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QCD instanton vacuum

Quarks in tinstanton

Light quarks in the instanton background Light quark determinant

partition function Heavy quarks in the instanton

Heavy quark propagator in the instanton vacuum with light quarks

interactions at any number of light quarks N_f Heavy–light quarks interactions at

interactions a $N_f = 1$ Heavy quarks-light

QCD Vacuum generated Light-Heavy Quarks Interaction

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National University of Uzbekistan

9th APCTP-BLTP JINR Joint Workshop in Kazakhstan Almaty, June 27 – July 4, 2015

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unction Heavy quarks the instanton

Heavy quark propagator in the instanton vacuum with light quarks

quarks interactions at any number of light quarks N_f Heavy-light quarks interactions at $N_f = 1$

Heavy quarks-light mesons

Outline

- QCD vacuum
- QCD instanton vacuum
- 2 Quarks in the instanton vacuum
 - Light quarks in the instanton background
 - Light quark determinant
 - Light quarks partition function
 - Heavy quarks in the instanton vacuum
 - Heavy quark propagator in the instanton vacuum with light quarks
 - Heavy–light quarks interactions at any number of light quarks N_f
 - lacktriangle Heavy–light quarks interactions at $N_f=1$
 - Heavy quarks-light mesons interactions at $N_f = 2$.
- 3 Heavy quark-antiquark system
 - Heavy quark–antiquark potential V_{lq} , generated by light quarks.
 - Quarkonium light hadron transitions.
- 4 Discussion

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Interactions at any number of light quarks N_f Heavy–light quarks interactions at $N_c = 1$

QCD vacuum on the lattice

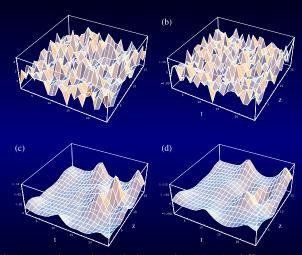


Figure : Action and topological charge densities in different configurations on the lattice.

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quarks-light

QCD instantons

Instantons –self-dual classical solutions of the equations of motion in Euclidean space (Belavin *et.al.*, 1975):

$$A_{\mu}^{I,a}(x) = \frac{2\rho^2 \bar{\eta}_{\mu a}^{\nu}(x-z)_{\nu}}{(x-z)^2 [\rho^2 + (x-z)^2]}, \ G_{\mu \nu}^a = \tilde{G}_{\mu \nu}^a.$$

- The topological charge $Q=rac{1}{32\pi^2}\int d^4x\,G^a_{\mu
 u} ilde{G}^a_{\mu
 u}=1.$
- For the antiinstanton the t'Hooft symbol $\bar{\eta} \to \eta$, $G_{\mu\nu}^a = -\tilde{G}_{\mu\nu}^a$, Q = -1.
- Chern-Simons number collective coordinate

$$N_{CS} = rac{1}{16\pi^2}\int d^3x\, \epsilon^{ijk} \left(A^a_i\partial_jA^a_k + rac{1}{3}\epsilon^{abc}A^a_iA^b_jA^c_k
ight),$$

- Large gauge transformations $N_W = \frac{1}{24\pi^2} \int d^3x \epsilon_{ijk} \left\langle \left(U^{\dagger} \partial_i U \right) \left(U^{\dagger} \partial_j U \right) \left(U^{\dagger} \partial_k U \right) \right\rangle$. $N_{CS} \Rightarrow N_{CS} + N_W$.
- Number of collective coordinates:

$$4 (centre) + 1 (size) + (4N_c-5) (orientations) = 4N_c$$

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Dependence on N_{CS}

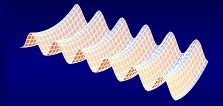


Figure : Vacuum gluon energy vs Chern-Simons number collective coordinate N_{CS} . The amplitude of quantum tunneling $\sim \exp{(-S_I)}$ between the states with $|\Delta N_{CS}| = 1$. Here the action $S_I = \frac{8\pi^2}{\sigma^2}$.

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OCD Vacuum

Instanton vacuum model

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- Quarks in instanton
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- vacuum with light quarks Heavy-light quarks interactions at any number of light quarks No.
- any number of light quarks N_f Heavy–light quarks interactions at $N_f=1$ Heavy quarks-light

- Sum ansatz $A = \sum_I A^I + \sum_{\bar{I}} A^{\bar{I}}$ for dilute gas approximation.
- Inter-instantons interactions have to stabilize instanton size ρ and inter-instanton distance R.

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$S\chi SB$ in QCD instanton vacuum

- Correct description of the spontaneous breaking of the chiral symmetry ($S\chi SB$), which is responsible for properties of most light hadrons.
- $S\chi SB$ is due to the delocalization of single-instanton quark zero modes in the instanton medium.
- Only two parameters:
 - average instanton size $\rho \sim$ 0.3 fm,
 - ullet average inter-instanton distance $R\sim 1\,\mathrm{fm}$,
 - suggested phenomenologically (Shuryak1981),
 - derived variationally from $\Lambda_{\overline{\rm MS}}$ (Diakonov,Petrov1983)
 - confirmed by lattice measurements (Negele et al1998, DeGrand et al2001, Faccioli et al2003, Bowman etal2004).
- The model provided a consistent description of the light quark physics (Diakonov et al, Goeke et al, Musakhanov et al).

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Heavy quarks-light

Parameters of instanton vacuum

- Averaged instanton size $\bar{\rho}$;
- Averaged inter-instanton distance \bar{R} .
- Results:
 - Lattice estimate: $\bar{R} \approx 0.89 \, \text{fm}$, $\bar{\rho} \approx 0.36 \, \text{fm}$,
 - Phenomenological estimate: $\bar{R} \approx 1$ fm, $\bar{\rho} \approx 0.33$ fm,
 - Our estimate (with account of $1/N_c$ corrections): $\bar{R} \approx 0.76 \, fm$, $\bar{\rho} \approx 0.32 \, fm$, correspond

$$R \approx 0.76$$
 fm, $\rho \approx 0.32$ fm, correspond $F_{\pi,m=0} = 88 MeV, \langle \bar{q}q \rangle_{m=0} = -(255 MeV)^3$

Thus within 10 - 15% uncertainty different approaches give similar estimates

- Packing parameter $\pi^2(\frac{\bar{\rho}}{R})^4 \sim 0.1-0.3$
 - ⇒ Independent averaging over instanton positions and orientations.

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Light quarks in the single instanton background

Very strong modification of the light quark propagator

$$S(x,y) pprox rac{\ket{\Phi_0(x,\zeta)}\bra{\Phi_0(y,\zeta)}}{im} + rac{1}{i\hat{\partial}},$$

due to the zero mode

$$(i\hat{\partial}+g\hat{A})\Phi_0(x,\zeta)=0$$
.

Here collective coordinates ζ : a instanton position z and color orientation U.

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Heavy-light quarks interactions at

 $N_f=1$ Heavy quarks-light Spontaneous Breaking of the Chiral Symmetry

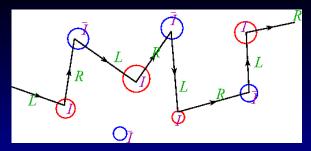


Figure : Light quark in the instanton vacuum. $N_f = 1$.

Sum-up of multi-scattering series \Rightarrow full light quark propagator:

$$S - S_0 = -S_0 \sum_{i,j} \hat{p} |\Phi_{0i}\rangle \left\langle \Phi_{0i} \left| \left(\frac{1}{B(m)} \right) \right| \Phi_{0j} \right\rangle \langle \Phi_{0j} | \hat{p} S_0,$$
 $B(m) = \hat{p} S_0 \hat{p}$

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vacuum Heavy quark propagator in the instanton vacuum with

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Low-frequency part of the light quark determinant with the quark sources

We was able to find $\mathrm{Det}_{low}(\hat{P}+im)e^{(-\xi^+S\xi)}$:

$$= \int \prod_{f} D\psi_{f} D\psi_{f}^{\dagger} \exp \int \left(\psi_{f}^{\dagger} (\hat{p} + im_{f}) \psi_{f} + \psi_{f}^{\dagger} \xi_{f} + \xi_{f}^{+} \psi_{f} \right)$$

$$\prod_{f} \prod_{f} \psi_{f} \psi_{f}^{\dagger} = \prod_{f} \psi_{f}^{\dagger} \left(\prod_{f} \psi_{f} (\hat{p} + im_{f}) \psi_{f} + \psi_{f}^{\dagger} \xi_{f} + \xi_{f}^{+} \psi_{f} \right)$$

$$\times \prod_{f} \left\{ \prod_{+}^{N_{+}} V_{+,f}[\psi^{\dagger}, \psi] \prod_{-}^{N_{-}} V_{-,f}[\psi^{\dagger}, \psi] \right\} ,$$

where $V_{\pm,f}[\psi^\dagger,\psi]=$

$$=i\int dx \left(\psi_f^\dagger(x)\,\hat{p}\Phi_{\pm,0}(x;\zeta_\pm)
ight)\int dy \left(\Phi_{\pm,0}^\dagger(y;\zeta_\pm)(\hat{p}\,\psi_f(y)
ight).$$

Fermionic fields ψ^\dagger, ψ has a meaning of constituent quarks.

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Partition function

Averaging over instantons collective coordinates \Rightarrow partition function $Z[\xi_f, \xi_f^+] =$

$$= \int \prod_f D\psi_f D\psi_f^\dagger \exp \int \left(\psi_f^\dagger (\hat{p} + im_f) \psi_f + \psi_f^\dagger \xi_f + \xi_f^+ \psi_f \right) \\ \times \int D\zeta \prod_{+}^{N_+} \prod_f V_{+,f} [\psi^\dagger, \psi] \prod_{-}^{N_-} \prod_f V_{-,f} [\psi^\dagger, \psi] \;,$$

Small packing parameter provided here independent averaging:

$$\overline{\prod_f V_{\pm,f}[\psi^\dagger,\psi]} = \int extit{d}\zeta_\pm \prod_f V_{\pm,f}[\psi^\dagger,\psi]$$

 \Rightarrow non-local t'Hooft-like vertex with $2N_f$ -legs. Nonlocality range $\sim \rho$.

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Heavy quark propagator in the instanton vacuum with

Heavy-light quarks interactions at any number of light quarks N_f Heavy-light quarks interactions at Dynamical quark mass M(q)

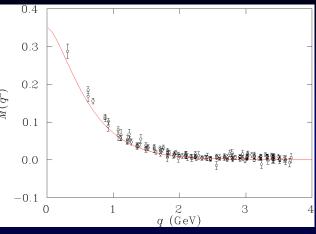


Figure : Light quark dynamical mass M(q) (red line) in comparison with lattice results (Bowman et al 2004). M(q)-dependence is entirely defined by zero-mode!!! $M(0) \approx 365 \, MeV$ gives a strength of the light quark interactions with QCD vacuum instantons.

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Heavy quarks in the instanton vacuum

$$L_{\Psi} = \Psi^{+}(\hat{P} + im_{H})\Psi \Rightarrow Q^{+}\gamma_{4}P_{4}Q + Q^{+}Q_{1}Q,$$

$$Q_{1} = \frac{\vec{P}^{2}}{2m_{H}} - \frac{\vec{\sigma}\vec{B}}{2m_{H}}, \quad P = p - gA, \quad \vec{B} = rot\vec{A}.$$

(Infinitely) heavy quark propagator (Wilson line) in instanton vacuum defined as

$$w = \int D\zeta \frac{1}{\theta^{-1} - \sum_{i} a_{i}}$$

where $a_i(t) = iA_{i,\mu}(x(t))\frac{d}{dt}x_{\mu}(t), w_{\pm} = \frac{1}{\theta^{-1}-2}, < t|\theta|t'> =$ $\theta(t-t')$.

Pobylica Eq. for the w^{-1} has a solution

$$w^{-1} = \theta^{-1} - rac{N}{2} \sum_{\perp} \theta^{-1} (w_{\pm} - \theta) \theta^{-1} + O(N^2/V^2),$$

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Heavy quark interaction with instanton vacuum

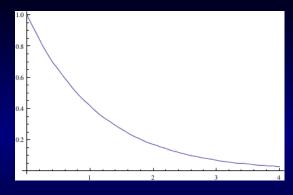


Figure : Form-factor $i_0[q\rho]/i_0[0]$, $i_0[0]=0.55197$. The split of heavy quark mass in instanton vacuum is $\Delta M=16\pi i_0[0](\rho^4/R^4)\rho^{-1}/N_c$. At $\rho=0.32 fm$, R=0.76 fm (ChPT) $\Delta M=148 MeV$ gives a strength of the heavy quark interactions with QCD vacuum instantons at the range $\sim \rho$.

Heavy quark propagator at light quark number generated Light-Heavy Quarks $N_{\rm f}=1$ Interaction Extension of DPP89 solution (planar graphs) is $w^{-1}[\psi, \psi^{\dagger}] =$

 $=\overline{ heta^{-1}}-rac{N}{2}\sum_{ar{}}rac{1}{\overline{V_{+}[\psi^{\dagger},\psi]}}\Delta_{H,\pm}[\psi^{\dagger},\psi]+O(N^{2}/V^{2}),$

 $\Delta_{H,\pm}[\psi^{\dagger},\psi] = \int d\zeta_{\pm} V_{\pm}[\psi^{\dagger},\psi]\theta^{-1}(w_{\pm}-\theta)\theta^{-1}.$

 $S_{H} = \frac{1}{\theta^{-1} - \lambda \sum_{+} \Delta_{H,\pm} \left[\frac{\delta}{\delta \xi}, \frac{\delta}{\delta \xi^{+}}\right]} \exp \left[-\xi^{+} \left(\hat{p} + iM(p)\right)^{-1} \xi\right] |_{\xi = \xi^{+}}$

 $S_H^{-1} pprox heta^{-1} - \lambda \sum_{k} \Delta_{H,\pm} [rac{\delta}{\delta \xi}, rac{\delta}{\delta \xi^+}] \exp \left[-\xi^+ \left(\hat{p} + iM(p)
ight)^{-1} \xi
ight] |_{\xi=0}$

At any N_f and in saddle-point approximation no an essential

Then heavy quark propagator at light quark number $N_f = 1$

DPP89 solution is reproduced at the approximation:

difference with $N_f = 1$.

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Heavy quark propagator in the instanton vacuum with light quarks

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Heavy–light quarks interactions at any number of light quarks N_f

is given by the expression $-\lambda \sum_{\pm} Q^{\dagger} \Delta_{H,\pm} [\psi^{\dagger},\psi] Q$

$$=-i\lambda\sum_{\pm}\int d^{4}z_{\pm}dU_{\pm}\prod_{f=1}^{N_{f}}\frac{d^{4}k_{f}}{(2\pi)^{4}}\frac{d^{4}q_{f}}{(2\pi)^{4}}\exp(i(q_{f}-k_{f})z_{\pm})$$

$$\psi_{f,a_{f}\alpha_{f}}^{+}(k_{f})(\gamma_{\mu_{f}}\gamma_{\nu_{f}}\frac{1\pm\gamma_{5}}{2})_{\alpha_{f}\beta_{f}}(U_{\pm,i_{f}}^{a_{f}}(\tau_{\mu_{f}}^{\mp}\tau_{\nu_{f}}^{\pm})_{j_{f}}^{i_{f}}U_{\pm,b_{f}}^{\dagger j_{f}}\psi_{f,\beta_{f}}^{b_{f}}(q_{f})$$

$$\frac{(2\pi\rho)^{2}F(k_{f})F(q_{f})}{8}Q_{a_{3}}^{+}U_{\pm,i_{3}}^{a_{3}}\left(\theta^{-1}(w_{\pm}-\theta)\theta^{-1}\right)_{j_{3}}^{i_{3}}U_{\pm,b_{3}}^{\dagger j_{3}}Q^{b_{3}}$$

At any N_f the interaction term have 2 heavy and $2N_f$ light quark legs. The actual structure is defined by the color orientation integration and it have to have $SU_L(N_f) \times SU_R(N_f)$ symmetry.

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Heavy–light quarks interactions at $N_f=1$

is

$$-\lambda \sum_{\pm} Q^{\dagger} \Delta_{H,\pm} [\psi^{\dagger}, \psi] Q$$

$$= i \int \frac{d^{4}k_{1}}{(2\pi)^{4}} \frac{d^{4}k_{2}}{(2\pi)^{4}} \frac{d^{3}q}{(2\pi)^{3}} (2\pi)^{4} \delta^{3} (\vec{k}_{2} + \vec{k}_{1} - \vec{q}) \delta(k_{2,4} - k_{1,4})$$

$$(M(k_{1})M(k_{2}))^{1/2} 8\pi \rho^{3} i_{0}(q\rho) \left[\frac{2N_{c} - 1}{N_{c}^{2} - 1} \psi^{+}(k_{1}) \psi(k_{2}) Q^{+} Q + \frac{N_{c} - 2}{N_{c}^{2} - 1} (\psi^{+}(k_{1})QQ^{+}\psi(k_{2}) + \psi^{+}(k_{1})\gamma_{5}QQ^{+}\gamma_{5}\psi(k_{2})) \right]$$

Bosonizaion of the first term provide the light-quark exchange QQ potential V_{lq} , while second and third terms – Qq mesons degenerated on parity.

QCD Vacuum generated Light-Heavy Quarks

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Heavy auarks-light Heavy quarks-light mesons interactions at $N_f = 2$. Heavy-light quarks interactions at $N_f = 2$ has an essential

part—the co-product of colorless Q^+Q -factor and the colorless light-quarks one. This term at $N_f = 2$ leads to the effective action for the mesons $\Phi' = \sigma' + i\gamma_5 \vec{\tau} \vec{\phi}' + i\vec{\tau} \vec{\sigma}' + \gamma_5 \eta'$, interacting with heavy quarks

$$S[\sigma', \vec{\phi'}, \eta', \vec{\sigma'}, Q^{+}Q] = \frac{1}{2} \int d^{4} \left(\sigma'^{2} + \vec{\phi'}^{2} + \vec{\sigma'}^{2} + \eta'^{2}\right)$$

$$- \operatorname{Tr} \ln \left[1 + \frac{iM/\sigma_{0}}{\hat{p} + i(m + M(p))} F(p) \Phi' F(p)\right]$$

$$- \operatorname{Tr} \frac{1}{\hat{p} + i(m + M(p)) + \frac{iM}{\sigma_{0}} F(p) \Phi' F(p)} i \left(M(p) + \frac{iM}{\sigma_{0}} F(p) \Phi' F(p)\right) \left(\frac{i}{N_{0}} \int e^{-ipx} \frac{d^{4}p_{1}}{(2\pi)^{4}} \frac{d^{4}p_{2}}{(2\pi)^{4}} J_{0}(p\rho) Q^{+}Q\right).$$

The first and second lines describe mesons and their interactions, while the third and fourth one give the renormalization of the heavy quark mass and heavy-light quark meson interactions terms.

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Heavy quarks–light mesons interactions at $N_f = 2$ and LO.

The renormalization of heavy quark mass is given by

$$S_Q = -rac{2i}{N_c}rac{1}{R^4}J_0(0)\left(\intrac{d^3p_1d\omega_1}{(2\pi)^4}Q^+(ec{p}_1,\omega_1)Q(ec{p}_1,\omega_1)
ight).$$

So, the instanton media contribution to the heavy quark mass is

$$\Delta M = -2J_0(0)/N_c R^4 = 16\pi i_0(0)(\rho^4/R^4)\rho^{-1}/N_c,$$

as we expected.

With $\rho=0.35$ fm, R=0.856 fm we obtain at LO M=570 MeV and $\Delta M=148$ MeV. These factors define the coupling between heavy and light quarks and certainly between heavy quarks and light mesons.

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Charmonium size $r_c \sim 0.4$ fm, bottomonium size $r_b \sim 0.2$ fm. Cornell potential V = a/r + br – rather well description of the charmonium and bottomonium spectrum. What are an instanton corrections?

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Heavy quark-antiquark system correlator

The correlator $C(L_1, L_2) =$

$$\frac{1}{Z} \int D\psi D\psi^{\dagger} \prod_{\pm}^{N_{\pm}} \bar{V}_{\pm}[\psi^{\dagger}, \psi] \exp \int \left(\psi^{\dagger}(p + im)\psi\right) W[\psi, \psi^{\dagger}],$$

$$< T|W[\psi, \psi^{\dagger}]|0> = \left(\prod_{\pm}^{N_{\pm}} \bar{V}_{\pm}[\psi^{\dagger}, \psi]\right)^{-1} \int D\zeta \prod_{\pm}^{N_{\pm}} V_{\pm}[\psi^{\dagger}, \psi]$$
 $\times < T|\frac{1}{\theta^{-1} - \sum_{i} a_{i}^{(1)}}|0> < 0|\frac{1}{\theta^{-1} - \sum_{i} a_{i}^{(2)}}|T>.$

is a Wilson loop along the rectangular contour $L \times r$. The sides $L_1 = (0, T), L_2 = (T, 0)$ are parallel to x_4 axes and separated by the distance r. The $a^{(1)}, a^{(2)}$ are the projections of the instantons onto the lines L_1, L_2 .

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Eq. for heavy quark-antiquark system correlator

The extension of DPP89 solution is $W^{-1}[\psi,\overline{\psi^{\dagger}}]=$

$$= w_1^{-1}[\psi, \psi^{\dagger}] \times w_2^{-1, T}[\psi, \psi^{\dagger}] - \frac{N}{2} \sum_{\pm} \bar{V}_{\pm}^{-1}[\psi^{\dagger}, \psi] \int d\zeta_{\pm}$$

$$\times V_{\pm}[\psi^{\dagger},\psi]\theta^{-1}\left(w_{\pm}^{(1)}-\theta\right)\theta^{-1}(\times)\left(\theta^{-1}\left(w_{\pm}^{(2)}-\theta\right)\theta^{-1}\right)^{T}$$

where, superscript ${\mathcal T}$ means the transposition, (\times) – tensor product and

$$w^{(1,2)^{-1}}[\psi,\psi^{\dagger}] = \theta^{-1} - \frac{N}{2} \sum_{\pm} \frac{1}{\bar{V}_{\pm}[\psi^{\dagger},\psi]} \Delta_{H,\pm}^{(1,2)}[\psi^{\dagger},\psi] + O(\frac{N^2}{V^2}).$$

The integration of the first term in $W^{-1}[\psi, \psi^{\dagger}]$ over ψ, ψ^{\dagger} leads to heavy quark–antiquark potential V_{lq} , generated by light quarks.

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Heavy quark–antiquark potential V_{lq} , generated by light quarks, $N_f=1$

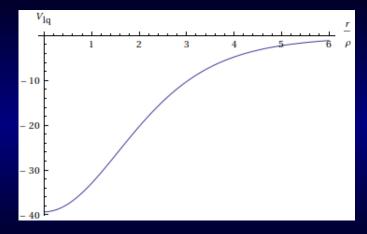


Figure : Heavy quark–antiquark potential $V_{lq}(r/\rho)$ (in MeV), generated by light quarks, its range is controlled by dynamical light quark mass $M \sim 0.36$ GeV.

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partition function Heavy quarks

the instanton vacuum

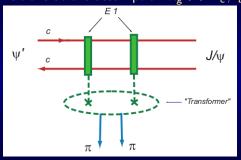
Heavy quark propagator in the instanton vacuum with light quarks

Heavy-light quarks interactions at any number of light quarks N

light quarks N_f
Heavy-light
quarks
interactions at

 $N_f = 1$ Heavy quarks-light Quarkonium light hadron transitions.

Charmonium size $r_c \sim 0.4$ fm, bottomonium size $r_b \sim 0.2$ fm. Hadronic transitions at the assumption $\lambda_e >> r_c, r_b$:



But $\lambda_{\rm g} pprox
ho = 0.35 \, {\it fm} \sim {\it r_c}, {\it r_b}.$

What are an instanton corrections?

Yousuf Musakhanov

background

function

Heavy quark the instanton

any number of light quarks Ne quarks-light

Quarkonium pion transitions. $\psi' \to J/\psi \ \pi\pi$.

Neglect by heavy mesons and rearrange light mesons as $(\sigma_0 + \sigma' + i\gamma_5 \vec{\tau} \phi')/\sigma_0 \Rightarrow U^{\gamma_5} = \exp(i\vec{\tau} \vec{\pi}(x)\gamma_5).$

Then the interaction term $S_{Q\pi}$ between heavy quark Q and pions is

$$S_{Q\pi} = C_{Q\pi} \int d^4x \operatorname{tr} \partial_\mu U(x) \partial_\mu U^+(x)$$

$$\times \left(i \int e^{-ipx} \frac{d^3p_1 d\omega_1}{(2\pi)^4} \frac{d^3p_2 d\omega_2}{(2\pi)^4} \frac{J_0(p\rho)}{J_0(0)} Q^+(\vec{p}_1, \omega_1) Q(\vec{p}_2, \omega_2)\right)$$

$$C_{Q\pi} = -R^4 \Delta M F_{\pi}^2$$

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vacuum Quarks in th

instanton vacuum

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determinant Light quarks partition

partition function Heavy quarks the instanton

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Discussion

- Light quarks strongly interact with QCD vacuum instantons due to zero-modes. These one is responsible for the dynamical quark mass $M \sim 400 \, MeV$ together with $S\chi {\rm SB}$ and the most important properties of light hadrons and nuclei.
- Heavy quarks interact with these instantons moderately and it leads to heavy quark mass shift $\Delta M \sim 150~MeV$.
- QCD vacuum instantons generate also heavy-light quarks interactions, responsible for the traces of the light quarks chiral dynamics in heavy quarks physics.
- There is a consistent way to estimate the couplings in the phenomenological chiral lagrangian for heavy-heavy, heavy-light and light-light quarks mesons, accounting $S\chi SB$ and heavy quark symmetries.
- The calculations are on the progress in the collaboration with Hyun-Chul Kim and his group.