QCD Vacuum generated Light-Heavy Quarks Interaction

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Outline

QCD vacuum on the lattice

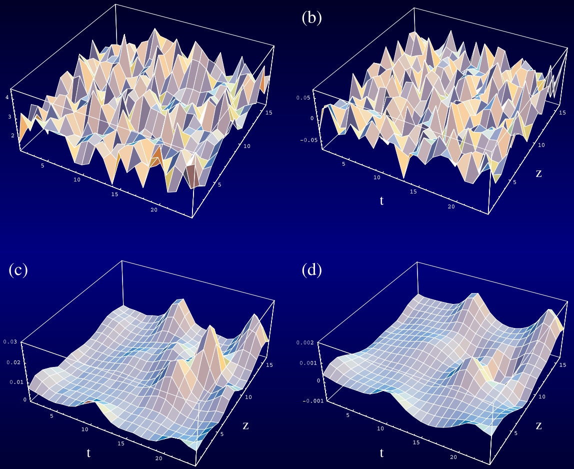


Figure : Action and topological charge densities in different configurations on the lattice.

QCD instantons

Instantons –self-dual classical solutions of the equations of motion in Euclidean space (Belavin *et.al.*, 1975):

$$A^{I,a}_{\mu}(x) = \frac{2\rho^2 \bar{\eta}^{\nu}_{\mu a}(x-z)_{\nu}}{(x-z)^2[\rho^2+(x-z)^2]},\ \ G^a_{\mu\nu} = \tilde{G}^a_{\mu\nu}.$$

- The topological charge $Q = \frac{1}{32\pi^2}\int d^4x\, G^a_{\mu\nu}\tilde{G}^a_{\mu\nu} = 1.$
- For the antiinstanton the t'Hooft symbol $\bar{\eta} \to \eta$, $G^a_{\mu\nu} = -\tilde{G}^a_{\mu\nu},\ Q = -1.$
- Chern-Simons number collective coordinate
$$N_{CS} = \frac{1}{16\pi^2}\int d^3x\, \epsilon^{ijk}\left(A^a_i\partial_j A^a_k + \frac{1}{3}\epsilon^{abc}A^a_i A^b_j A^c_k\right),$$
- Large gauge transformations
$N_W = \frac{1}{24\pi^2}\int d^3x \epsilon_{ijk}\left\langle \left(U^\dagger\partial_i U\right)\left(U^\dagger\partial_j U\right)\left(U^\dagger\partial_k U\right)\right\rangle.$
$N_{CS} \Rightarrow N_{CS} + N_W.$
- Number of collective coordinates:

$$4\,(centre)\ +\ 1\,(size)\ +\ (4N_c-5)\,(orientations)\ =\ 4N_c$$

Dependence on N_{CS}

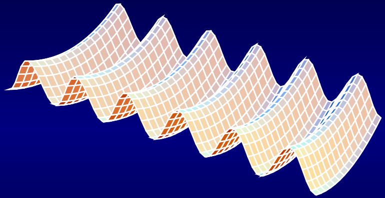


Figure : Vacuum gluon energy vs Chern-Simons number collective coordinate N_{CS}.The amplitude of quantum tunneling $\sim \exp(-S_I)$ between the states with $|\Delta N_{CS}| = 1$. Here the action $S_I = \frac{8\pi^2}{g^2}$.

Instanton vacuum model

- Sum ansatz $A = \sum_I A^I + \sum_{\bar{I}} A^{\bar{I}}$ for dilute gas approximation.
- Inter-instantons interactions have to stabilize instanton size ρ and inter-instanton distance R.

$S\chi$SB in QCD instanton vacuum

- Correct description of the spontaneous breaking of the chiral symmetry ($S\chi$SB), which is responsible for properties of most light hadrons.
- $S\chi$SB is due to the delocalization of single-instanton quark zero modes in the instanton medium.
- Only two parameters:
  - average instanton size $\rho \sim 0.3\,\mathrm{fm}$,
  - average inter-instanton distance $R \sim 1\,\mathrm{fm}$,

  – suggested phenomenologically (Shuryak1981),
  – derived variationally from $\Lambda_{\overline{\mathrm{MS}}}$ (Diakonov,Petrov1983)
  – confirmed by lattice measurements (Negele et al1998, DeGrand et al2001, Faccioli et al2003, Bowman etal2004).
- The model provided a consistent description of the light quark physics (Diakonov et al, Goeke et al, Musakhanov et al).

Parameters of instanton vacuum

- Averaged instanton size $\bar{\rho}$;
- Averaged inter-instanton distance $\bar{R}$.
- Results:
  - Lattice estimate: $\bar{R} \approx 0.89\, fm$, $\bar{\rho} \approx 0.36\, fm$,
  - Phenomenological estimate: $\bar{R} \approx 1\, fm$, $\bar{\rho} \approx 0.33\, fm$,
  - Our estimate (with account of $1/N_c$ corrections): $\bar{R} \approx 0.76\, fm$, $\bar{\rho} \approx 0.32\, fm$, correspond $F_{\pi,m=0} = 88 MeV$, $\langle \bar{q}q \rangle_{m=0} = -(255 MeV)^3$

  Thus within $10 - 15\%$ uncertainty different approaches give similar estimates
- Packing parameter $\pi^2 (\frac{\bar{\rho}}{\bar{R}})^4 \sim 0.1 - 0.3$

  $\Rightarrow$ Independent averaging over instanton positions and orientations.

Light quarks in the single instanton background

Very strong modification of the light quark propagator

$$S(x,y) \approx \frac{|\Phi_0(x,\zeta)\rangle \langle\Phi_0(y,\zeta)|}{im} + \frac{1}{i\hat{\partial}}\,,$$

due to the zero mode

$$(i\hat{\partial} + g\hat{A})\Phi_0(x,\zeta) = 0\,.$$

Here collective coordinates ζ: a instanton position z and color orientation U.

Spontaneous Breaking of the Chiral Symmetry

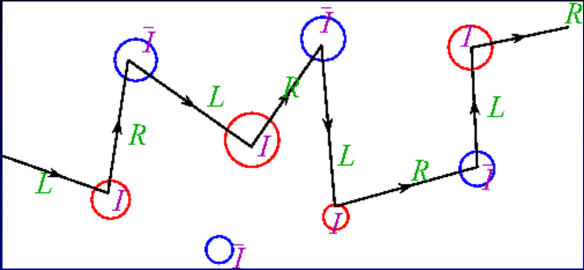


Figure : Light quark in the instanton vacuum. $N_f = 1$.

Sum-up of multi-scattering series ⇒ full light quark propagator:

$$S - S_0 = -S_0 \sum_{i,j} \hat{p} |\Phi_{0i}\rangle \left\langle \Phi_{0i} \left| \left(\frac{1}{B(m)} \right) \right| \Phi_{0j} \right\rangle \langle \Phi_{0j} | \hat{p} S_0,$$

$$B(m) = \hat{p} S_0 \hat{p}$$

Low-frequency part of the light quark determinant with the quark sources

We was able to find $\mathrm{Det}_{low}(\hat{P} + im)e^{(-\xi^+ S\xi)}$:

$$= \int \prod_f D\psi_f D\psi_f^\dagger \exp \int \left(\psi_f^\dagger (\hat{p} + im_f)\psi_f + \psi_f^\dagger \xi_f + \xi_f^+ \psi_f \right)$$

$$\times \prod_f \left\{ \prod_{+}^{N_+} V_{+,f}[\psi^\dagger, \psi] \prod_{-}^{N_-} V_{-,f}[\psi^\dagger, \psi] \right\} ,$$

where $V_{\pm,f}[\psi^\dagger, \psi] =$

$$= i \int dx \left(\psi_f^\dagger(x)\, \hat{p}\Phi_{\pm,0}(x;\zeta_\pm) \right) \int dy \left(\Phi_{\pm,0}^\dagger(y;\zeta_\pm)(\hat{p}\,\psi_f(y) \right) .$$

Fermionic fields $\psi^\dagger, \psi$ has a meaning of constituent quarks.

Partition function

Averaging over instantons collective coordinates $\Rightarrow$ partition function $Z[\xi_f, \xi_f^+] =$

$$= \int \prod_f D\psi_f D\psi_f^\dagger \exp \int \left(\psi_f^\dagger (\hat{p} + im_f)\psi_f + \psi_f^\dagger \xi_f + \xi_f^+ \psi_f \right)$$

$$\times \int D\zeta \prod_+^{N_+} \prod_f V_{+,f}[\psi^\dagger, \psi] \prod_-^{N_-} \prod_f V_{-,f}[\psi^\dagger, \psi] \,,$$

Small packing parameter provided here independent averaging:

$$\overline{\prod_f V_{\pm,f}[\psi^\dagger, \psi]} = \int d\zeta_\pm \prod_f V_{\pm,f}[\psi^\dagger, \psi]$$

$\Rightarrow$ non-local t'Hooft-like vertex with $2N_f$-legs. Nonlocality range $\sim \rho$.

Dynamical quark mass $M(q)$

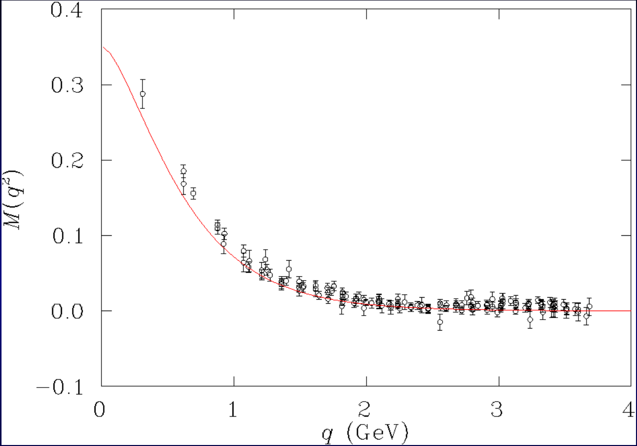


Figure : Light quark dynamical mass M(q) (red line) in comparison with lattice results (Bowman et al 2004). M(q)-dependence is entirely defined by zero-mode!!! $M(0) \approx 365\,MeV$ gives a strength of the light quark interactions with QCD vacuum instantons.

Heavy quarks in the instanton vacuum

$L_\Psi = \Psi^+(\hat{P} + im_H)\Psi \Rightarrow Q^+\gamma_4 P_4 Q + Q^+ Q_1 Q,$
$Q_1 = \frac{\vec{P}^2}{2m_H} - \frac{\vec{\sigma}\vec{B}}{2m_H},\ \ P = p - gA,\ \ \vec{B} = rot\vec{A}.$
(Infinitely) heavy quark propagator (Wilson line) in instanton vacuum defined as

$$w = \int D\zeta \frac{1}{\theta^{-1} - \sum_i a_i}$$

where $a_i(t) = iA_{i,\mu}(x(t))\frac{d}{dt}x_\mu(t), w_\pm = \frac{1}{\theta^{-1} - a_\pm},\ < t|\theta|t' >= \theta(t - t')$.
Pobylica Eq. for the w^{-1} has a solution

$$w^{-1} = \theta^{-1} - \frac{N}{2}\sum_\pm \theta^{-1}(w_\pm - \theta)\theta^{-1} + O(N^2/V^2),$$

Heavy quark interaction with instanton vacuum

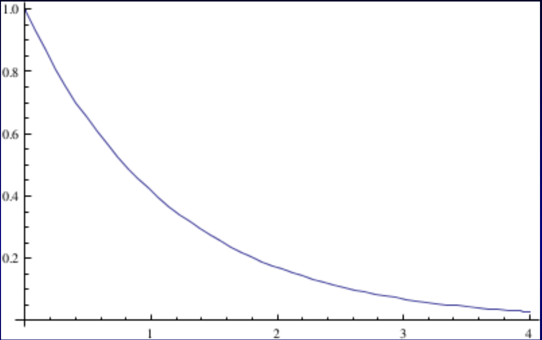


Figure : Form-factor $i_0[q\rho]/i_0[0]$, $i_0[0] = 0.55197$. The split of heavy quark mass in instanton vacuum is $\Delta M = 16\pi i_0[0](\rho^4/R^4)\rho^{-1}/N_c$. At $\rho = 0.32fm, R = 0.76fm$ (ChPT) $\Delta M = 148MeV$ gives a strength of the heavy quark interactions with QCD vacuum instantons at the range $\sim \rho$.

Heavy quark propagator at light quark number $N_f = 1$

Extension of DPP89 solution (planar graphs) is $w^{-1}[\psi, \psi^\dagger] =$

$$= \theta^{-1} - \frac{N}{2}\sum_{\pm}\frac{1}{V_\pm[\psi^\dagger,\psi]}\Delta_{H,\pm}[\psi^\dagger,\psi] + O(N^2/V^2),$$

$$\Delta_{H,\pm}[\psi^\dagger,\psi] = \int d\zeta_\pm \; V_\pm[\psi^\dagger,\psi]\theta^{-1}(w_\pm - \theta)\theta^{-1}.$$

Then heavy quark propagator at light quark number $N_f = 1$

$$S_H = \frac{1}{\theta^{-1} - \lambda\sum_\pm \Delta_{H,\pm}[\frac{\delta}{\delta\xi},\frac{\delta}{\delta\xi^+}]}\exp\left[-\xi^+\left(\hat{p} + iM(p)\right)^{-1}\xi\right]|_{\xi=\xi^+}$$

DPP89 solution is reproduced at the approximation:

$$S_H^{-1} \approx \theta^{-1} - \lambda\sum_\pm \Delta_{H,\pm}[\frac{\delta}{\delta\xi},\frac{\delta}{\delta\xi^+}]\exp\left[-\xi^+\left(\hat{p} + iM(p)\right)^{-1}\xi\right]|_{\xi=}$$

At any N_f and in saddle-point approximation no an essential difference with $N_f = 1$.

Heavy–light quarks interactions at any number of light quarks N_f

is given by the expression $-\lambda \sum_{\pm} Q^{\dagger}\Delta_{H,\pm}[\psi^{\dagger},\psi]Q$

$$
\begin{aligned}
&= -i\lambda \sum_{\pm} \int d^4 z_{\pm} dU_{\pm} \prod_{f=1}^{N_f} \frac{d^4 k_f}{(2\pi)^4}\frac{d^4 q_f}{(2\pi)^4} \exp(i(q_f - k_f)z_{\pm}) \\
&\psi^{+}_{f,a_f\alpha_f}(k_f)(\gamma_{\mu_f}\gamma_{\nu_f}\frac{1\pm\gamma_5}{2})_{\alpha_f\beta_f}(U^{a_f}_{\pm,i_f}(\tau^{\mp}_{\mu_f}\tau^{\pm}_{\nu_f})^{i_f}_{j_f} U^{\dagger j_f}_{\pm,b_f}\psi^{b_f}_{f,\beta_f}(q_f) \\
&\frac{(2\pi\rho)^2 F(k_f)F(q_f)}{8} Q^{+}_{a_3} U^{a_3}_{\pm,i_3}\left(\theta^{-1}(w_{\pm}-\theta)\theta^{-1}\right)^{i_3}_{j_3} U^{\dagger j_3}_{\pm,b_3} Q^{b_3}
\end{aligned}
$$

At any N_f the interaction term have 2 heavy and $2N_f$ light quark legs. The actual structure is defined by the color orientation integration and it have to have $SU_L(N_f)\times SU_R(N_f)$ symmetry.

Heavy–light quarks interactions at $N_f = 1$

is

$$
-\lambda \sum_{\pm} Q^\dagger \Delta_{H,\pm}[\psi^\dagger, \psi] Q
$$

$$
= i \int \frac{d^4 k_1}{(2\pi)^4} \frac{d^4 k_2}{(2\pi)^4} \frac{d^3 q}{(2\pi)^3} (2\pi)^4 \delta^3(\vec{k}_2 + \vec{k}_1 - \vec{q}) \delta(k_{2,4} - k_{1,4})
$$

$$
(M(k_1)M(k_2))^{1/2} 8\pi\rho^3 i_0(q\rho) \left[\frac{2N_c - 1}{N_c^2 - 1} \psi^+(k_1)\psi(k_2) Q^+ Q \right.
$$

$$
\left. + \frac{N_c - 2}{N_c^2 - 1} (\psi^+(k_1) Q Q^+ \psi(k_2) + \psi^+(k_1)\gamma_5 Q Q^+ \gamma_5 \psi(k_2)) \right]
$$

Bosonizaion of the first term provide the light-quark exchange QQ potential V_{lq}, while second and third terms – Qq mesons degenerated on parity.

Heavy quarks–light mesons interactions at $N_f = 2$.

Heavy–light quarks interactions at $N_f = 2$ has an essential part–the co-product of colorless Q^+Q-factor and the colorless light-quarks one. This term at $N_f = 2$ leads to the effective action for the mesons $\Phi' = \sigma' + i\gamma_5\vec{\tau}\vec{\phi}' + i\vec{\tau}\vec{\sigma}' + \gamma_5\eta'$, interacting with heavy quarks

$$S[\sigma', \vec{\phi}', \eta', \vec{\sigma}', Q^+Q] = \frac{1}{2}\int d^4\left(\sigma'^2 + \vec{\phi}'^2 + \vec{\sigma}'^2 + \eta'^2\right)$$

$$-\mathrm{Tr}\ln\left[1 + \frac{iM/\sigma_0}{\hat{p} + i(m + M(p))}F(p)\,\Phi'\,F(p)\right]$$

$$-\mathrm{Tr}\frac{1}{\hat{p} + i(m + M(p)) + \frac{iM}{\sigma_0}F(p)\,\Phi'\,F(p)}\,i\,(M(p) +$$

$$+\frac{M}{\sigma_0}F(p)\Phi' F(p)\Big)\left(\frac{i}{N_c}\int e^{-ipx}\frac{d^4p_1}{(2\pi)^4}\frac{d^4p_2}{(2\pi)^4}J_0(p\rho)Q^+Q\right).$$

The first and second lines describe mesons and their interactions, while the third and fourth one give the renormalization of the heavy quark mass and heavy-light quark meson interactions terms.

Heavy quarks–light mesons interactions at $N_f = 2$ and LO.

The renormalization of heavy quark mass is given by

$$S_Q = -\frac{2i}{N_c}\frac{1}{R^4}J_0(0)\left(\int \frac{d^3p_1 d\omega_1}{(2\pi)^4} Q^+(\vec{p}_1,\omega_1)Q(\vec{p}_1,\omega_1)\right).$$

So, the instanton media contribution to the heavy quark mass is

$$\Delta M = -2J_0(0)/N_c R^4 = 16\pi i_0(0)(\rho^4/R^4)\rho^{-1}/N_c,$$

as we expected.

With $\rho = 0.35$ fm, $R = 0.856$ fm we obtain at LO $M = 570$ MeV and $\Delta M = 148$ MeV. These factors define the coupling between heavy and light quarks and certainly between heavy quarks and light mesons.

Heavy quarkonium

Charmonium size $r_c \sim 0.4\, fm$, bottomonium size $r_b \sim 0.2\, fm$.
Cornell potential $V = a/r + br$ – rather well description of the charmonium and bottomonium spectrum.
What are an instanton corrections?

Heavy quark–antiquark system correlator

The correlator $C(L_1, L_2) =$

$$\frac{1}{Z}\int D\psi D\psi^\dagger \prod_{\pm}^{N_\pm} \bar{V}_\pm[\psi^\dagger,\psi]\exp\int\left(\psi^\dagger(p\hat{+}im)\psi\right)W[\psi,\psi^\dagger],$$

$$< T|W[\psi,\psi^\dagger]|0 >= \left(\prod_{\pm}^{N_\pm}\bar{V}_\pm[\psi^\dagger,\psi]\right)^{-1}\int D\zeta\prod_{\pm}^{N_\pm}V_\pm[\psi^\dagger,\psi]$$

$$\times < T|\frac{1}{\theta^{-1}-\sum_i a_i^{(1)}}|0 >< 0|\frac{1}{\theta^{-1}-\sum_i a_i^{(2)}}|T > .$$

is a Wilson loop along the rectangular contour $L \times r$. The sides $L_1 = (0, T), L_2 = (T, 0)$ are parallel to x_4 axes and separated by the distance r. The $a^{(1)}, a^{(2)}$ are the projections of the instantons onto the lines L_1, L_2.

Eq. for heavy quark–antiquark system correlator

The extension of DPP89 solution is $W^{-1}[\psi,\psi^\dagger] =$

$$= w_1^{-1}[\psi,\psi^\dagger]\times w_2^{-1,T}[\psi,\psi^\dagger] - \frac{N}{2}\sum_\pm \bar{V}_\pm^{-1}[\psi^\dagger,\psi]\int d\zeta_\pm$$

$$\times V_\pm[\psi^\dagger,\psi]\theta^{-1}\left(w_\pm^{(1)}-\theta\right)\theta^{-1}(\times)\left(\theta^{-1}\left(w_\pm^{(2)}-\theta\right)\theta^{-1}\right)^T$$

where, superscript T means the transposition, $(\times)$ – tensor product and

$$w^{(1,2)^{-1}}[\psi,\psi^\dagger] = \theta^{-1} - \frac{N}{2}\sum_\pm \frac{1}{\bar{V}_\pm[\psi^\dagger,\psi]}\Delta_{H,\pm}^{(1,2)}[\psi^\dagger,\psi] + O(\frac{N^2}{V^2}).$$

The integration of the first term in $W^{-1}[\psi,\psi^\dagger]$ over $\psi,\psi^\dagger$ leads to heavy quark–antiquark potential V_{lq}, generated by light quarks.

Heavy quark–antiquark potential V_{lq}, generated by light quarks, $N_f = 1$

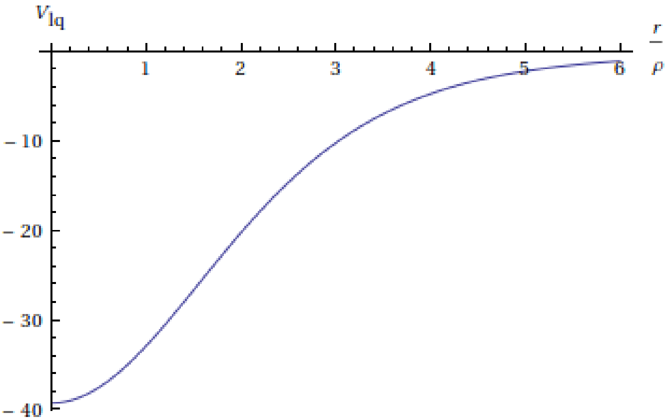
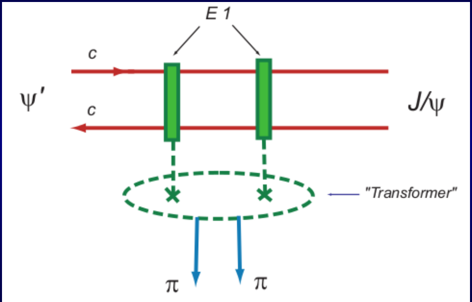


Figure : Heavy quark–antiquark potential $V_{lq}(r/\rho)$ (in MeV), generated by light quarks, its range is controlled by dynamical light quark mass $M \sim 0.36$ GeV.

Quarkonium light hadron transitions.

Charmonium size $r_c \sim 0.4\, fm$, bottomonium size $r_b \sim 0.2\, fm$.
Hadronic transitions at the assumption $\lambda_g >> r_c, r_b$:



But $\lambda_g \approx \rho = 0.35\, fm \sim r_c, r_b$.
What are an instanton corrections?

Quarkonium pion transitions. $\psi' \to J/\psi\ \pi\pi$.

Neglect by heavy mesons and rearrange light mesons as $(\sigma_0 + \sigma' + i\gamma_5\vec{\tau}\vec{\phi}')/\sigma_0 \Rightarrow U^{\gamma_5} = \exp(i\vec{\tau}\vec{\pi}(x)\gamma_5)$.
Then the interaction term $S_{Q\pi}$ between heavy quark Q and pions is

$$S_{Q\pi} = C_{Q\pi} \int d^4x \, \mathrm{tr}\, \partial_\mu U(x) \partial_\mu U^+(x)$$

$$\times \left(i \int e^{-ipx} \frac{d^3p_1 d\omega_1}{(2\pi)^4} \frac{d^3p_2 d\omega_2}{(2\pi)^4} \frac{J_0(p\rho)}{J_0(0)} Q^+(\vec{p}_1, \omega_1) Q(\vec{p}_2, \omega_2) \right)$$

$$C_{Q\pi} = -R^4 \Delta M \, F_\pi^2$$

Discussion

- Light quarks strongly interact with QCD vacuum instantons due to zero-modes. These one is responsible for the dynamical quark mass $M \sim 400\,MeV$ together with $S\chi$SB and the most important properties of light hadrons and nuclei.
- Heavy quarks interact with these instantons moderately and it leads to heavy quark mass shift $\Delta M \sim 150\,MeV$.
- QCD vacuum instantons generate also heavy-light quarks interactions, responsible for the traces of the light quarks chiral dynamics in heavy quarks physics.
- There is a consistent way to estimate the couplings in the phenomenological chiral lagrangian for heavy-heavy, heavy-light and light-light quarks mesons, accounting $S\chi$SB and heavy quark symmetries.
- The calculations are on the progress in the collaboration with Hyun-Chul Kim and his group.