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The 9th APCTP-BLTP JINR Joint Workshop

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Almaty, Kazakhstan

Collective Hamiltonian for chiral and wobbling modes

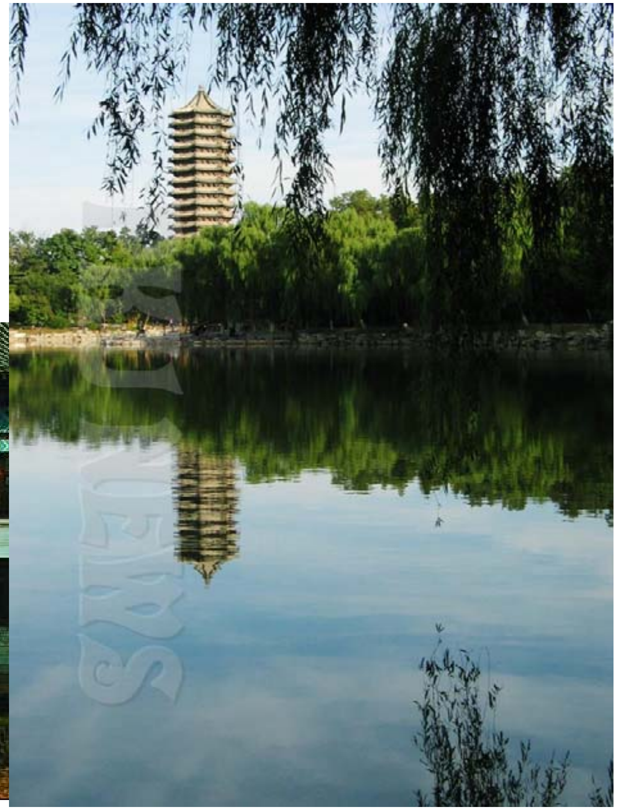
Jie Meng

School of Physics, Peking University

Collaborators:

Qibo Chen, Rostilav Jolos, Shuangquan Zhang, Pengwei Zhao

Founded in 1898, Peking University was originally known as the Imperial University of Peking. It was the first national university covering comprehensive disciplines in China, and has been a leading institution of higher education in China since its establishment.



北京大學
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A Glimpse of Nuclear Structure in China

First National Nuclear Structure Conference 1986

会议名称为“全国高自旋与IBM研讨会 ”旗号

Ninth National Nuclear Structure Conference 2002

核结构专业委员会调整 (2002.4-2004.7)

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Fourteenth National Nuclear Structure Conference 2012

中国核物理学会核结构专业委员会

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第一届北京亚原子物理国际暑期学校1999年7月25-29日

INTERNATIONAL SUMMER SCHOOL ON SUBATOMIC PHYSICS

1st Course: NUCLEAR AND ASTRO-PHYSICS WITH RI BEAMS

1st Course: Nuclear and Astro-physics with RI Beams   6 institutes, 10 lecturers

第二届北京亚原子物理国际暑期学校2001年8月21日-25日
INTERNATIONAL SUMMER SCHOOL ON SUBATOMIC PHYSICS
2nd Course: Nuclear structure and reaction in astrophysics

2nd Course: Nuclear structure and reaction in astrophysics 7 institutes, 11 lecturers

北京大學 PEKING UNIVERSITY

第三届北京亚原子物理国际暑期学校 **2004年8月26-30日**

INTERNATIONAL SUMMER SCHOOL ON SUBATOMIC PHYSICS

3rd Course: Nuclear far from stability, Hypernuclei,Quark-gluon plasama and Nuclear astrophysics

3rd Course: Frontiers of Nuclear Physics 8 institutes, 14 lecturers

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第四届 北京亚原子物理国际暑期学校 **2006年8月21-25日**

INTERNATIONAL SUMMER SCHOOL ON SUBATOMIC PHYSICS 2006

4th Course: Physics with new generation of RIBF August 21-25, 2006

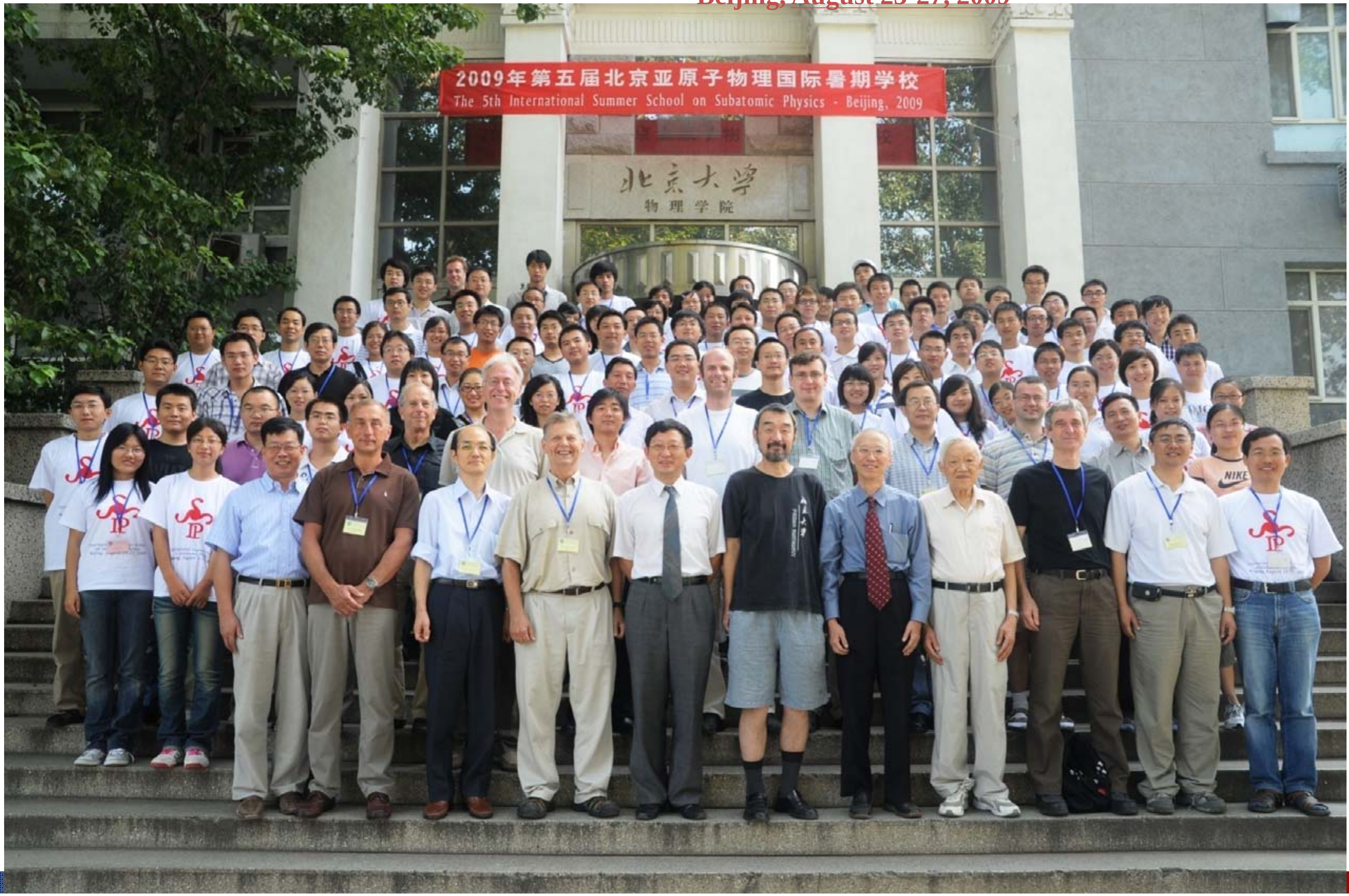
4th Course: Physics with new generation of RI Beams 10 institutes, 19 lecturers

第五届 北京亚原子物理国际暑期学校 2009年8月23日— 27日

INTERNATIONAL SUMMER SCHOOL ON SUBATOMIC PHYSICS

5th Course: Some New Facets of Nuclear Physics

Beijing, August 23-27, 2009

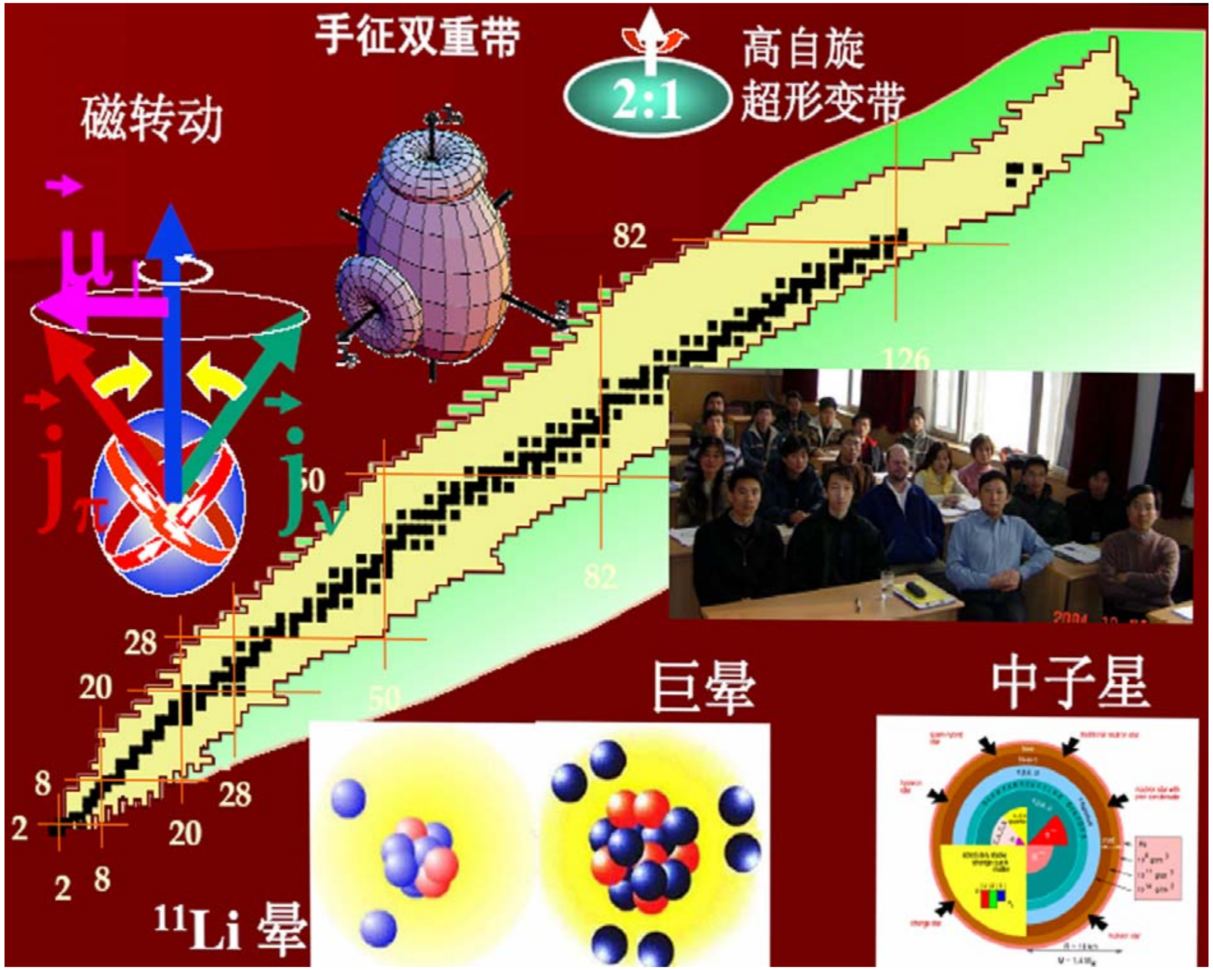


INTERNATIONAL SUMMER SCHOOL ON SUBATOMIC PHYSICS

6th Course: New Frontiers of Nuclear Physics

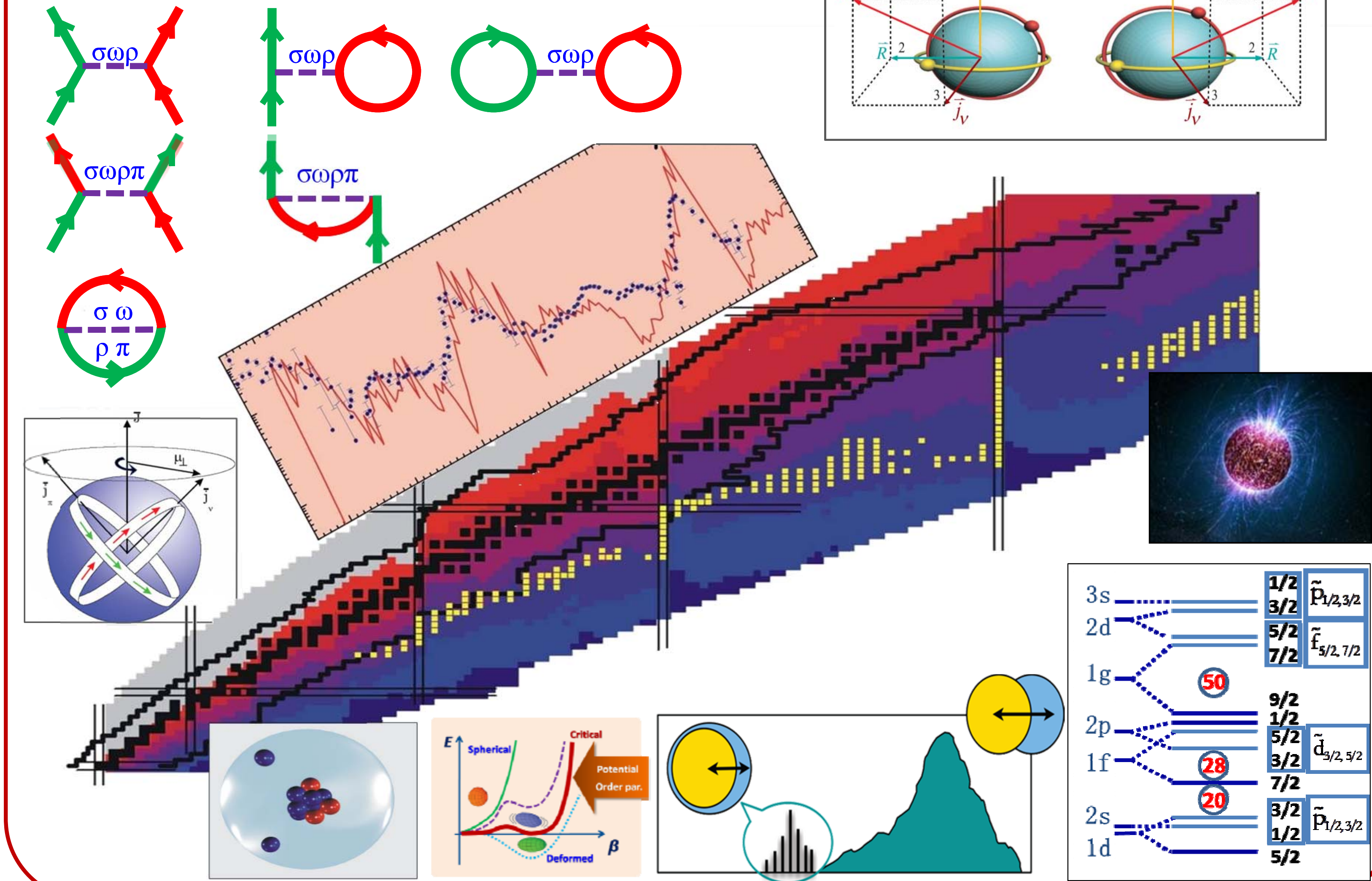
Beijing, August 27-31, 2011

Research team：Since December 1997



Covariant density functional

Progress in Physics. 31(2011)199-336.



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Progress in Particle and Nuclear Physics 57 (2006) 470–563

Progress in Particle and Nuclear Physics

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Review

Relativistic continuum Hartree Bogoliubov theory for ground-state properties of exotic nuclei

J. Meng[a,b,c,*], H. Toki[d], S.G. Zhou[b,c], S.Q. Zhang[a], W.H. Long[a], L.S. Geng[a,d]

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第31卷 第4期
2011年12月

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PROGRESS IN PHYSICS

Vol.31 No.4
Dec. 2011

原子核物理中的协变密度泛函理论

孟 杰[1,2]*，郭建友[3]，李 剑[1]，李志攀[4]，梁豪兆[1]，龙文辉[5]，牛一斐[1]，牛中明[1]，
尧江明[4]，张 颖[1]，赵鹏巍[1]，周善贵[6,7]

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3. 安徽大学物理与材料科学学院，合肥，230039
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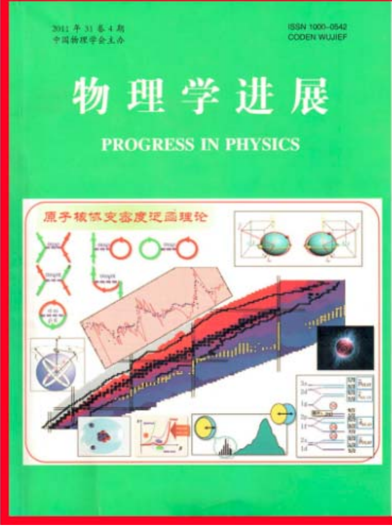
5. 兰州大学核科学与技术学院，兰州，730000
6. 中国科学院理论物理研究所，北京，100190
7. 兰州重离子加速器国家实验室原子核理论中心，兰州，730000

摘要： 文章介绍了原子核协变密度泛函理论的历史发展
描述以及在一些交叉学科领域的应用。首先，通过回顾原
合二十一世纪原子核物理面临的机遇和挑战，对当前核物
随后系统介绍了原子核协变密度泛函理论，内容包括协变
式、介子交换模型、点耦合模型、交换项、张量相互作用、
函理论的应用包括原子核基态性质和激发态性质的描述以
用。其中，基态性质包括原子核结合能、半径、单粒子能
质包括原子核磁转动、低激发态性质、集体转动、量子相
型检验的应用中，主要以核纪年法测算宇宙年龄和 Cabib
检验等为例，介绍协变密度泛函理论在交叉学科领域的应



Covariant Density Functional Theory in Nuclear Physics

Meng Jie[1,2], Guo Jian-You[3], Li Jian[1], Li Zhi-Pan[4], Liang Hao-Zhao[1], Long Wen-Hui[5], Niu Yi-Fei[1], Niu Zhong-Ming[1], Yao Jiang-Ming[4], Zhang Ying[1], Zhao Peng-Wei[1], Zhou Shan-Gui[6,7]

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Front. Phys., 2013, 8(1): 55–79
DOI 10.1007/s11467-013-0287-y

REVIEW ARTICLE

Progress on tilted axis cranking covariant density functional theory for nuclear magnetic and antimagnetic rotation

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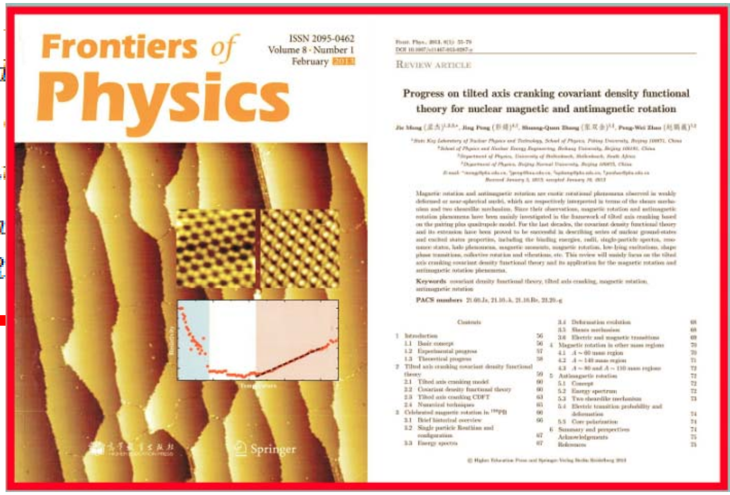

Physics Reports 570 (2015) 1–84

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Physics Reports

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Hidden pseudospin and spin symmetries and their origins in atomic nuclei

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[c] *School of Physics and Nuclear Energy Engineering, Beihang University, Beijing 100191, China*

[d] *Department of Physics, University of Stellenbosch, Stellenbosch, South Africa*

[e] *State Key Laboratory of Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sci*

[f] *Center of Theoretical Nuclear Physics, National Laboratory of Heavy Ion Accelerator, Lanzhou 7300*

IOP Publishing Journal of Physics G: Nuclear and Particle Physics

J. Phys. G: Nucl. Part. Phys. **00** (2015) 000000 (52pp)

Topical Review

Halos in medium-heavy and heavy nuclei with covariant density functional theory in continuum

J Meng[1,2,3] **and S G Zhou**[4,5,6]

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Collective Hamiltonian for chiral and wobbling modes

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Outline

- Introduction
- Theoretical framework
- Chiral modes
- Wobbling modes
- Summary and perspective

Chirality

- **Chirality was originally suggested in 1997 and firstly observed in 2001.**

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Nuclear Physics A 617 (1997) 131–147

NUCLEAR PHYSICS A

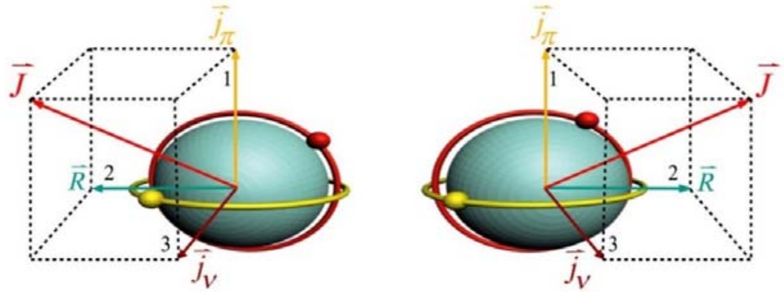
Tilted rotation of triaxial nuclei

S. Frauendorf, Jie Meng [1]

Institut für Kern- und Hadronenphysik, Forschungszentrum Rossendorf e.V., PF 510119, 01314 Dresden, Germany

Received 14 November 1996

Originally suggested in 1997
chiral doublet bands



Firstly Observed in 2001

VOLUME 86, NUMBER 6 PHYSICAL REVIEW LETTERS 5 FEBRUARY 2001

Chiral Doublet Structures in Odd-Odd $N = 75$ Isotones: Chiral Vibrations

K. Starosta,[1,*] T. Koike,[1] C. J. Chiara,[1] D. B. Fossan,[1] D. R. LaFosse,[1] A. A. Hecht,[2] C. W. Beausang,[2] M. A. Caprio,[2] J. R. Cooper,[2] R. Krücken,[2] J. R. Novak,[2] N. V. Zamfir,[2,†] K. E. Zyromski,[2] D. J. Hartley,[3] D. L. Balabanski,[3,‡] Jing-ye Zhang,[3] S. Frauendorf,[4] and V. I. Dimitrov[4,‡]

[1]*Department of Physics and Astronomy, SUNY at Stony Brook, Stony Brook, New York 11794*
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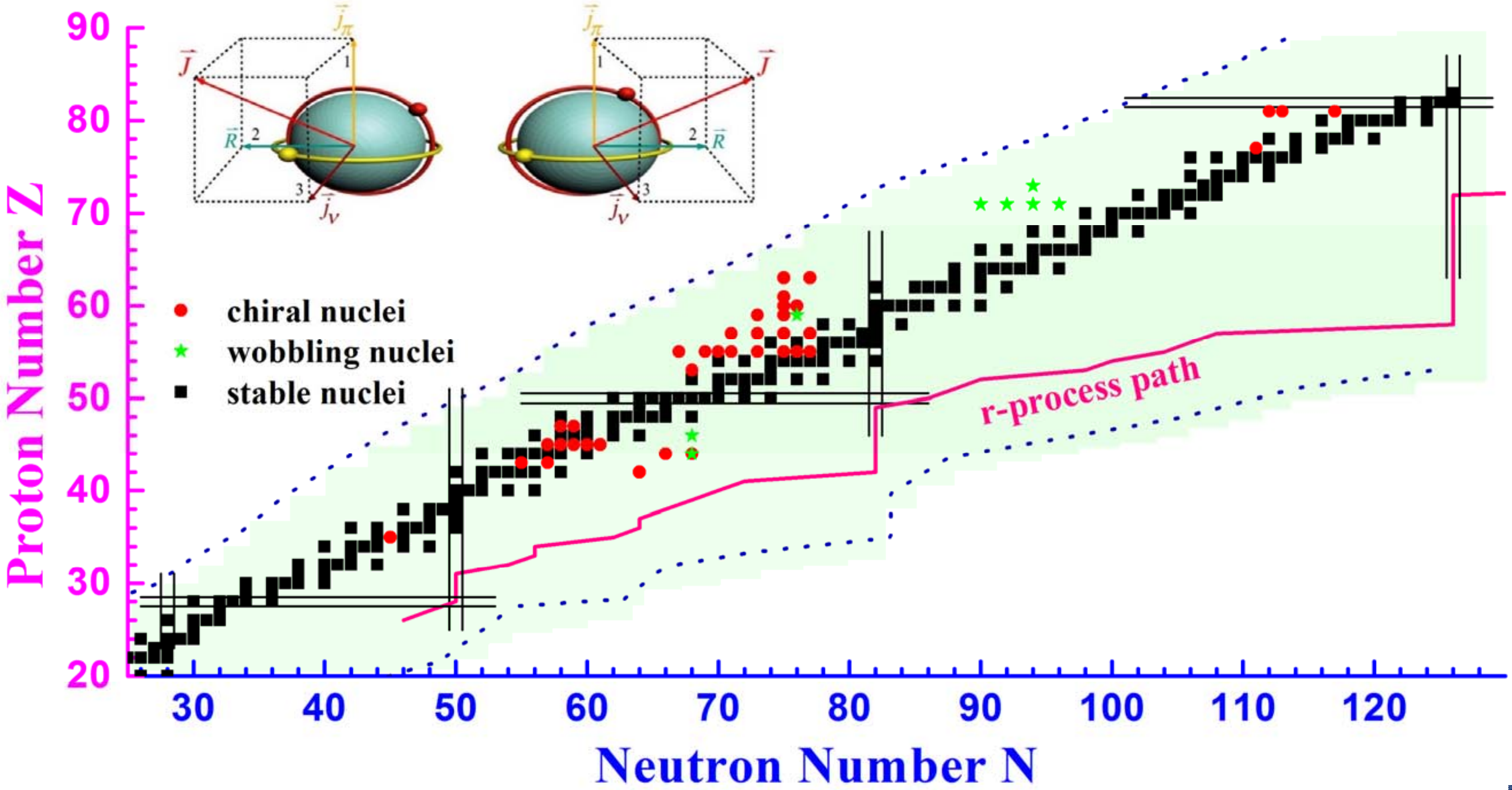
(Received 24 July 2000)

Chiral and wobbling modes

- **The investigation of chiral and wobbling modes in atomic nuclei has become one of the hottest topics in nuclear physics.**



"Standard" models

- **Triaxial PRM**
  - ✔ Lab frame; quantal model; with quantum tunneling;
  - ✖ Phenomenological

Frauendorf & Meng, NPA617, 131 (1997)
Peng et al., PRC 68, 044324 (2003)
Koike et al., PRL 93, 172502 (2004)
Zhang et al., PRC 75, 044307 (2007)
Qi et al., PLB 675, 175 (2009)
Lawrie & Shirinda, PLB 689, 66 (2010)
Hamamoto, PRC 65, 044305 (2002)
Hamamoto & Hagemann PRC 67, 014319 (2003)
Frauendorf & Dönau, PRC 89, 014322 (2014)

- **Tilted axis cranking (TAC)**
  - ✔ Mean-field approximation; intrinsic frame; microscopic; self-consistent;
  - ✖ Semi-classical; no quantum tunneling;

Frauendorf & Meng, NPA617, 131 (1997)
Dimitrov et al., PRL 84, 5732(2000)
Olbratowski et al., PRL 93, 052501 (2004)
Olbratowski et al., PRC 73, 054308 (2006)
Matta et al., PRL 114, 082501 (2015)

- **Other models**
  - ■ Projected shell model

Sheikh & Hara, PRL 82, 3968(1999),
Dar et al NPA 933, 123 (2015)

  - ■ IBFFM

S. Brant et al., PRC 69, 017304 (2004)
S. Brant et al., PRC 78, 034301 (2008)
Tonev et al., PRL 96, 052501 (2006)

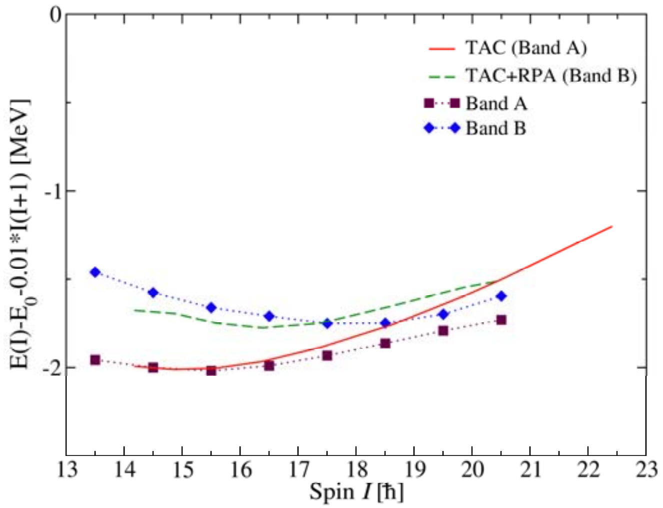
  - ■ Pairing truncated shell model

K. Higashiyama et al, PRC 72, 024315 (2005)

Beyond mean field approximation: RPA

- **TAC + RPA for chiral mode**
  - ✔ **Beyond mean field; chiral vibration**
  - ✖ **Chiral rotation;**

Mukhopadhyay et al., PRL 99, 172501 (2007)
Almehed et al., PRC 83, 054308 (2011)



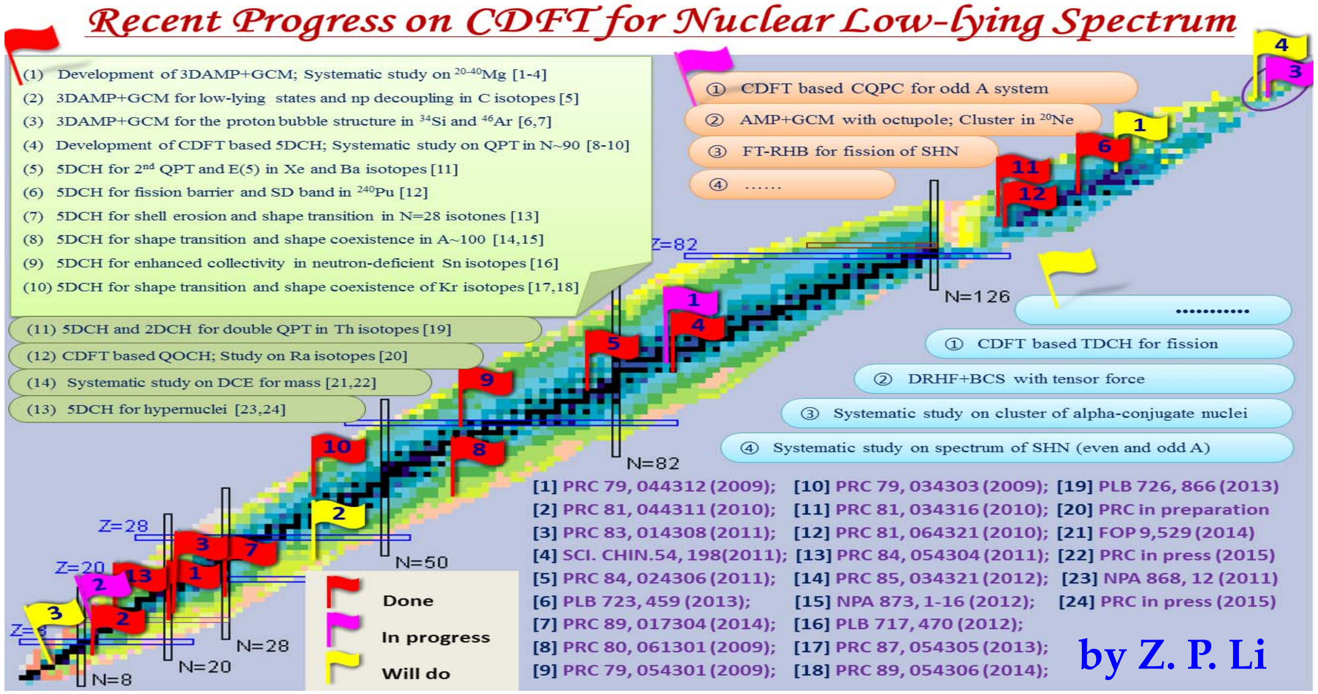
- **Cranking + RPA for wobbling mode**
  - ✔ **Beyond mean field; wobbling excitation**
  - ✖ **Anharmonic wobbling bands;**

Mikhailov & Janssen, PLB 72, 303 (1978)
Marshalek, NPA 331, 429 (1979)
Shimizu & Matsuyanagi, PTP 70, 144 (1983)
Matsuzaki et al., PRC 65, 041303(R) (2002)
Matsuzaki et al., PRC 69, 034325 (2004)
Matsuzaki et al., PRC 69, 064317 (2004)
Almehed et al., PS 125, 139 (2006)
Shoji et al., PTP 121, 139 (2009)

It is thus imperative to search a unified method for studying both chiral and wobbling modes.

Collective Hamiltonian

- **Collective Hamiltonian, e.g. based on CDFT, has achieved great success on applications for shape evolution/transition.**



In present talk, the collective Hamiltonian based on cranking mean field is reported and applications for chiral rotation and wobbling motion are demonstrated.

Outline

- Introduction
- Theoretical framework
- Chiral modes
- Wobbling modes
- Summary and perspective

Microscopic basis

- **Microscopic basis** Collective Hamiltionian, which aims to describe large amplitude collective motions, can be obtained by
  - **Generate coordinate method (GCM)** *Hill&Wheeler, PR 89, 1102 (1953); Ring&Schuck1980*
  - **Adiabatic time-dependent Hartree-Fock (ATDHF) method** *Baranger&Kumar, NPA 122, 241 (1968); Ring&Schuck1980*
  - **Adiabatic self-consistent coordinate method (ASCC)** *Marumori et al., PTP 64, 1294 (1980); Matsuo et al., PTP 103, 959 (2000); Hinohara et al., PRC 82, 064313 (2010); Matsuyanagi et al., JPG 37, 064018 (2010)*
    - **Starting point:** time-dependent Hartree-Fock (TDHF) equation
    - **Assumptions:** adiabatic approximation, i.e., the collective motion is slow or collective momenta are small (can be large)
    - **Procedure:** expand the TDHF equations with respect to the collective momenta up to second order

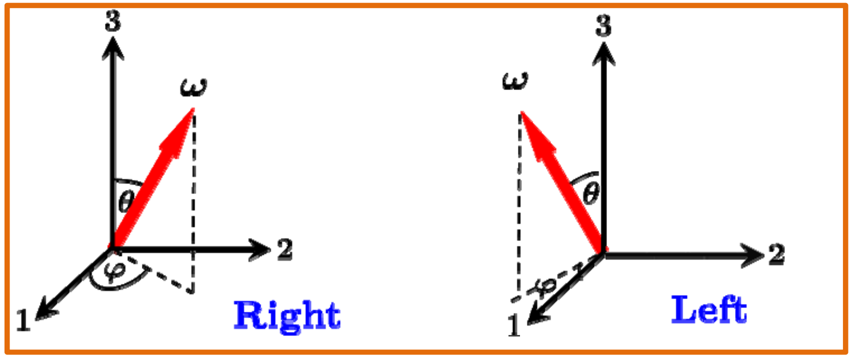
$$\mathcal{H}(q,p) = \langle \phi(q,p)|\hat{H}|\phi(q,p)\rangle = \frac{1}{2}\sum_{ij} B^{ij}(q)p_i p_j + V(q)$$

$$B^{ij}(q) = \frac{\partial^2 \mathcal{H}}{\partial p_i \partial p_j}\Big|_{p=0} \qquad V(q) = \mathcal{H}(q,p)|_{p=0}$$

Coll. coordinate & Coll. Hamiltonian

- For chiral and wobbling modes, the orientation angles of angular momentum can be chosen as collective coordinates.

$$(\theta, \varphi)$$



- For simplicity, only one collective coordinate is considered here,

$$(\theta, \varphi) \rightarrow \varphi$$

- **The classical form of a collective Hamiltonian in terms of φ as,**

$$\hat{H}_{\text{coll}} = \hat{T}_{\text{kin}}(\varphi) + V(\varphi) = \frac{1}{2} B\dot{\varphi}^2 + V(\varphi)$$

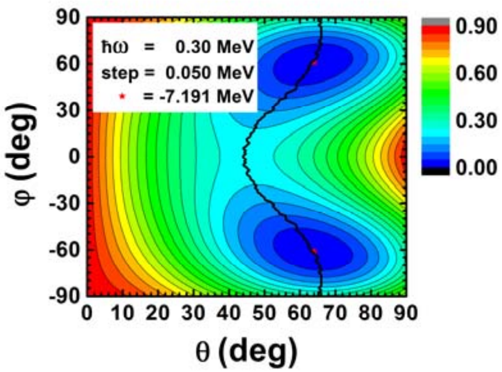
- According to general Pauli quantization *Pauli1933*

$$\hat{H}_{\text{coll}} = \hat{T}_{\text{kin}}(\varphi) + V(\varphi) = -\frac{\hbar^2}{2\sqrt{B(\varphi)}} \frac{\partial}{\partial \varphi} \frac{1}{\sqrt{B(\varphi)}} \frac{\partial}{\partial \varphi} + V(\varphi)$$

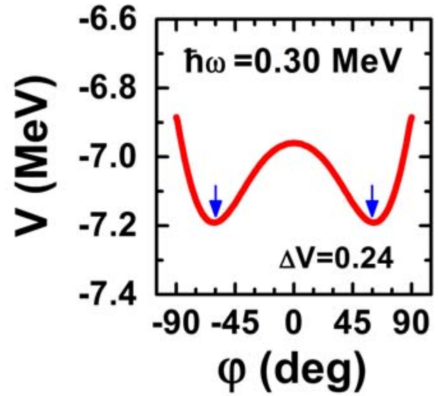
Coll. potential & Mass parameter

- **The collective potential V(φ) could be extracted by minimizing the total Routhian surface, obtained by any TAC calculation, with respect to θ for given φ.**

$V(\theta, \varphi)$



$V(\varphi)$



- **Mass parameter *B*(φ) could be obtained from TAC calculations by cranking formula**

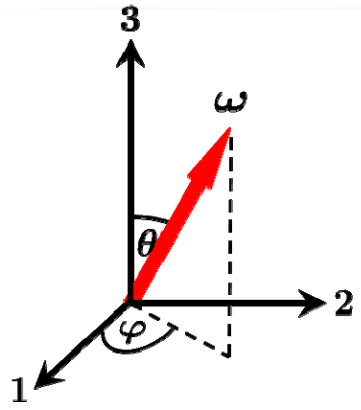
$$B(\varphi) = 2\hbar^2 \sum_{l \neq 0} \frac{(E_l - E_0)^3 |\langle l | \frac{\partial}{\partial \varphi} | 0 \rangle|^2}{[(E_l - E_0)^2 - \hbar^2 \Omega^2]^2}$$

$$= 2\hbar^2 \sum_{l \neq 0} \frac{(E_l - E_0) \left| \langle l | [\hat{h}', \frac{\partial}{\partial \varphi}] | 0 \rangle \right|^2}{[(E_l - E_0)^2 - \hbar^2 \Omega^2]^2}.$$

Basis space

- **Symmetry**

  The collective Hamiltonian keeps invariant with respect to $\varphi \to -\varphi$.



- **Basis states**

  - **Box boundary condition**

$$\psi_n(\pi/2) = \psi_n(-\pi/2) = 0$$

$$\psi_n^{(+)}(\varphi) = \sqrt{\frac{2}{\pi}} \frac{\cos(2n-1)\varphi}{B^{1/4}(\varphi)}, \qquad n \geq 1$$

$$\psi_n^{(-)}(\varphi) = \sqrt{\frac{2}{\pi}} \frac{\sin 2n\varphi}{B^{1/4}(\varphi)}, \qquad n \geq 1$$

  - **Periodic boundary condition**

$$\psi_n(\varphi) = \psi_n(\varphi + \pi/2) = 0$$

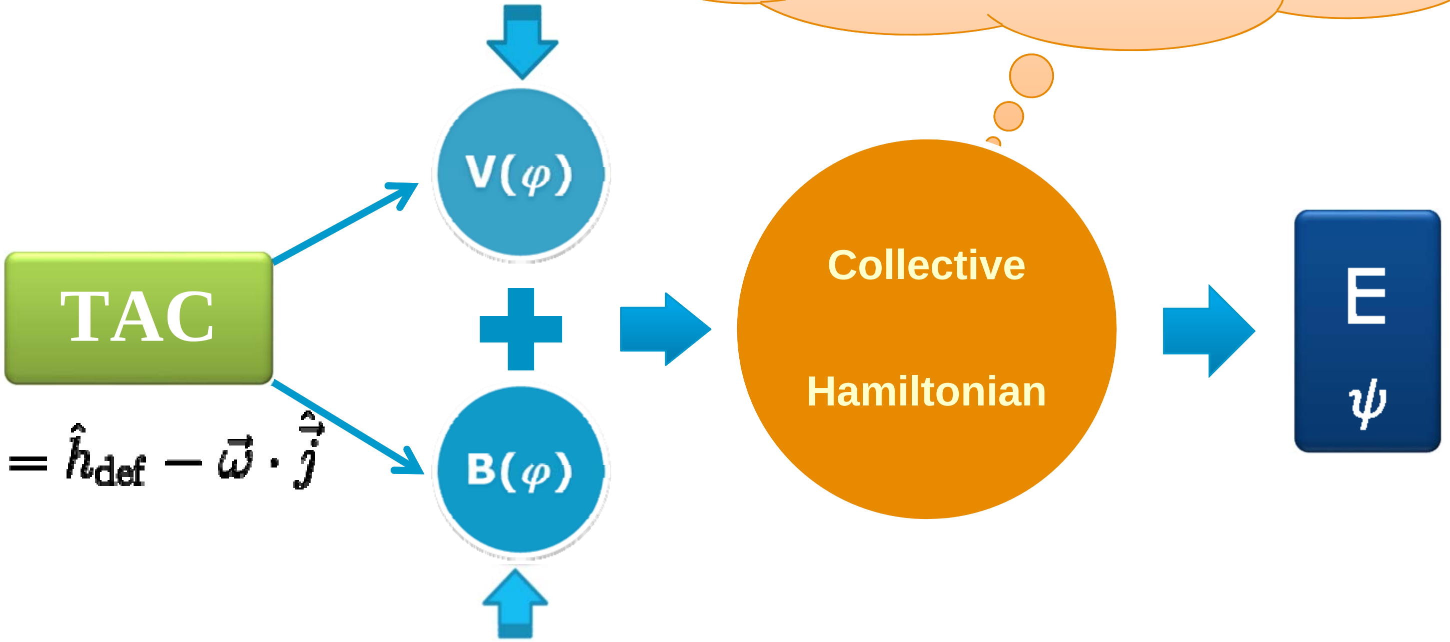
$$\psi_n^{(+)}(\varphi) = \sqrt{\frac{2}{\pi(1+\delta_{n0})}} \frac{\cos 2n\varphi}{B^{1/4}(\varphi)}, \qquad n \geq 0$$

$$\psi_n^{(-)}(\varphi) = \sqrt{\frac{2}{\pi}} \frac{\sin 2n\varphi}{B^{1/4}(\varphi)}, \qquad n \geq 1$$

A schematic illustration

$$\text{minimize } E' = \langle h' \rangle - \frac{1}{2}\sum_{k=1}^{3}\mathcal{J}_k\omega_k^2$$

$$\hat{H}_{\text{coll}} = -\frac{\hbar^2}{2\sqrt{B(\varphi)}}\frac{\partial}{\partial\varphi}\frac{1}{\sqrt{B(\varphi)}}\frac{\partial}{\partial\varphi}+V(\varphi)$$



$$\hat{h}' = \hat{h}_{\text{def}} - \vec{\omega}\cdot\hat{\vec{j}}$$

$$B = 2\hbar^2\sum_{l\neq 0}\frac{(E_l-E_0)\left|\frac{\partial\vec{\omega}}{\partial\varphi}\langle l|\hat{\vec{j}}|0\rangle\right|^2}{\left[(E_l-E_0)^2-\hbar^2\Omega^2\right]^2}$$

Q. B. Chen, S. Q. Zhang, P. W. Zhao, R. V. Jolos, J. Meng
Phys. Rev. C 87, 024314 (2013)

Q. B. Chen, S. Q. Zhang, P. W. Zhao, J. Meng
Phys. Rev. C 90, 044306 (2014)

Outline

- Introduction
- Theoretical framework
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- Wobbling modes
- Summary and perspective

Numerical details

- For chiral modes, we consider a system of a high-*j* proton particle and a high-*j* neutron hole coupled to a triaxial rotor.

$$\hat{h}' = \hat{h}_{\rm def} - \boldsymbol{\omega}\cdot\hat{\boldsymbol{j}}, \quad \hat{h}_{\rm def} = \hat{h}^{\pi}_{\rm def} + \hat{h}^{\nu}_{\rm def}, \quad \boldsymbol{j} = \boldsymbol{j}_\pi + \boldsymbol{j}_\nu,$$

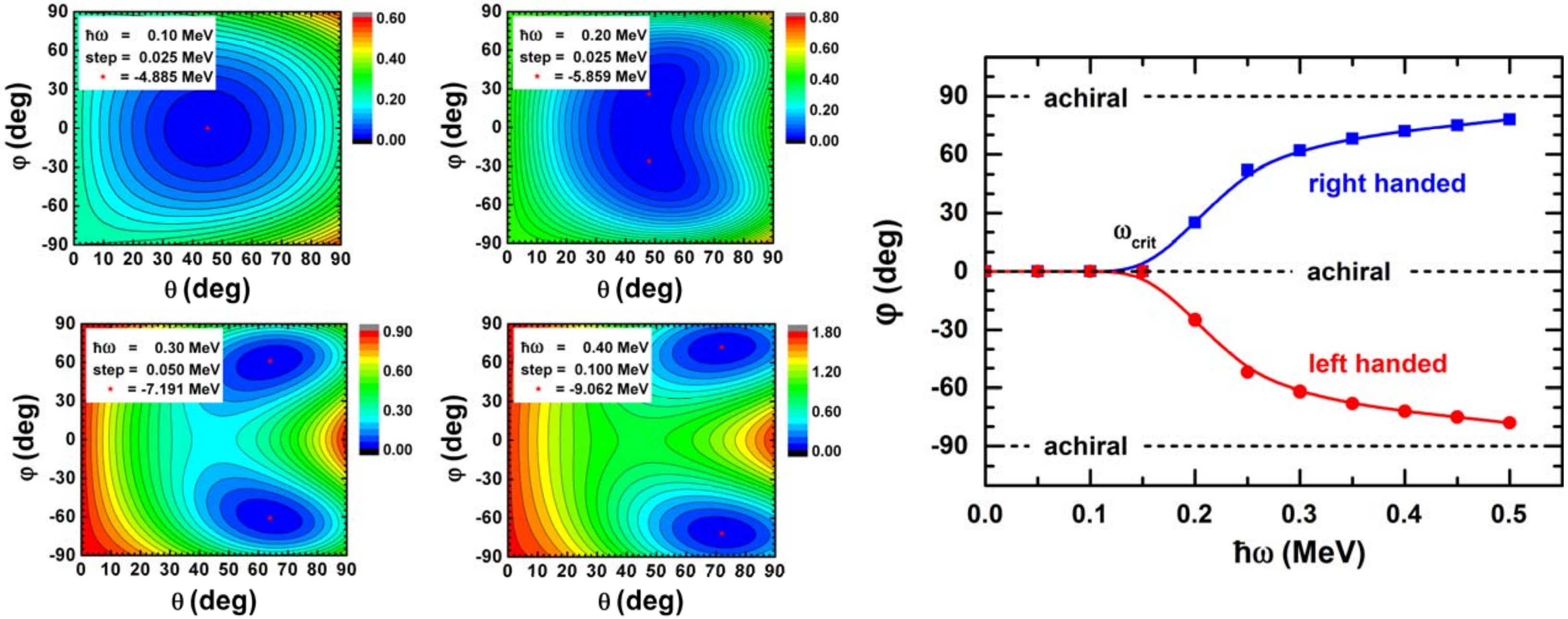
$$\boldsymbol{\omega} = (\omega\sin\theta\cos\varphi, \omega\sin\theta\sin\varphi, \omega\cos\theta),$$

$$\hat{h}^{\pi(\nu)}_{\rm def} = \frac{1}{2}C_{\pi(\nu)}\left\{(\hat{j}_3^2 - \frac{j(j+1)}{3})\cos\gamma + \frac{1}{2\sqrt{3}}(\hat{j}_+^2 + \hat{j}_-^2)\sin\gamma\right\},$$

$$E'(\theta,\varphi) = \langle\hat{h}'\rangle - \frac{1}{2}\sum_{k=1}^{3}\mathcal{J}_k\omega_k^2, \quad \mathcal{J}_k = \mathcal{J}_0\sin^2\left(\gamma - \frac{2\pi}{3}k\right),$$

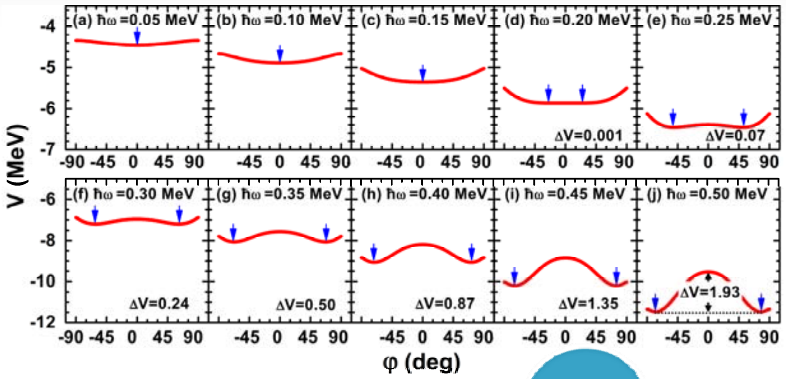
- System:
  - ✓ Configuration: $\pi(1h_{11/2})^1 \otimes \nu(1h_{11/2})^{-1}$
  - ✓ Single-*j* shell Hamiltonian coefficients: $C_\pi = 0.25$ MeV$, C_\nu = -0.25$ MeV
  - ✓ Triaxial deformation: $\gamma = -30^\circ$
  - ✓ Moment of inertia: $\mathcal{J}_0 = 40\ \hbar^2/\text{MeV}$

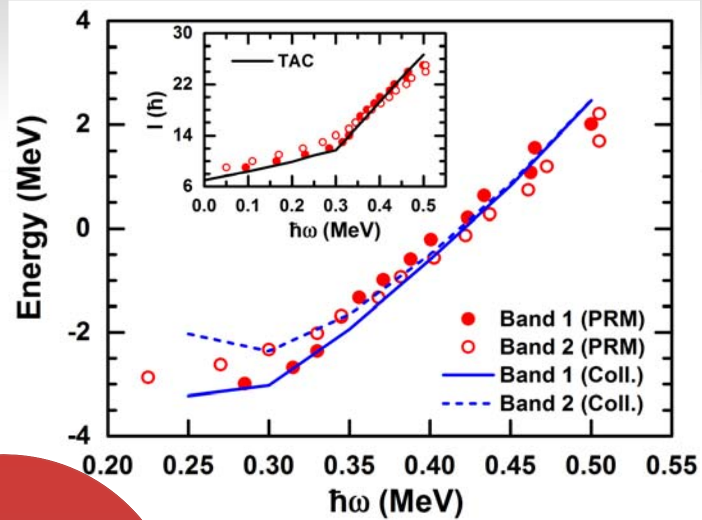
Total Routhian surfaces



- **Total Routhian surfaces**
  - Obtained by TAC in the rotating frame for $\pi(1h_{11/2})^1 \otimes \nu(1h_{11/2})^{-1}$.
  - Symmetrical about $\varphi = 0$; chiral solutions with $\pm|\varphi|$ are identical.
  - Minima: from $\varphi = 0$ to $\varphi \neq 0$; from one to two; critical at $\hbar\omega = 0.15$ MeV.
  - Rotating mode: from planar to aplanar to principal axis rotation.

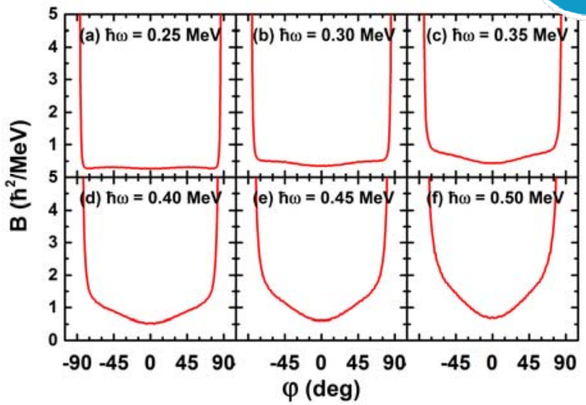
Application for chiral modes





TAC → $V(\varphi)$ + $B(\varphi)$ → Collective Hamiltonian → E, ψ

$$\hat{H}_{\rm coll} = -\frac{\hbar^2}{2\sqrt{B(\varphi)}}\frac{\partial}{\partial\varphi}\frac{1}{\sqrt{B(\varphi)}}\frac{\partial}{\partial\varphi} + V(\varphi)$$



Q. B. Chen, S. Q. Zhang, P. W. Zhao, R. V. Jolos, J. Meng

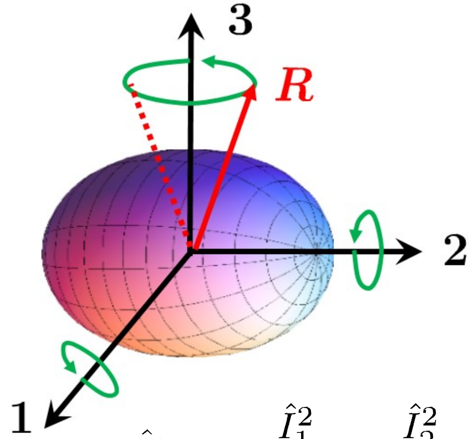
Phys. Rev. C 87, 024314 (2013)

Outline

- Introduction
- Theoretical framework
- Chiral modes
- Wobbling modes
- Summary and perspective

Wobbling

- In *Bohr & Mottelson1975, Vol. II*, the concept of wobbling motion was first proposed for a rotating triaxial nuclei.

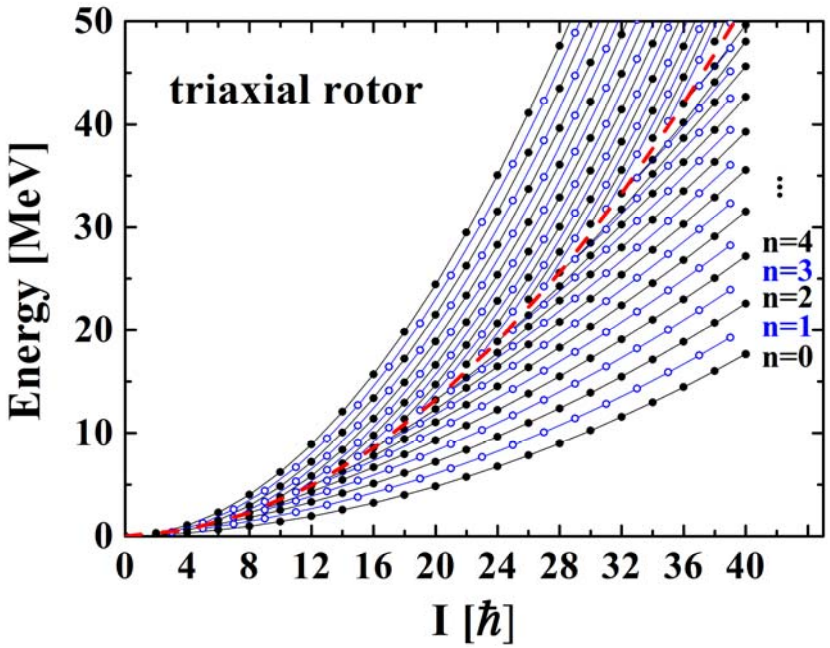


$$\hat{H}_{\rm rot} = \frac{\hat{I}_1^2}{2\mathcal{J}_1} + \frac{\hat{I}_2^2}{2\mathcal{J}_2} + \frac{\hat{I}_3^2}{2\mathcal{J}_3}$$

$$\mathcal{J}_3 \gg \mathcal{J}_1 \neq \mathcal{J}_2, I \gg 1$$

$$E(I, \boldsymbol{n}) = \frac{I(I+1)}{2\mathcal{J}_3} + (\boldsymbol{n} + \frac{1}{2})\hbar\Omega_{\rm wob}$$

$$\hbar\Omega_{\rm wob} = 2I\sqrt{\left(\frac{\hbar^2}{2\mathcal{J}_1} - \frac{\hbar^2}{2\mathcal{J}_3}\right)\left(\frac{\hbar^2}{2\mathcal{J}_2} - \frac{\hbar^2}{2\mathcal{J}_3}\right)} \propto I$$

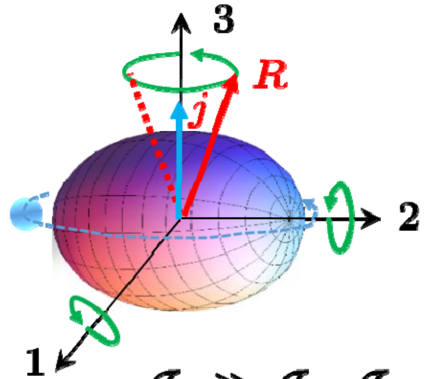


Shi & Chen, CPC 39, 054105 (2015)

- **Simple wobbler**

Wobbling

- **For a triaxial rotor coupled to a high-*j* quasiparticle**



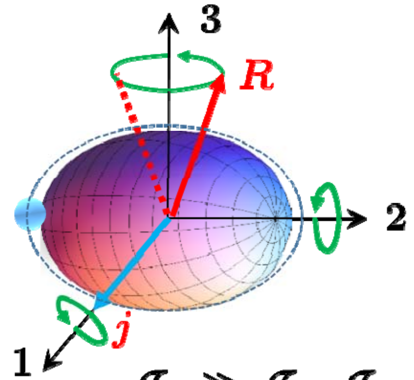
$\mathcal{J}_3 \gg \mathcal{J}_1, \mathcal{J}_2$

Frauendorf & Dönau, PRC 89, 014322 (2014)

- **Longitudinal wobbler:**

$$j // \mathcal{J}^{\max} \qquad I \uparrow, \quad \hbar\Omega_{\text{wob}} \uparrow$$

$$\hbar\Omega_{\text{wob}} = \frac{j}{\mathcal{J}_3}\left[\left(1+\frac{J}{j}\left(\frac{\mathcal{J}_3}{\mathcal{J}_1}-1\right)\right)\left(1+\frac{J}{j}\left(\frac{\mathcal{J}_3}{\mathcal{J}_2}-1\right)\right)\right]^{1/2}$$



$\mathcal{J}_3 \gg \mathcal{J}_1, \mathcal{J}_2$

- **Transverse wobbler:**

$$j \perp \mathcal{J}^{\max} \qquad I \uparrow, \quad \hbar\Omega_{\text{wob}} \downarrow$$

$$\hbar\Omega_{\text{wob}} = \frac{j}{\mathcal{J}_1}\left[\left(1+\frac{J}{j}\left(\frac{\mathcal{J}_1}{\mathcal{J}_3}-1\right)\right)\left(1+\frac{J}{j}\left(\frac{\mathcal{J}_1}{\mathcal{J}_2}-1\right)\right)\right]^{1/2}$$

Collective potential

- For longitudinal and transverse wobblers, we consider a system of a high-j proton particle coupled to a triaxial rotor.

$$\hat{h}' = \hat{h}_{\mathrm{def}} - \boldsymbol{\omega} \cdot \hat{\boldsymbol{j}}, \quad \hat{h}_{\mathrm{def}} = \hat{h}^{\pi}_{\mathrm{def}}, \quad \boldsymbol{j} = \boldsymbol{j}_{\pi}$$

$$\boldsymbol{\omega} = (\omega \sin\theta \cos\varphi, \omega \sin\theta \sin\varphi, \omega \cos\theta),$$

$$\hat{h}_{\mathrm{def}} = \frac{1}{2} C \Big\{ (\hat{j}_3^2 - \frac{j(j+1)}{3}) \cos\gamma + \frac{1}{2\sqrt{3}} (\hat{j}_+^2 + \hat{j}_-^2) \sin\gamma \Big\}$$

$$E'(\theta, \varphi) = \langle h' \rangle - \frac{1}{2} \sum_{k=1}^{3} \mathcal{J}_k \omega_k^2, \quad \mathcal{J}_k : \text{moments of inertia},$$

Minimizing the total Routhian with respect to θ for given φ, the collective potential $V(\varphi)$ is finally obtained.

- For simple wobbler, the simple triaxial rotor does not couple any particles, the total Routhian is degenerated to

$$E'(\theta, \varphi) = -\frac{1}{2} \sum_{k=1}^{3} \mathcal{J}_k \omega_k^2$$

and similarly the total Routhian is obtained by minimizing the total Routhian with respect to θ for given φ.

Mass parameter (HA)

For a harmonic oscillator: $\Omega = \sqrt{\frac{\text{stiffness}}{\text{mass}}} = \sqrt{\frac{C}{B}} \Rightarrow B = \frac{C}{\Omega^2}$

- **For simple wobbler, harmonic approximation (HA) adopted** *Bohr & Mottelson1975*

$$V(\varphi) = -\frac{1}{2}\omega^2(\mathcal{J}_1 \cos^2\varphi + \mathcal{J}_2 \sin^2\varphi)$$
$$\approx -\frac{1}{2}\mathcal{J}_1\omega^2 + \frac{1}{2}\omega^2(\mathcal{J}_1 - \mathcal{J}_2)\varphi^2, \qquad \text{for } \varphi \to 0^\circ$$

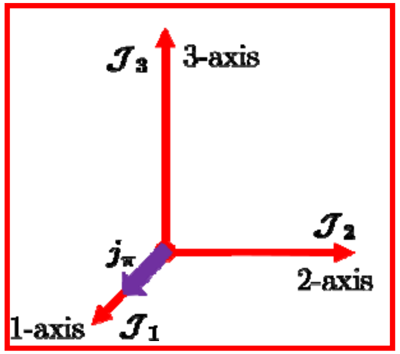
$$C = \omega^2(\mathcal{J}_1 - \mathcal{J}_2)$$

$$\hbar\Omega_{\text{wob}} = 2I\sqrt{\left(\frac{\hbar^2}{2\mathcal{J}_2} - \frac{\hbar^2}{2\mathcal{J}_1}\right)\left(\frac{\hbar^2}{2\mathcal{J}_3} - \frac{\hbar^2}{2\mathcal{J}_1}\right)}$$
$$= \frac{\hbar^2 I}{\mathcal{J}_1}\sqrt{\frac{(\mathcal{J}_1 - \mathcal{J}_2)(\mathcal{J}_1 - \mathcal{J}_3)}{\mathcal{J}_3\mathcal{J}_2}}$$
$$= \hbar\omega\sqrt{\frac{(\mathcal{J}_1 - \mathcal{J}_2)(\mathcal{J}_1 - \mathcal{J}_3)}{\mathcal{J}_3\mathcal{J}_2}}.$$

$$B = \frac{\mathcal{J}_2\mathcal{J}_3}{\mathcal{J}_1 - \mathcal{J}_3}$$

Mass parameter (HFA)

- **For longitudinal and transverse wobblers, harmonic frozen alignment (HFA) approximation adopted** *Frauendorf & Dönau2014PRC*



$$\mathcal{J}_1^*(\omega) = \frac{\mathcal{J}_1\omega + j}{\omega} = \mathcal{J}_1 + \frac{j}{\omega}$$ **effective moment of ineritia**

$$\begin{aligned} V(\varphi) &= \langle \hat{h}_{\mathrm{def}} \rangle - \omega j \cos\varphi - \frac{1}{2}\omega^2(\mathcal{J}_1 \cos^2\varphi + \mathcal{J}_2 \sin^2\varphi) \\ &\approx \langle \hat{h}_{\mathrm{def}} \rangle - \omega j (1 - \frac{\varphi^2}{2}) - \frac{1}{2}\mathcal{J}_1\omega^2 + \frac{1}{2}\omega^2(\mathcal{J}_1 - \mathcal{J}_2)\varphi^2, \quad \text{for } \varphi \to 0 \\ &= \langle \hat{h}_{\mathrm{def}} \rangle - \frac{1}{2}\omega j - \frac{1}{2}\Big(\mathcal{J}_1 + \frac{j}{\omega}\Big)\omega^2 + \frac{1}{2}\omega^2\Big[\Big(\mathcal{J}_1 + \frac{j}{\omega}\Big) - \mathcal{J}_2\Big]\varphi^2 \\ &= \langle \hat{h}_{\mathrm{def}} \rangle - \frac{1}{2}\omega j - \frac{1}{2}\mathcal{J}_1^*\omega^2 + \frac{1}{2}\omega^2\Big[\mathcal{J}_1^*(\omega) - \mathcal{J}_2\Big]\varphi^2 \end{aligned}$$

$$C = \omega^2(\mathcal{J}_1^*(\omega) - \mathcal{J}_2)$$

$$\begin{aligned} B(\omega) &= \frac{\mathcal{J}_2\mathcal{J}_3}{\mathcal{J}_1^*(\omega) - \mathcal{J}_3} \\ &= \frac{\mathcal{J}_2\mathcal{J}_3}{(\mathcal{J}_1 - \mathcal{J}_3) + \frac{j}{\omega}} \end{aligned}$$

$$\begin{aligned} \hbar\Omega_{\mathrm{wob}} &= \sqrt{\frac{\mathcal{J}_1^*(\omega) - \mathcal{J}_2}{B(\omega)}}\hbar\omega \\ &= \hbar\sqrt{\frac{\big[(\mathcal{J}_1 - \mathcal{J}_3)\omega + j\big]\big[(\mathcal{J}_1 - \mathcal{J}_2)\omega + j\big]}{\mathcal{J}_2\mathcal{J}_3}} \end{aligned}$$

Numerical details

- **Configuration for longitudinal and transverse wobblers:** $\pi(1h_{11/2})^1$
- **Deformation parameters:** $\beta = 0.25, \gamma = -30^\circ$

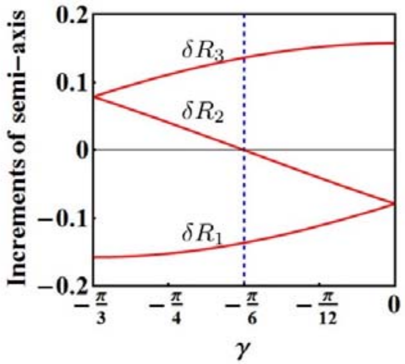
1, 2, and 3-axis correspond to short (s), intermediate (i), and long (l) axis

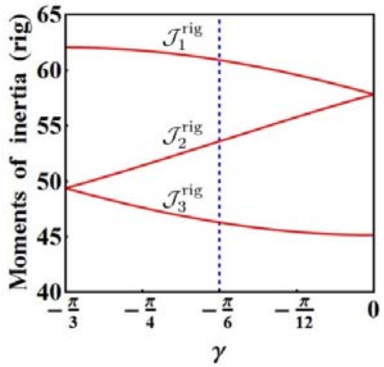
- **Moments of inertia:** *Ring&Schuck1980*

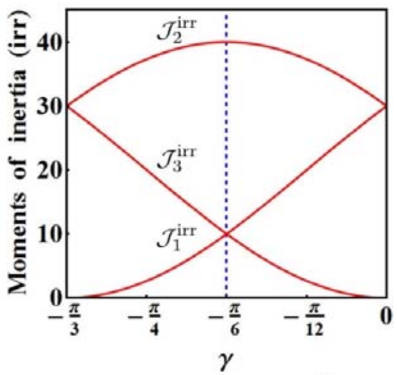
Increments of axis δR

Moments of inertia

Simple and longitudinal | **Transverse**







$$\delta R_k = R_0\sqrt{\frac{5}{4\pi}}\beta\cos(\gamma - \frac{2\pi}{3}k)$$

$$\mathcal{J}_k^{\rm rig} = \mathcal{J}_0^{\rm rig}\left[1 - \sqrt{\frac{5}{4\pi}}\beta\cos(\gamma - \frac{2\pi}{3}k)\right]$$

$$\mathcal{J}_k^{\rm irr} = \mathcal{J}_0^{\rm irr}\sin^2(\gamma - \frac{2\pi}{3}k)$$

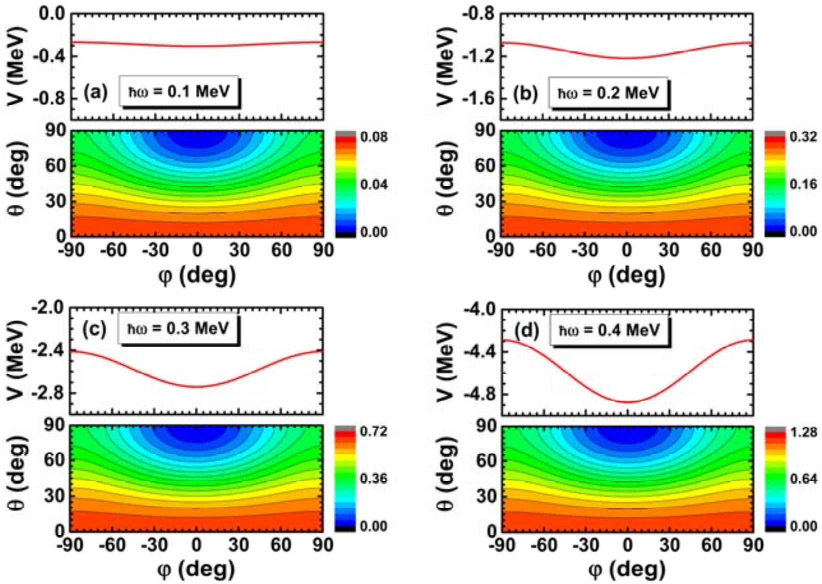
$j_\pi // 1$ $\quad$ $j_\pi // \mathcal{J}^{\rm max}$ $\quad$ $j_\pi \perp \mathcal{J}^{\rm max}$

Wobbling for a triaxial rotor

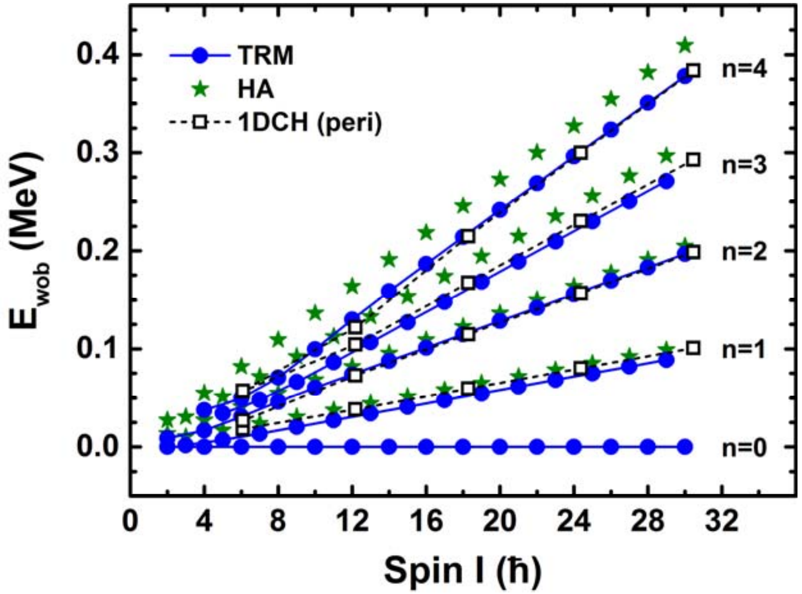
$$E'(\theta,\varphi) = -\frac{1}{2}\sum_{k=1}^{3}\mathcal{J}_k\omega_k^2$$

$$B = \frac{\mathcal{J}_2\mathcal{J}_3}{\mathcal{J}_1 - \mathcal{J}_3}$$

Chen, Zhang, Zhao, and Meng,
PRC 90, 044306 (2014)



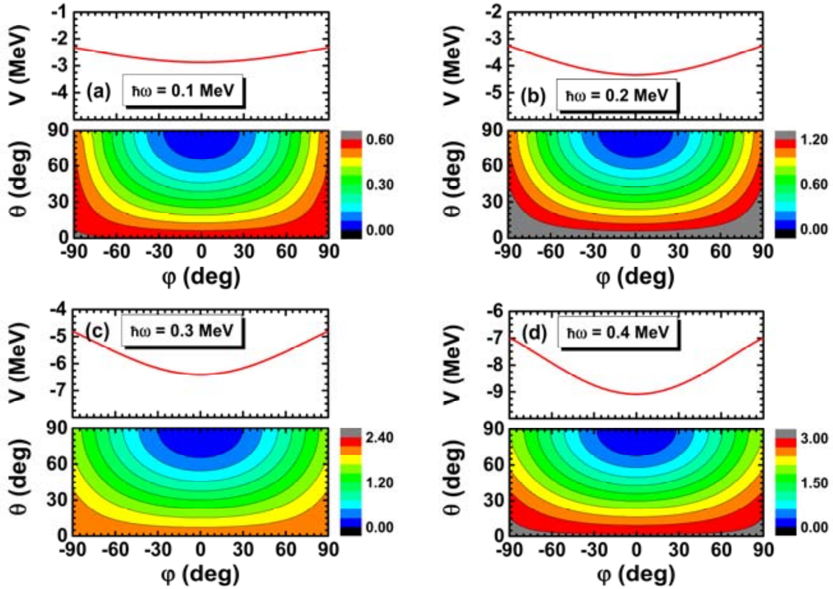
- symmetrical: $(\theta = 90^\circ, \varphi = 0^\circ)$
- minima: $\varphi = 0^\circ$
- stiffness: larger



- Increasing trend of wobbling frequency
- Collective Hamiltonian excellently reproduces the TRM results.

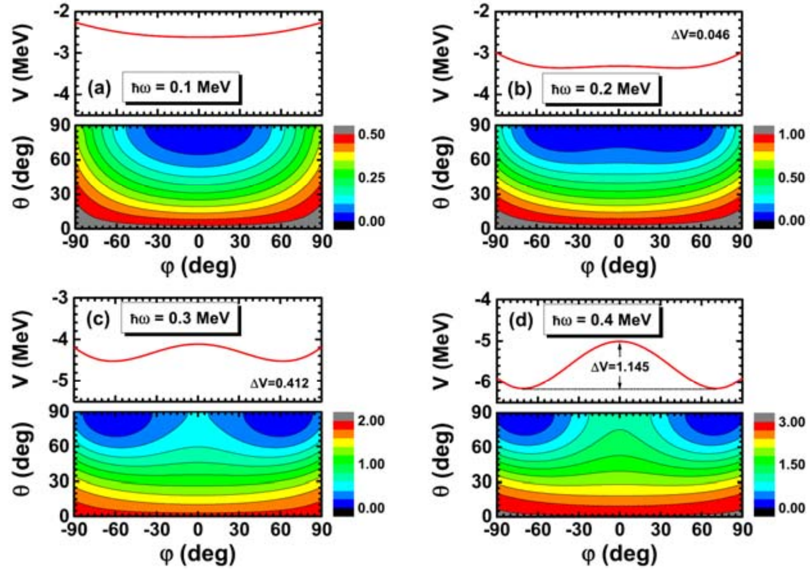
Collective potential

Longitudinal wobbler



- symmetrical: $\varphi = 0^\circ$
- minima: $(\theta = 90^\circ, \varphi = 0^\circ)$
- stiffness: larger

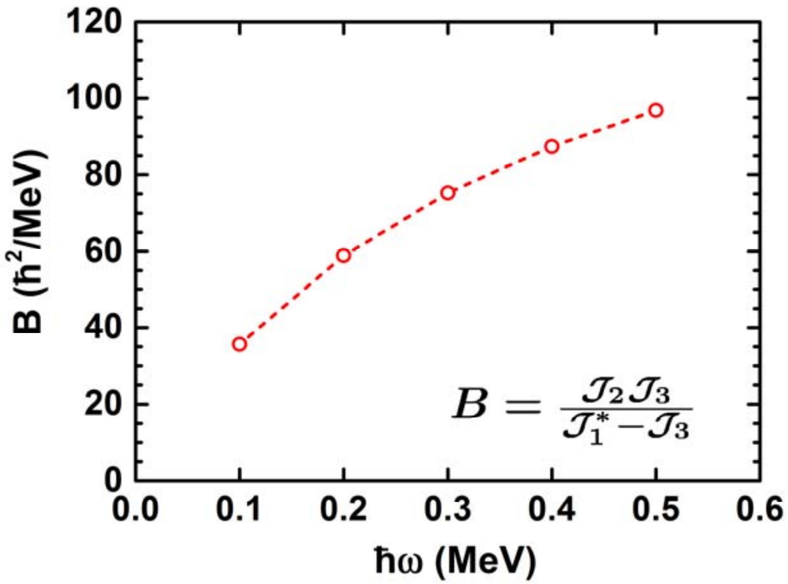
Transverse wobbler



- symmetrical: $\varphi = 0^\circ$
- minima: from $\varphi = 0^\circ$ to $\varphi \neq 0^\circ$
- potential barrier: increase

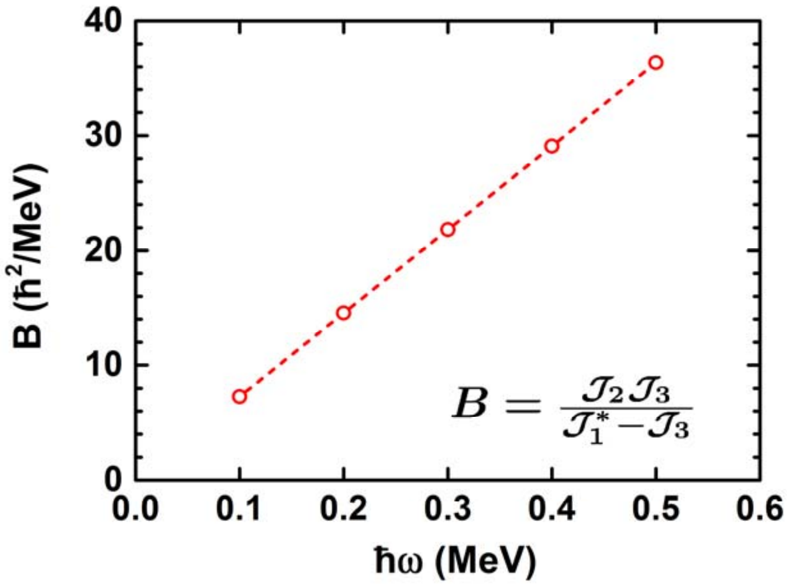
Mass parameter

Longitudinal wobbler



- increases with increasing rotational frequency since the effective moment of inertia for 1-axis decreases.

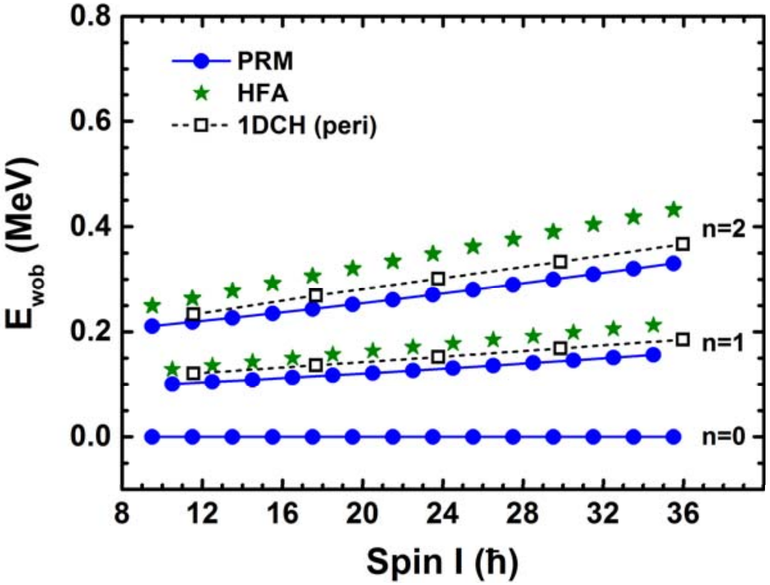
Transverse wobbler



- Linearly increases with increasing rotational frequency since $\mathcal{J}_1 = \mathcal{J}_3$.

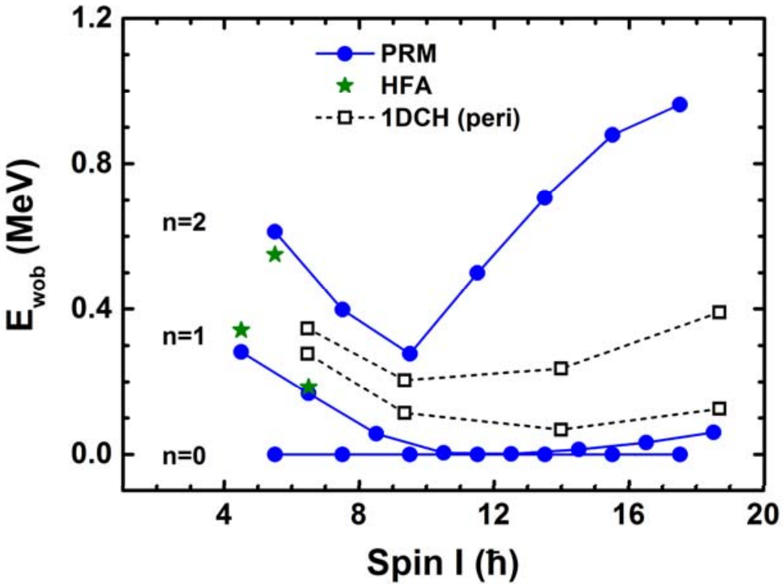
Comparison with PRM

Longitudinal wobbler



- Increasing trend of wobbling frequency
- HFA results gradually deviate from PRM with increasing *n*.
- Collective Hamiltonian excellently reproduces the PRM results.

Transverse wobbler



- Decreasing trend of wobbling frequency
- The onset of transitions from the transverse to longitudinal wobbling motions in PRM is reproduced.

Outline

- Introduction
- Theoretical framework
- Chiral modes
- Wobbling modes
- Summary and perspective

Summary and perspective

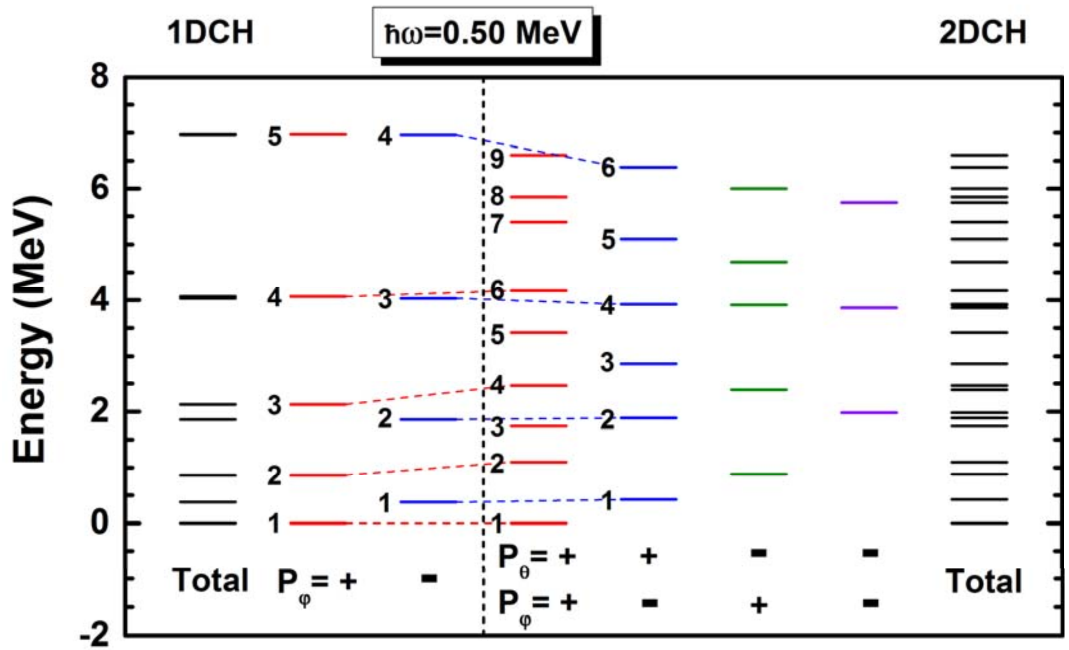
Summary

- **The nuclear chirality and wobbling are fields of broad interest in nuclear physics.**
- **The fruitful experimental results about chirality and wobbling in recent years stimulate us to search a unified method to microscopically describe the chiral doublet bands and wobbling bands.**
- **The collective Hamiltonian based on cranking mean field is thus developed. It goes beyond the mean-field approximation, includes the quantum fluctuation in the collective coordinate, and restores the broken symmetry in the mean-field approximation.**
- **The collective Hamiltonian can reproduce the PRM results very well for chiral and wobbling modes and also can well describe the wobbling modes in realistic nuclei.**

Summary and perspective

Perspective

- Combine to microscopic TAC.
- Two dimensional calculations: $\varphi \rightarrow (\theta, \varphi)$.



Thank you!