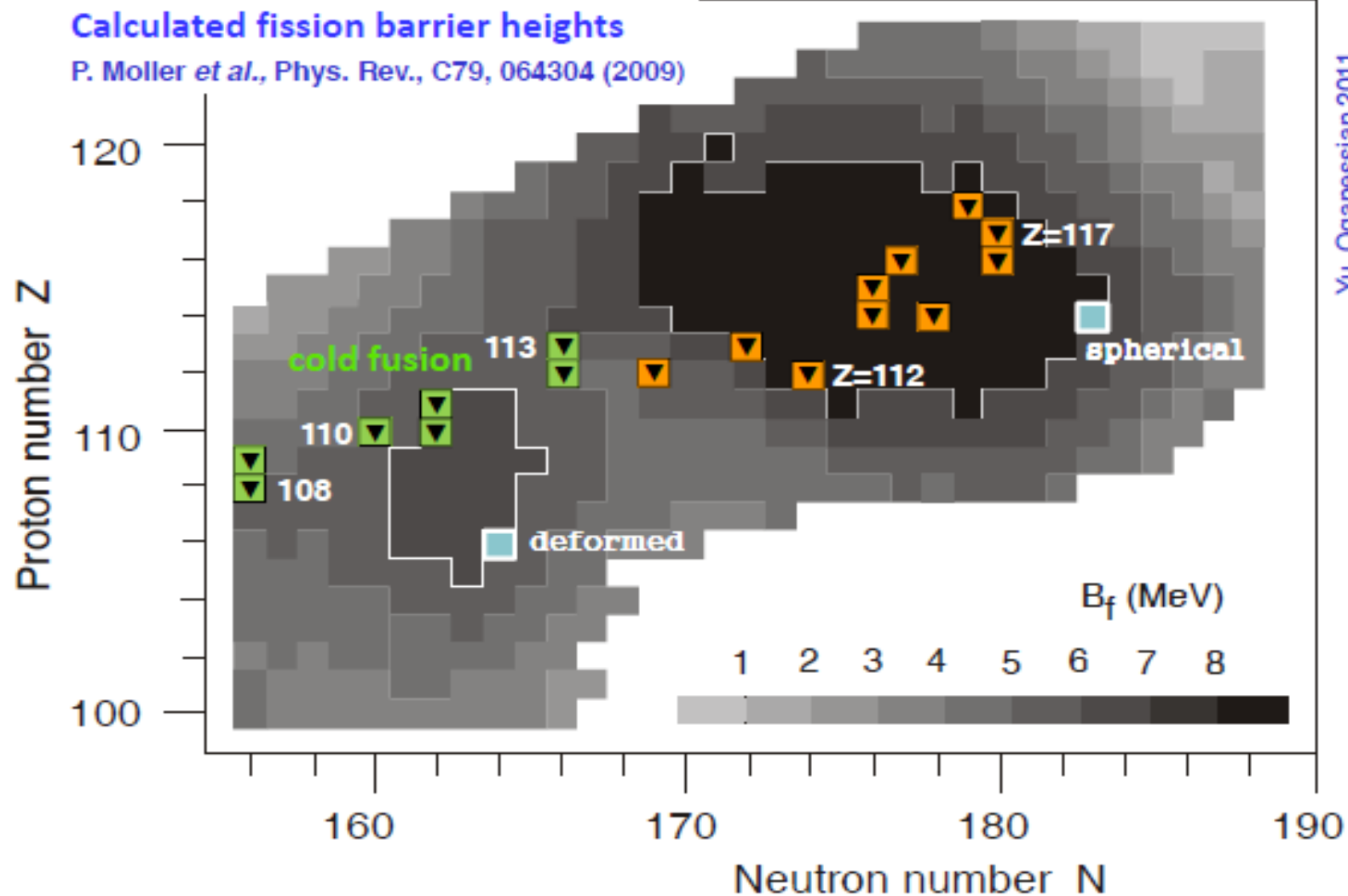
$$\tau_{\text{relax}} \sim 10^{-21} \text{ s (see, e.g., [24])}$$

$$f(t = 0) = 0, f(t \gg \tau_{\text{relax}}) = 1.$$

- [24] Bertsch G F 1978 *Z. Phys. A* **289** 103
 Cassing W and Nörenberg W 1983 *Nucl. Phys. A* **401** 467



Role of fission barrier in synthesis of the superheavy elements



Yu. Oganessian 2011

Fission barriers calculated by macroscopic-microscopic model:
 M. Kowal, P. Jachimowicz, and A. Sobiczewski, Phys. Rev. C 82, 014303 (2010)

$$B_{\text{fis}}(J, T) = cB_{\text{fis}}^{LD}(J) - h(T)q(J)\delta W,$$

$$h(T) = \frac{1}{1 + \exp[(T - T_0)/d]}$$

$$T_0 = 1.16 \text{ MeV}, \quad d = 0.3 \text{ MeV}$$

$$q(J) = \frac{1}{1 + \exp[(J - J_{1/2})/\Delta J]}$$

$$J_{1/2} = 20\hbar, \quad \Delta J = 3\hbar,$$

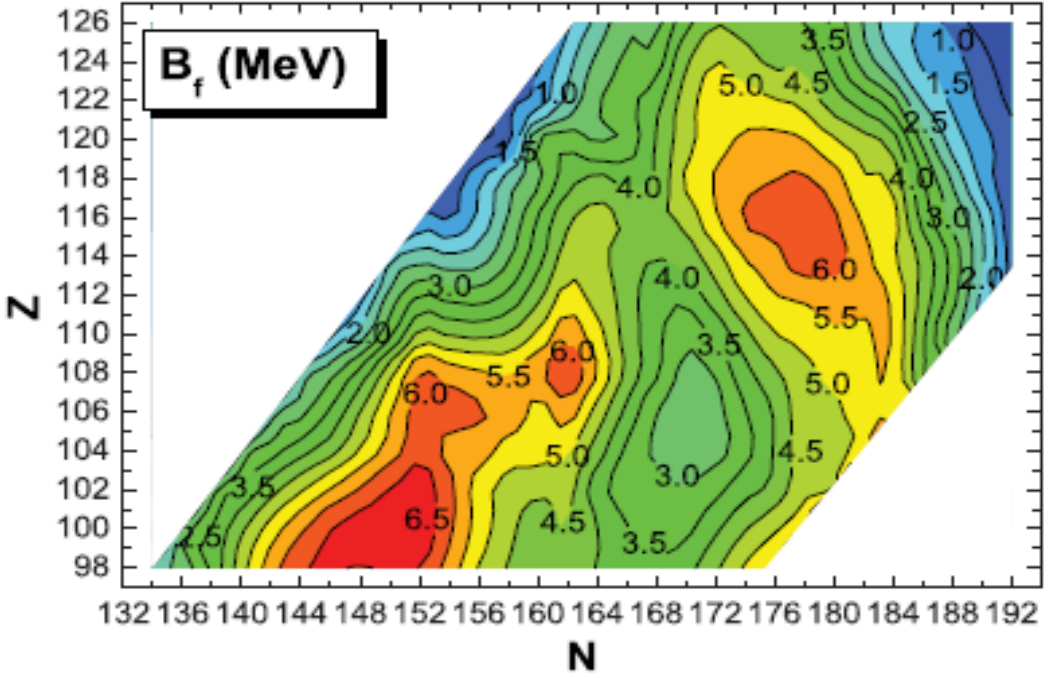
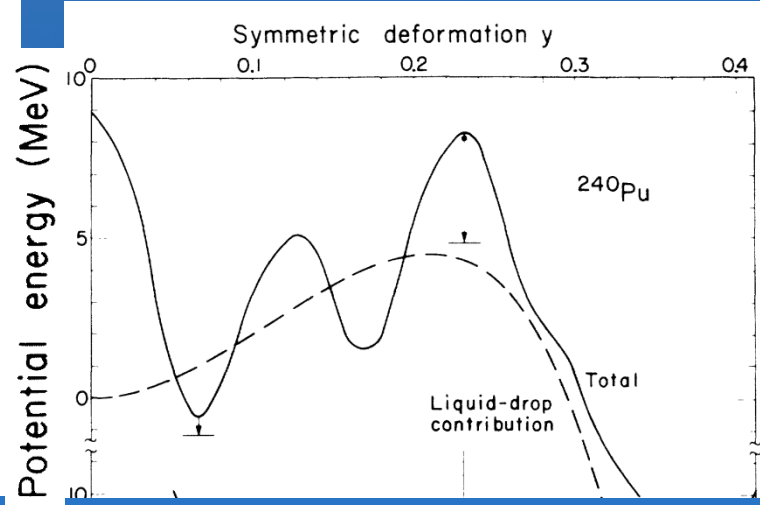


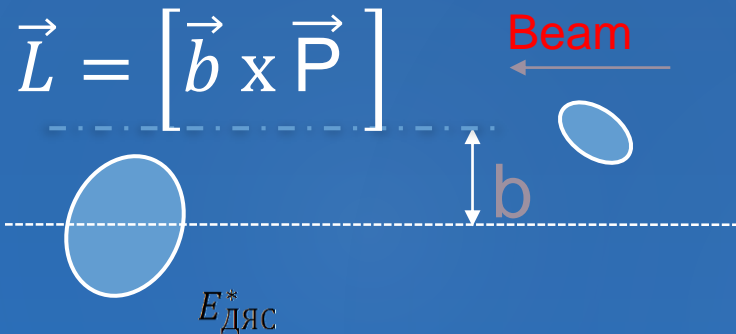
FIG. 6. (Color online) Contour map of calculated fission barrier heights B_f for even-even superheavy nuclei.



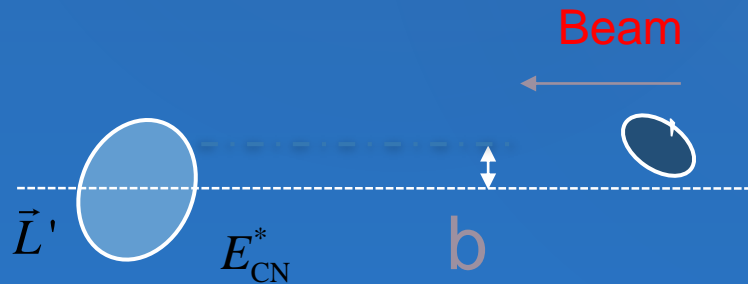


- I. Deep inelastic collisions:
- 1) Partial momentum transfer;
 - 2) There is not equilibrium of energy distribution and mass distribution;
 - 3) Anisotropic angular distribution

Formation of the dinuclear system (Capture reactions)

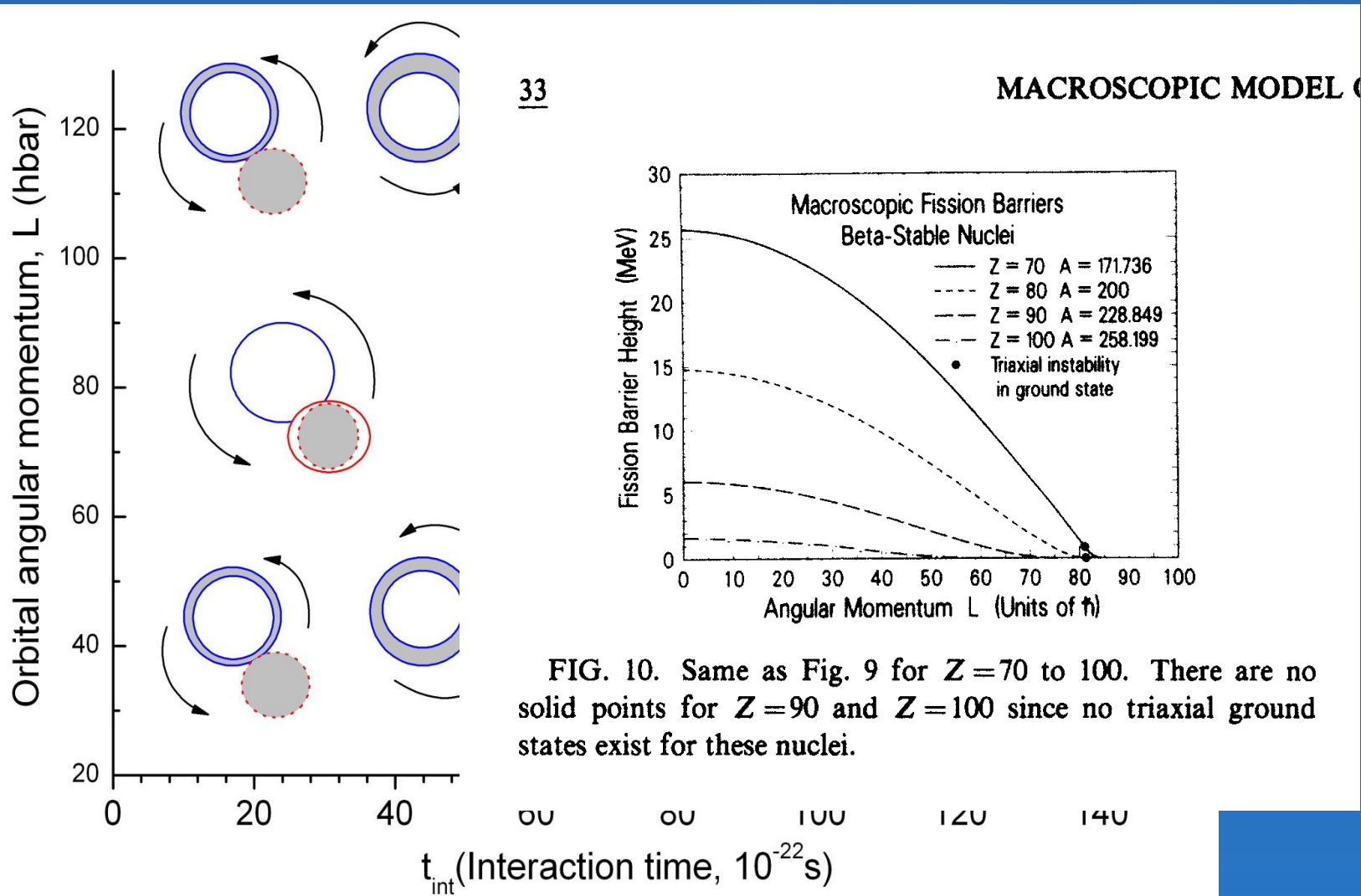


- II. Quasifission:
- 1) Full momentum transfer;
 - 2) Equilibrium of energy distribution and mass distribution;
 - 3) Anisotropic and isotropic angular distributions.



- III. Compound nucleus formation:
- 1) Full momentum transfer;
 - 2) Equilibrium of energy distribution and mass distribution;
 - 3) Isotropic angular distributions.


Mechanisms of the reaction following capture (capture means formation of dinuclear system): Fusion-fission, quasifission and fast-fission.




The methods of calculation of the capture and Fusion cross section in the dinuclear system approach.

Main assumptions:

- 1) the shell effects does not allow to fuse nuclei immediately;
- 2) The hindrance to fusion is determined by the intrinsic fusion barrier B_{fus}^* which is determined from the landscape of the potential energy surface of dinuclear system;
- 3) the interacting nuclei can be deformed and nucleon exchange between them takes place allowing dinuclear system to be transformed into compound nucleus or to populate shapes corresponding minimal values of the potential energy surface;
- 4) The lifetime of dinuclear system is determined by its ¹⁵excitation energy E_{DNS}^* and quasifission barrier B_{qf} .



Theoretical calculation of evaporation residue cross section (synthesis of superheavy element).



$$\sigma_{ER} = \sum_{l=0}^{l_d} (2l + 1) \sigma_l^{fus}(E, l) W_{surv}(E, l)$$

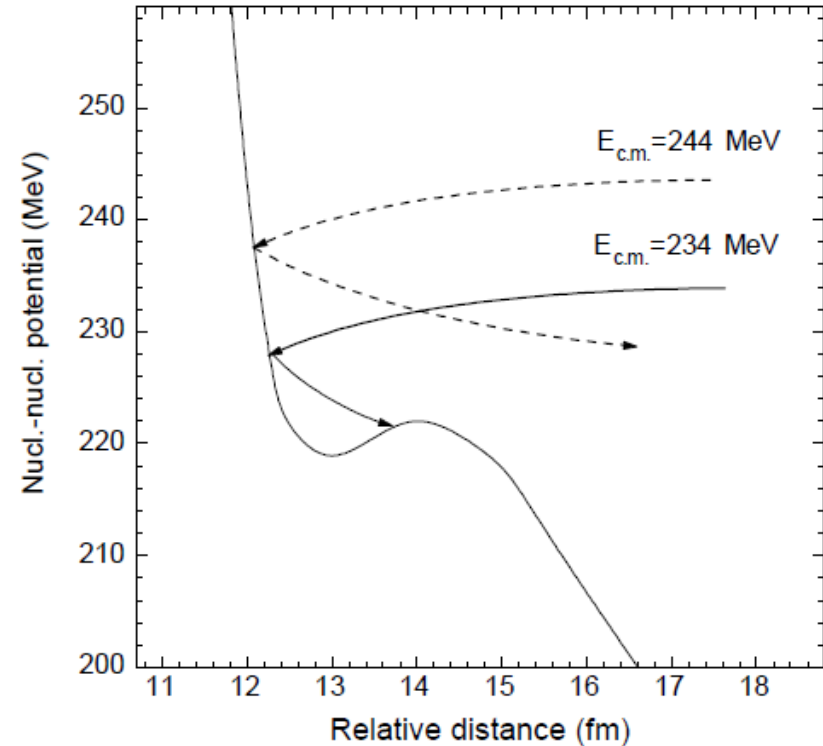
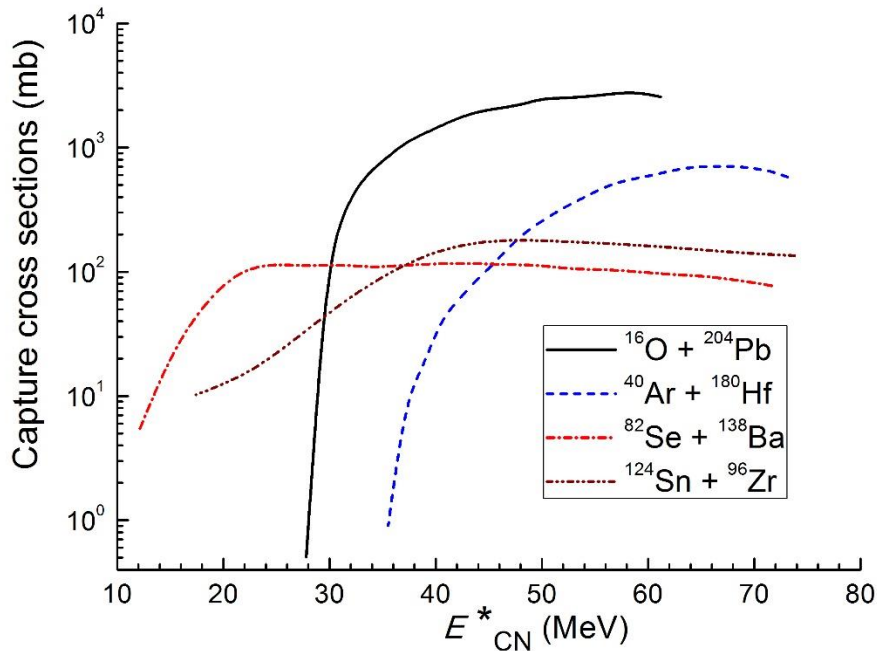
$$\sigma_l^{fus}(E, l) = \sigma_l^{capture}(E, l) P_{CN}(E, l)$$

$P_{CN}(E, l)$ is fusion probability which calculated by diffusion-dissipative method, Y. Aritomo, Phys.Rev.C65, 014607 (2001) or G.G. Adamian, N.V. Antonenko, and W. Scheid, Eur. Phys. J. A 41, 235 (2009);

A. K. Nasirov, G. Giardina, S.Hofmann, et al. Phys. Rev. C **79**, 024606 (2009).

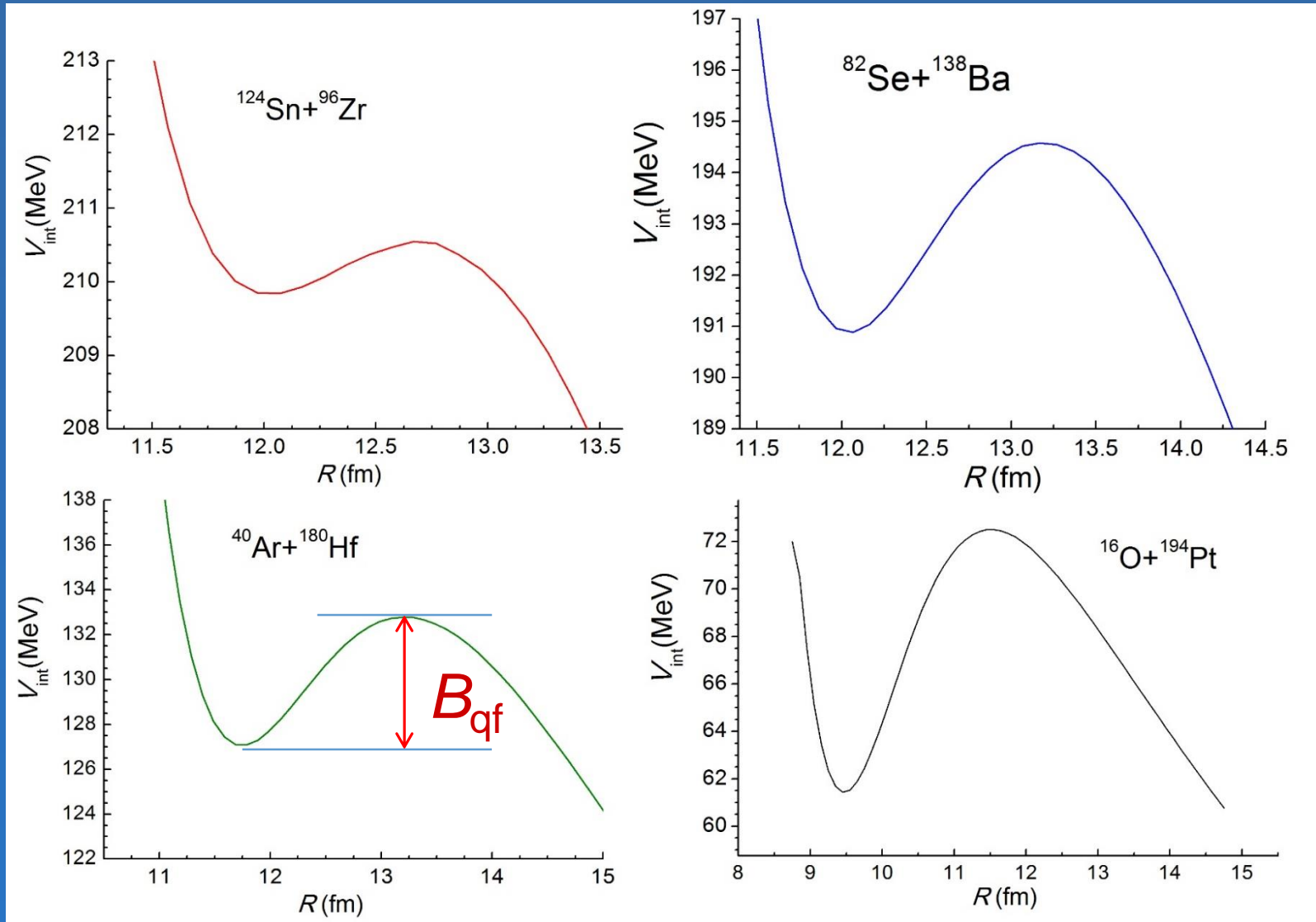
$\sigma_l^{capture}$ is capture probability, which calculated in different theoretical models by different way.

Comparison of the capture cross sections of the reactions leading to ^{220}Th



$$\mu(R)\ddot{R} + \gamma_R(R)\dot{R}(t) = -\frac{\partial V(R)}{\partial R} - \dot{R}^2 \frac{\partial \mu(R)}{\partial R}$$

Comparison of the potential wells of the nucleus-nucleus interaction for reactions leading to formation of ^{220}Th .





Equations of motion used to find the capture of projectile by target-nucleus



$$\mu(R)\ddot{R} + \gamma_R(R)\dot{R}(t) = -\frac{\partial V(R)}{\partial R} - \dot{R}^2 \frac{\partial \mu(R)}{\partial R}$$

$$\mu(R) = \delta\mu(R) + m_0 A_T A_P / A_{\text{tot}}$$

$$\times \left(1 - \frac{2}{A_{\text{tot}}} \int \frac{\rho_1^{(0)}(\mathbf{r} - \mathbf{r}_1) \rho_2^{(0)}(\mathbf{r} - \mathbf{r}_2)}{\rho_1^{(0)}(\mathbf{r} - \mathbf{r}_1) + \rho_2^{(0)}(\mathbf{r} - \mathbf{r}_2)} d^3\mathbf{r} \right),$$

$$\frac{dL}{dt} = \gamma_\theta(R)R(t) \left[\dot{\theta}R(t) - \dot{\theta}_1 R_{1\text{eff}} - \dot{\theta}_2 R_{2\text{eff}} \right]$$

$$L_0 = J_R \dot{\theta} + J_1 \dot{\theta}_1 + J_2 \dot{\theta}_2,$$

$$E_{\text{rot}} = \frac{J_R \theta^2}{2} + \frac{J_1 \theta_1^2}{2} + \frac{J_2 \theta_2^2}{2}$$



Hamiltonian for calculation of the transport coefficients of collective motion

$$H = H_{\text{coll}}(Z_1, A_1, Z_2, A_2, R, \alpha_1, \alpha_2, \beta_1, \beta_2) + H_{\text{micr}}\left(\{\varepsilon_{i_1}, n_{i_1}\}, \{\varepsilon_{i_2}, n_{i_2}\}\right) + \delta V \quad (1)$$

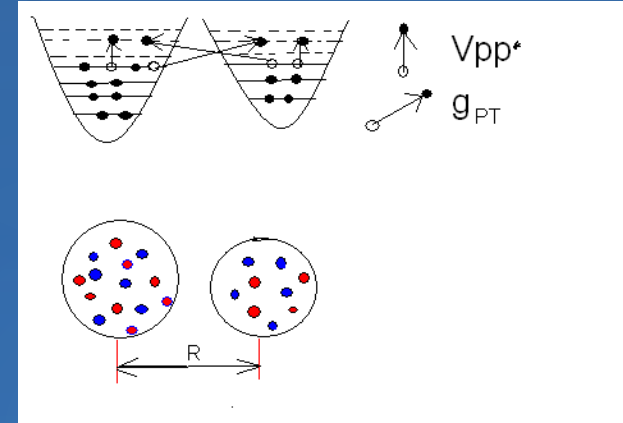
where

$$H_{\text{coll}} = \frac{P^2}{2\mu} + U(Z_1, A_1, Z_2, A_2, R, \alpha_1, \alpha_2, \beta_1, \beta_2) - \text{for the relative motion of nuclei}; \quad (2)$$

$$H_{\text{micr}} = \sum_{i_P} \varepsilon_{i_P} \hat{a}_{i_P}^+ \hat{a}_{i_P} + \sum_{i_T} \varepsilon_{i_T} \hat{a}_{i_T}^+ \hat{a}_{i_T} - \text{for nucleons of nuclei}; \quad (3)$$

$$\delta V = \sum_{i_P, j_T} g_{i_P j_T}(R) (\hat{a}_{i_P}^+ \hat{a}_{j_T} + \hat{a}_{j_T}^+ \hat{a}_{i_P}) + \sum_{i_P, j_T} \kappa_{i_P j_T}(R) (\hat{a}_{i_P}^+ \hat{a}_{j_T} + \hat{a}_{j_T}^+ \hat{a}_{i_P}) + \sum_{i_P, j_P} \Lambda_{i_P j_P}^{(T)}(R) \hat{a}_{i_P}^+ \hat{a}_{j_P} + \sum_{i_T, j_T} \Lambda_{i_T j_T}^{(P)}(R) \hat{a}_{i_T}^+ \hat{a}_{j_T} - \text{nucleon exchange between nuclei and particle-hole excitations in nuclei}; \quad (4)$$

$g_{i_P j_T}$, $\kappa_{i_P j_T}$ and $\Lambda_{i_T j_T}^{(P)}$ – matrix elements of nucleon exchange between nuclei and particle – hole excitations in them caused by meanfield of partner nucleus.



G.G. Adamian, et al. Phys. Rev. C56 No.2, (1997) p.373-380
A.K. Nasirov, Thesis of the Doctor of Science, 2004, INP, Tashkent

Nucleus-nucleus interaction potential

$$V_C(R, \alpha_1, \alpha_2) = \frac{Z_1 Z_2}{R} e^2 + \frac{Z_1 Z_2}{R^3} e^2 \left\{ \left(\frac{9}{20\pi} \right)^{1/2} \sum_{i=1}^2 R_{0i}^2 \beta_2^{(i)} P_2(\cos \alpha_i) + \frac{3}{7\pi} \sum_{i=1}^2 R_{0i}^2 \left[\beta_2^{(i)} P_2(\cos \alpha_i) \right]^2 \right\}$$

$$V_{nucl}(R, \alpha_1, \alpha_2) = \int \rho_1^{(0)}(\vec{r} - \vec{R}) f_{eff} \left[\rho_1^{(0)} + \rho_2^{(0)} \right] \rho_2^{(0)}(\vec{r}) d^3 \vec{r}$$

$$\rho_i^{(0)}(\vec{r}, \vec{R}_i, \alpha_i, \theta_i, \beta_2^{(i)}) = \left\{ 1 + \exp \left[\frac{|\vec{r} - \vec{R}_i(t)| - R_{oi} (1 + \beta_2^{(i)} Y_{20}(\theta_i, \alpha_i))}{a} \right] \right\}^{-1}$$

$$21 \quad V_{rot} = \hbar^2 \frac{l(l+1)}{2\mu [R(\alpha_1, \alpha_2)]^2 + J_1 + J_2}$$

Density dependent effective nucleon-nucleon forces

$$f_{eff}(r) = C_0 \left(f + f' \vec{\tau}_1 \vec{\tau}_2 + (g + g' \vec{\tau}_1 \vec{\tau}_2) \vec{\sigma}_1 \vec{\sigma}_2 \right)$$

$$f(r) = f^{ex} + (f^{in} - f^{ex}) \frac{\rho(r)}{\rho(0)}$$

Constants	Versions	
	I	II
	1	2
f_{in}	-0,09	+0,09
f_{ex}	-2,23	-2,59
f'_{in}	0,89	0,42
f'_{ex}	0,06	0,54
g	0,7	0,7
g'	0,83	0,83

$C_0 = 300 \text{ MeV fm}^{-3}$

The values of the constants of the effective nucleon-nucleon forces from the textbook A.B. Migdal, “*Theory of the Finite Fermi-Systems and properties of Atomic Nuclei*”, Moscow, Nauka, 1983. The constants of version II were used in our calculations.

Expressions for the friction coefficients

$$\gamma_R(R(t)) = \sum_{i,i'} \left| \frac{\partial V_{ii'}(R(t))}{\partial R} \right|^2 B_{ii'}^{(1)}(t), \quad (\text{B.1})$$

$$\gamma_\theta(R(t)) = \frac{1}{R^2} \sum_{i,i'} \left| \frac{\partial V_{ii'}(R(t))}{\partial \theta} \right|^2 B_{ii'}^{(1)}(t), \quad (\text{B.2})$$

and the dynamic contribution to the nucleus-nucleus potential

$$\delta V(R(t)) = \sum_{i,i'} \left| \frac{\partial V_{ii'}(R(t))}{\partial R} \right|^2 B_{ii'}^{(0)}(t), \quad (\text{B.3})$$

$$B_{ik}^{(n)}(t) = \frac{2}{\hbar} \int_0^t dt' (t - t')^n \exp\left(\frac{t' - t}{\tau_{ik}}\right) \times \sin[\omega_{ik}(\mathbf{R}(t'))(t - t')] [\tilde{n}_k(t') - \tilde{n}_i(t')], \quad (\text{B.4})$$

$$\hbar\omega_{ik} = \epsilon_i + \Lambda_{ii} - \epsilon_k - \Lambda_{kk}. \quad (\text{B.5})$$

Calculation of the competition between complete fusion and quasifission: $P_{cn}(E_{DNS}, L)$

$$P_{CN}(E_{DNS}^*, \ell) = \sum_{Z_{sym}}^{Z_{max}} Y_Z(E_{DNS}^*, \ell) P_{CN}^{(Z)}(E_{DNS}^*, \ell)$$

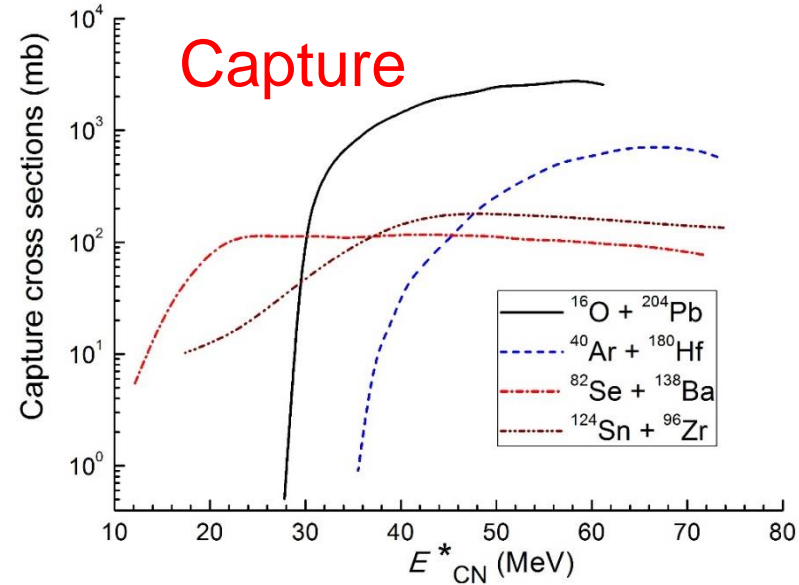
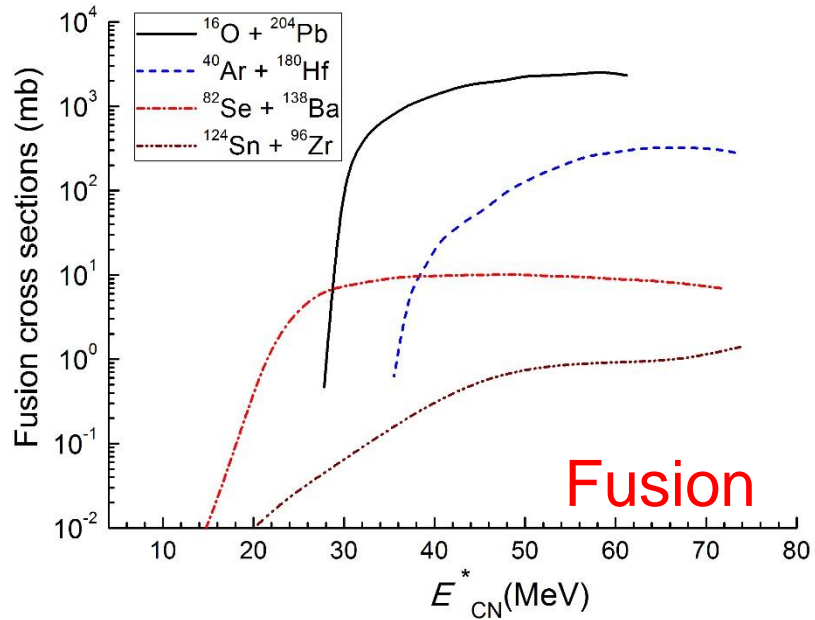
where

$$P_{CN}^{(Z)}(E_{DNS}^*, \ell) = \frac{\rho(E_{DNS}^*(Z) - B_{fus}^*(Z), \ell)}{\rho(E_{DNS}^*(Z) - B_{fus}^*(Z), \ell) + \rho(E_{DNS}^*(Z) - B_{qf}^*(Z), \ell) + \rho(E_{DNS}^*(Z) - B_{sym}^*(Z), \ell)}$$

$$\begin{aligned} \frac{\partial}{\partial t} Y_Z(E_Z^*, \ell, t) &= \Delta_{Z+1}^{(-)} Y_{Z+1}(E_Z^*, \ell, t) + \Delta_{Z-1}^{(+)} Y_{Z-1}(E_Z^*, \ell, t) \\ &\quad - (\Delta_Z^{(-)} + \Delta_Z^{(+)} + \Lambda_Z^{qf}) Y_Z(E_Z^*, \ell, t) \end{aligned}$$

for $Z = 2, 3, \dots, Z_{tot} - 2$

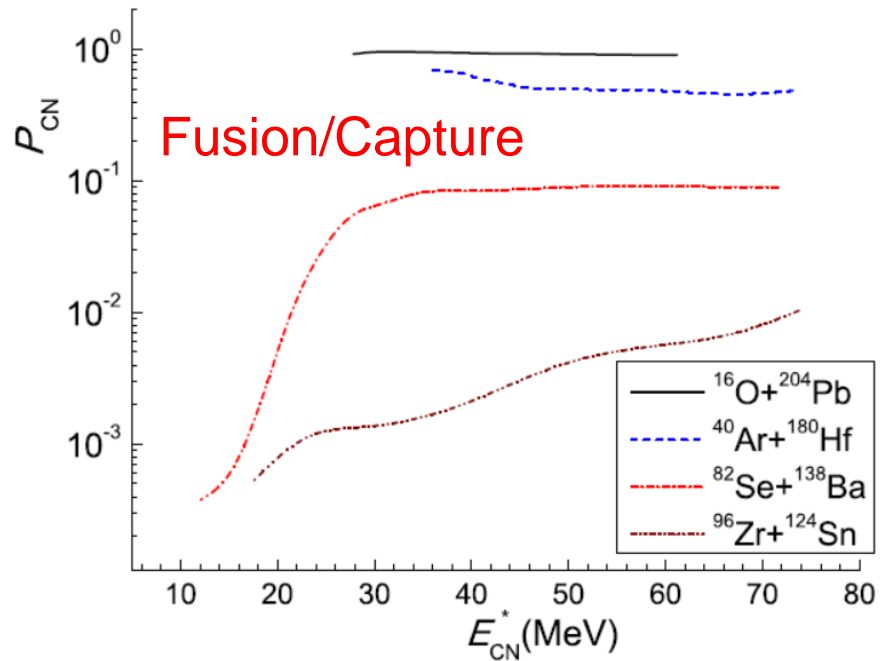
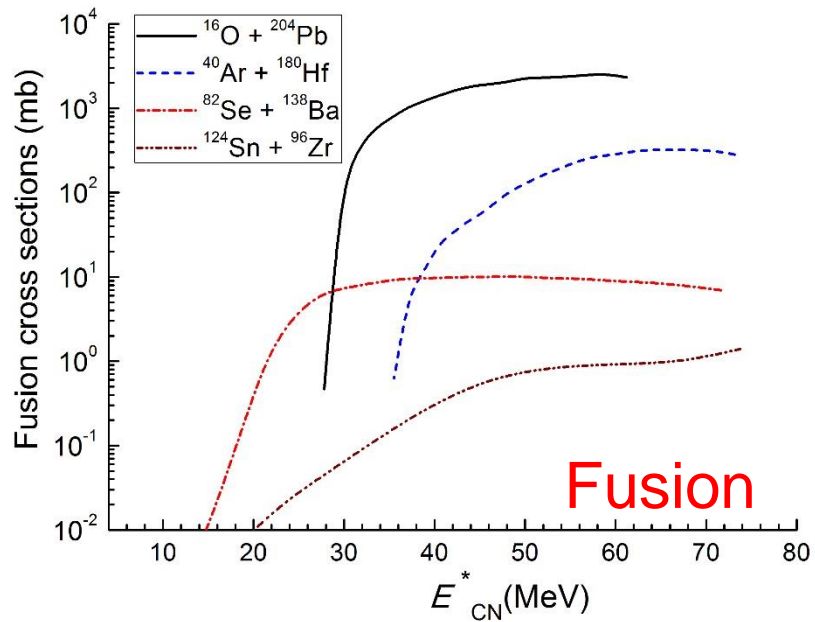
Comparison of the complete fusion cross sections of the 4 reactions



$$\sigma_l^{fus}(E, l) = \sigma_l^{capture}(E, l) P_{CN}(E, l)$$

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Comparison of the complete fusion cross sections of the 4 reactions

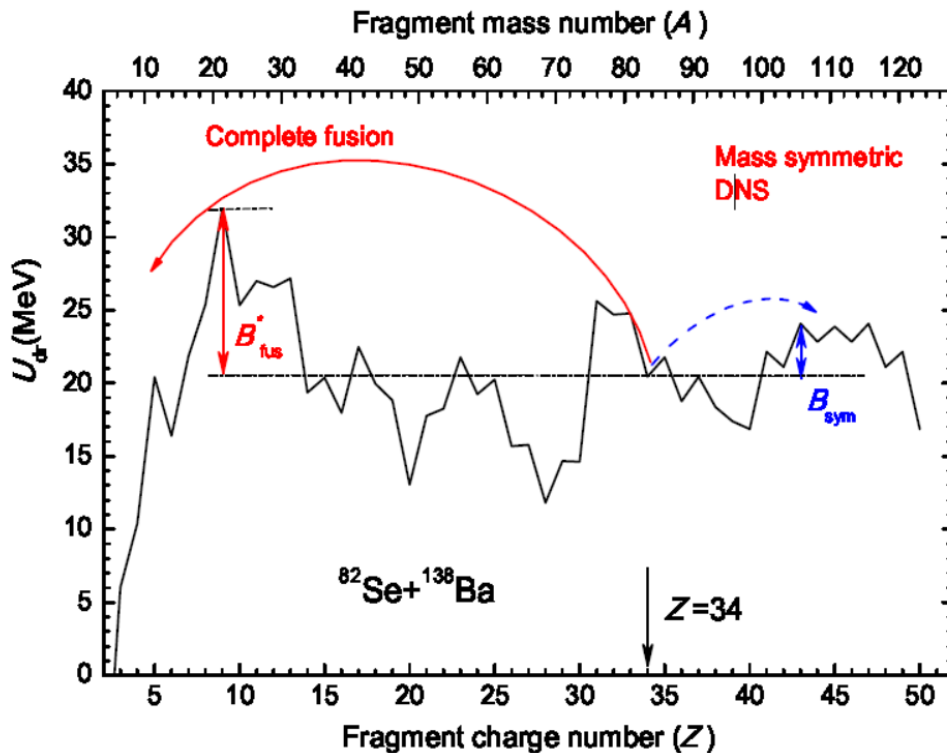
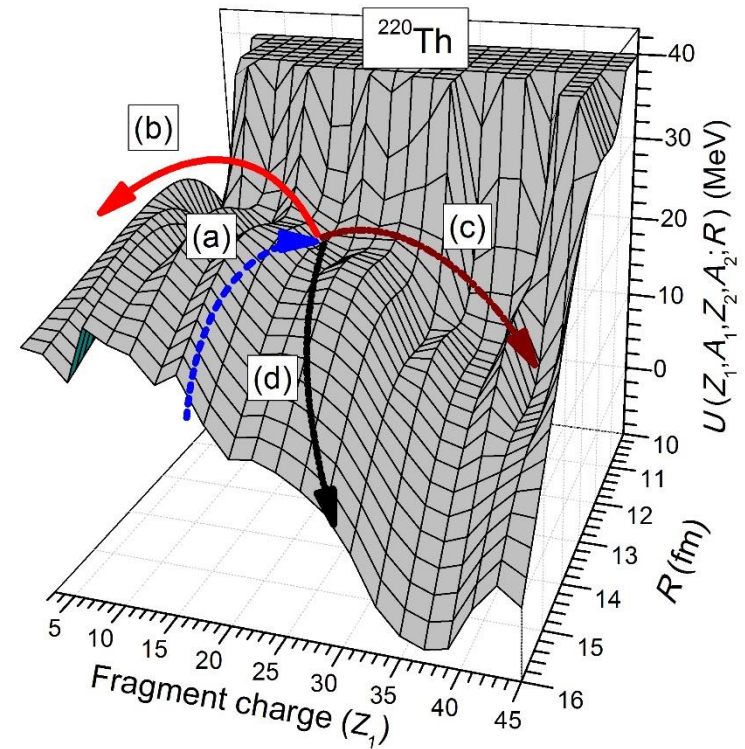


$$\sigma_l^{fus}(E, l) = \sigma_l^{capture}(E, l) P_{CN}(E, l)$$

Potential energy surface

$$U(Z_1, A_1, Z_2, A_2, R, Qgg) = B_1(Z_1, A_1) + B_2(Z_2, A_2) - B_{CN} + V_{int}(Z_1, A_1, Z_2, A_2, R)$$

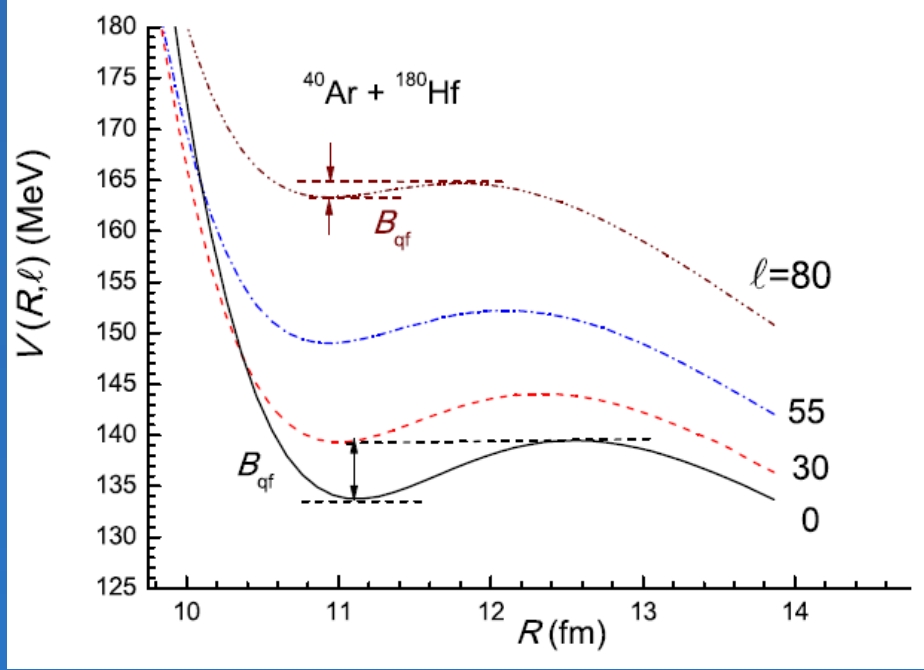
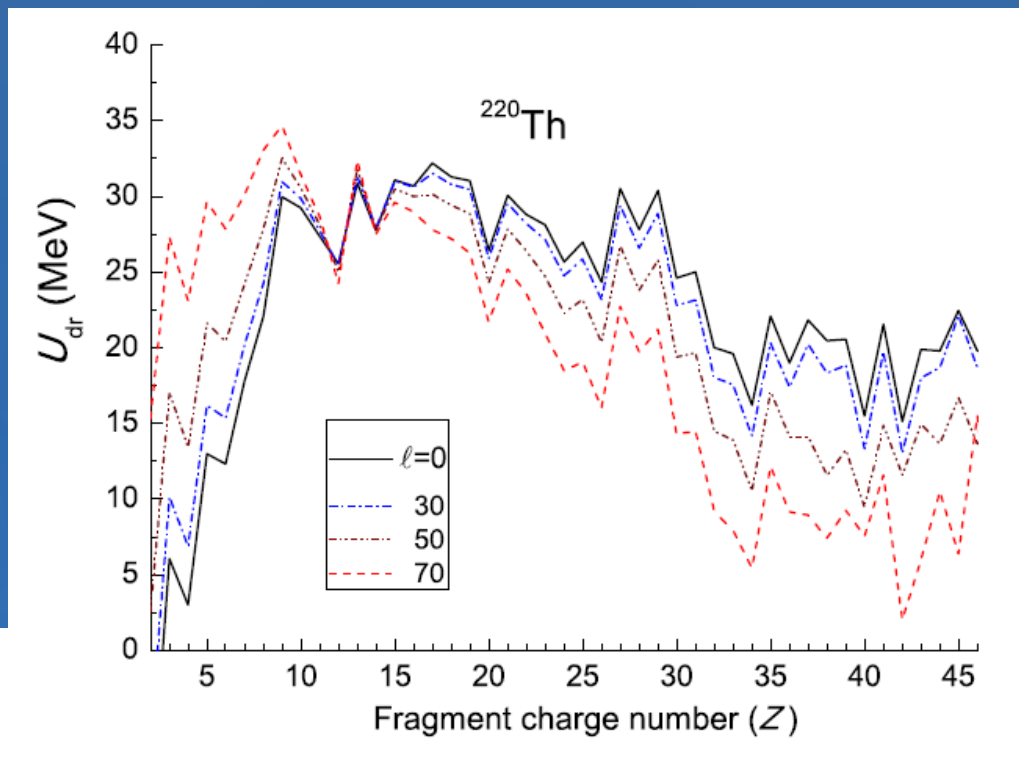
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Driving potential

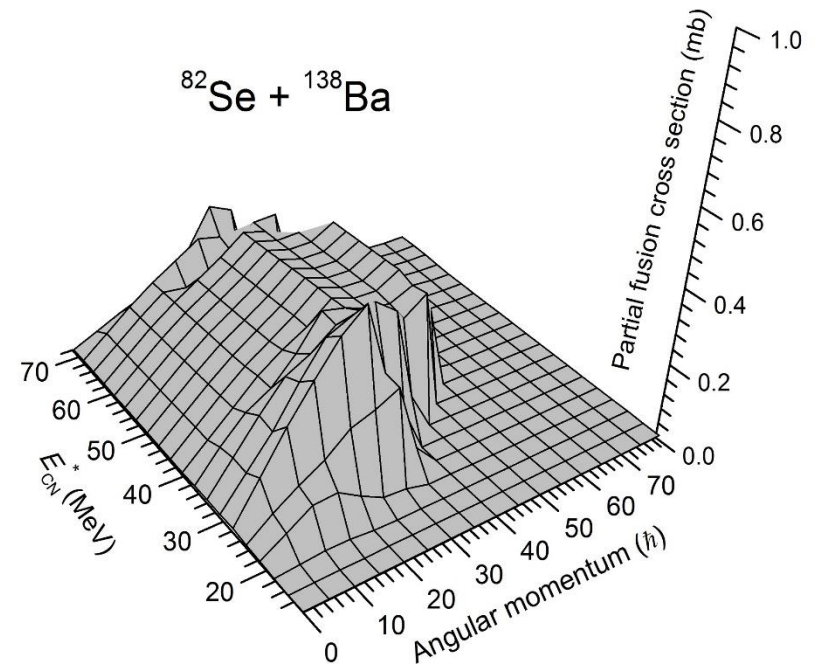
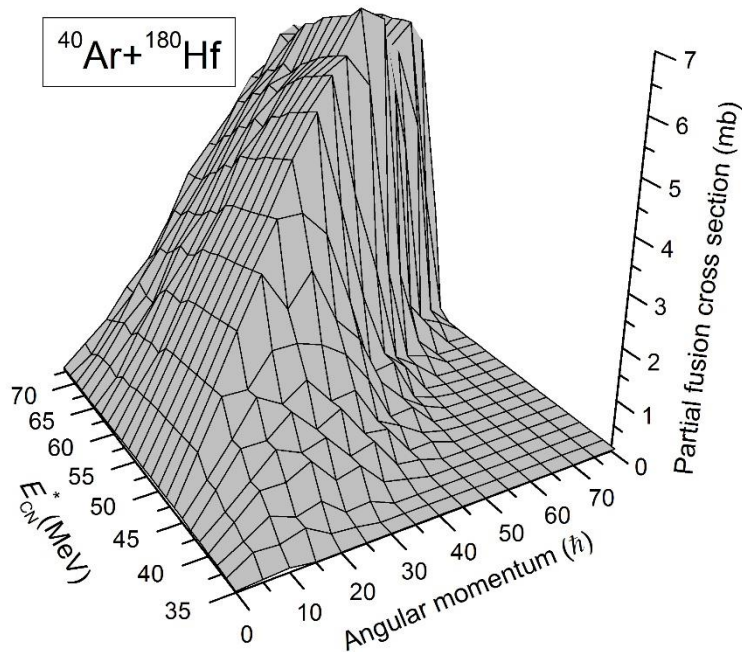
$$U_{dr}(Z_1, A_1, Z_2, A_2, Rm, Qgg) = B_1(Z_1, A_1) + B_2(Z_2, A_2) - B_{CN} + V_{int}(Z_1, A_1, Z_2, A_2, Rm)$$

Dependence of the driving potential on the angular momentum



Dependence of the quasifission barrier on the angular momentum

Partial fusion cross section of the $^{40}\text{Ar}+^{180}\text{Hf}$ and $^{82}\text{Se}+^{138}\text{Ba}$ reactions

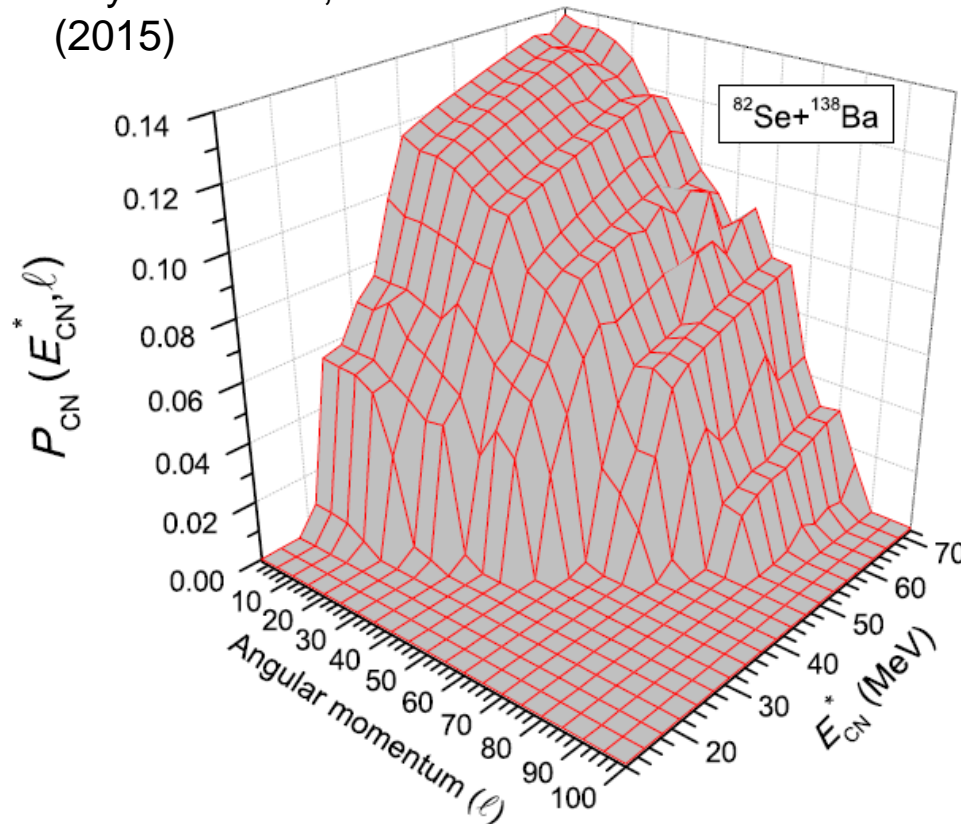


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Probability of fusion as a function of the energy and angular momentum.

KIM, KIM, NASIROV, MANDAGLIO, AND GIARDINA

Phys.Rev.C91, 064608
(2015)

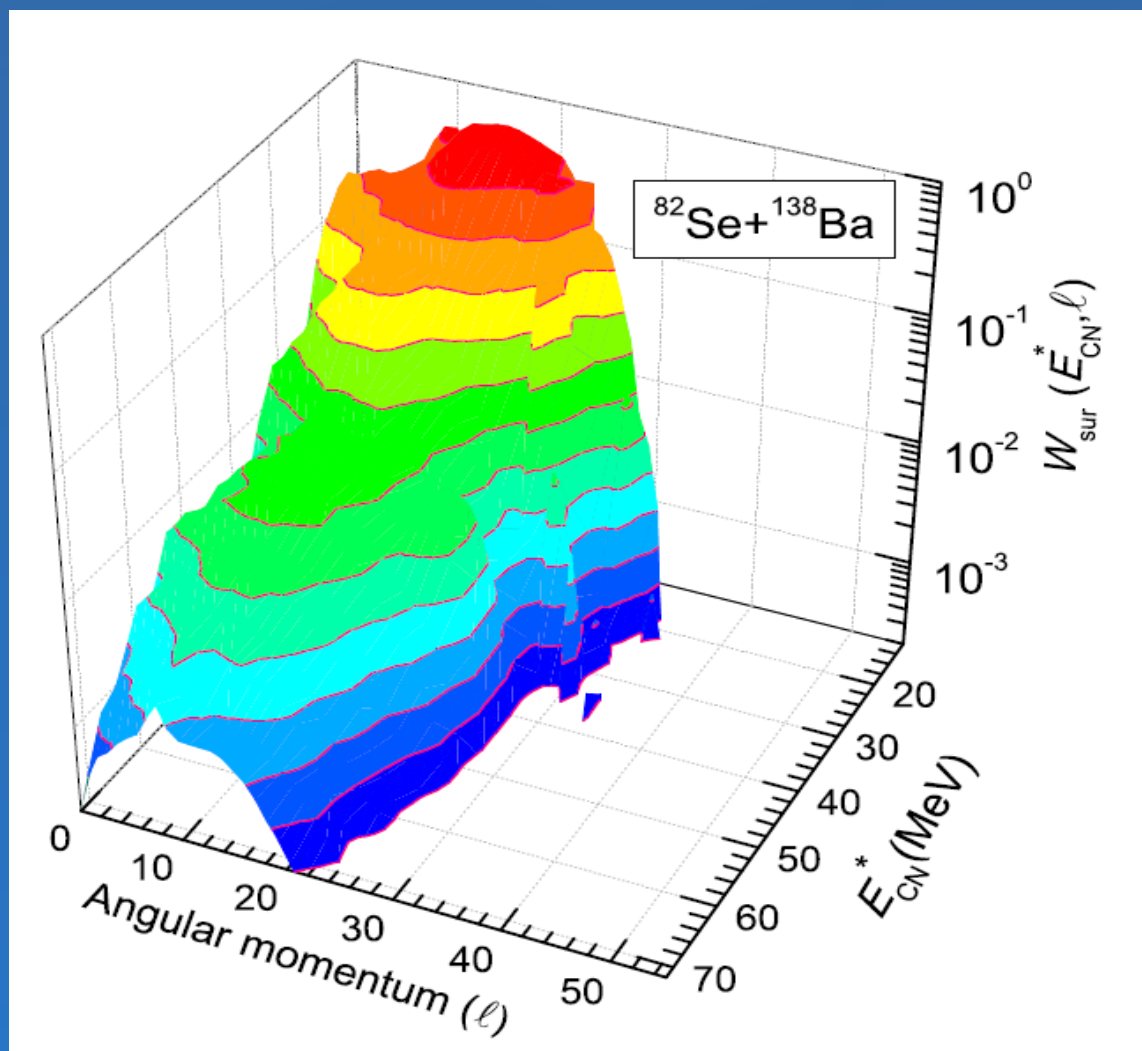


$$P_{\text{CN}}(E_{\text{CN}}, l) = \frac{\sigma_{\text{fus}}(E_{\text{CN}}, l)}{\sigma_{\text{cap}}(E_{\text{CN}}, l)}$$

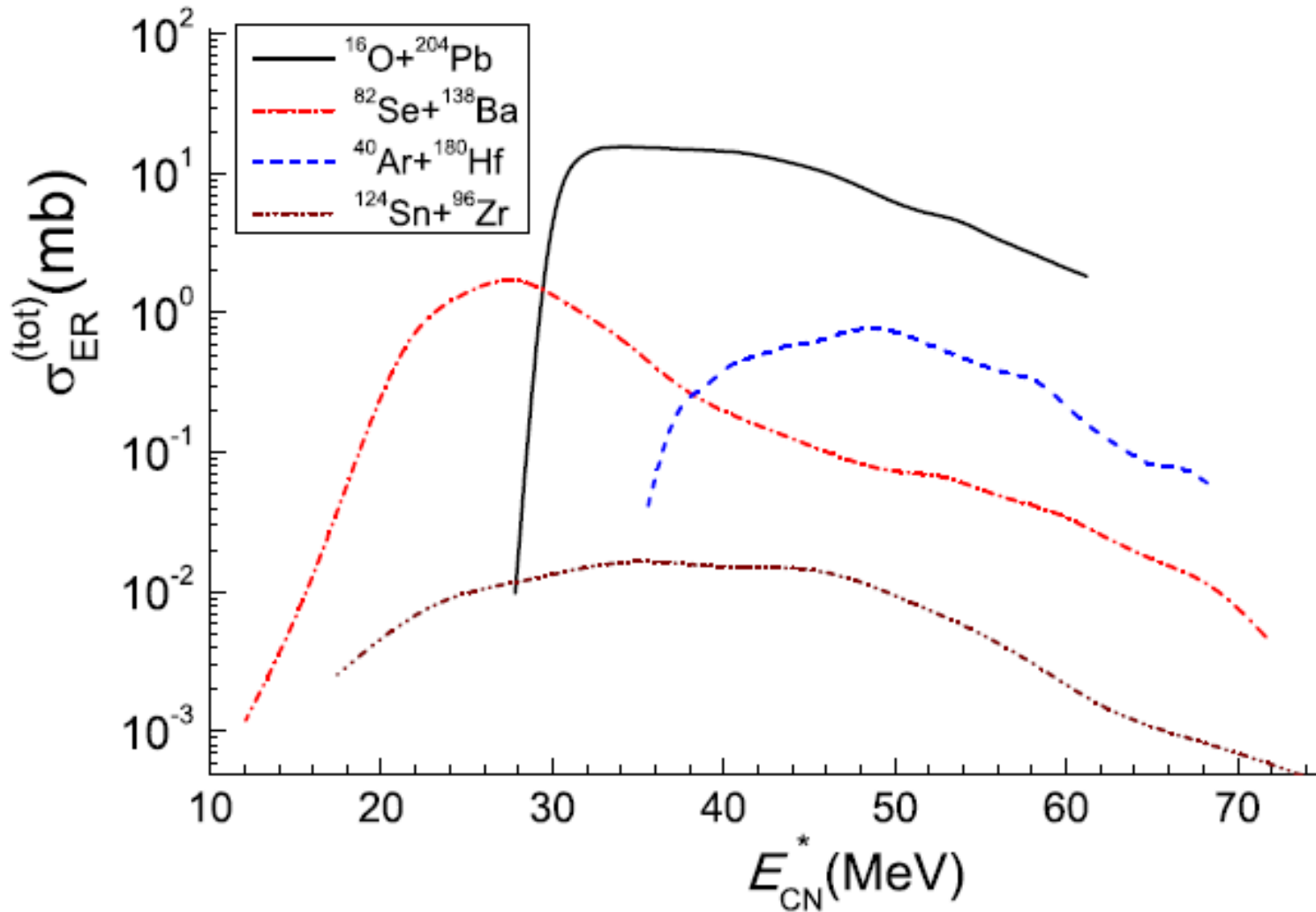
The partial fusion probability decreases by the increase of angular momentum l but total cross section is proportional to l .

$$\sigma_{\text{ER}} = \sum_{l=0}^{l_d} (2l + 1) \sigma_l^{\text{fus}}(E, l) W_{\text{surv}}(E, l)$$

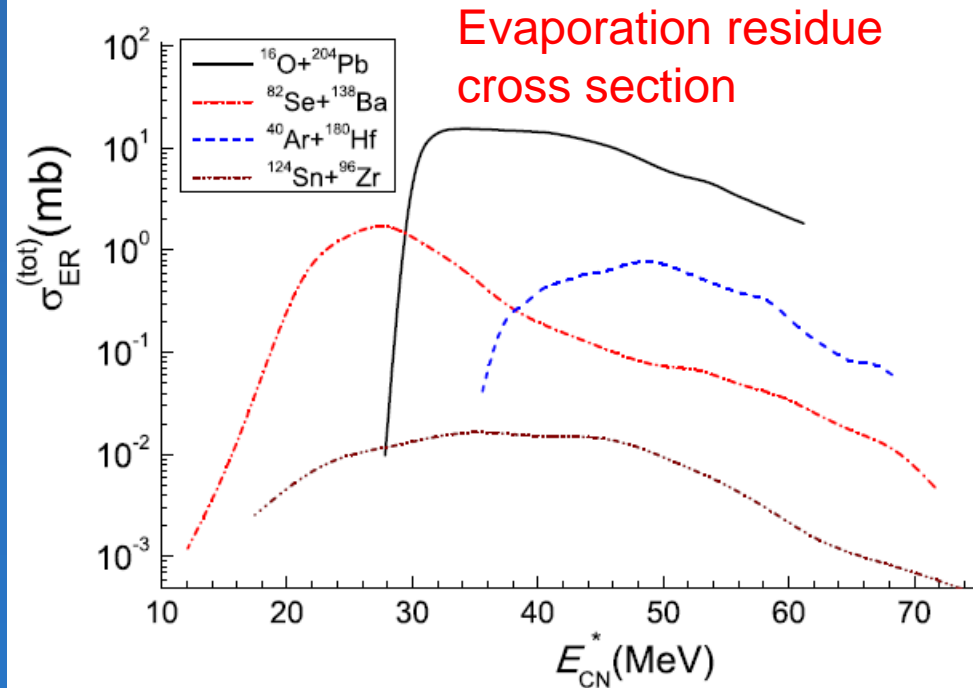
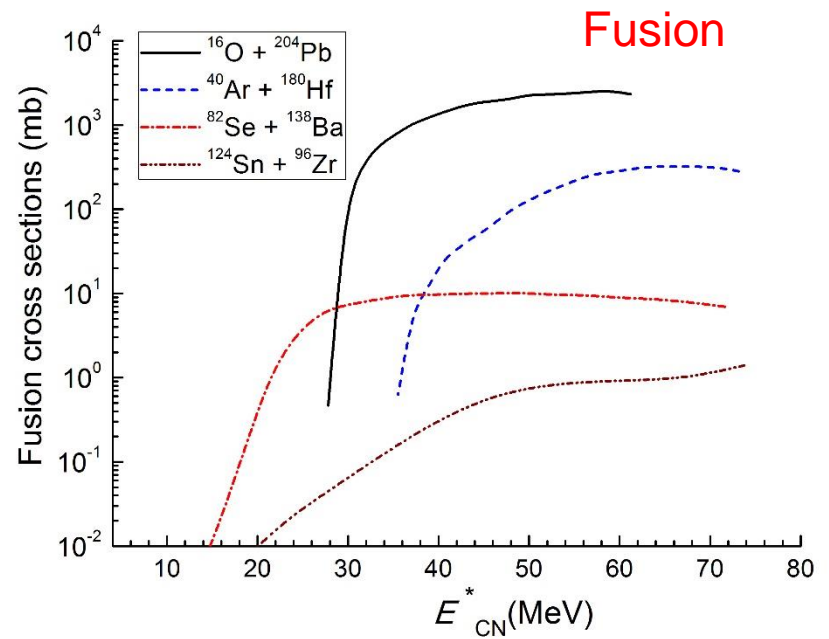
Probability of surviving the heated and rotating compound nucleus against fission as a function of the energy and angular momentum.



Comparison of the evaporation residue cross sections calculated for the 4 reactions leading to ^{220}Th .



The survived part of the compound nucleus against to fission.



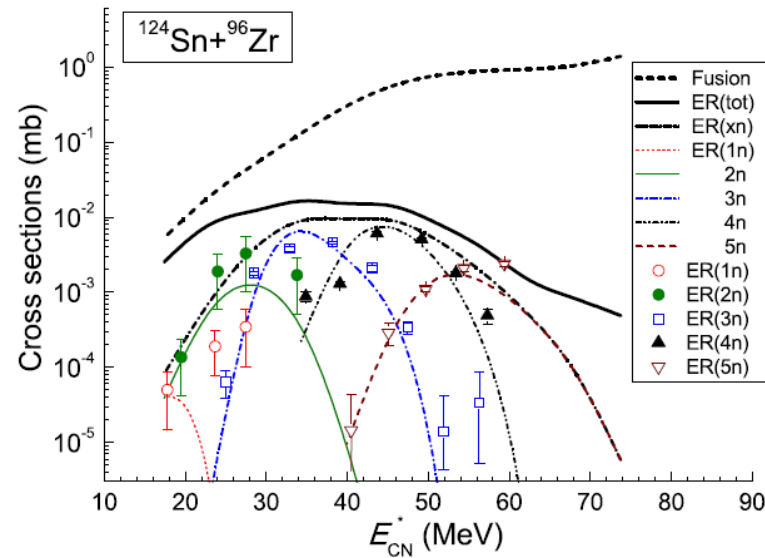
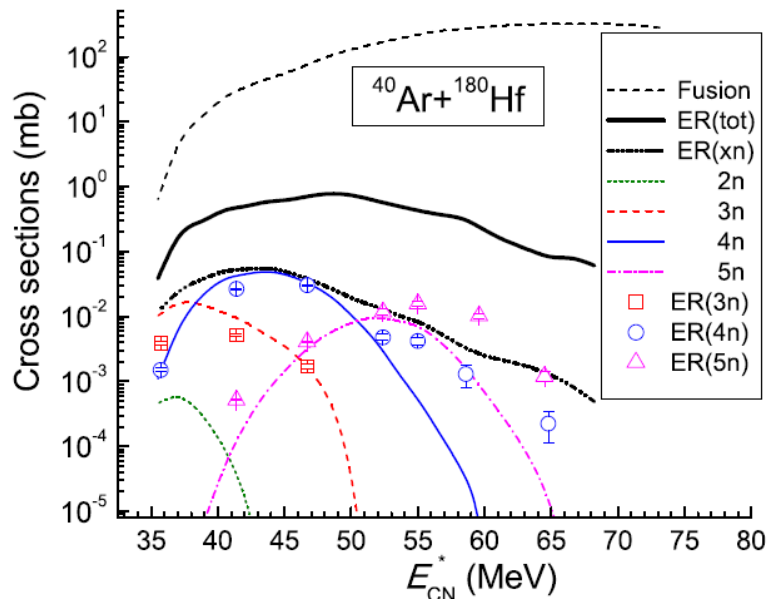
Total cross section of the evaporation residues formation.

Kim et al. Phys.Rev.C91, 064608 (2015)

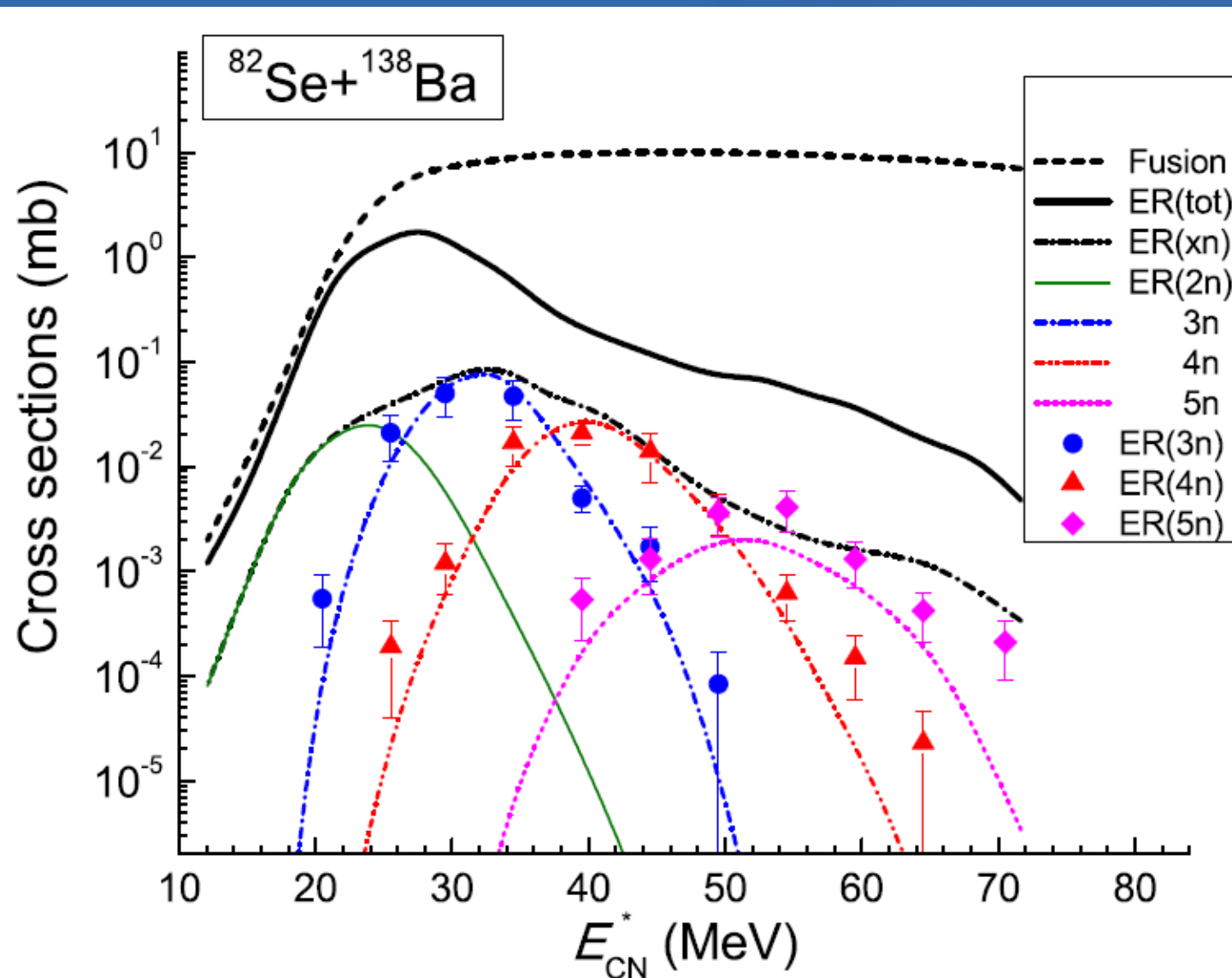
Comparison of the cross sections of the different neutron emission channels with the corresponding experimental data.

EFFECTS OF ENTRANCE CHANNELS ON THE ...

PHYSICAL REVIEW C 91, 064608 (2015)



Comparison of the cross sections of the different neutron emission channels with the corresponding experimental data.



Conclusions

The experiments of synthesis of superheavy elements were successful cold and hot fusion reactions due to using favorable conditions for the entrance channel and properties of the being formed compound nucleus. The strong hindrance to complete fusion increases in mass symmetric reactions. This hindrance is increase of the intrinsic fusion barrier which is determined by the landscape of the potential energy surface.

The hindrance to complete fusion is not so strong in hot fusion reactions because the mass asymmetry of those reactions is small. Initial system is already close to be fused. But large excitation energy can decrease of the survival probability of the compound nucleus against fission.

Therefore, it is important to analyze the fusion probability of colliding nuclei and the survival probability of the compound nucleus in order to choice reaction partners and beam energy.

Thank you for attention !

Thank you for warm hospitality !



About description of the events of the synthesis of superheavy elements



The measured evaporation cross section can be described by the formula:

$$\sigma_{ER}(E^*) = \sum_{\ell=0}^{\ell=\ell_f} \sigma_{\text{cap}}(E_{\text{c.m.}}, \ell) P_{\text{CN}}(E^*, \ell) W_{\text{surv}}(E^*, \ell)$$

where

$$\sigma_{\text{fus}}(E_{\text{c.m.}}, \ell) = \sigma_{\text{cap}}(E_{\text{c.m.}}, \ell) P_{\text{CN}}(E^*, \ell)$$

is considered as the cross section of compound nucleus formation; W_{surv} is the survival probability of the heated and rotating nucleus. The smallness of P_{CN} means hindrance to fusion caused by huge contribution of quasifission process:

$$38 \quad \sigma_{\text{qfis}}(E_{\text{c.m.}}, \ell) = \sigma_{\text{cap}}(E_{\text{c.m.}}, \ell) (1 - P_{\text{CN}}(E^*, \ell))$$

Collective enhancement of level density of DNS



$$K_{rot}(E_{DNS}) = \begin{cases} (\sigma_{\perp}^2 - 1)f(E_{DNS}) + 1, & \text{if } \sigma_{\perp} > 1, \\ 1, & \text{if } \sigma_{\perp} \leq 1, \end{cases}$$

where $\sigma_{\perp} = J_{(DNS)}T/\hbar^2$; $f(E) = (1 + \exp[(E - E_{cr})/d_{cr}])^{-1}$; $E_{cr} = 120\tilde{\beta}_2^2 A^{1/3}$ MeV; $d_{cr} = 1400\tilde{\beta}_2^2 A^{2/3}$. $\tilde{\beta}$ is the effective quadrupole deformation for the dinuclear system. We find it from the calculated $\mathcal{J}_{\perp}^{(DNS)}$.

Dependence of the fission barrier on the excitation energy and angular momentum of compound nucleus.

$$B_{\text{fis}}(J, T) = c B_{\text{fis}}^m(J) - h(T) q(J) \delta W,$$

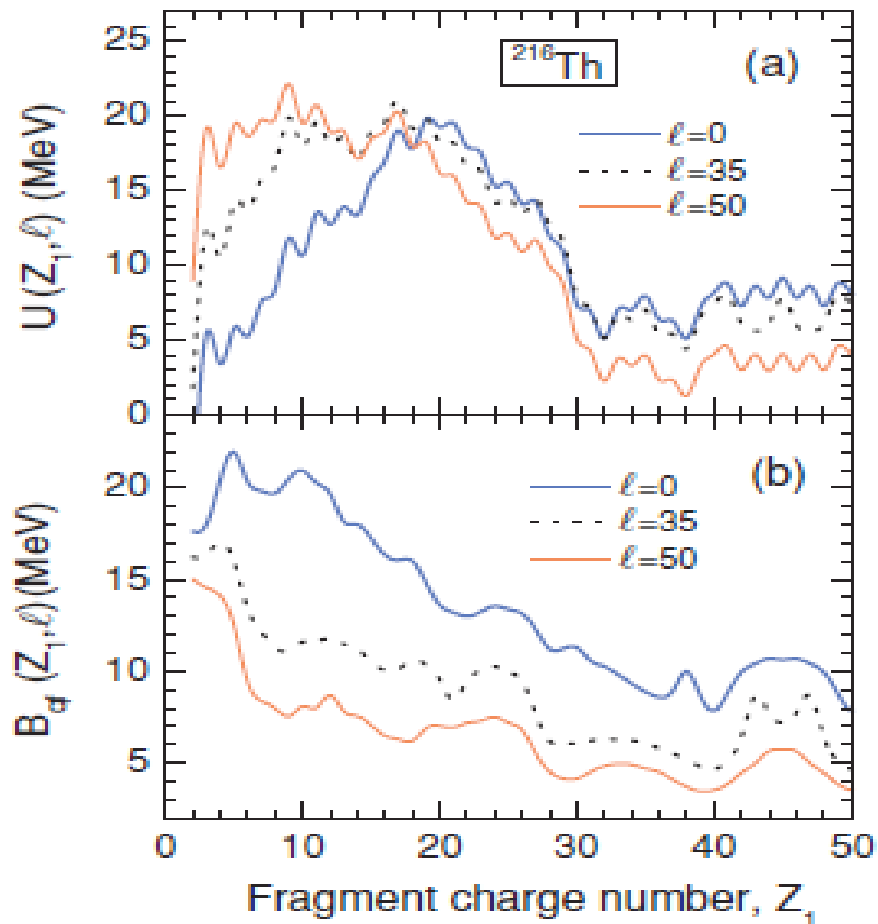
with

$$h(T) = \begin{cases} 1, & T \leq 1.65 \text{ MeV}, \\ k \exp(-mT), & T > 1.65 \text{ MeV}, \end{cases}$$

and

$$q(J) = \{1 + \exp[(J - J_{1/2})/\Delta J]\}^{-1},$$

where $B_{\text{fis}}^m(J)$ is the parameterized macroscopic fission barrier [15] depending on angular momentum J , $\delta W = \delta W_{\text{sad}} - \delta W_{\text{gs}} \simeq -\delta W_{\text{gs}}$ is the microscopic (shell) correction to the fission barrier taken from the tables [8] and the constants for the macroscopic fission barrier scaling, temperature and angular momentum dependencies of the microscopic correction are chosen to be as follows: $c = 1.0$, $k = 5.809$, $m = 1.066 \text{ MeV}^{-1}$, $\Delta J = 3\hbar$; for nuclei with $Z > 102$ we use $J_{1/2} = 20\hbar$. This procedure let the shell corrections become dynamical quantities, too.



Dependence of the driving potential and quasifission barrier on the angular momentum of dinuclear system formed in reactions leading to formation of compound nucleus ^{216}Th .
 decreases by increasing the angular momentum.