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Modern problems of nuclear and elementary particle physics

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Holographic Baryons in Dense Matter

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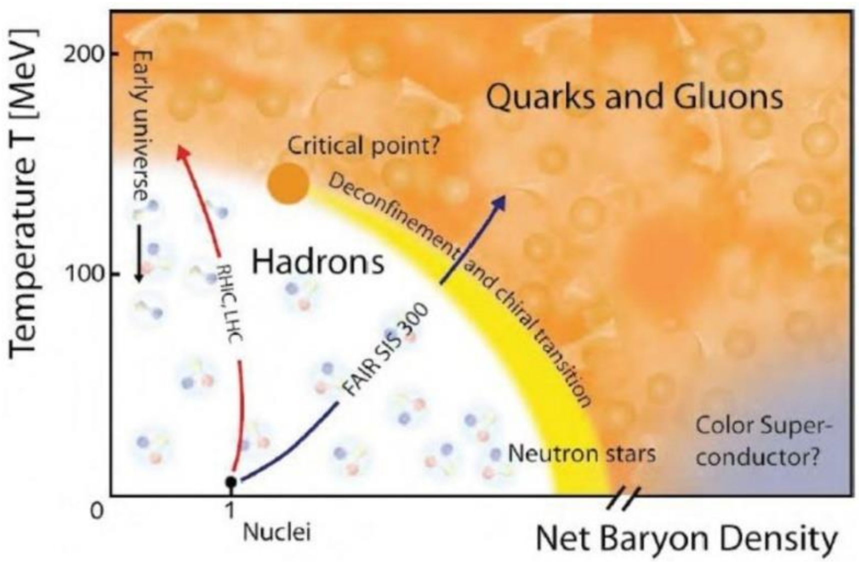
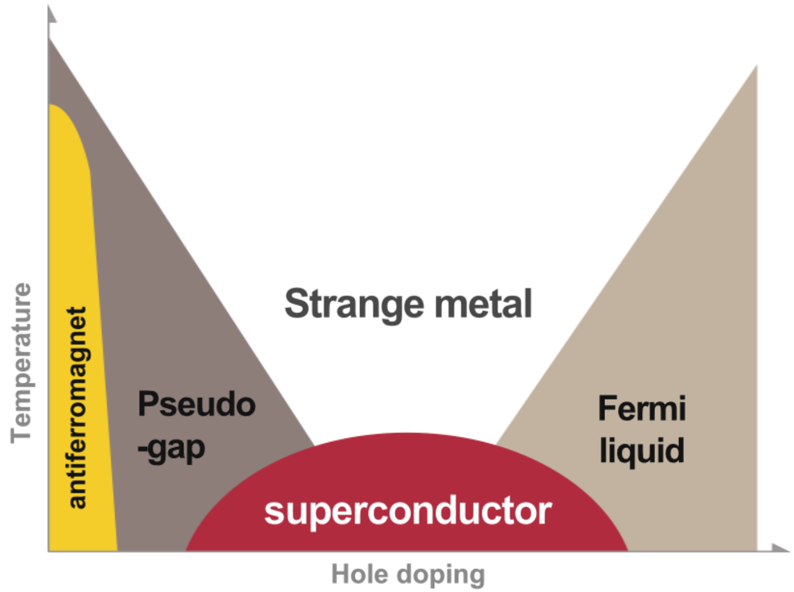
I. Introduction : Motivation & Basics

Q : How to understand the nonperturbative physics of the strongly interacting systems ?

Ex) In QCD, how to explain confinement, chiral symmetry breaking, phases (with or w/o chemical potential), meson spectra etc. ?

Ex) How to understand the phenomena in the Strongly correlated condensed matter systems?





AdS-CFT Holography : 3+1 dim. QFT ⇔ 4+1 Classical Gravity Theory

- Useful tool for strongly interacting systems
  such as QCD, Composite (Higgs) particles?, Condensed Matter, etc.

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String theory as a tool for the strongly interacting systems

II. Holography Principle – AdS/CFT

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IV. Summary

II. Holography Principle (AdS/CFT Correspondence)

"2nd revolution of the string theory (1994)

「Quantum Field Theory
in a given space time dimension (Ex):3+1=4 dim)

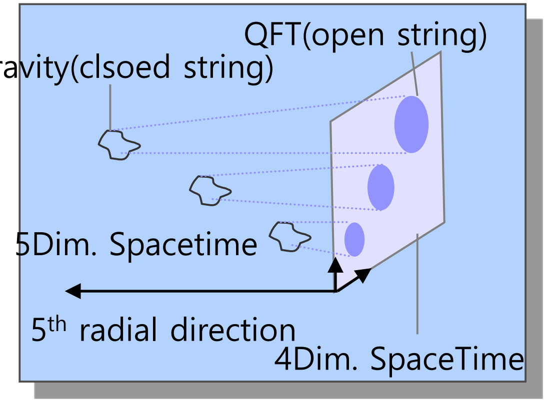
can be equivalently described by

the classical gravity theory
in one higher dimensional spacetime (Ex): 4+1=5dim).

(Ex: p = 3)

(Classical) Gravity Theory
Anti de Sitter (AdS) Space
in (p+1)+1 dimension

time radial



(Quantum) Field Theory
Conformal Field Theory (CFT)
in p+1 dimension

space time

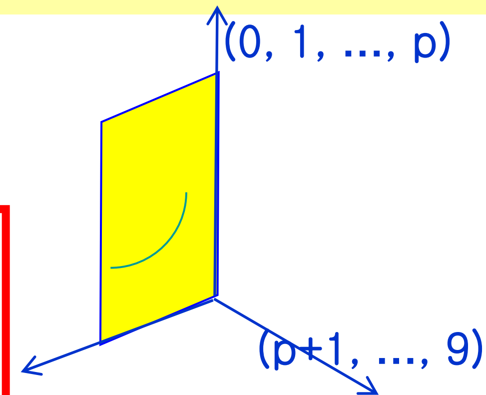
Main idea on holography through the Dp branes

Dp-brane carry tension(energy) and charges (for p+2 form)

Dp brane's low E dynamics by fluctuating Open Strings

⇒ SUGRA on AdS (p+2) space ⟺ Dp−branes ⟺ **=** (p+1)dim. SU(Nc) SYM

#Dp−branes = Nc



Question : 4 = 5 ?

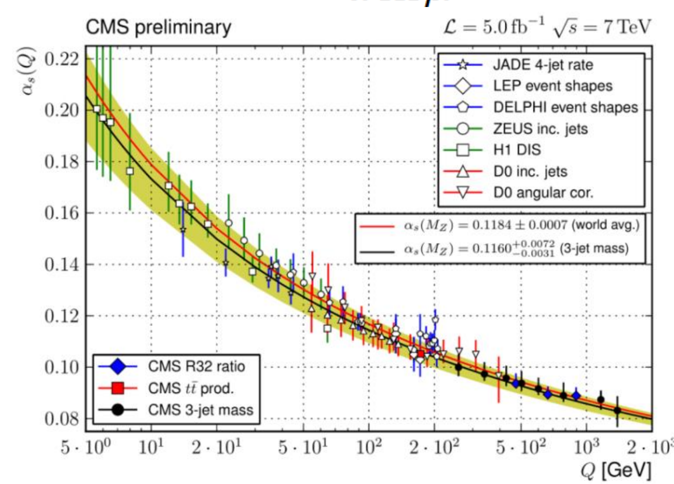
3+1 dim. QFT (large Nc)
⇔
4+1 dim. Effective Gravity description

Naïve Answer : Coupling constants are running in QFT !

$$\beta(g^2) = \frac{dg^2(\mu)}{d\ln\mu}$$

Energy scale in QFT corresponds to the parameter in extra "dimension" or radial direction in AdS5 space

$$g_s = e^{\phi(r)} = g_{YM}^2(\mu)$$



New Paradigm for the Strongly Interacting Quantum System

'Size' L of the 5dim is proportional to the coupling constant λ of the 4 dim.

4Dim QFT	Perturbative : Easy	Nonperturbative : Hard
Coupling constant λ	$\lambda \ll 1$	$\lambda \gg 1$
Size of the paramenter L	$L \ll 1$	$L \gg 1$
5Dim parameter	Quantum Gravity : Hard	Classical Gravity : "Easy"

· Strongly Interacting Quantum System ($\lambda >> 1$) $\leftrightarrow$ Classical Gravity ($L >> 1$)

New Methodolgoy : can use the 5 dim. classical gravity description for the 4dim. strongly interacting system.

4d QFT(on"boundary") ⇔ 5d Gravity (in"bulk")

Parameter (g_{YM}^2, N) $\quad 4\pi g_s N = \frac{R^4}{\alpha'^2} = \lambda = g_{YM}^2 N \quad$ (g_s, R)

Ex) Nc of D3 branes

N=4 SU(Nc) SYM ⟷ SUGRA on AdS5 x S5

Comments

- With $\beta(g^2) = 0$ (conf. inv.) — In Anti-deSitter Space

N=4 SU(Nc) SYM ⟷ SUGRA on AdS5 x S5

Nc of D3 branes

(g_{YM}^2, N) $\quad 4\pi g_s N = \frac{R^4}{\alpha'^2} = \lambda = g_{YM}^2 N \quad$ (g_s, R)

- With $\beta(g^2) \to 0$ (conf. inv.) — In asymptotic AdS Space

(ex) QCD — ?? (in asymptotic AdS Space)

Intersecting D-Branes – Flavors, mesons, etc.

Ex) D3-D7 Low energy dynamics → N=2 SYM with #M hyper

Strings

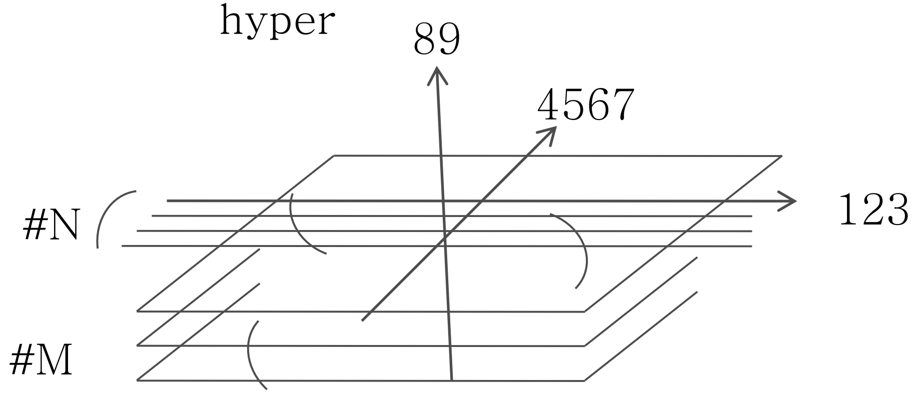
CPX 0,1,2,3

3-3 : Aμ, Φ, λ, χ

(N=2 Vector multiplet in adjoint

3-7 : Q, $\tilde{Q}$, ψ, $\tilde{\psi}$

(N=2 Hypermultiplet matter in fundamental)



7-7 open strings : Low energy dynamics for D7 branes (DBI action)

$$S_{D7} = -\mu_7 \int d^8\xi \sqrt{-\det\left(P[G]_{ab} + 2\pi\alpha' F_{ab}\right)} + \frac{(2\pi\alpha')^2}{2}\mu_7 \int P[C^{(4)}] \wedge F \wedge F$$

$$\mu_7 = [(2\pi)^7 g_s \alpha'^4]^{-1}$$

Still far from QCD !

Extension of the AdS/CFT

- the gravity theory on the asymptotically AdS space
  -> modified boundary quantum field theory (nonconf, less SUSY, etc.)

- Gravity w/ black hole background corresponds to the finite T field theory

AdS/CFT Dictionary

Witten 98;
Gubser,Klebanov,Polyakov 98

Parameters (g_s , R) $4\pi g_s N = \frac{R^4}{\alpha'^2} = \lambda = g_{YM}^2 N$ (g_{YM}^2 , N)

Partition function of bulk gravity theory (semi-classial)

$$Z_{str}[\phi_0(x)] = \int_{\phi_0} \mathcal{D}\phi \exp(-S[\phi, g_{\mu\nu}]) = e^{-S(\phi_{cl}[\phi_0])}$$

$$\phi(t, \mathbf{x};\, u = \infty) = u^{\Delta-4}\phi_0(t, \mathbf{x}) \quad \phi$$

ϕ_0 bdry value of the bulk field

Generating functional of bdry QFT for operator $\mathcal{O}$

$$Z[\phi_0(x)] = \left\langle \exp \int_{boundary} d^d x \phi_0 \mathcal{O} \right\rangle = \int \mathcal{D}[\Phi] \exp\{iS_4 + i\int \phi_0(x)\mathcal{O}\}$$

ϕ_0 : source of the bdry op. $\mathcal{O}$

- ϕ: scalar → $S = \int d^4x du \sqrt{-g}\,(g^{ab}\partial_a\phi\partial_b\phi - m^2\phi^2)$, $\phi(u) \sim u^{4-\Delta}\phi_0 + u^{\Delta}\langle\mathcal{O}\rangle$
- Correlation functions by $\frac{\delta^n Z_{\text{string}}}{\delta\phi_0(t_1, \mathbf{x}_1)\cdots\delta\phi_0(t_n, \mathbf{x}_n)} = \left\langle T\,\mathcal{O}(t_1, \mathbf{x}_1)\cdots\mathcal{O}(t_n, \mathbf{x}_n)\right\rangle_{\text{field theory}}$
- 5D bulk field Φ(t, x, u) ←→ Operator $\mathcal{O}$ (t, x)
  w/ 5D mass $E(\lambda, J_1, J_2, \cdots)$ ←→ w/ Operator dimesion $\Delta(\lambda, J_1, J_2, \cdots)$
- 5D gauge symmetry ←→ Current (global symmetry)
- Radial coord. r in the bulk is proportional to the energy scale E of QFT

$\mathcal{O}$(Operator in QFT) <-> ϕ (p-form Field in 5D)

$$(\Delta - p)(\Delta + p - 4) = m_5^2$$

Δ : Conformal dimension
m_5^2 : mass (squared)

Ex)

4D: $\mathcal{O}(x)$		5D: $\phi(x, z)$	p	Δ	$(m_5)^2$
$\bar{q}_L \gamma^\mu t^a q_L$		$A^a_{L\mu}$	1	3	0
$\bar{q}_R \gamma^\mu t^a q_R$		$A^a_{R\mu}$	1	3	0
$\bar{q}^\alpha_R q^\beta_L$		$(2/z)X^{\alpha\beta}$	0	3	-3
$\langle \mathrm{Tr} G^2 \rangle$	Gluon cond .	dilaton	0	4	0
$\bar{q}_L \gamma^\mu q_L$, $\bar{q}_R \gamma^\mu q_R$	baryon density	vector w/ U(1)	1	3	0
	Baryon			9/2	$(5/2)^2$

fields in gravity

- massless dilaton
- scalar field with $m^2 = -\frac{3}{R^2}$
- m=0 vector field A_μ in the $SU(N_f)$ gauge group

<-> dual <->

operators of QCD

- gluon condensation $\langle \mathrm{Tr} G^2 \rangle$
- chiral condensation $\bar{q}_R q_L$
- mesons in the $SU(N_f)$ flavor group

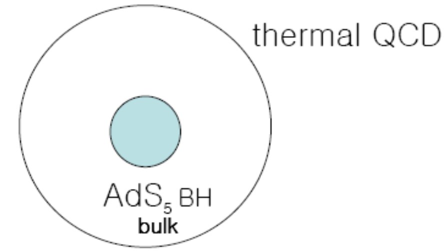
Temperatue

- Black hole gemometry
- $T = \frac{r_T}{\pi R^2}$

E. Witten (1998)

$$ds_5^2 = \frac{1}{z^2}\Big(f^2(z)dt^2 - (dx^i)^2 - \frac{1}{f^2(z)}dz^2\Big),$$

$$f^2(z) = 1 - (\frac{z}{z_T})^4 \qquad T = \frac{1}{\pi z_T}$$



Flavor degrees of freedom

- Adding probe brane
- $y(\rho) = M_q + \frac{<\bar{\psi}\psi>}{\rho^2} + \cdots \qquad (\rho >> 1)$

Chemical potential or Density

- Turning on $U(1)$ gauge field on probe brane
- $A_\mu \leftrightarrow < J^\mu > = \bar{\psi}\gamma^\mu\psi$
- $A_t = \mu + \frac{Q}{\rho^2} + \cdots \qquad (\rho >> 1)$

Source of gauge field

- End point of fundamental strings
- Physical object which carry $U(1)$ baryon charge
- Fundamental strings which connect probe brane and black hole $\rightarrow$ Quarks
- Fundamental strings which connect probe brane and baryon vertex $\rightarrow$ Baryons

III. Application – AdS/QCD (Holographic QCD)

Witten '98

Goal : Using the 5 dim. dual gravity, study 4dim. strongly interacting QCD such as spectra & Phases, etc.

parameters (Nc, Nf, m_q, T and μ, χ–symm., gluon condensation, etc.)

Ex) finte temperature for the pure Yang-Mills theory without quark matters

Needs the dual geometry !.

Approaches :

- Top-down Approach : rooted in string theory
  Find brane config. or SUGRA solution giving
  the gravity dual (May put the probe brane)

  Ex) Nc of D3(D4) + M of D7(D8), 10Dim. SUGRA solution etc.

- Bottom-up Approach:phenomenological
  Introduce fields, etc. (as needed based on AdS/CFT)
  5D setup → 4D effective Lagrangian

Low T QCD Phase transition High T

	Confinement	Deconfinement
QCD (4Dim)	Hadron	Quark-Gluon
Gravity (5Dim)	Thermal AdS	AdS Black Hole

Hawking-Page transition
=Transition of bulk geometry

Hawking-Page phase transition

[Herzog , Phys.Rev.Lett.98:091601,2007]

The geometry is described by the following action

$$S = \frac{1}{2\kappa^2}\int d^5x\sqrt{g}\,(-\mathcal{R}+2\Lambda)$$

$\Lambda = -\frac{6}{R^2}$: cosmological constant

The geometry with smaller action is the stable one for given T.

$$\Delta S = \lim_{\epsilon\to 0}(S_{AdSBH} - S_{tAdS}) = \frac{\pi z_h R^3}{\kappa^2}\left(\frac{1}{z_{IR}^4} - \frac{1}{2z_h^4}\right)$$

> 0 for T
< Tc
< 0 for T>Tc

Ex) Confinement -Deconfinement

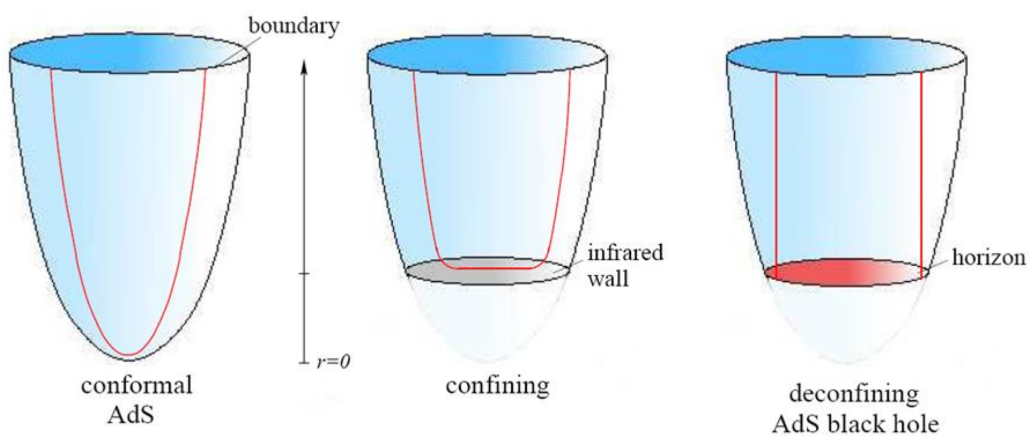


Figure from
Erdmenger et.al,
EPJA (2008)

Holographic QCD for finite chemical potential

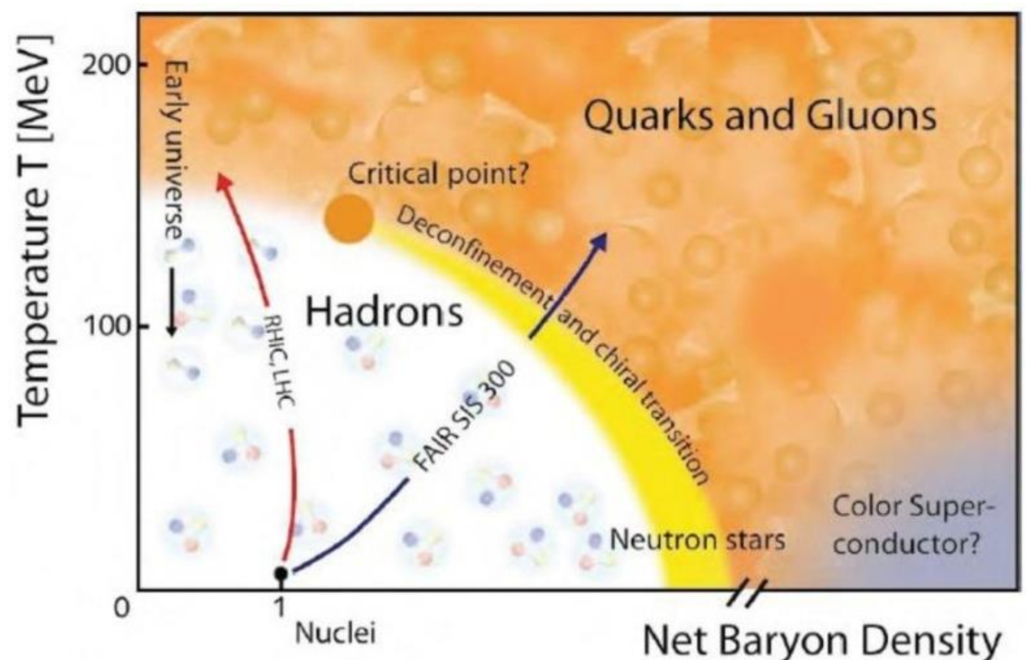
Low T QCD Phase transition High T

	Confinement	Deconfinement
QCD (4Dim)	Hadron	Quark-Gluon
Gravity (5Dim)	thermal & charged AdS	RNAdS Black Hole

Hawking−Page transition (BHL, Park ,Sin JHEP 0907,(2009))

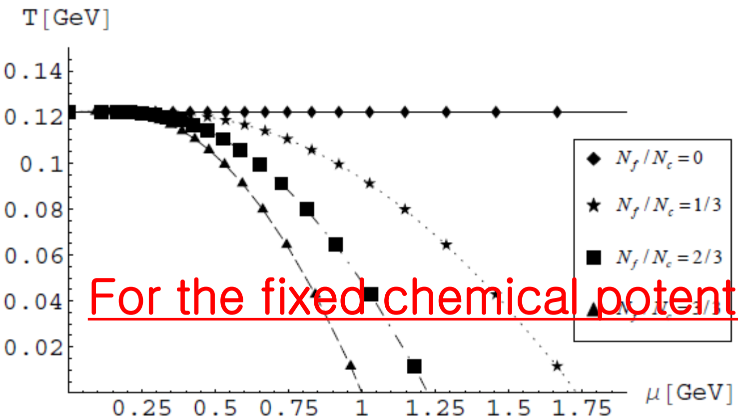
(q = 0)

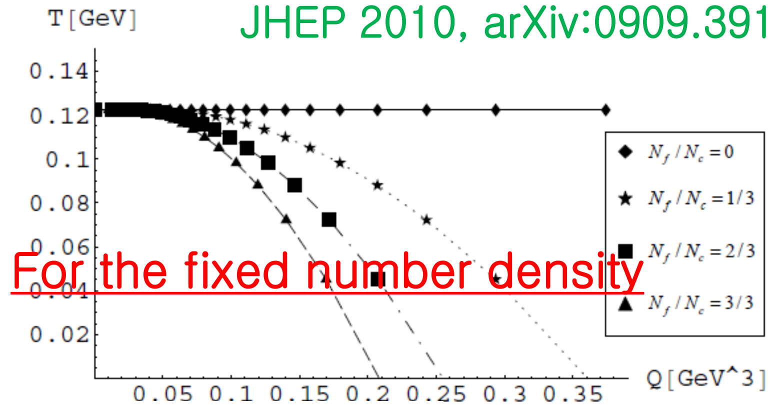
Gravity (5Dim)	Thermal AdS	AdS Black Hole

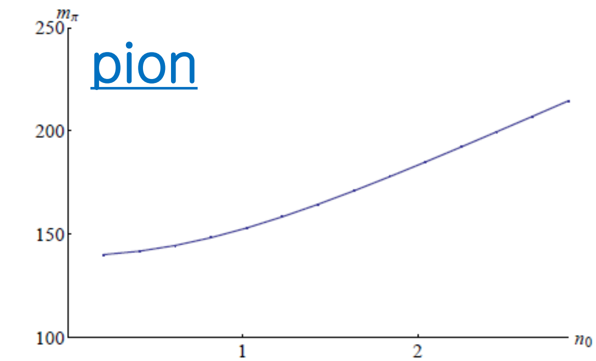


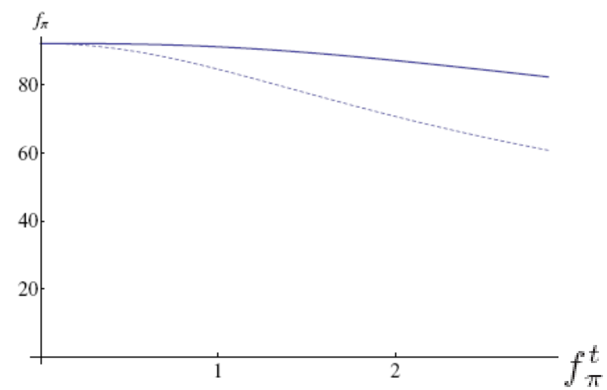
$$\Delta S = \int d^5x\sqrt{G}\ \mathrm{Tr}\left[|DX|^2 - \frac{3}{R^2}|X|^2 + \frac{1}{4g_5^2}\left(F_L^2 + F_R^2\right)\right] \quad M_X^2 = -3/l^2$$

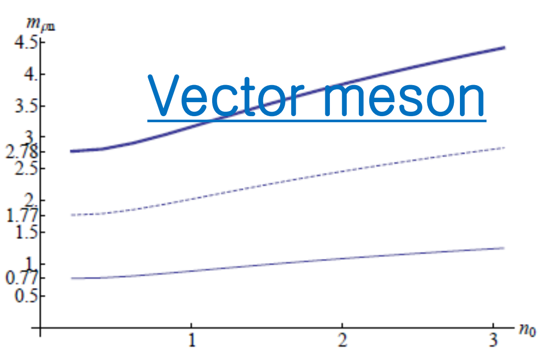
Jo, BHL, Park,Sin
JHEP 2010, arXiv:0909.3914

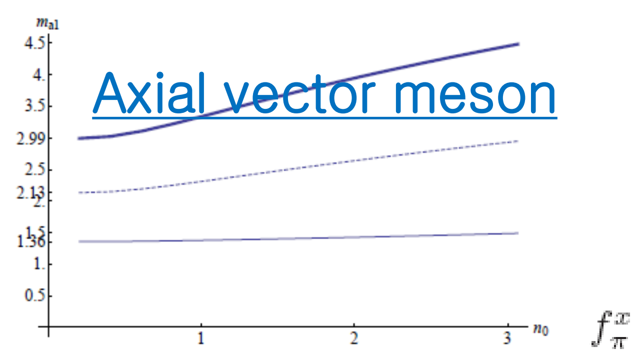












Nucleons in a nuclear medium

BHL, C. Park
Phys.Lett. B746 (2015)

Action (Hard Wall Model, U(2) x U(2)

$$S = \int d^5x\sqrt{-G}\left[\frac{1}{2\kappa^2}\left(\mathcal{R}-2\Lambda\right)-\frac{1}{4g^2}\left(F^{(L)}_{MN}F^{(L)MN}+F^{(R)}_{MN}F^{(R)MN}\right)\right] \qquad \Lambda=-6/R^2$$

+ complex scalar Φ + fermion fields (Ψ_1, Ψ_2)

$$F^{(L)}_{MN} = \partial_M L_N - \partial_N L_M - i\left[L_M, L_N\right],$$
$$F^{(R)}_{MN} = \partial_M R_N - \partial_N R_M - i\left[R_M, R_N\right].$$

The nuclear medium :

classified by baryon and isospin charges

→ turn on only diagonal time components of the gauge field

$V^0{}_t$ $\quad$ $V^3{}_t$ $\quad$ $L_M = R_M = -V_M$

tcAdS background geometry

$$ds^2 = \frac{R^2}{z^2}\left(-f(z)dt^2 + \frac{1}{f(z)}dz^2 + d\vec{x}^2\right)$$

$$f(z) = 1 + \frac{3Q^2\kappa^2}{g^2R^2}z^6 + \frac{D^2\kappa^2}{3g^2R^2}z^6,$$
$$V^0_t = \frac{Q}{\sqrt{2}}\left(2z_{IR}^2 - 3z^2\right),$$
$$V^3_t = \frac{D}{3\sqrt{2}}\left(2z_{IR}^2 - 3z^2\right),$$

$Q = Q_P + Q_N$ $\qquad$ $D = Q_P - Q_N$

For chiral symmetry, introduce a complex scalar field Φ with a negative mass , −3 (in R=1 unit).

$$\Phi = \phi \mathbb{1} \; e^{i\sqrt{2}\pi},$$

Equation of motion

$$0 = \frac{1}{\sqrt{-g}}\partial_z\left(\sqrt{-g}g^{zz}\partial_z\phi\right) + 3\phi,$$

Solution

$$\phi(z) = m_q \; z \; {}_2F_1\left(\frac{1}{6},\frac{1}{2},\frac{2}{3},-\frac{\left(D^2+9Q^2\right)z^6}{3\;N_c}\right) + \sigma \; z^3 \; {}_2F_1\left(\frac{1}{2},\frac{5}{6},\frac{4}{3},-\frac{\left(D^2+9Q^2\right)z^6}{3\;N_c}\right)$$

To describe nucleons , introduce corresponding bulk fermion fields in the tcAdS .

$$\begin{aligned} S &= i\int d^5x\sqrt{-G}\Big[\overline{\Psi}^1\Gamma^M\nabla_M\Psi^1 + \overline{\Psi}^2\Gamma^M\nabla_M\Psi^2 - m_1\overline{\Psi}^1\Psi^1 - m_2\overline{\Psi}^2\Psi^2 \\ &\qquad - g_Y\left(\overline{\Psi}^1\Phi\Psi^2 + \overline{\Psi}^2\Phi^+\Psi^1\right)\Big], \end{aligned}$$

$$\nabla_M \Psi^1 = \left(\partial_M - \frac{i}{4}\omega_M - iL_M\right)\Psi^1,$$
$$\nabla_M \Psi^2 = \left(\partial_M - \frac{i}{4}\omega_M - iR_M\right)\Psi^2.$$

the mass of bulk fermions must be ± 5/2 related to the conformal dimension of nucleons,9/2, $m_1 = -m_2 = 5/2.$

Equation of motion

$$0 = \left[e^M_C \Gamma^C \left(\partial_M - \frac{i}{4}\omega^{AB}_M \Gamma_{AB} + iV_M\right) - m_1\right]\Psi^1 - g_Y \phi \Psi^2,$$
$$0 = \left[e^M_C \Gamma^C \left(\partial_M - \frac{i}{4}\omega^{AB}_M \Gamma_{AB} + iV_M\right) - m_2\right]\Psi^2 - g_Y \phi \Psi^1,$$

$$\Gamma^{AB} = \frac{i}{2}\left[\Gamma^A, \Gamma^B\right]$$

The boundar y boundary condition

$$\delta \overline{\Psi}^{(1,2)} \Gamma^M \Psi^{(1,2)} \Big|_{\epsilon}^{z_{IR}} = 0,$$

the Fourier mode expansion of 5-dimensional fermions

$$\Psi(z,t,\vec{x}) = \sum_{\omega_n} \int \frac{d^3p}{(2\pi)^4}\ \Psi(z,\omega_n,\vec{p})\ e^{-i(\omega_n t - \vec{p}\cdot\vec{x})},$$

$$\Psi^1(z,\omega_n,\vec{p}) = \begin{pmatrix} f_L^{1(n,\pm,\pm)}\ \psi_L^{(n,\pm,\pm)} \\ f_R^{1(n,\pm,\pm)}\ \psi_R^{(n,\pm,\pm)} \end{pmatrix} \qquad \Psi^2(z,\omega_n,\vec{p}) = \begin{pmatrix} f_L^{2(n,\pm,\pm)}\ \psi_L^{(n,\pm,\pm)} \\ f_R^{2(n,\pm,\pm)}\ \psi_R^{(n,\pm,\pm)} \end{pmatrix}$$

where n denotes the n -th resonance and the first and second sign imply the parity and isospin quantum number respectively.

the normalizable mode functions to be f1L and f 2R
To be associated with the 4-dimensional chirality.

Using the previous Fourier mode decomposition, the 5-dimensional Dirac equation is reduced to

$$\begin{pmatrix} \mathcal{D}_-\ \mathbb{1} & -\frac{g_Y\phi}{z}\ \mathbb{1} \\ -\frac{g_Y\phi}{z}\ \mathbb{1} & \mathcal{D}_+\ \mathbb{1} \end{pmatrix} \begin{pmatrix} f_L^{1(n,\pm,\pm)} \\ f_L^{2(n,\pm,\pm)} \end{pmatrix} = -\begin{pmatrix} \mathbb{E}_+ & 0 \\ 0 & \mathbb{E}_+ \end{pmatrix} \begin{pmatrix} f_R^{1(n,\pm,\pm)} \\ f_R^{2(n,\pm,\pm)} \end{pmatrix},$$

$$\begin{pmatrix} \mathcal{D}_+\ \mathbb{1} & \frac{g_Y\phi}{z}\ \mathbb{1} \\ \frac{g_Y\phi}{z}\ \mathbb{1} & \mathcal{D}_-\ \mathbb{1} \end{pmatrix} \begin{pmatrix} f_R^{1(n,\pm,\pm)} \\ f_R^{2(n,\pm,\pm)} \end{pmatrix} = \begin{pmatrix} \mathbb{E}_- & 0 \\ 0 & \mathbb{E}_- \end{pmatrix} \begin{pmatrix} f_L^{1(n,\pm,\pm)} \\ f_L^{2(n,\pm,\pm)} \end{pmatrix},$$

$$\mathcal{D}_{\pm} = \sqrt{f(z)}\left[\partial_z - \frac{2}{z}\left(1 - \frac{zf'}{8f(z)}\right)\right] \pm \frac{5}{2z},$$

$$\mathbb{E}_{\pm} = \frac{1}{\sqrt{f(z)}}(\omega_n - V_t)\ \mathbb{1} \pm \vec{\sigma}\cdot\vec{p}\,.$$

Proton with the isospin charge 1/2 is governed by

$$\begin{pmatrix} \mathcal{D}_- \mathbb{1} & \frac{g_Y\phi}{z}\mathbb{1} \\ \frac{g_Y\phi}{z}\mathbb{1} & \mathcal{D}_+ \mathbb{1} \end{pmatrix}\begin{pmatrix} f_L^{1(1,+,+)} \\ f_R^{1(1,+,+)} \end{pmatrix}$$
$$= \begin{pmatrix} -\left\{\frac{1}{\sqrt{f(z)}}\left(\omega - \frac{V_t^0+V_t^3}{2}\right) + p\right\} & 0 \\ 0 & \frac{1}{\sqrt{f(z)}}\left(\omega - \frac{V_t^0+V_t^3}{2}\right) - p \end{pmatrix}\begin{pmatrix} f_R^{1(1,+,+)} \\ f_L^{1(1,+,+)} \end{pmatrix}$$

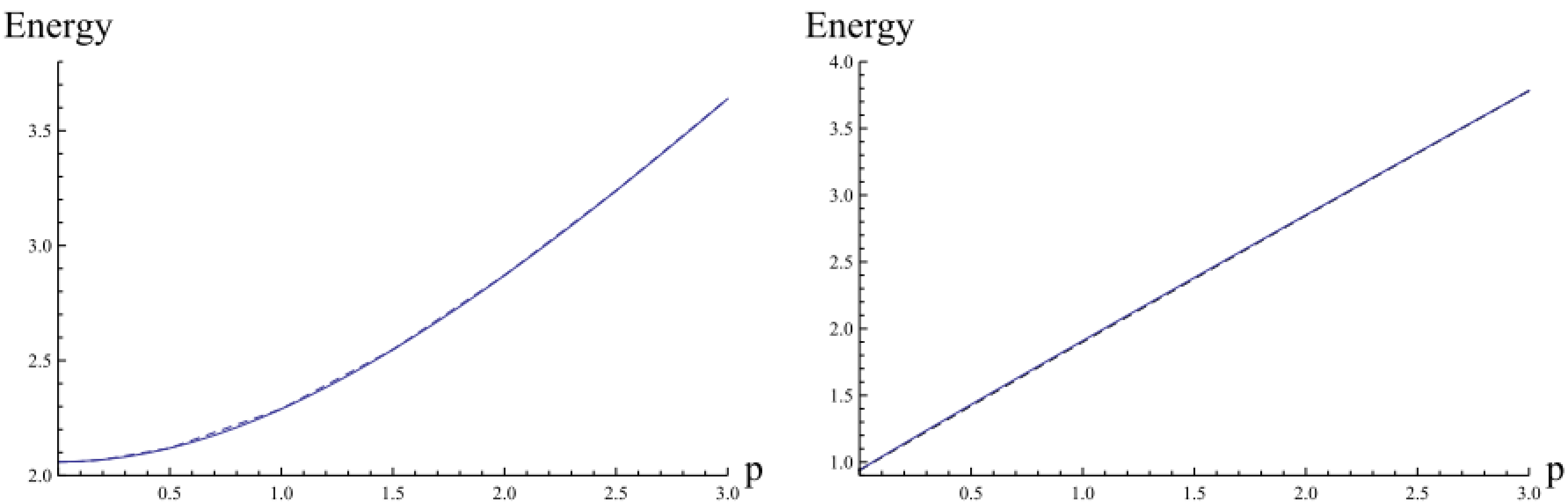
Neutron with the isospin charge −1/2 is governed by

$$\begin{pmatrix} \mathcal{D}_- \mathbb{1} & \frac{g_Y\phi}{z}\mathbb{1} \\ \frac{g_Y\phi}{z}\mathbb{1} & \mathcal{D}_+ \mathbb{1} \end{pmatrix}\begin{pmatrix} f_L^{1(1,+,-)} \\ f_R^{1(1,+,-)} \end{pmatrix}$$
$$= \begin{pmatrix} -\left\{\frac{1}{\sqrt{f(z)}}\left(\omega - \frac{V_t^0-V_t^3}{2}\right) + p\right\} & 0 \\ 0 & \frac{1}{\sqrt{f(z)}}\left(\omega - \frac{V_t^0-V_t^3}{2}\right) - p \end{pmatrix}\begin{pmatrix} f_R^{1(1,+,-)} \\ f_L^{1(1,+,-)} \end{pmatrix}$$

two boundary conditions $\quad f_L^{1(n,\pm,\pm)}(0) = 0 \quad \text{and} \quad f_R^{1(n,\pm,\pm)}(z_{IR}) = 0,$

Dispersion relation in the vacuum

$Q = D = 0$



(a) $\phi = 0$ (b) $\phi \neq 0$

Figure 1: The nucleon's mass in the vacuum
(a) for mq = σ =0 and
(b) with mq =2 . 38MeV, σ =(304MeV)3
and gY =4 . 699 which reproduce the correct nucleon's mass in the vacuum.

Nucleon's rest mass in the nuclear medium

$m_q = 2.38\text{MeV}, \ , \sigma = (304\text{MeV})^3$ and $g_Y = 4.699$.

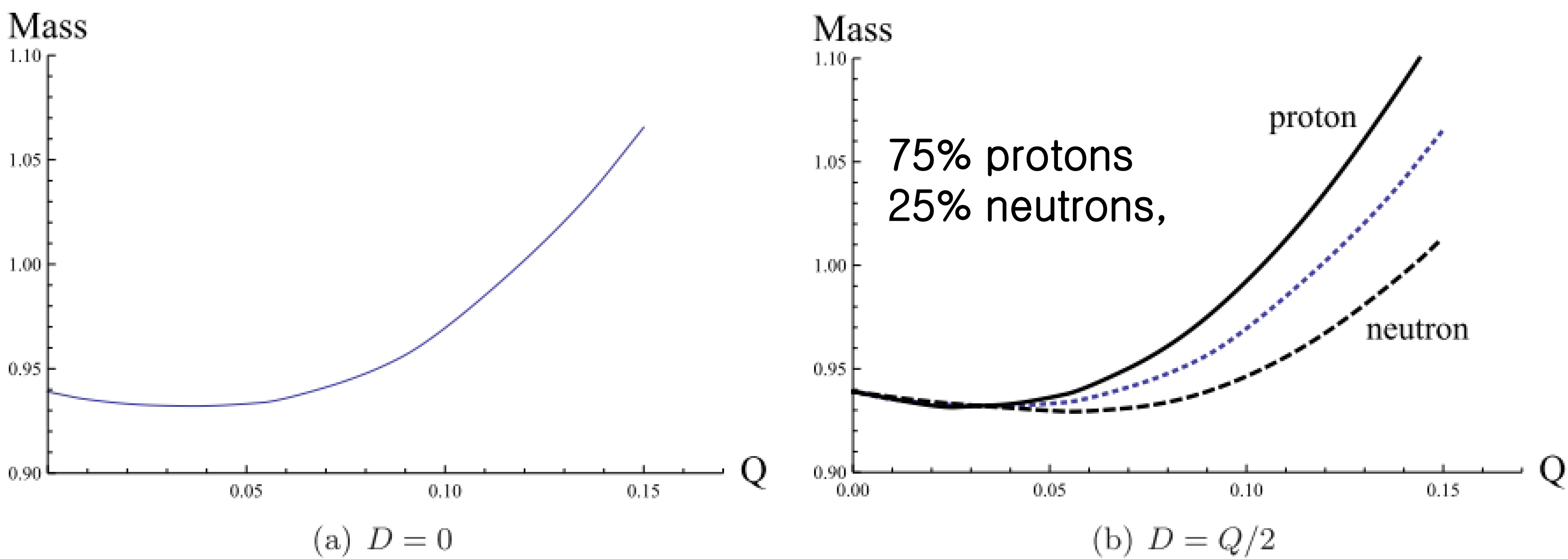
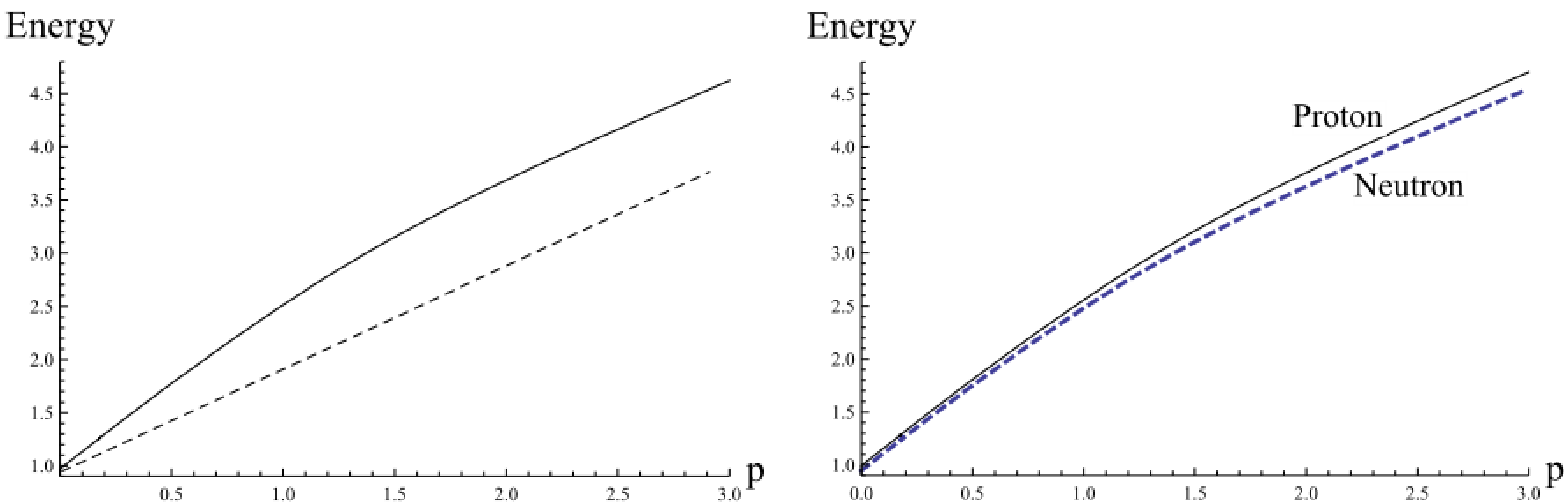


Figure 2: The nucleon's mass spectrum in the nuclear medium.
(a) For D =0, proton and neutron are degenerate.
(b) For D = Q/ 2, the masses of proton and neutron are splitted due to the isospin interaction, where the dotted line denotes the nucleon mass for D =0.

Dispersion relations



(a) $Q = 0.1$ and $\alpha = 0$

(b) $Q = 0.1$ and $\alpha = 1/2$

Figure 3: The nucleon's dispersion relation in the nuclear medium.
(a) The dashed and solid line indicate the dispersion relations in the vacuum and in the nuclear medium.
(b)The isospin interaction splits the degeneracy of nucleons. The energy of proton (neutron) slightly increases (decreases).

IV. Summary

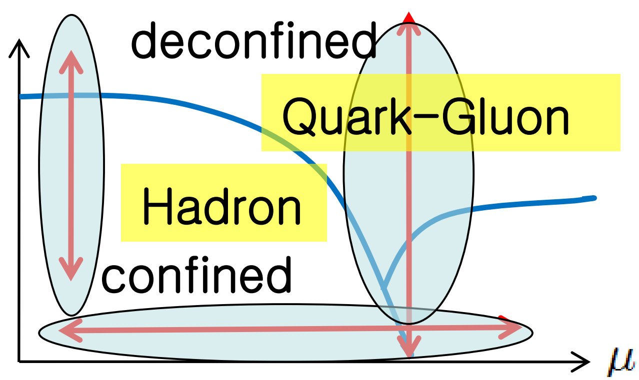
- Holographic Principles (through the D-brane configuration)

(d+1 dim.) (classical) gravity ↔ (d dim.) (quantum) YM theories

- BH geometry ↔ Finite Temperature field theory

- Constructing the dual geometry :

Top-down & Bottom-up



- Holographic QCD

– w/o chemical potential –

phase : confined phase ↔ deconfined phase transition

Geometry : thermal AdS ↔ AdS BH

Hawking-Page transition

– in dense matter – (U(1) chemical potential→ baryon density)

deconfined phase by RNAdS BH ↔ hadronic phase by tcAdS

Hawking-Page phase transition

IV. Summary – continued

- In the hadronic phase, the quark density dependence of the light meson masses has been investigated.
- Holographic study on the Baryon Properties in Dense Matter
- Holographic principle can also be applied to the nonequilibrium physics
- Holographic Principle may provide a new methodology for the strongly interacting phenomena!

Thank You !