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Modern problems of nuclear and elementary particle physics

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Mass Spectrum and Decay Constants of Conventional Mesons within an Infrared Confinement Model

Gurjav Ganbold (BLTP, JINR; IPT MAS (Mongolia))

in collaboration with: **T. Gutsche** (Tuebingen)
M. A. Ivanov (Dubna)
V. E. Lyubovitskij (Tuebingen)

Outline

- Model
  - Compositeness condition
  - Infrared confinement
- Meson ground states, definition of mass
- Numerical results:
  - Electromagnetic decay widths of mesons
  - Leptonic decay constants of mesons
  - Mass spectrum of mesons
  - Fermi coupling - Smoothing
- Summary and outlook

Motivation

Hadron physics today:

- Experiments: big quantity of high-precision data ⇒ Challenge to theory
- Quark bound states: low momenta ⇒ pQCD looses its applicability
- Lattice QCD: results promising but still not good established
- Standard Model (QCD) operates only with fundamental particles
  It is not yet clear how to explain the appearance of the numerous number of the observed hadrons and elucidate the generation of their masses.
  The origin of the hadron masses = one of the puzzles.

➢ ***Models needed by theory & experiment***

Covariant Confined Quark Model (Short Review)

- Lagrangian-based formulation $\Rightarrow$ full Lorentz invariance
- Direct inclusion of higher-number quark states (baryons, tetraquarks,...)
- One free parameter per hadron (for large number of hadrons)
- Wide application and convincing results:

  - calculation of the leptonic decay constants,

  - estimation of basic form factors needed for semi-leptonic,

  - non-leptonic and rare decays of *B* mesons and Λ_b baryons and etc.

Basic Assumptions:

T. Branz et al., Phys. Rev. **D81**, 034010 (2010).

- Hadrons *H(x)* interact by *quark exchanges* with hadron-quark coupling g_H.

$$L_{\text{int}} = g_H H(x) J_H(x)$$

- The matrix element <*physical state|bare state*> is determined by renormalization:

$$Z_H = \left\langle H_{bare} \mid H_{phys} \right\rangle^2 , \qquad H_{bare} = Z_H^{1/2} H_{phys}$$

- The **compositeness condition** eliminates the bare fields from consideration.

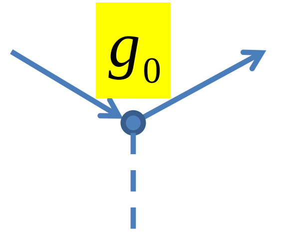
$$Z_H = 1 - g_{ren}^2 \tilde{\Pi}'_H (M_H^2) = 0$$

- **Infrared confinement** is introduced to guarantee the absence of all possible *thresholds* corresponding to quark production. It allows to use the same values for the constituent quark masses.

Compositeness Condition

- **Yukawa-type model**

$$L_Y = \bar{q}\,(i\hat{\partial} - m)\,q + \phi_0\,(-\partial^{\mu}\partial_{\mu} - M_0^2)\,\phi_0 + g_0\phi_0\,(\bar{q}\,\Gamma\, q)$$



$$g_r \equiv Z^{1/2} g_0$$

Generating functional + explicit integrations over quark fields

$$Z_Y = \int \delta\phi_0 \int \delta\bar{q} \int \delta q \exp\left\{ i \int dx\, L_Y(x) \right\}$$

$$\Pi(x-y) \equiv i < T\left\{ (\bar{q}\Gamma q)_x (\bar{q}\Gamma q)_y \right\} >= -i \cdot tr\left\{ \Gamma\, S(x-y)\,\Gamma\, S(y-x) \right\}$$

$$\tilde{\Pi}(p^2) = \tilde{\Pi}(M^2) + (p^2 - M^2)\,\tilde{\Pi}'(M^2) + \tilde{\Pi}^{ren}(p^2), \qquad \tilde{\Pi}'(p^2) = \frac{d}{dp^2}\tilde{\Pi}(p^2)$$

Renormalization

$$M^2 \equiv M_0^2 - g_0^2\tilde{\Pi}(M^2)\,; \quad \phi_r \equiv Z^{-1/2}\phi_0\,; \quad g_r \equiv Z^{1/2} g_0\,;$$

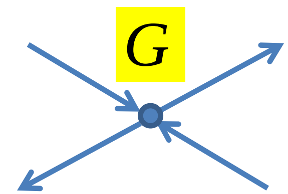
$$Z \equiv [1 - g_0^2\tilde{\Pi}'(M^2)]^{-1} = 1 - g_r^2\tilde{\Pi}'(M^2)$$

$$Z_Y^{ren} = \int \delta\phi_r \exp\left\{ \frac{i}{2}\Big(\phi_r(\Box - M^2)\phi_r\Big) + \frac{i\, g_r^2}{2}\Big(\phi_r \tilde{\Pi}^{ren}\phi_r\Big) \right\}$$

$$\cdot \exp\left\{ -\sum_{n=3}^{\infty} \frac{i^n g_r^n}{n} \int dx_1 \ldots \int dx_n\, \phi_r(x_1)\ldots\phi_r(x_n) \cdot \mathrm{tr}\left\{ \Gamma\, S(x_1 - x_2)\ldots\Gamma\, S(x_n - x_1) \right\} \right\}$$

- **Fermi-type model**

$$L_F = \bar{q}\,(i\hat{\partial} - m)\,q + \frac{G}{2}(\bar{q}\,\Gamma\,q)^2$$



Gaussian representation:

$$\exp\left\{ i\,\frac{G}{2}(\bar{q}\,\Gamma\,q)^2 \right\} = \int \delta\phi \exp\left\{ -i\,\frac{1}{2G}(\phi\,\phi) + i\left(\phi(\bar{q}\,\Gamma\,q)\right) \right\}$$

$$Z_F = \int \delta\bar{q} \int \delta q \exp\left\{ i \int dx\, L_F(x) \right\}$$

$$= \int \delta\phi \exp\left\{ -i\,\frac{1}{2G}(\phi\,\phi) - \sum_{n=2}^{\infty} \frac{i^n}{n} \int dx_1 ... \int dx_n\, \phi(x_1)...\phi(x_n) \cdot tr\left\{ \Gamma\, S(x_1 - x_2) ... \Gamma\, S(x_n - x_1) \right\} \right\}$$

Bi-linear terms in boson fields:

$$L_F^{(2)} = \frac{1}{2}\left(\phi \left[-\frac{1}{G} + \Pi(M^2) + (\Box - M^2)\Pi'(M^2) \right] \phi \right) + \frac{1}{2}\left(\phi\, \Pi^{ren} \phi \right)$$

Condition and renormalization

$$-\frac{1}{G} + \tilde{\Pi}(M^2) = 0$$

$$\phi_{ren} = \left[\tilde{\Pi}'(M^2) \right]^{-1/2} \phi$$

$$Z_F^{ren} = \int \delta\phi_{ren} \exp\left\{ \frac{i}{2}\left(\phi_{ren} (\Box - M^2) \phi_{ren} \right) + \frac{i}{2} \frac{1}{\tilde{\Pi}'(M^2)} \left(\phi_{ren} \tilde{\Pi}^{ren} \phi_{ren} \right) \right\}$$

$$\cdot \exp\left\{ -\sum_{n=3}^{\infty} \frac{i^n}{n} \left[\frac{1}{\tilde{\Pi}'(M^2)} \right]^{n/2} \int dx_1 ... \int dx_n\, \phi_{ren}(x_1) ... \phi_{ren}(x_n) \cdot \mathrm{tr}\left\{ \Gamma\, S(x_1 - x_2) ... \Gamma\, S(x_n - x_1) \right\} \right\}$$

$$Z_Y^{ren} \Leftrightarrow Z_F^{ren}$$

$$L_F^{\text{int}} = \frac{G}{2} J_H^2(x) \quad \Leftrightarrow \quad L_Y^{\text{int}} = g_H H(x) J_H(x)$$

$$g_r \equiv \left[\tilde{\Pi}'(M^2) \right]^{-1/2}$$

$$-\frac{1}{G} + \tilde{\Pi}(M^2) = 0$$

Compositeness Condition

$$Z \equiv 1 - g_r^2 \cdot \tilde{\Pi}'(M^2) = 0$$

$$\phi_0 \equiv Z^{1/2} \phi_r = 0$$

The vanishing of the wave function renormalization constant (Z=0) in the Yukawa theory may be interpreted as the condition that the bare (unrenormalized) field vanishes for a composite boson.

B. Jouvet, Nuovo Cim. 3, 1133 (1956)
A. Salam, Nuovo Cim. 25, 224 (1962)
S. Weinberg, Phys.Rev. 130, 776 (1963)

Equation for Meson Mass

$$1 - G \cdot \tilde{\Pi}(M^2) = 0$$

Model: Meson-Quark Interaction

- **Interaction Lagrangian**:

$$L_{\rm int} = g_H H(x) J_H(x)$$

- **Quark currents (for mesons):**

$$J_H(x) = \int dx_1 \int dx_2 \, F_H(x; x_1, x_2) \left(\bar{q}(x_2) \Gamma_H \, q(x_1) \right)$$

$$\Gamma_P = \gamma^5; \quad \Gamma_V = \gamma^\mu$$

- **Vertex function (translational invariant):**

$$F_H(x; x_1, x_2) = \delta(x - \omega_1 x_1 - \omega_2 x_2) \cdot \Phi_H\left(|x_1 - x_2|^2 \right)$$

$$\omega_j = \frac{m_j}{m_1 + m_2}; \quad \omega_1 + \omega_2 = 1$$

- **Vertex** in Gaussian form (its Fourier transformation) :

$$\tilde{\Phi}_H(-p^2) = \exp\left(\frac{p^2}{\Lambda_H^2} \right)$$

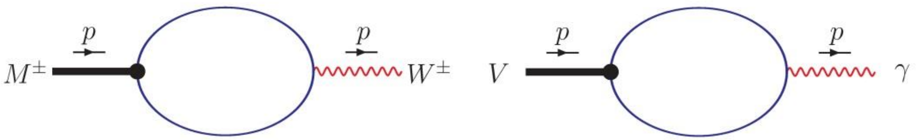
$1/\Lambda_H$ ~ hadron "size"

- **Quark propagator (**in the Schwinger representation**):**

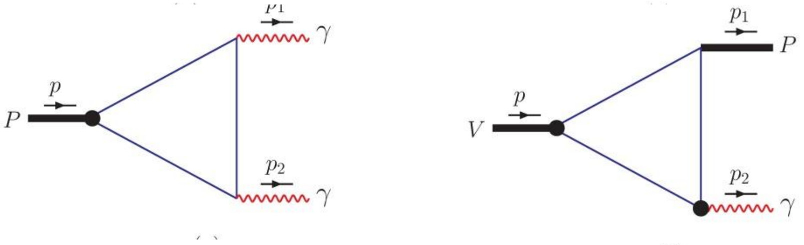
$$\tilde{S}_{m1}(\hat{p}) = \frac{m_1 + \hat{p}}{m_1^2 - p^2} = (m_1 + \hat{p}) \cdot \int_0^\infty ds_1 \exp\left[-s\left(m_1^2 - p^2 \right) \right]$$

- Matrix elements are combinations of propagators and vertices:

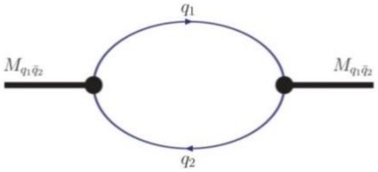
- **Leptonic decay constants**



- **Electromagnetic decay widths**



- **Mass (polarization) function**



- Loop integrals over ***k*** (and external momenta ***p***, too) are taken in Euclidean space:

$$k^0 \to ik_4; \quad p^0 \to ip_4; \quad k^2 \to -k_E^2 \le 0; \quad p^2 \to -p_E^2 \le 0$$

- **Loop integrals are absolutely convergent** (*Gaussian exponentials*).

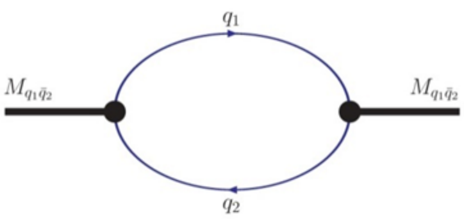
- Loop momenta ***k*** (in the numerator) may be expressed by exponentials:

$$k_\mu \cdot \exp(2kp) = \frac{1}{2}\frac{\partial}{\partial p_\mu}\exp(2kp)$$

Meson Ground-State Spectrum

- **Mass function (operator) for Pseudoscalar and Vector mesons:**

$$\Gamma_P = \gamma^5; \quad \Gamma_V = \gamma^\mu$$



$$\tilde{\Pi}_{PP}(p^2) = \mathrm{N}_c \int \frac{d^4k}{(2\pi)^4 i} \tilde{\Phi}_P^2(-k^2)\,\mathrm{tr}\left\{\gamma^5 \tilde{S}_{m1}(k_1 + p\omega_1)\gamma^5 \tilde{S}_{m2}(k_2 - p\omega_2)\right\}$$

$$\tilde{\Pi}_{VV}^{\mu\nu}(p^2) = \mathrm{N}_c \int \frac{d^4k}{(2\pi)^4 i} \tilde{\Phi}_V^2(-k^2)\,\mathrm{tr}\left\{\gamma^\mu \tilde{S}_{m1}(k_1 + p\omega_1)\gamma^\nu \tilde{S}_{m2}(k_2 - p\omega_2)\right\}$$

- **Meson mass equation**:

$$1 - G \cdot \tilde{\Pi}(M^2) = 0$$

$$\tilde{\Pi}_H(p^2) = \frac{3}{4\pi^2} \int_0^\infty \frac{dt \cdot t}{a_H^2} \int_0^1 ds \exp\left(-t \cdot z_0 + z_H\right) \cdot \left[\frac{n_H}{a_H} + m_1 m_2 + p^2\left(\omega_1 - \frac{b}{a_H}\right)\left(\omega_2 + \frac{b}{a_H}\right)\right]$$

$$a_H \equiv t + 2/\Lambda_H^2 \; ; \; z_0 \equiv s\,m_1^2 + (1-s)\,m_2^2 - s\,(1-s)\,p^2; \; z_H \equiv \frac{2t}{2 + t \cdot \Lambda_H^2}(s - \omega_2)^2\,p^2; \; n_P = 2; \; n_V = 1; \; b \equiv t\left(s - \omega_2\right).$$

- **Branching point**:

$$if: \quad p^2 = (m_1 + m_2)^2 \qquad then: \quad z_0 = p^2\left[s\,\omega_1^2 + (1-s)\,\omega_2^2 - s\,(1-s)\right] \xrightarrow[s=\omega_2]{} 0$$

$$\text{Integral} \int_0^\infty \frac{dt \cdot t}{a_H^2} \ldots = \text{diverges and a threshold singularity appears!}$$

Removing Singularity by Infrared Cut-off

Cut off the upper bound of t-integral (**infrared cut-off in terms of k-integral**)

for $\lambda > 0:$ no threshold singularity: $\int_0^{1/\lambda^2} \frac{dt\, t}{a_H^2} \ldots =$ converges!

T. Branz et al., Phys. Rev. **D81**, 034010 (2010).

$$\tilde{\Pi}_H(p^2) = \frac{3}{4\pi^2} \int_0^{1/\lambda^2} \frac{dt \cdot t}{a_H^2} \int_0^1 ds \exp\left(-t \cdot z_0 + z_H\right) \cdot \left[\frac{n_H}{a_H} + m_1 m_2 + p^2 \left(\omega_1 - \frac{b}{a_H}\right)\left(\omega_2 + \frac{b}{a_H}\right)\right]$$

A meson in the interaction Lagrangian is characterized by parameters

- the coupling constant **g_H**
- the size parameter **Λ_H**
- two constituent quark masses **m_1** & **m_2**
- the infrared confinement parameter **λ** universal for all hadrons.

Hereby, the Yukawa couplings g_H for all mesons H are removed by $Z = 1 - g_H^2 \tilde{\Pi}'(M^2) = 0$

- **Model parameters**: constituent quark masses, hadron size parameters, a universal infrared cut-off (totally **4+N+1** parameters for **N** hadrons → **1+5/N** per hadron)

Numerical results for Decay constants and Widths

- **Fixing** model parameters by fitting the ***electromagnetic decay widths*** and ***leptonic decay constants.***

- Fixed parameters:

$$\lambda = 0.181\,\mathrm{GeV},$$
$$m_{ud} = 0.235\,\mathrm{GeV},$$
$$m_s = 0.442\,\mathrm{GeV},$$
$$m_c = 1.61\,\mathrm{GeV},$$
$$m_b = 5.07\,\mathrm{GeV}$$

M. A. Ivanov et al,
Phys. Rev. D **85**, 034004 (2012).

TABLE III: The fitted values of the size parameters Λ_H in GeV.

π	K	D	D_s	B	B_s	B_c	η_c	η_b	
0.87	1.02	1.71	1.81	1.90	1.94	2.50	2.06	2.95	
ρ	ω	ϕ	J/ψ	K^*	D^*	D_s^*	B^*	B_s^*	Υ
0.61	0.50	0.91	1.93	0.75	1.51	1.71	1.76	1.71	2.96

G.Ganbold, T.Gutsche, M.Ivanov, V.Lubovitsky
J.Phys. G 42, 075002 (2015).

- ***Electromagnetic decay widths:***

Process	Fit Values	Data [24]
$\pi^0 \to \gamma\gamma$	5.07×10^{-3}	$(7.7 \pm 0.4) \times 10^{-3}$
$\eta_c \to \gamma\gamma$	3.47	5.0 ± 0.4
$\rho^\pm \to \pi^\pm\gamma$	76.3	67 ± 7
$\omega \to \pi^0\gamma$	687	703 ± 25
$K^{*\pm} \to K^\pm\gamma$	57.7	50 ± 5
$K^{*0} \to K^0\gamma$	129	116 ± 10
$D^{*\pm} \to D^\pm\gamma$	0.59	1.5 ± 0.5
$J/\psi \to \eta_c\gamma$	1.90	1.58 ± 0.37

- ***Leptonic decay constants:***

	Fit Values	Data
f_π	128.4	130.4 ± 0.2
f_K	156.0	156.1 ± 0.8
f_D	206.7	206.7 ± 8.9
f_{D_s}	257.5	257.5 ± 6.1
f_B	189.7	192.8 ± 9.9
f_{B_s}	235.3	238.8 ± 9.5
f_{η_c}	386.6	438 ± 8
f_{B_c}	445.6	489 ± 5
f_{η_b}	609.1	801 ± 9

f_ρ	221.2	221 ± 1
f_ω	204.2	198 ± 2
f_ϕ	228.2	227 ± 2
$f_{J/\psi}$	415.0	415 ± 7
f_{K^*}	215.0	217 ± 7
f_{D^*}	223.0	245 ± 20
$f_{D_s^*}$	272.0	272 ± 26
f_{B^*}	196.0	196 ± 44
$f_{B_s^*}$	229.0	229 ± 46
f_Υ	661.3	715 ± 5

+ Agreement between our **fit values** and the **PDG data** is quite satisfactory.

+ The constituent quark masses and the values of $\boldsymbol{\Lambda_H}$ fall into the expected range.

+ The meson “size” ~1/ $\boldsymbol{\Lambda_H}$ shrinks as the mass grows.

Numerical results for Fermi coupling G

G.Ganbold, T.Gutsche, M.Ivanov, V.Lubovitsky
J.Phys. G 42, 075002 (2015).

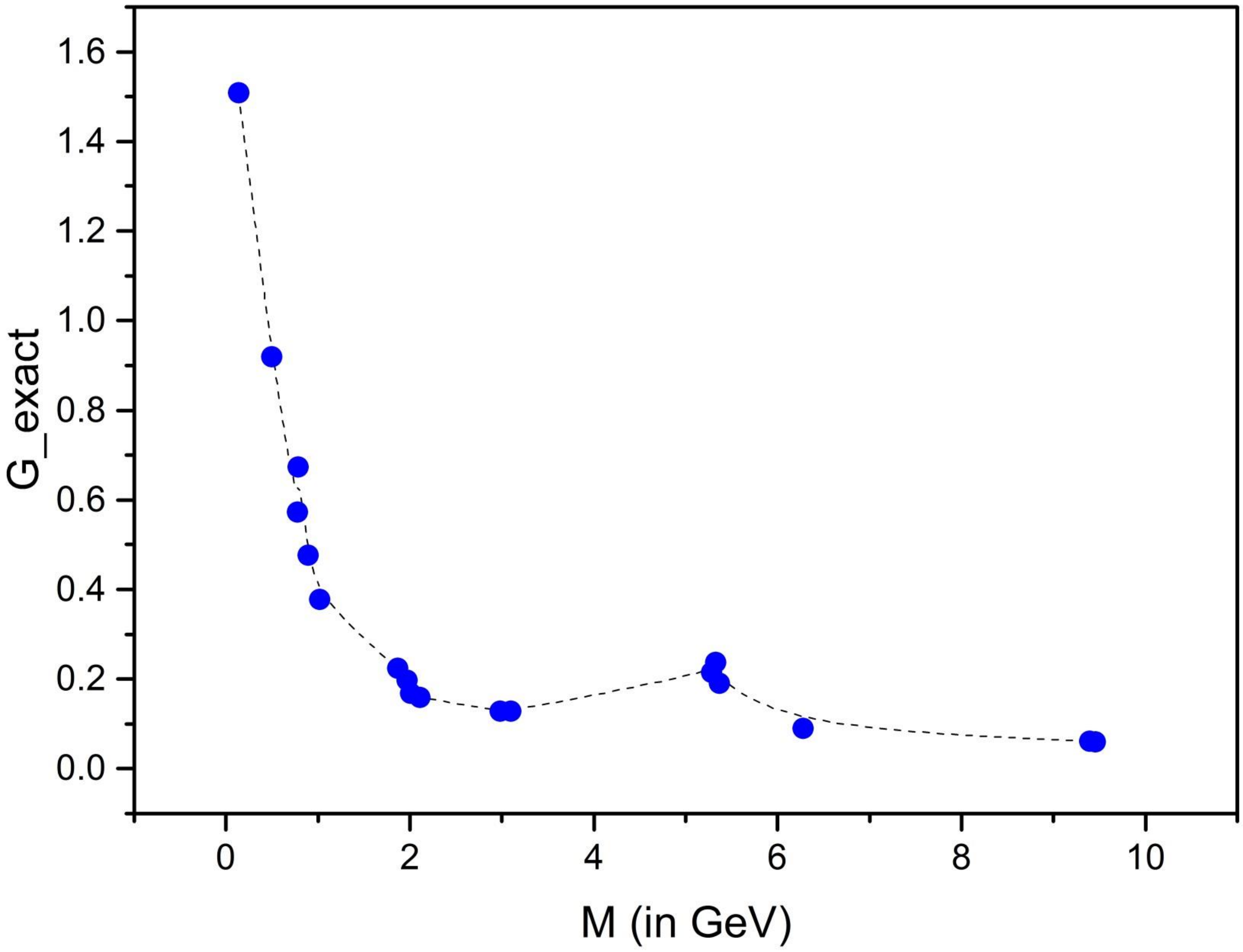
- Estimation of the Fermi coupling:

$$G = 1/\tilde{\Pi}_H(M_{\exp}^2)$$

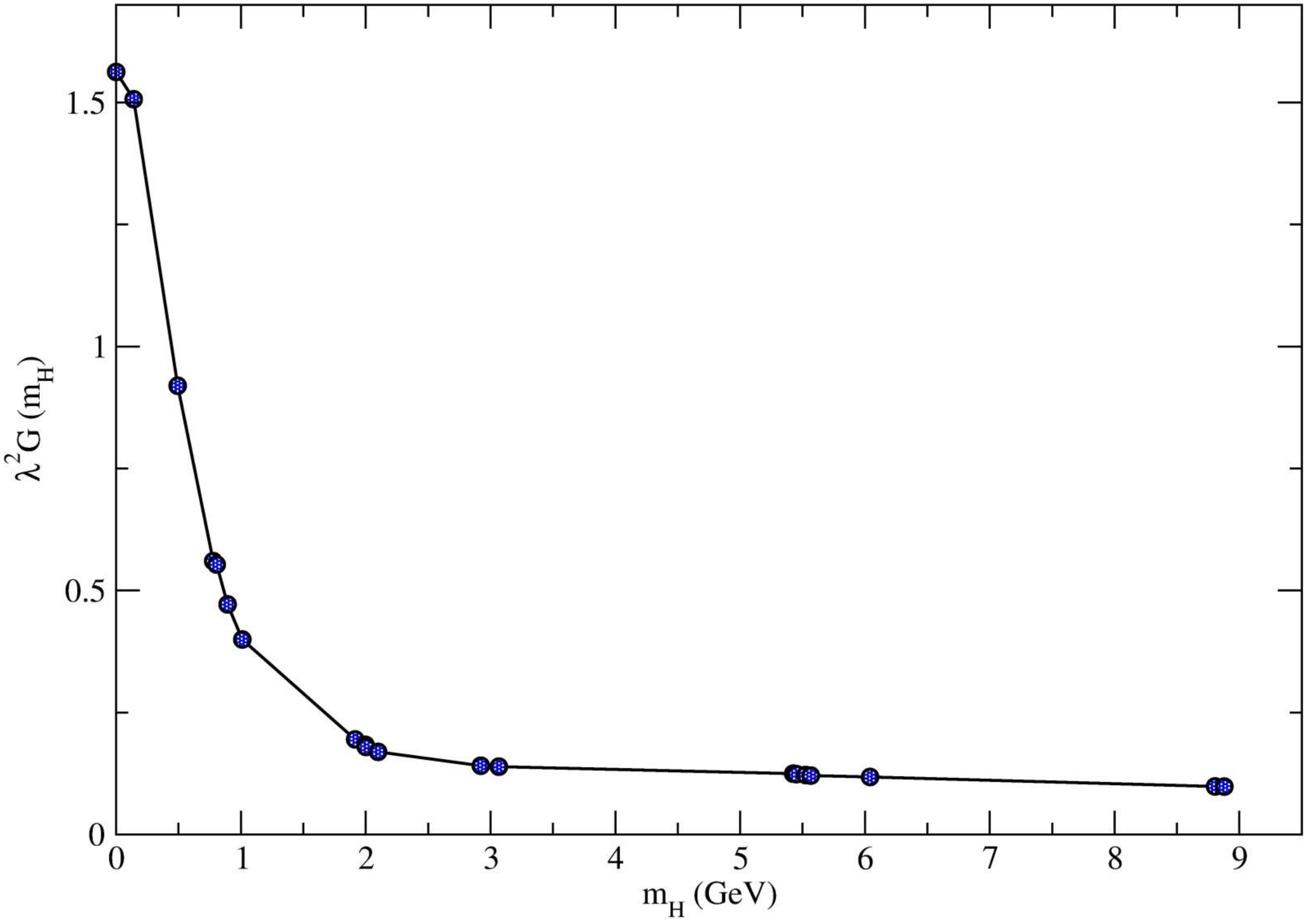
$$\tilde{\Pi}_H(M^2) \sim [\mathrm{GeV}^2] \quad \Rightarrow \quad \lambda^2 G(M) \sim [\mathrm{dimensionless}]$$

$J^{PC} = 0^{-+}$	PDG (MeV)	λ^2 G	$J^{PC} = 1^{--}$	PDF (MeV)	λ^2 G
π	139.57	1.508	**ρ**	775.26	0.576
K	493.68	0.919	**ω**	782.65	0.673
D	1869.62	0.224	**K***	891.66	0.476
D_s	1968.50	0.197	**Φ**	1019.45	0.377
H_c	2983.7	0.128	**D***	2010.29	0.168
B	5279.26	0.215	**D*s**	2112.3	0.158
B_s	5366.77	0.191	**J/ψ**	3096.92	0.129
B_c	6274.5	0.0906	**B***	5325.2	0.237
η_b	9398.0	0.0612	**B*s**	5415.8	0.231
			ϒ	9460.3	0.0601

Plot of $\lambda^2 G$ by fitting meson physical masses



$\lambda^2 G$ after smoothing



Numerical results for meson masses

G.Ganbold, T.Gutsche, M.Ivanov, V.Lubovitsky
J.Phys. G 42, 075002 (2015).

- Estimation of Meson Mass:

$$1 - G_{smooth} \cdot \tilde{\Pi}_H(M^2) = 0$$

$J^{PC} = 0^{-+}$ | $J^{PC} = 1^{--}$

	PDG (MeV)	λ^2 G (smooth)	M (MeV)		PDG (MeV)	λ^2 G (smooth)	M (MeV)
π	139.57	1.507	141	**ρ**	775.26	0.560	778
K	493.68	0.920	493	**ω**	782.65	0.554	806
D	1869.62	0.195	1915	**K***	891.66	0.472	893
D_s	1968.50	0.184	1998	**Φ**	1019.45	0.401	1011
η_c	2983.7	0.141	2922	**D***	2010.29	0.180	2001
B	5279.26	0.125	5425	**D*s**	2112.3	0.170	2099
B_s	5366.77	0.122	5524	**J/ψ**	3096.92	0.139	3067
B_c	6274.5	0.118	6041	**B***	5325.2	0.124	5450
η_b	9398.0	0.0986	8806	**B*s**	5415.8	0.121	5566
				ϒ	9460.3	0.0984	8880

Fermi coupling G: comparison with α_s

- It is interesting to compare **G** with the effective QCD coupling $\boldsymbol{\alpha_s}$ obtained in the relativistic models with specific forms of analytically confined propagators.

$$L = -\frac{1}{4}\left(\partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g f^{abc} A_\mu^b A_\nu^c\right)^2 + \sum_f \left(\bar{q}_f^a \left[\gamma_\alpha \partial_\alpha - m_f + ig\,\gamma_\alpha t^C A_\alpha^C\right]^{ab} q_f^b\right)$$

$$\tilde{S}(\hat{p}) = (i\hat{p} + m) \cdot \int_0^{1/\Lambda^2} dt \exp\left\{-t \cdot (p^2 + m^2)\right\};$$

$$D(x) = \int_{\Lambda^2/4}^{\infty} ds\, e^{-sx^2} = \frac{e^{-x^2\Lambda^2/4}}{4\pi^2 x^2} \quad .$$

G.Ganbold,
Phys. Rev. D 79, 034034 (2009).
Phys. Part. Nucl. **43**, 79, (2012)
Phys. Part. Nucl. **45**, 10, (2014)

- In that models the four-quark nonlocal interaction is induced by one-gluon exchange between bi-quark currents. Since the confined gluon propagator has the dimension of **~ 1/GeV^2** , the resulting coupling $\boldsymbol{\alpha_s}$ is dimensionless.

$$1 - \alpha_s \cdot \frac{8C_J}{3\pi^3} \int d^4k\, V_J(k) \cdot \Pi_J(p,k) \cdot V_J(-k) = 0; \qquad (p^2 = -M_J^2)$$

$$V_J(k) = \int dx \sqrt{D(x)}\, U_J(x)\, e^{ikx}$$

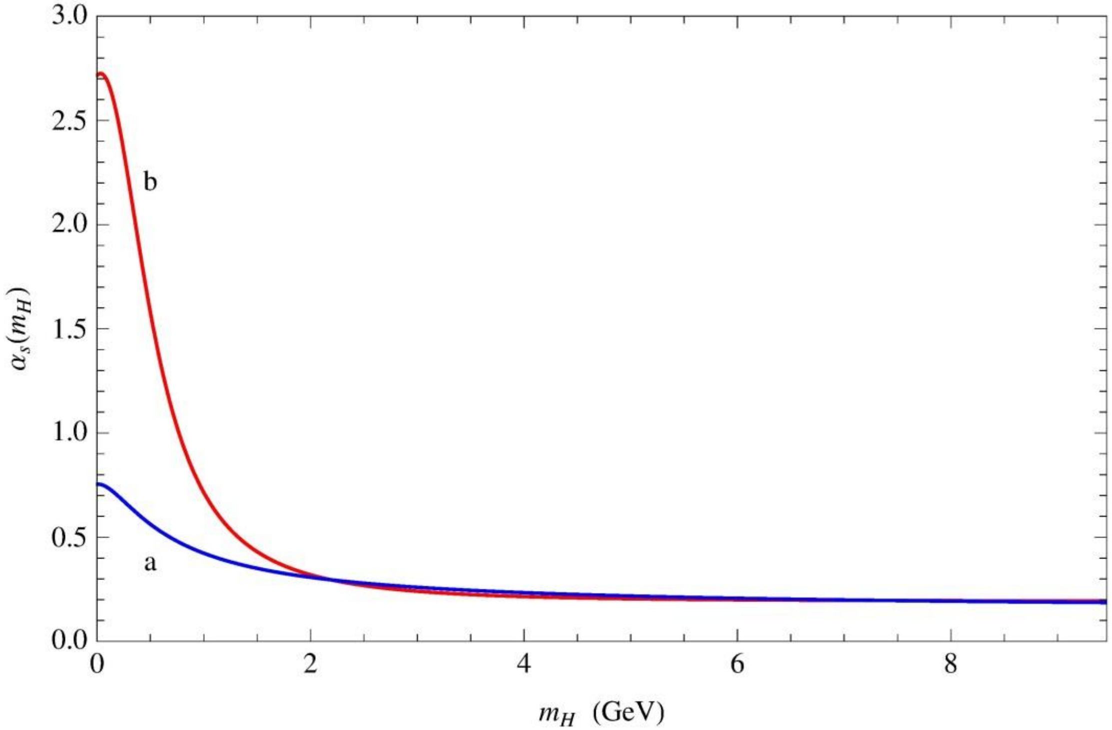
λ^2*G (red curve, rescaled by **~1.74**)
in comparison with $\boldsymbol{\alpha_s}$ (blue curve)

G.Ganbold, T.Gutsche, M.Ivanov, V.Lubovitsky J.Phys. G 42, 075002 (2015).

$$\lambda = 0.181\,\text{GeV},\quad m_{ud} = 0.235\,\text{GeV},\quad m_s = 0.442\,\text{GeV},\quad m_c = 1.61\,\text{GeV},\quad m_b = 5.07\,\text{GeV}$$

G.Ganbold, Phys. Rev. D 81, 094008 (2010).

$$\Lambda = 345\ \text{MeV},\quad m_{ud} = 193\ \text{MeV},\quad m_s = 293\ \text{MeV},\quad m_c = 1848\ \text{MeV},\quad m_b = 4693\ \text{MeV}$$



- Despite the different model origins and input parameter values, the behaviors of two curves are very similar each other in the region above ~ 2 GeV.

- Their values at origin are mostly determined by the confinement mechanisms realized in different ways. This could explain their different behaviors in the region below 2 GeV.

Summary and Outlook

- ♣ A brief sketch of an approach to the bound state problem in QFT based on the ***compositeness condition*** is represented.

- ♣ We have explicitly demonstrated that the four-fermion theory with the Fermi coupling ***G*** is equivalent to the Yukawa-type theory **if,**
  - the wave function renormalization constant in the Yukawa theory is equal to zero,
  - ***G*** is inversely proportional to the meson mass function calculated at physical mass.

- ♣ The mass spectrum and decay constants of conventional mesons has been estimated. We calculated ***G*** as a function of physical mass.

- ♣ A *smoothness criterion* – by varying the meson masses in such a way to obtain the smooth behavior of the Fermi coupling ***G***.
  The meson mass spectrum obtained in this manner is found to be in good agreement with the recent experimental data (*from* ***π(140)*** *up to* ***ϒ(9460)***.

- ♣ We have compared the behavior of ***G*** with the strong QCD coupling α_s calculated in a QCD-inspired models.

- ♣ The approach may be extended to other sections of hadron physics (Ex: charmonium radial excitations, X-Y-Z mesons, baryons, glueball-like states, ….).