Analysis of the Light Scalar Mesons within QCD sum rule

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Outline

- QCD Sum Rule (SR)
- Scalar meson nonet of mass <1GeV
- Our analysis
- Summary and Discussion

QCD sum rule (SR)

- Correlator of the interpolating current J_S with the quantum number of the hadron under consideration

$$\Pi_S(q^2) = i\int d^4x\, e^{iq\cdot x}\langle 0|TJ_S(x)J_S^\dagger(0)|0\rangle$$

Nonperturbative QCD Vacuum

- Calculating it in deeply Euclidean region by the perturbative OPE

$\Pi_S^{OPE}(q^2)$: Condensates from the nonperturbative vacuum

- $\Pi^{\mathrm{OPE}}(q^2)$ is related to physical region by the dispersion relation

$$\Pi_S^{OPE}(q^2) = \frac{1}{\pi}\int_0^\infty ds^2\, \frac{\mathrm{Im}\,\Pi_S(s^2)}{s^2 - q^2}$$

Narrow resonance approx. in the phen. side

$$\langle 0 | J_S | S \rangle = \sqrt{2} f_S M_S^4$$

Quark-hadron duality

$$\mathrm{Im}\,\Pi_S(s^2) = 2\pi f_S^2 M_S^8 \delta(s^2 - M_S^2) + \theta(s^2 - s_0^2)\,\mathrm{Im}\,\Pi_S^{OPE}(s^2)$$

threshold

- $\mathrm{Im}\,\Pi_S(q^2) = \pi \sum_n \delta(q^2 - m_n^2) \langle 0 | J_S(0) | n \rangle \langle n | J_S^\dagger(0) | 0 \rangle$

- Borel transform makes the contributions from the continuum suppressed exponentially.

- QCD sum rules :

$$\boxed{\frac{1}{\pi}\int_0^{s_0^2} ds^2 e^{-s^2/M^2}\ \mathrm{Im}\,\Pi_S^{OPE}(s^2)} = 2 f_S^2 M_S^8 e^{-M_S^2/M^2}$$

$\tilde{\Pi}_S(M^2)$: Must be POSITIVE

M : Borel Mass

- Mass of Particle can be determined by

$$M_S = \sqrt{(\partial_M \tilde{\Pi}_S / 2\tilde{\Pi}_S) M^3}$$

- Generally, including all contributions from OPE, the mass must be independent on the Borel mass.
- Actually, we cannot do it. Up to a certain energy dimension operators, mass plateau appears in some region of the Borel mass.

$\Rightarrow$ Borel window

- Borel window must be opened in $M < s_0$.

Light scalar meson nonet

- Members :

$$I = 1 \ : \ a_0^0, a_0^\pm(980)$$

$$I = 1/2 \ : \ \kappa^\pm, \kappa^0, \bar{\kappa}^0(800)$$

$$I = 0 \ : \ \sigma(500), f_0(980)$$

- Large decay widths :

$$\Gamma_{a_0} = 50 \sim 100\text{MeV}, \Gamma_{f_0} = 40 \sim 100\text{MeV}$$

$$\Gamma_\sigma = 400 \sim 700\text{MeV}$$

Refs. : PDG, Chin. Phys. C, 38(2014) 09001

$q\bar{q}$ interpretation

- With ideal mixing : $L=1$ for P=+1

$$a_0^+(980) = u\bar{d},\ a_0^0 = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d}),\ a_0^- = d\bar{u}$$

$$\kappa^+(800) = u\bar{s},\ \kappa^0 = d\bar{s},\ \bar{\kappa}^0 = s\bar{d},\ \kappa^- = s\bar{u}$$

$$\sigma(600) = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d}),\ f_0(980) = s\bar{s}$$

- (?1)Decays of a_0 : fraction of $s\bar{s}$?

$$\frac{\Gamma[a_0(980) \to \eta\pi]}{\Gamma[a_0(980) \to \eta\pi + K\bar{K}]} = 0.85 \pm 0.02$$

Amsler et al,
Phys. Rep.
384(2004)61

- (?2) Mass degeneracy in a_0, f_0

  1. From number of strange quarks

$$m_{f_0} > m_\kappa > m_{a_0},\ m_\sigma$$

  2. $L=1$ gives 400MeV more mass :

  from the mass formula in a quark model

  (Kochelev, H.-J. Lee, Vento, PLB 594 (2004) 87),

  for example : $f_0(980)$

$$M_{f_0} = E_{conf} + 2m_s + E_{OGE} + E_I + E_{L=1}$$

$$\simeq 214 + 2\times 407 - 2 + 0 + 400 = 1425\mathrm{MeV}$$

$[qq][\overline{qq}]$ interpretation

- One gluon exchange & instanton :
  strongest attraction in two quarks
  of $|\bar{3}_F, \bar{3}_C, 1_S\rangle$: scalar (S) diquark
  in two antiquarks of $|3_F, 3_C, 1_S\rangle$: S antidiquark

  −Jaffe & Wilczek, Shuryak & Zahed

- In flavor space :

$$3_f \otimes 3_f = \bar{3}_A \oplus 6_S, \ \bar{3}_f \otimes \bar{3}_f = 3_A \oplus \bar{6}_S$$

$$\Rightarrow \bar{3}_A \otimes 3_A = 1 \oplus 8$$

Explicitly

$$[ud]_A \leftrightarrow \bar{s}, \ [us]_A \leftrightarrow \bar{d}, \ [ds]_A \leftrightarrow \bar{u}$$

$$[\bar{u}\bar{d}]_A \leftrightarrow s, \ [\overline{us}]_A \leftrightarrow d, \ [\bar{d}\bar{s}]_A \leftrightarrow u$$

- In terms of S diquark & S antidiquark : L=0

$$a_0^+(980) = [\bar{d}\bar{s}][us],\ a_0^0 = \frac{1}{\sqrt{2}}([\bar{d}\bar{s}][ds] - [\bar{u}\bar{s}][us]),\ a_0^- = [\bar{u}\bar{s}][ds]$$

$$\kappa^+(800) = [\bar{d}\bar{s}][ud],\ \kappa^0 = [\bar{u}\bar{s}][ud],\ \bar{\kappa}^0 = [\bar{u}\bar{d}][us],\ \kappa^- = [\bar{u}\bar{d}][ds]$$

$$\sigma(600) = [\bar{u}\bar{d}][ud],\ \ f_0(980) = \frac{1}{\sqrt{2}}([\bar{d}\bar{s}][ds] + [\bar{u}\bar{s}][us])$$

- Number of strange quark :

$$m_{f_0} = m_{a_0} > m_\kappa > m_\sigma$$: Inverted mass spectrum

- Strange quark component in $f_0,\ a_0$:

$$f_0,\ a_0 \to K\bar{K}$$

SRs for light scalar nonet

- Interpolating currents : energy dim.=6

$$
\begin{aligned}
J_\sigma &= \epsilon_{abc}\epsilon_{ade}(u_b^T C\gamma_5 d_c)(\bar{u}_d C\gamma_5 \bar{d}_e) \\
J_{f_0} &= \frac{1}{\sqrt{2}}\epsilon_{abc}\epsilon_{ade}\Big((u_b^T C\gamma_5 s_c)(\bar{u}_d C\gamma_5 \bar{s}_e) + (u \rightarrow d)\Big) \\
J_{a_0^0} &= \frac{1}{\sqrt{2}}\epsilon_{abc}\epsilon_{ade}\Big((u_b^T C\gamma_5 s_c)(\bar{u}_d C\gamma_5 \bar{s}_e) - (u \rightarrow d)\Big) \\
J_{\kappa^+} &= \epsilon_{abc}\epsilon_{ade}(u_b^T C\gamma_5 d_c)(\bar{d}_d C\gamma_5 \bar{s}_e)
\end{aligned}
$$

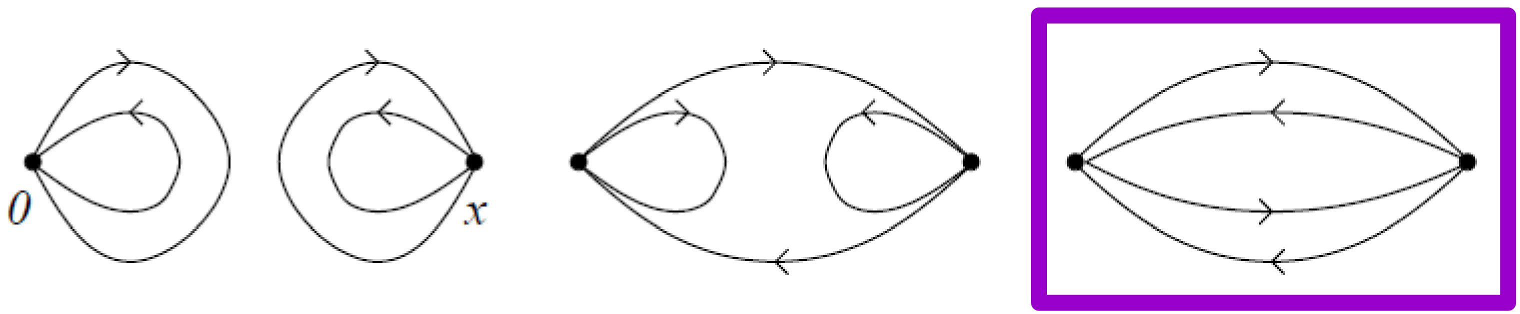
- After Borel transform :

  Energy dimension of the correlator =10

Some details for sigma :

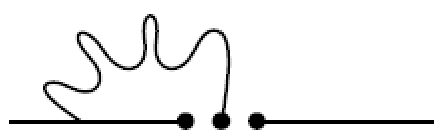
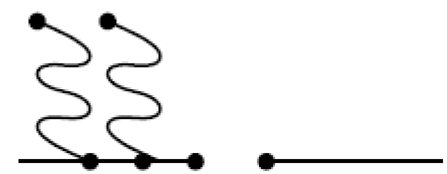
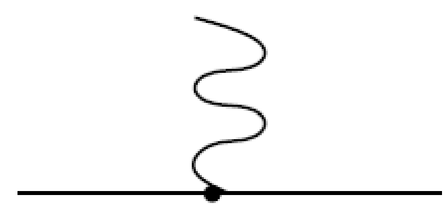
- Vacuum expectation value of currents :

$$
\begin{aligned}
\bullet \quad & \langle 0 | T J_S^{\sigma}(x) J_S^{\sigma\dagger}(0) | 0 \rangle \\
= \quad & \epsilon_{abc}\epsilon_{ade}\epsilon_{a'b'c'}\epsilon_{a'd'e'} \Big(\mathrm{Tr}[S^u_{bd}(x,x)\overline{\Gamma}_{S,2} S^{d,T}_{ce}(x,x)\Gamma^T_{S,1}]\,\mathrm{Tr}[S^u_{d'b'}(0,0)\overline{\Gamma}_{S,3} S^{d,T}_{e'c'}(0,0)\Gamma^T_{S,4}] \\
& -\mathrm{Tr}[S^u_{bd}(x,x)\overline{\Gamma}_{S,2} S^{d,T}_{e'e}(0,x)\Gamma^T_{S,4} S^u_{d'b'}(0,0)\overline{\Gamma}_{S,3} S^{d,T}_{cc'}(x,0)\Gamma^T_{S,1}] \\
& -\mathrm{Tr}[S^u_{bb'}(x,0)\overline{\Gamma}_{S,3} S^{d,T}_{e'c'}(0,0)\Gamma^T_{S,4} S^u_{d'd}(0,x)\overline{\Gamma}_{S,2} S^{d,T}_{ce}(x,x)\Gamma^T_{S,1}] \\
& \boxed{+\mathrm{Tr}[S^u_{bb'}(x,0)\overline{\Gamma}_{S,3} S^{d,T}_{cc'}(x,0)\Gamma^T_{S,1}]\,\mathrm{Tr}[S^u_{d'd}(0,x)\overline{\Gamma}_{S,2} S^{d,T}_{e'e}(0,x)\Gamma^T_{S,4}] \Big) .}
\end{aligned}
$$



- Quark propagator :

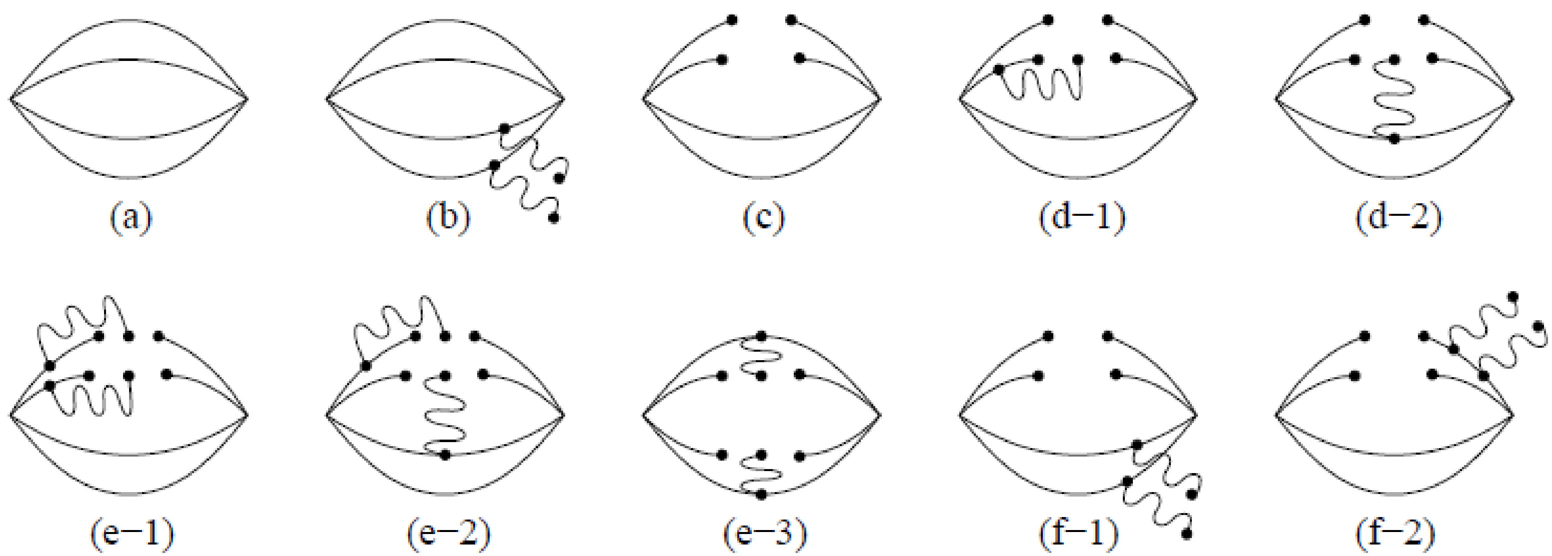
$$
\begin{aligned}
S^q_{ab}(x) &= -i\langle 0|T q_a(x)\bar{q}_b(0)|0\rangle \\
&= \delta_{ab}\Big(\frac{\hat{x}}{2\pi^2 x^4} + i\frac{\langle\bar{q}q\rangle}{12} - \frac{x^2}{192}\langle g\bar{q}\sigma\cdot Gq\rangle + i\frac{x^4}{2^9\cdot 3^3}\langle\bar{q}q\rangle\langle g^2G^2\rangle\Big) \\
&\quad -i\frac{g}{32\pi^2}G^{\mu\nu}_{ab}\frac{1}{x^2}(\hat{x}\sigma_{\mu\nu}+\sigma_{\mu\nu}\hat{x}),
\end{aligned}
$$

- 1st :
- 2nd : 
- 3rd : 
- 4th : 
- 5th : 

- QCD SR for sigma :

$$\frac{M^{10}E_4}{2^9\cdot 5\pi^6} + \frac{g_c^2\langle G^2\rangle M^6 E_2}{2^{10}\cdot 3\pi^6} + \frac{\langle\bar{q}q\rangle^2}{12\pi^2}M^4E_1 - \frac{\langle\bar{q}q\rangle i g_c\langle\bar{q}\sigma\cdot Gq\rangle}{12\pi^2}M^2E_0$$

$$+\,59\frac{(ig_c\langle\bar{q}\sigma\cdot Gq\rangle)^2}{2^{10}\cdot 3^2\pi^2} + 7\,\frac{g_c^2\langle G^2\rangle\langle\bar{q}q\rangle^2}{2^6\cdot 3^3\pi^2} = 2f_1^2 m_1^8 e^{-m_1^2/M^2}$$



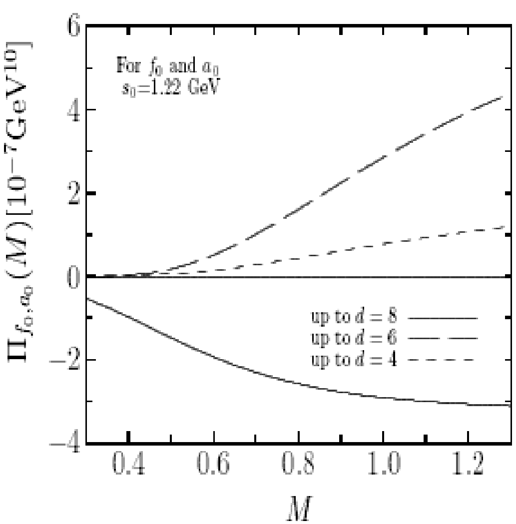
LHS of SRs with scalar Diquark

- Values of condensates and mass :

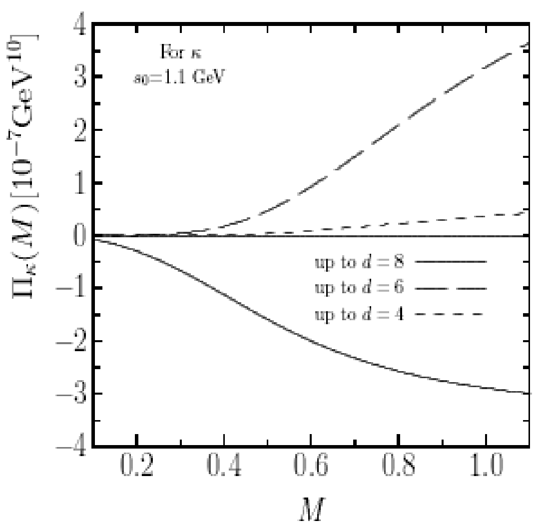
$$\langle \bar{u}u \rangle = -(0.25)^3 \text{ GeV}^3,\ \langle \bar{s}s \rangle = f_s \langle \bar{u}u \rangle,\ \langle g_c^2 G^2 \rangle = 0.5 \text{ GeV}^4,$$

$$ig_c \langle \bar{u}\sigma \cdot Gu \rangle = 0.8 \text{ GeV}^2 \langle \bar{u}u \rangle,\ \ ig_c \langle \bar{s}\sigma \cdot Gs \rangle = f_s ig_c \langle \bar{u}\sigma \cdot Gu \rangle,$$

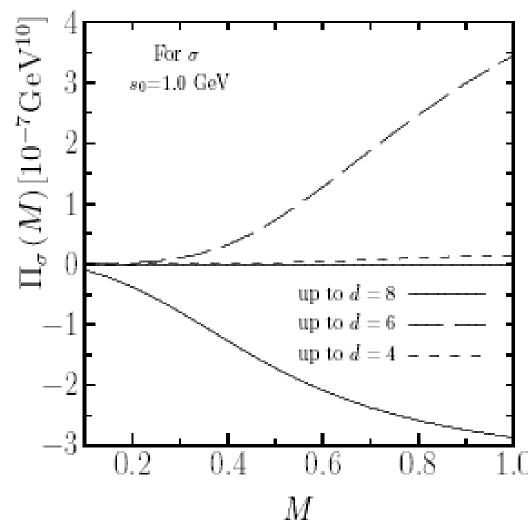
$$m_s = 0.15 \text{ GeV},\ \ f_s = 0.8$$



$f_0,\ a_0$



κ



σ

What we have seen…

- Large negative contribution from d=8 ops. :

  ➡ no physical meaning in SR.

- Any other structure for the light scalar mesons?

- Effect from Instanton?

- Generally, five types of relativistic currents :

$$\bar{3}_C \otimes 3_C \ : \ J_S^i = \varepsilon_{abc}[q_{1,b}^T \Gamma_i^A q_{2,c}]\varepsilon_{ade}[\bar{q}_{3,d}^T \bar{\Gamma}_i^A \bar{q}_{4,e}]$$

$$6_C \otimes \bar{6}_C \ : \ J_S^i = \{[q_{1,a}^T \Gamma_i^S q_{2,b}] + (a \leftrightarrow b)\}\{[\bar{q}_{3,a}^T \bar{\Gamma}_i^S \bar{q}_{4,b}] + (a \leftrightarrow b)\}$$

$$\text{with } \bar{\Gamma} = \gamma_0 \Gamma^\dagger \gamma_0, \text{ and } \Gamma_i^{A,T} = -\Gamma_i^A, \ \Gamma_i^{S,T} = \Gamma_i^S$$

$$\boxed{\begin{aligned} \Gamma_i^A &= C\gamma_5 (S), \ C(PS), \ C\gamma_5\gamma_\mu (V) \\ \Gamma_i^S &= C\gamma_\mu (AV), \ C\sigma_{\mu\nu} (T) \end{aligned}}$$

- General interpolating currents could be :

$$\boxed{J_S = \alpha J_S^S + \beta J_S^{PS} + v J_S^V + v' J_S^{AV} + t J_S^T}$$

Chen et al., Phys.Lett.B650:369-372,2007

SR for sigma again

- 't Hooft instanton induced interaction for u,d :

$$\mathcal{L} = \frac{G}{4(N_c^2-1)}\left[\frac{2N_c-1}{2N_c}\left((\bar\psi\tau_\alpha^-\psi)^2+(\bar\psi\gamma_5\tau_\alpha^-\psi)^2\right)+\frac{1}{4N_c}(\bar\psi\sigma_{\rho\sigma}\tau_\alpha^-\psi)^2\right]$$

Fierz trans.

$$\begin{aligned}\mathcal{L} &= \frac{G}{2N_c(N_c-1)}\,\epsilon_{abc}\epsilon_{ade}\left[(u_b^T\Gamma_S d_c)(\bar u_d\Gamma_S\bar d_e^T)-(u_b^T\Gamma_{PS}d_c)(\bar u_d\Gamma_{PS}\bar d_e^T)\right]\\ &+\frac{G}{4N_c(N_c+1)}(u_a^T\Gamma_{T,\rho\sigma}d_{a'})\left((\bar u_a\Gamma_T^{\rho\sigma}\bar d_{a'}^T)+(\bar u_{a'}\Gamma_T^{\rho\sigma}\bar d_a^T)\right),\end{aligned}$$

$$\boxed{\alpha=1,\ \beta=-1,\ v=0,\ v'=0,\ t=1/4 \ \text{ for } \ N_C=3}$$

- From PDG:

$f_0(500)$ DECAY MODES

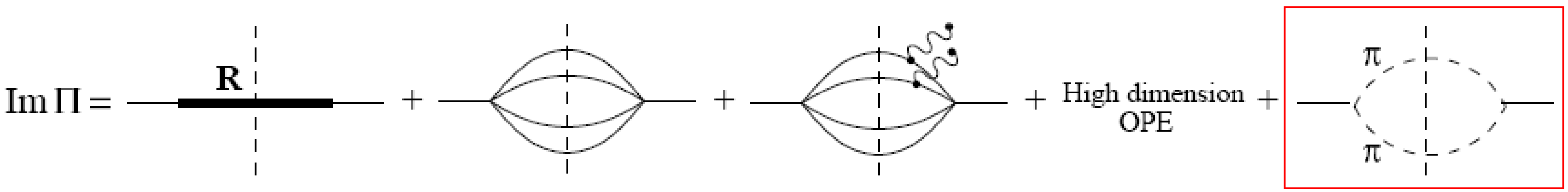
	Mode	Fraction (Γ_i/Γ)
Γ_1	$\pi\pi$	dominant
Γ_2	$\gamma\gamma$	seen

- Interpolating current of the tetraquark can couple to the two pion state : Fierz transf.

$$J^{\pi}_{f_0} = -\frac{1}{16}\Big[(\alpha-\beta)\Big((\bar{u}\gamma^5 d + \bar{d}\gamma^5 u)^2 - (\bar{u}\gamma^5 d - \bar{d}\gamma^5 u)^2 + (\bar{u}\gamma^5 u - \bar{d}\gamma^5 d)^2\Big)$$
$$+(\alpha+\beta)\Big((\bar{u}\gamma^5\gamma_\mu d + \bar{d}\gamma^5\gamma_\mu u)^2 - (\bar{u}\gamma^5\gamma_\mu d - \bar{d}\gamma^5\gamma_\mu u)^2 + (\bar{u}\gamma^5\gamma_\mu u - \bar{d}\gamma^5\gamma_\mu d)^2\Big)\Big]$$

- We need to modify the phenomenological side.
  - $\mathrm{Im}\,\Pi_S(q^2) = \pi \sum_n \delta(q^2 - m_n^2)\langle 0 | J_S(0) | n\rangle\langle n | J_S^\dagger(0) | 0\rangle$

- Narrow resonance + two pion state in the phen. side :

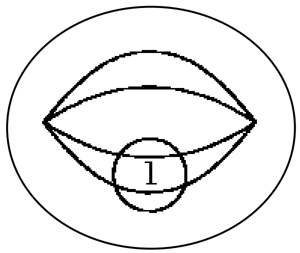


- PCAC gives :

$$\frac{1}{\pi}\Pi^{2\pi}(q^2)=\frac{6}{16^2\pi^2}\left[(\alpha-\beta)^2\left(\frac{\langle\bar{q}q\rangle^2}{4f_\pi^2}\right)^2+(\alpha+\beta)^2\left(\frac{f_\pi^2}{4}\right)(q^2-2m_\pi^2)^2\right]$$

$$\times\sqrt{1-\frac{4m_\pi^2}{q^2}}\,\theta(q^2-4m_\pi^2)$$

- Instanton effects：

$\boxed{\alpha^2-\beta^2} \longleftarrow$ 

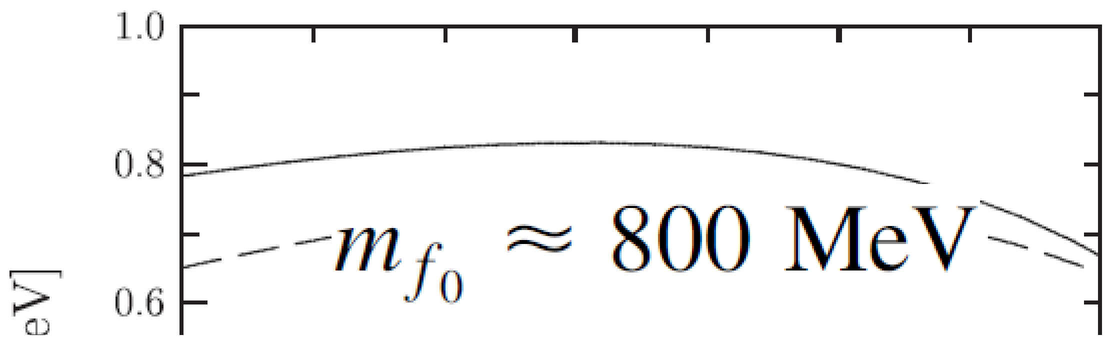
$$\Pi^{I+\bar{I}}(q) = (\alpha^2-\beta^2)\frac{32 n_{\text{eff}}\rho_c^4}{\pi^8 m_q^{*2}} f_6(q) + [19(\alpha^2+\beta^2)-6\alpha\beta]\frac{n_{\text{eff}}\rho_c^4\langle\bar{q}q\rangle^2}{18\pi^4 m_q^{*2}} f_0(q)$$

- QCD sum rules：

$$\frac{1}{\pi}\int_0^{s_0^2} ds^2\, e^{-s^2/M^2}\mathrm{Im}\Pi^{OPE}(s^2) + \hat{B}[\Pi^{I+\bar{I}}(q)] - \frac{1}{\pi}\int_{4m_\pi^2}^{s_0^2} ds^2\, e^{-s^2/M^2}\mathrm{Im}\Pi^{2\pi}(s^2)$$
$$= 2f_{f_0}^2 m_{f_0}^8 e^{-m_{f_0}^2/M^2},$$

$$\begin{aligned}\frac{1}{\pi}\mathrm{Im}\Pi^{\text{OPE}}(q^2) = {} & (\alpha^2+\beta^2)\Big[\frac{(q^2)^4}{2^{12}\cdot 5\cdot 3\pi^6} + \frac{\langle g^2G^2\rangle}{2^{11}\cdot 3\pi^6}(q^2)^2\Big] \\ & + (\alpha^2-\beta^2)\Big[\frac{\langle\bar{q}q\rangle^2}{12\pi^2}q^2 - \frac{\langle\bar{q}q\rangle\langle ig\bar{q}\sigma\cdot Gq\rangle}{12\pi^2} \\ & + \frac{59(\langle ig\bar{q}\sigma\cdot Gq\rangle)^2}{2^9\cdot 3^2\pi^2}\delta(q^2) + \frac{7\langle g^2G^2\rangle\langle\bar{q}q\rangle^2}{2^5\cdot 3^3\pi^2}\delta(q^2)\Big]\end{aligned}$$

- For a=−b=1, there could be stable result!



f_0(600) T-MATRIX POLE $\sqrt{s}$

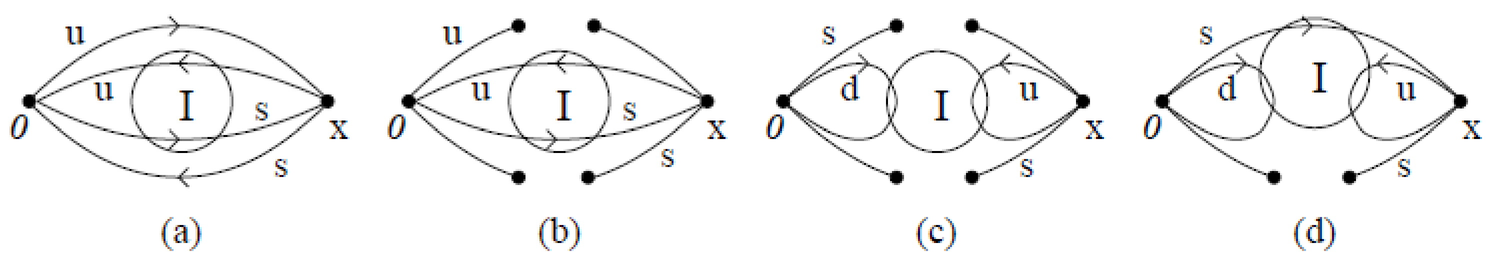
Note that $\Gamma \approx 2\ \mathrm{Im}(\sqrt{s_{\mathrm{pole}}})$.

VALUE (MeV)	*DOCUMENT ID*	*TECN*	*COMMENT*
(400–1200)−*i*(250–500) OUR ESTIMATE			
• • • We do not use the following data for averages, fits, limits, etc. • • •			
$(441^{+16}_{-8})-i(272^{+9}_{-12.5})$	[1] CAPRINI 06	RVUE	$\pi\pi \to \pi\pi$
$(470\pm50)-i(285\pm25)$	[2] ZHOU 05	RVUE	
$(541\pm39)-i(252\pm42)$	[3] ABLIKIM 04A	BES2	$J/\psi \to \omega\pi^+\pi^-$
$(528\pm32)-i(207\pm23)$	[4] GALLEGOS 04	RVUE	Compilation
$(440\pm8)-i(212\pm15)$	[5] PELAEZ 04A	RVUE	$\pi\pi \to \pi\pi$
$(533\pm25)-i(247\pm25)$	[6] BUGG 03	RVUE	

Other members with diquarks

- For f0(980) and a0(980)

$$L^{OPE}_{f_0,a_0}(M) = (\alpha^2+\beta^2)\left(\frac{M^{10}E_4}{2^9\cdot 5\pi^6} + \frac{g^2\left\langle G^2\right\rangle M^6E_2}{2^{10}\cdot 3\pi^6} + \frac{m_s\left\langle \bar{s}s\right\rangle M^6E_2}{2^5\cdot 3\pi^4} + \frac{m_s ig\left\langle \bar{s}\sigma\cdot Gs\right\rangle M^4E_1}{2^7\cdot 3\pi^4}\right.$$
$$\left. + \frac{m_s g^2\left\langle G^2\right\rangle\left\langle \bar{s}s\right\rangle M^2E_0}{2^8\cdot 3\pi^4} - \frac{m_s\left\langle \bar{q}q\right\rangle^2\left\langle \bar{s}s\right\rangle}{9}\right) - (\alpha^2-\beta^2)\left(\frac{m_s\left\langle \bar{q}q\right\rangle M^6E_2}{2^4\cdot 3\pi^4} - \frac{\left\langle \bar{q}q\right\rangle\left\langle \bar{s}s\right\rangle M^4E_1}{12\pi^2}\right.$$
$$\frac{m_s ig\left\langle \bar{q}\sigma\cdot Gq\right\rangle M^4}{\ } \quad \frac{M^2E_0}{\ }$$



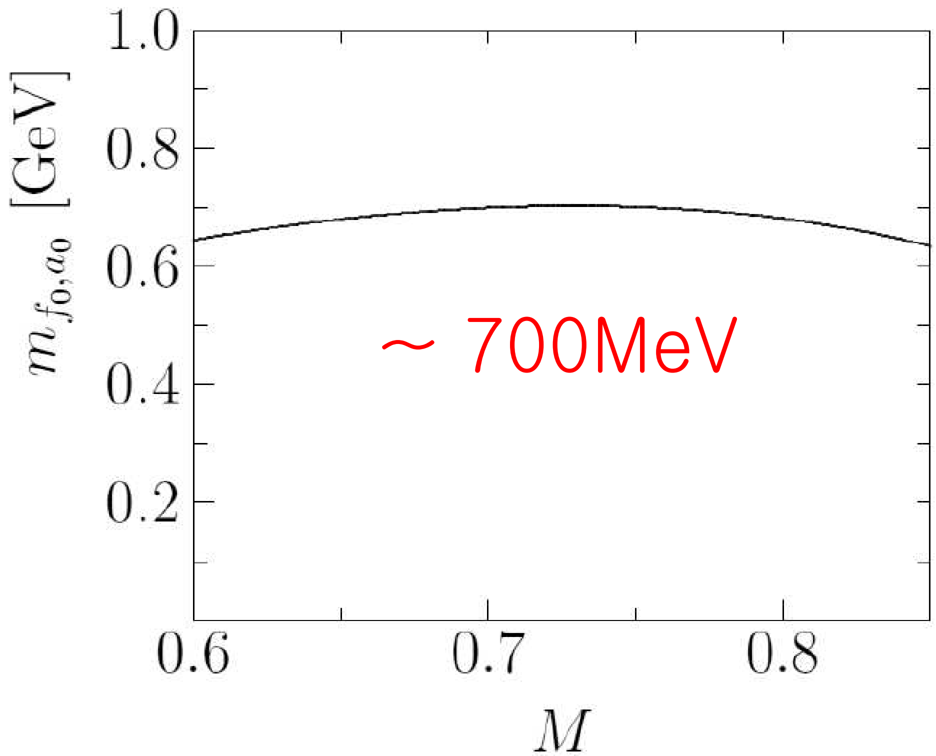
$$L^{Inst}_{f_0,a_0}(M) = (\alpha^2-\beta^2)\frac{32n_{eff}\rho_c^4}{\pi^8 m_q^* m_s^*}\hat{B}[I_6(Q)] + (19\alpha^2+19\beta^2-6\alpha\beta)\frac{n_{eff}\rho_c^4\left\langle \bar{q}q\right\rangle\left\langle \bar{s}s\right\rangle}{18\pi^4 m_q^* m_s^*}\hat{B}[I_0(Q)]$$
$$\mp(\alpha-\beta)^2\frac{n_{eff}\rho_c^4\left\langle \bar{s}s\right\rangle^2}{12\pi^4 m_q^*2}\hat{B}[I_0(Q)] \pm (\alpha-\beta)^2\frac{8n_{eff}\rho_c^6\left\langle \bar{s}s\right\rangle}{3\pi^6 m_q^* 2m_s^*}\hat{B}[g_0(Q)].$$

Upper sign : f0(980)

- Mass degeneracy in f0(980) and a0(980)

$$\alpha = \beta$$

- Mass fitting



Mass of $f_0(980), a_0(980)$ from the QCD sum rule with $s_0 = 1.37$ GeV.

- For kappa(800)

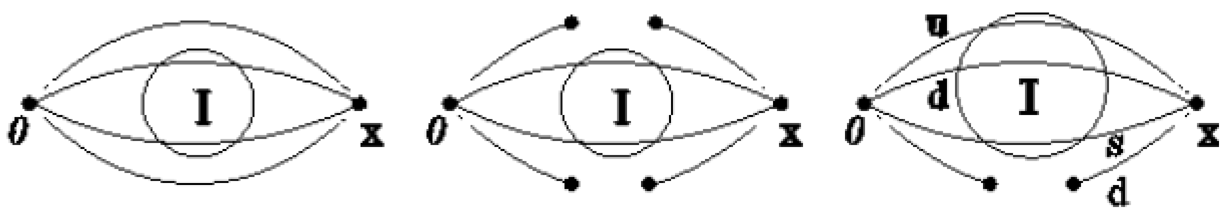
$$L_\kappa^{OPE}(M) = (\alpha^2+\beta^2)\left(\frac{M^{10}E_4}{2^9\cdot 5\pi^6} + \frac{g^2\langle G^2\rangle M^6 E_2}{2^{10}\cdot 3\pi^6} + \frac{m_s\langle\overline{s}s\rangle M^6 E_2}{2^6\cdot 3\pi^4} + \frac{m_s ig\langle\overline{s}\sigma\cdot Gs\rangle M^4 E_1}{2^8\cdot 3\pi^4}\right.$$

$$\left.+\frac{m_s g^2\langle G^2\rangle\langle\overline{s}s\rangle M^2 E_0}{2^9\cdot 3\pi^4} - \frac{m_s\langle\overline{q}q\rangle^3}{18}\right) - (\alpha^2-\beta^2)\left(\frac{m_s\langle\overline{q}q\rangle m^6 E_2}{2^5\cdot 3\pi^4} - \frac{\langle\overline{q}q\rangle\left(\langle\overline{q}q\rangle+\langle\overline{s}s\rangle\right)M^4E_1}{24\pi^2}\right.$$

$$-\frac{m_s ig\langle\overline{q}\sigma\cdot Gq\rangle M^4}{2^7\pi^4}\left(E_1+\overline{W}_1\right) + \frac{M^2E_0}{2^4\cdot 3\pi^2}\left(\langle\overline{q}q\rangle ig\langle\overline{s}\sigma\cdot Gs\rangle + \langle\overline{s}s\rangle ig\langle\overline{q}\sigma\cdot Gq\rangle + 2\langle\overline{q}q\rangle ig\langle\overline{q}\sigma\cdot Gq\rangle\right)$$

$$+\frac{m_s g^2\langle G^2\rangle\langle\overline{q}q\rangle M^2}{2^8 \cdots} \quad \cdots \quad ig\langle\overline{q}\sigma\cdot Gq\rangle(ig\langle\overline{q}\sigma\cdot Gq\rangle + ig\langle\overline{s}\sigma\cdot Gs\rangle)$$

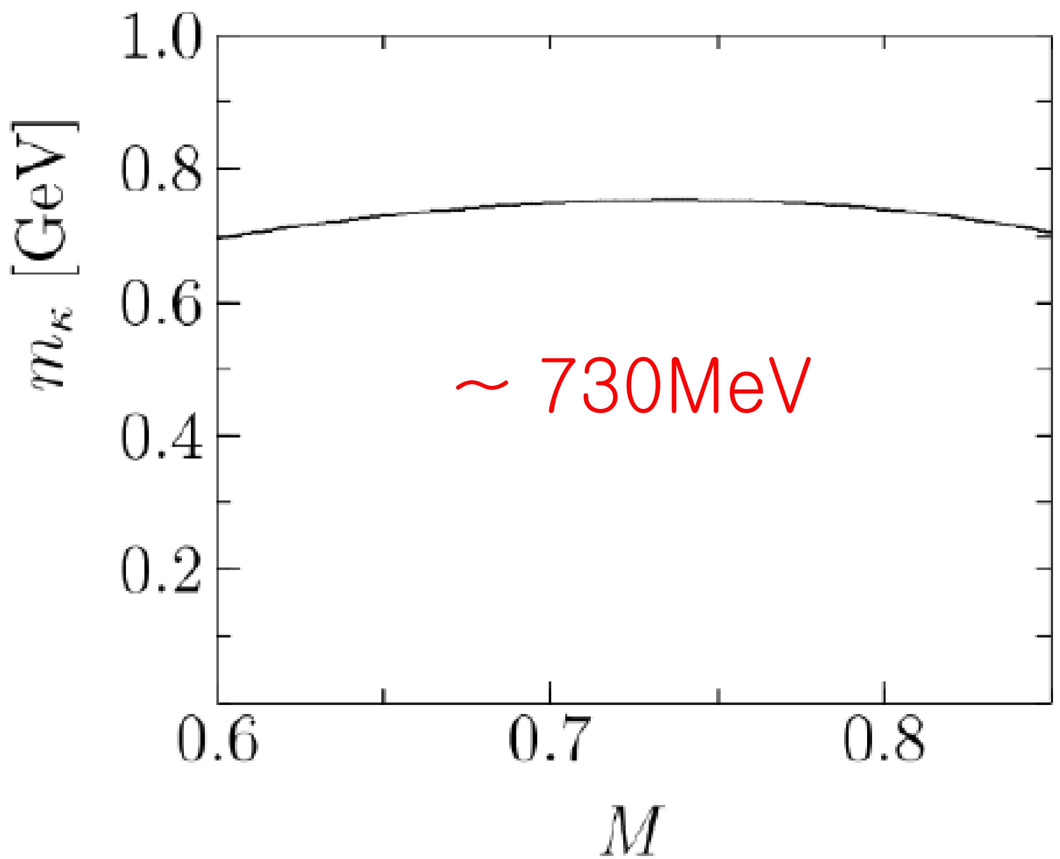
$$-7\frac{g^2\langle G \cdots}{\cdots}$$



$$L_\kappa^{Inst}(M) = (\alpha^2-\beta^2)\frac{16n_{eff}\rho_c^4}{\pi^8 m_q^{*2}}\left(1+\frac{m_q^*}{m_s^*}\right)\hat{B}[I_6(Q)] + (\alpha^2+\beta^2)\frac{n_{eff}\rho_c^4\langle\overline{q}q\rangle}{36\pi^4 m_q^{*2}}\left(19\langle\overline{s}s\rangle + 22\frac{m_q^*}{m_s^*}\langle\overline{q}q\rangle\right)\hat{B}[I_0(Q)]$$

$$-\alpha\beta\frac{n_{eff}\rho_c^4\langle\overline{q}q\rangle}{6\pi^4 m_q^{*2}}\left(\langle\overline{s}s\rangle + 2\frac{m_q^*}{m_s^*}\langle\overline{q}q\rangle\right)\hat{B}[I_0(Q)] - (\alpha-\beta)^2\frac{8n_{eff}\rho_c^6\langle\overline{q}q\rangle}{3\pi^6 m_q^{*2}m_s^*}\hat{B}[g_0(Q)]$$

- With $\alpha = \beta$, mass fitting :



Mass of κ as a function of M with $s_0 = 1.43$ GeV

Bound state of two psedoscalar mesons?

- For f0(980) : bound state of $\boxed{\text{two etas?}}$

  Y.U. Surovtsev et al., Int. J. Mod. Phys. A **26**, 610 (2011)

  –From analysis of resonances appearing in

$$\pi\pi \to \pi\pi,\ K\bar{K},\ \eta\eta$$

$$J/\psi \to \pi\pi,\ K\bar{K}$$

- Interpolating current :

$$J = J_\eta J_\eta = \alpha^2 J_8 J_8 + 2\alpha\beta J_8 J_1 + \beta^2 J_1 J_1$$

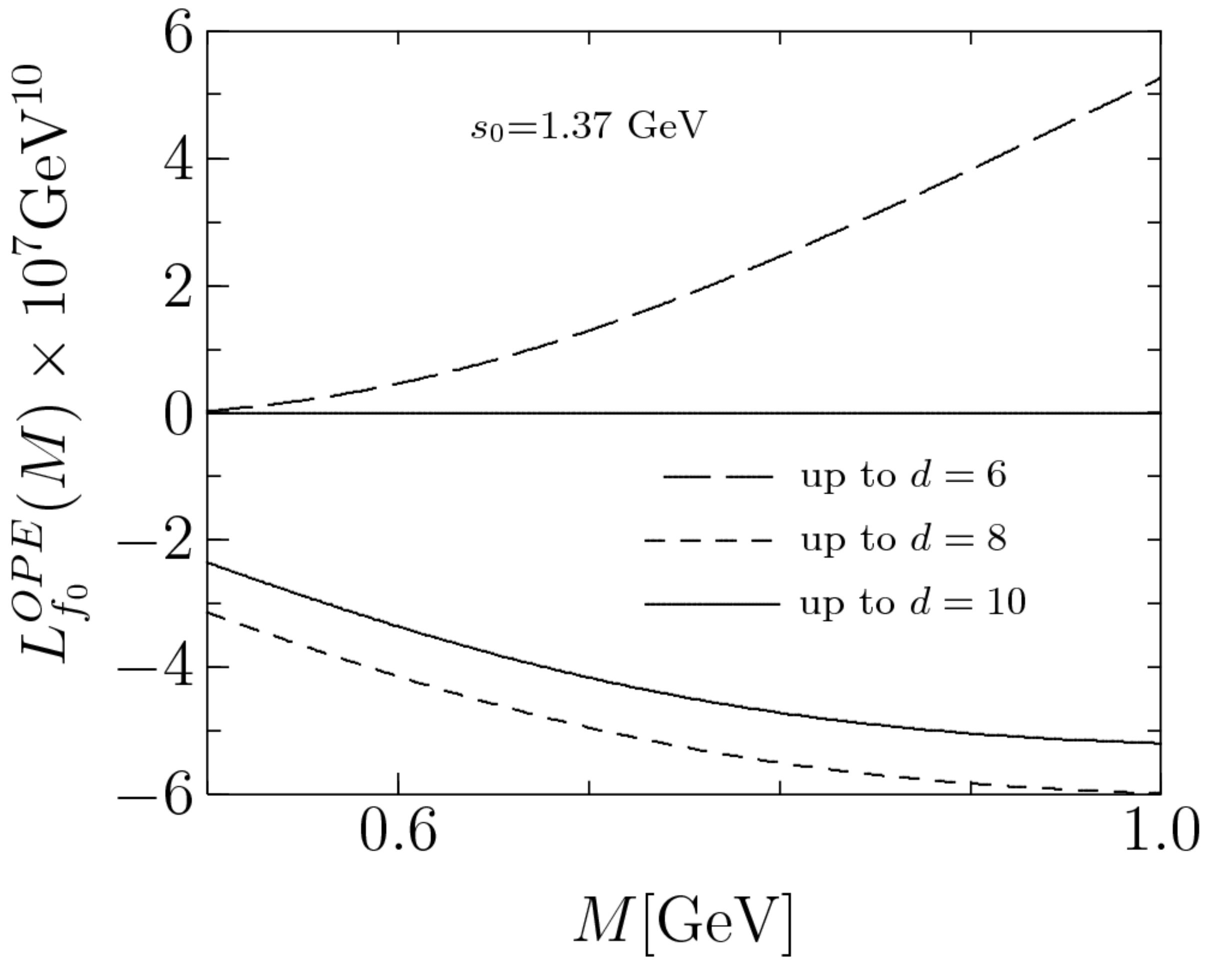
$$J_8 = i(\bar{u}\gamma_5 u + \bar{d}\gamma_5 d - 2\bar{s}\gamma_5 s)\ ,\ \ J_1 = i(\bar{u}\gamma_5 u + \bar{d}\gamma_5 d + \bar{s}\gamma_5 s)$$

$$\theta_p = -11.5^\circ$$

$$\psi_8 = u\bar{u} + d\bar{d} - 2s\bar{s}\ ,\quad \psi_1 = u\bar{u} + d\bar{d} + s\bar{s}$$

- Left Hand side of SR

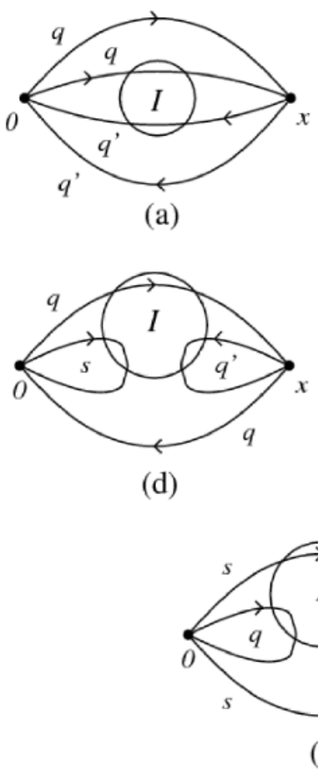
$$L_{f_0}^{OPE}(M) = \left(69(c+s)^4 + 72(c+s)^2(2c-s)^2 + \frac{33}{\cdots}(2c-s)^4\right)\frac{M^{10}E_4(M)}{\cdots}$$



$$+ \left(312(c+s)^2(2c-s)^2 - 73(2c-s)^4\right)\frac{\cdots}{2^{11}\cdot 3^2\pi^2}$$

$$- 4(c+s)^2(2c-s)^2\frac{m_s\langle\overline{q}q\rangle^2\langle\overline{s}s\rangle}{12} - 13(2c-s)^4\frac{m_s\langle\overline{s}s\rangle^3}{72}$$

Contributions from the instanton :



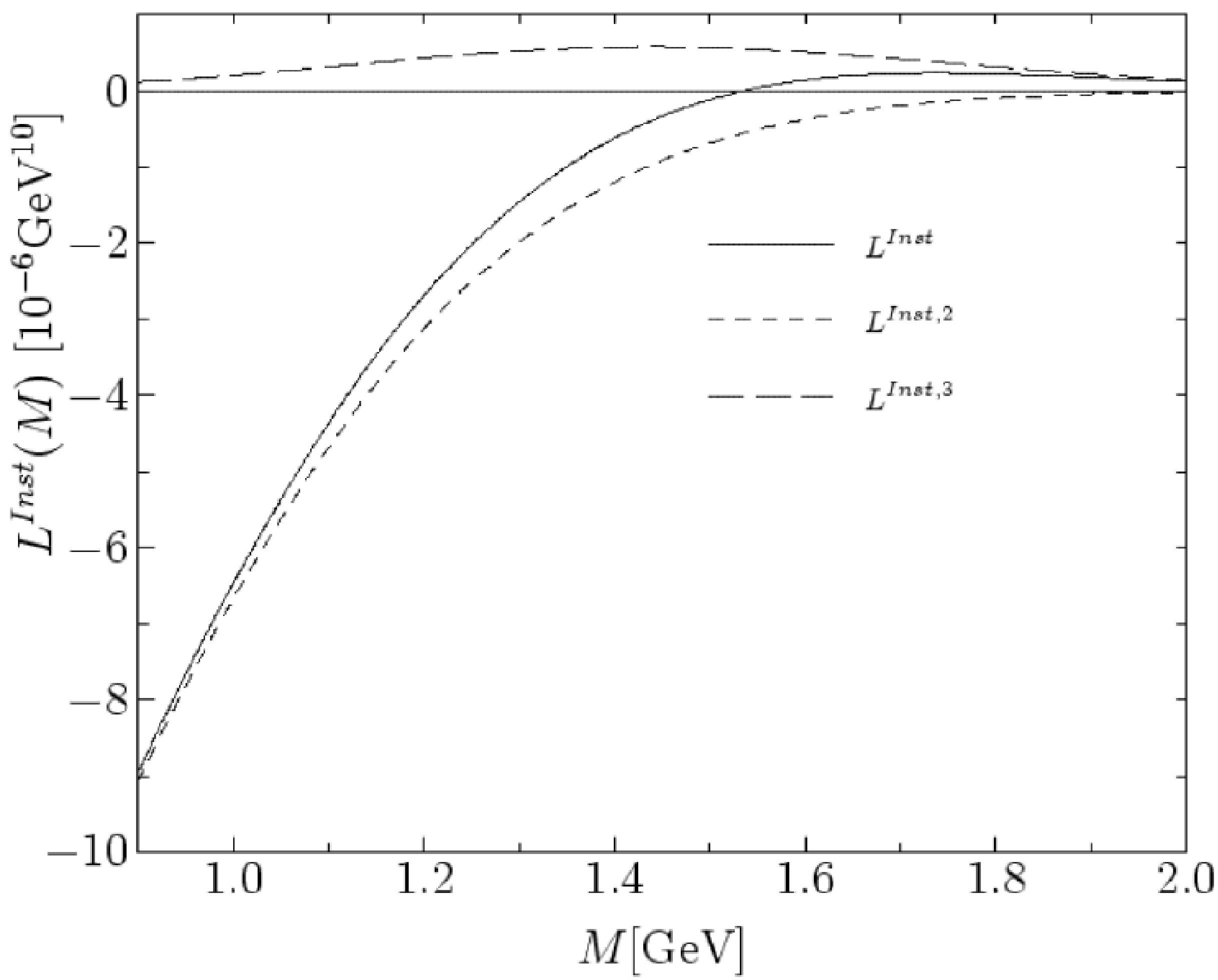


Fig. 1. Diagram
effects. q, q' can b
quark at the same
quark propagator
QCD vacuum. Li
propagators of the quarks on the instanton.

Another possibility :

- For f0(980) : bound state of two Kaons?
  - Weinstein and Isgur, PRL **48**, 659 (1982), PRD **27**, 588 (1983) : Using the color hyperfine and harmonic oscillator potentials.
  - T. Branz, et. Al. , Eur. Phys. J. A **37**, 303 (2008) : Using a phenomenological Lagrangian.

- Interpolating current :

$$|f_0(980)\rangle = \alpha|K^+K^-\rangle + \beta|K^0\bar{K}^0\rangle$$

$$\begin{aligned} J_{f_0} &= \alpha J_{K^+}J_{K^-} + \beta J_{K^0}J_{\bar{K}^0} \\ &= -[\alpha(\bar{s}\gamma_5 u)(\bar{u}\gamma_5 s) + \beta(\bar{s}\gamma_5 d)(\bar{d}\gamma_5 s)] \end{aligned}$$

Left hand side of SR :

$(\alpha^2 + \beta^2$ … $M^6 E_2(M^2)$

$+ \frac{m_s}{2^8\pi^4}\{$ … (M^2)

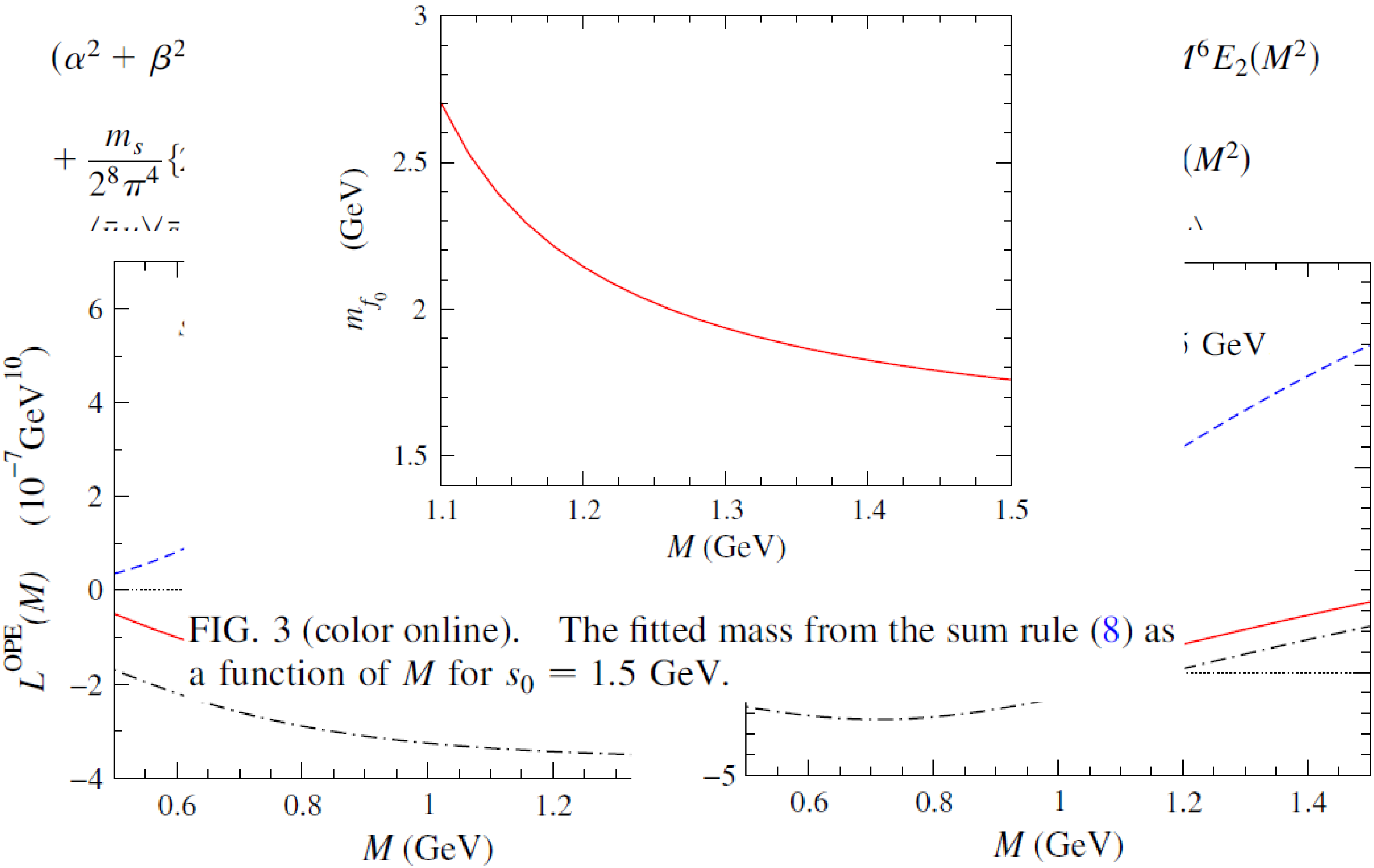
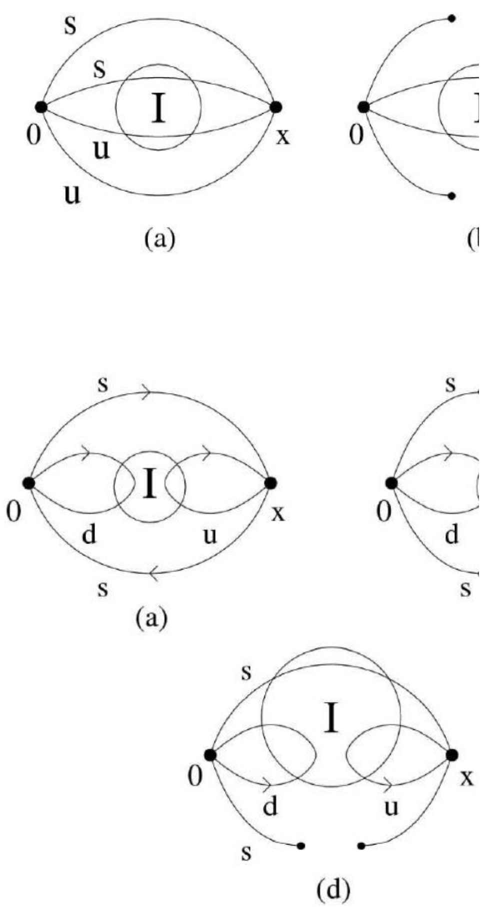
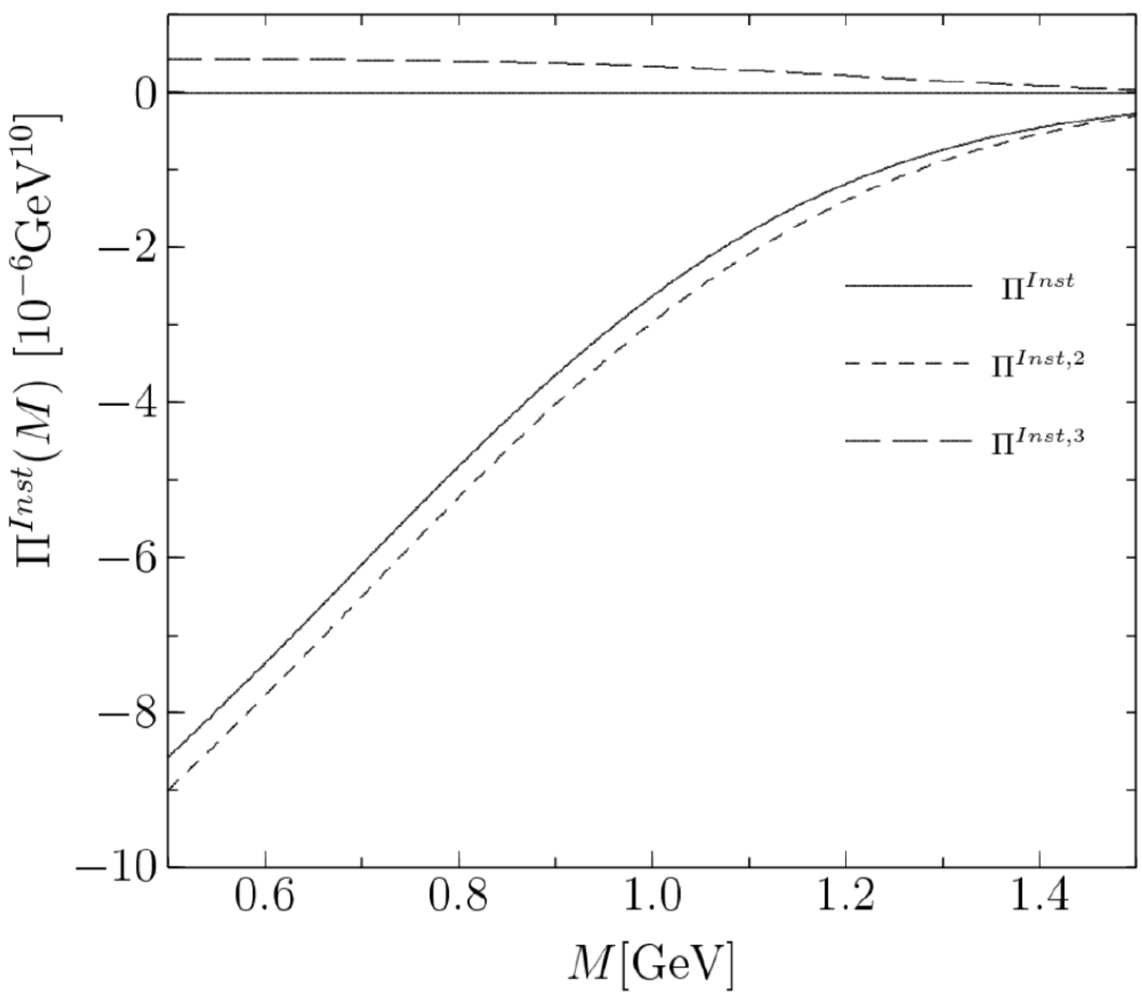


FIG. 3 (color online). The fitted mass from the sum rule (8) as a function of M for $s_0 = 1.5$ GeV.

Contributions from the instantons :





Summary and discussion

- f0(500) : S and PS diquark-antidiquark bound state?
  800MeV

- Are other members diquark-antidiquark bound states?
  - Mass splitting from sigma is too small.
    f0(980), a0(980) : 700MeV , k(800) : 730MeV

- Can f0(980) be a bound state of two mesons?
  - we could not see a signal which f0(980) is a bound state of the two etas or the two kaons even if contributions from instanton are included.

- Mixing tetraquarks and two quark state, or glueballs…

Thank you! !

I could get near the sun mostly yesterday!!!

$f_0(500)$ or σ
was $f_0(600)$

$I^G(J^{PC}) = 0^+(0^{++})$

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$f_0(500)$ T-MATRIX POLE $\sqrt{s}$

Note that $\Gamma \approx 2\,\mathrm{Im}(\sqrt{s_{\mathrm{pole}}})$.

VALUE (MeV)	*DOCUMENT ID*		*TECN*	*COMMENT*
(400–550)−*i*(200–350) OUR ESTIMATE				
• • • We do not use the following data for averages, fits, limits, etc. • • •				
$(440 \pm 10) - i(238 \pm 10)$	[1] ALBALADEJO	12	RVUE	Compilation
$(445 \pm 25) - i(278^{+22}_{-18})$	[2,3] GARCIA-MAR..	11	RVUE	Compilation
$(457^{+14}_{-13}) - i(279^{+11}_{-7})$	[2,4] GARCIA-MAR..	11	RVUE	Compilation
$(442^{+5}_{-8}) - i(274^{+6}_{-5})$	[5] MOUSSALLAM	11	RVUE	Compilation
$(452 \pm 13) - i(259 \pm 16)$	[6] MENNESSIER	10	RVUE	Compilation
$(448 \pm 43) - i(266 \pm 43)$	[7] MENNESSIER	10	RVUE	Compilation
$(455 \pm 6^{+31}_{-13}) - i(278 \pm 6^{+34}_{-43})$	[8] CAPRINI	08	RVUE	Compilation
$(463 \pm 6^{+31}_{-17}) - i(259 \pm 6^{+33}_{-34})$	[9] CAPRINI	08	RVUE	Compilation
$(552^{+84}_{-106}) - i(232^{+81}_{-72})$	[10] ABLIKIM	07A	BES2	$\psi(2S) \to \pi^+\pi^- J/\psi$
$(466 \pm 18) - i(223 \pm 28)$	[11] BONVICINI	07	CLEO	$D^+ \to \pi^-\pi^+\pi^+$
$(472 \pm 30) - i(271 \pm 30)$	[12] BUGG	07A	RVUE	Compilation
$(484 \pm 17) - i(255 \pm 10)$	GARCIA-MAR..	07	RVUE	Compilation