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“Modern problems of nuclear and elementary particle physics”
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EXPLORATIONS OF THE FEW-BODY CONTINUUM

Remarkable phenomena are observed in nuclei *near driplines:*
one- and two-neutron halos, two-proton radioactivity etc.

Characteristic features of halo systems → ***extreme few-body clusterization*** **extraordinary large sizes**

A general prerequisite for halo formation is a low relative orbital angular momentum (*l* = 0,1) *between the cluster constituents*

Studies of *correlations in relative motions* between the three fragments open a way for extended exploration of halo structure, its formation and how it dissolves

This demands a clear understanding of both *nuclear structure* and the *reaction mechanism* inducing the breakup

the challenge of *continuum spectroscopy* → ***the extraction* of *main excitation modes* and their *quantum numbers* in a finite region of excitation energies**

Three – body breakup reaction induced by the *halo* projectile a

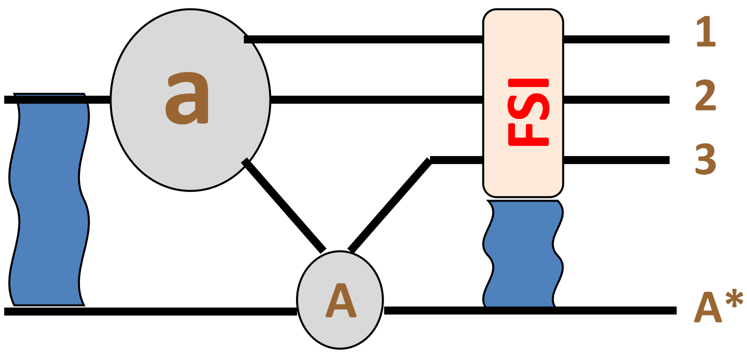
$$a + A \Rightarrow 1 + 2 + 3 + A^*$$

$$\sigma = \frac{(2\pi)^4}{\hbar v_i} \int dk_1\ dk_2\ dk_3\ dk_A\ \delta(E_f - E_i)\ \delta(P_f - P_i)\ |\ T_{fi}\ |^2$$

$$dk_1\ dk_2\ dk_3 = dk_x\ dk_y\ dk_{cm}$$

The exact reaction amplitude T_{fi} *(prior representation)*

$$T_{fi} = \langle \Psi_f^{(-)}(p_f, k_x, k_y)\ |\ \sum_{p,t} V_{p,t} - U_{aA}\ |\ \Psi_a\ \Phi_A\ \chi_i^{(+)}(p_i)\ \rangle$$



Kinematically complete experiments

- sensitivity to **3-body** correlations (*halo*)
- selection of halo excitation energy
- variety of observables
- **elastic & inelastic breakup**

The four-body *distorted wave approach*

low-energy halo excitations ⇒ ***small*** k_x & k_y ***; large*** p_i & p_f

(**no spectators**, three-body continuum, full scale **FSI**)

$$T_{fi}^{DW} = \langle \chi_i^{(-)}(p_f)\ \Phi_A\ \Psi^{(-)}(k_x, k_y)\ |\ \sum_{p,t} V_{p,t}\ |\ \Psi_a\ \Phi_A\ \chi_i^{(+)}(p_i)\ \rangle$$

In collisions we explore the *transition* properties of nuclei : from *ground* state to *continuum* states

$$\langle \Psi^{(-)}(\mathrm{k}_x, \mathrm{k}_y) \parallel \sum_p \frac{\delta(r - r_p)}{r r_p} [Y_L(\hat{r}_p) \times \sigma_p^S]_J \parallel \Psi_a \rangle$$

Reaction mechanism serves as a *filter* for nuclear excitations

↓

Variations of *nuclear targets* and *collision energies* allow to change the balance between *coulomb* and *nuclear* forces that break the nucleus

↓

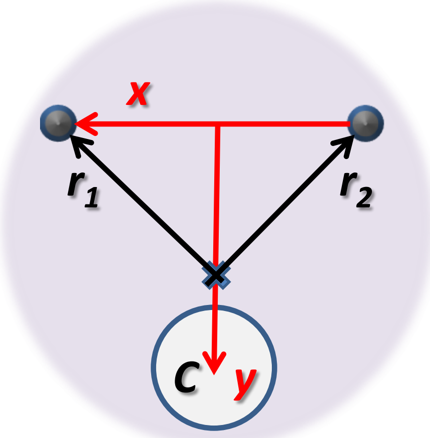
change the *weights* of different multipole excitations at the fixed excitation energy

↓

demands the *consistent* treatment both *nuclear structure* and *reaction mechanism*

Total hamiltonian of the three-body cluster models

$$H_A = H_{A_C} + T_{x,y} + V(r_1, r_2) + \sum_{i=1}^{A_C} V(r_1, r_i) + \sum_{i=1}^{A_C} V(r_2, r_i)$$



$$\rho^2 = \mu_x \bar{x}^2 + \mu_y \bar{y}^2$$
$$\alpha_\rho = \arctan\left(\frac{\sqrt{\mu_x}\, x}{\sqrt{\mu_y}\, y}\right)$$
$$\Omega_5^\rho = \{\alpha_\rho, \hat{x}, \hat{y}\}$$

$$\frac{\kappa^2}{2m} = \frac{\bar{k}_x^2}{2\mu_x} + \frac{\bar{k}_y^2}{2\mu_y}$$
$$\alpha_\kappa = \arctan\left(\frac{\sqrt{\mu_y}\, k_x}{\sqrt{\mu_x}\, k_y}\right)$$
$$\Omega_5^\kappa = \{\alpha_\kappa, \hat{k}_x, \hat{k}_y\}$$

$$\{\bar{x},\, \bar{y}\} \Rightarrow \{\rho\,,\, \Omega_5^\rho\}$$

$$\{\bar{k}_x,\, \bar{k}_y\} \Rightarrow \{\kappa\,,\, \Omega_5^\kappa\}$$

Calculations of the *bound* states and *continuum* wave functions

Borromean nature of halo nuclei
(no bound states between pairs of clusters) ⇒ **one type of the wave function *asymptotic behavior***

The bound state wave function ($\gamma = \{K, l_x, l_y, L, s, I, J\}$

$$\Psi_{JM}(\mathrm{r}_1, \cdots, \mathrm{r}_A) = \frac{1}{\rho^{5/2}} \sum_{\gamma} \chi_{\gamma}^{J}(\rho) \left[\Upsilon_{KL}^{l_x l_y}(\Omega_5^{\rho}) \otimes [\chi_s \otimes \phi_{nI}]_j\right]_{JM}$$

The continuum wave function at the positive energy

$$\Psi_{\nu,nIM_I}{}^{(\pm)}(\kappa; \mathrm{r}_1, \cdots, \mathrm{r}_A) = \sum_{\gamma} \imath^K (s\nu IM_I \mid jm_j)(LM_L\, jm_j \mid JM_J)\, \Upsilon_{KL}^{l_x l_y *}(\Omega_5^{\kappa}) \times$$
$$\times \frac{1}{\rho^{5/2}} \sum_{\gamma'} \chi_{\gamma',\gamma}^{J}(\kappa, \rho) \left[\Upsilon_{K'L'}^{l'_x l'_y}(\Omega_5^{\rho}) \otimes [\chi_{s'} \otimes \phi_{n'I'}]_{j'}\right]_{JM}$$

Set of coupled Schrodinger equations
for radial wave functions

$$\left(-\frac{\hbar^2}{2\mu}\left[\frac{d^2}{dr^2} - \frac{l(l+1)}{r^2}\right] + \epsilon_{\gamma} - E\right) \chi_{\gamma,\gamma'}^{J}(r)$$
$$= -\sum_{\gamma''} V_{\gamma,\gamma''}^{J}(r)\, \chi_{\gamma'',\gamma'}^{J}(r)$$

$$\Upsilon_{KLM}^{l_x l_y}(\Omega_5^{\rho}) = \psi_K^{l_x l_y}(\alpha_{\rho}) \left[Y_{l_x}(\hat{\mathrm{x}}) \otimes Y_{l_y}(\hat{\mathrm{y}})\right]_{LM}$$

$$\Upsilon_{\mathrm{KLM}}^{\mathrm{l_x l_y}}(\Omega_5^{\kappa}) = \psi_K^{l_x l_y}(\alpha_{\kappa}) \left[Y_{l_x}(\hat{\mathrm{k}}_{\mathrm{x}}) \otimes Y_{l_y}(\hat{\mathrm{k}}_{\mathrm{y}})\right]_{LM}$$

Consistent description of the whole set of observables simultaneously in different coordinate systems starting from the most inclusive ones to less inclusive

Hierarchy of observables

$$\frac{d\sigma}{dE_\kappa}$$

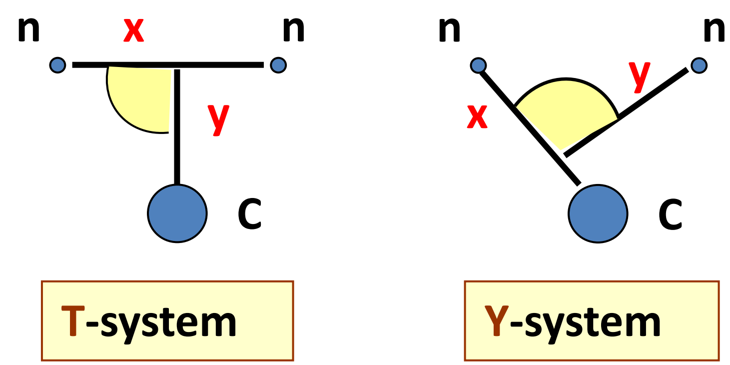
$$\frac{d^2\sigma}{d\varepsilon_y dE_\kappa}\ ;\ \frac{d^2\sigma}{d\varepsilon_x dE_\kappa}\ ;\ \frac{d^2\sigma}{d\cos\theta_{xy} dE_\kappa}$$

$$\frac{d^3\sigma}{d\cos\theta_{xy}d\varepsilon_y dE_\kappa}\ ;\ \cdots$$

$$\frac{d^4\sigma}{d\bar{k}_y dE_\kappa}\ ;\ \frac{d^4\sigma}{d\bar{k}_x dE_\kappa}\ ;\ \cdots$$

$$\cdots$$

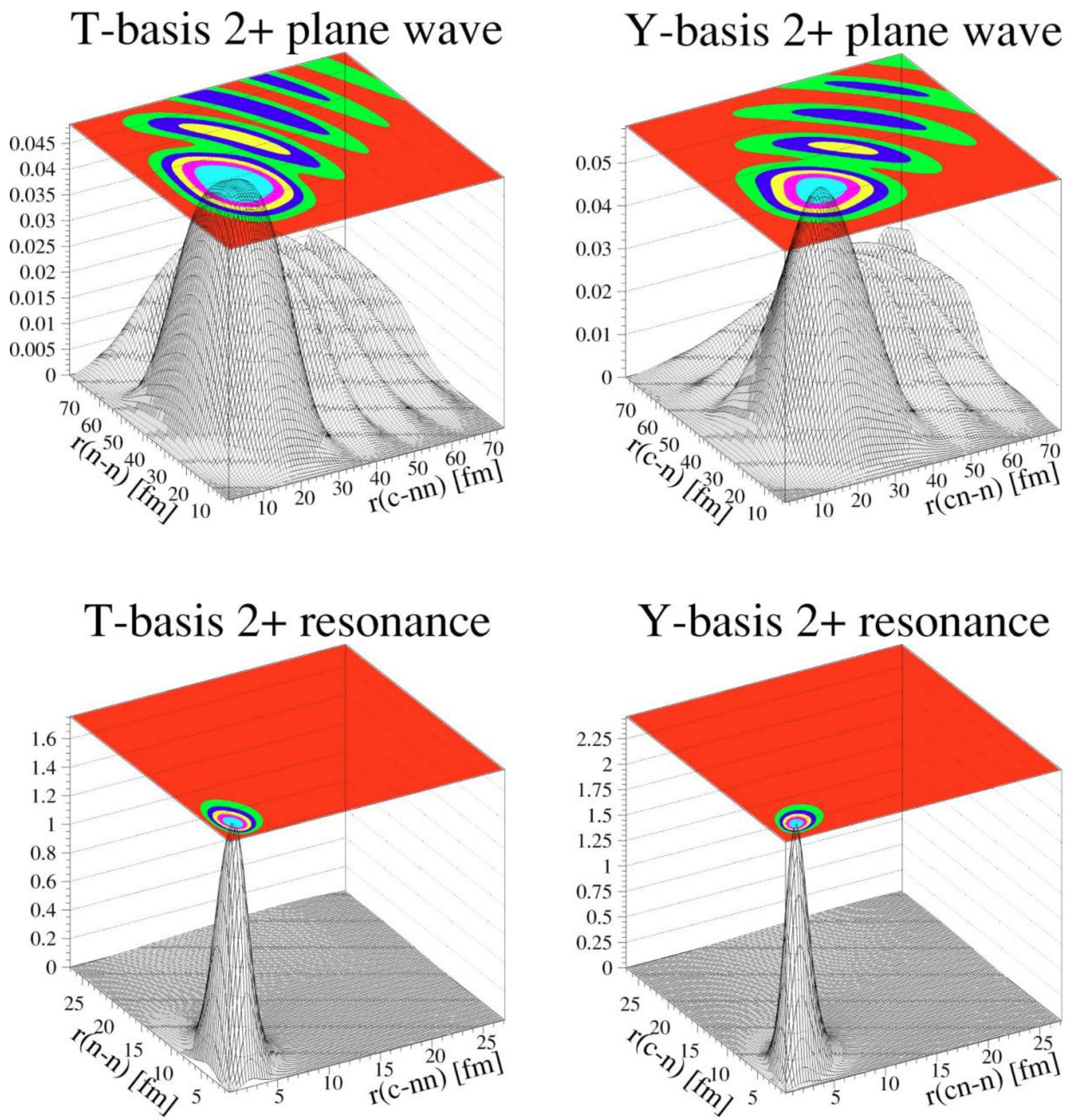
$$\frac{d^8\sigma}{d\hat{p}_f d\hat{k}_x d\hat{k}_y d\varepsilon_y dE_\kappa}$$



$$A_{\delta K}^{l_x l_y L}(E_\kappa, Y) = \sum_{l'_x l'_y} \langle l'_x l'_y | l_x l_y \rangle_{KL} \times A_{\delta K}^{l'_x l'_y L}(E_\kappa, T)$$

The three-body spatial correlations in ^{6}He

$$P_J(\kappa, x, y) = \sum_M \int |\Psi_{JM}(\bar{x}, \bar{y}; \bar{k}_x \bar{k}_y)|^2 \, d\hat{x} \, d\hat{y} \, d\Omega_5^\kappa$$

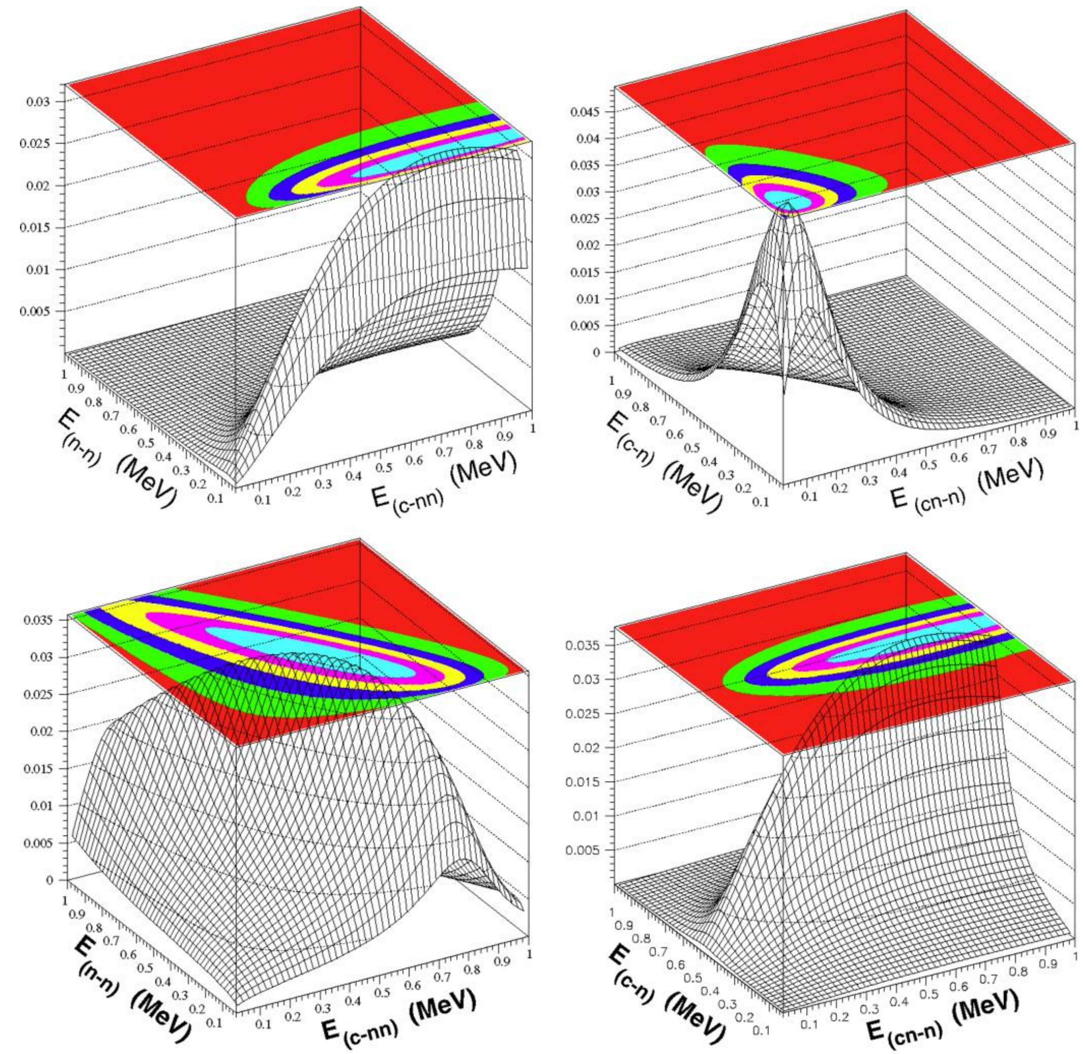


The three-body energy correlations in ^{6}He continua ($J^\pi = 1^-$)

T-system | **Y-system**

virtual ($s_{1/2}$) state in *nn*-subsystem

resonance ($p_{3/2}$) state in *cn* (^{5}*He*)-subsystem



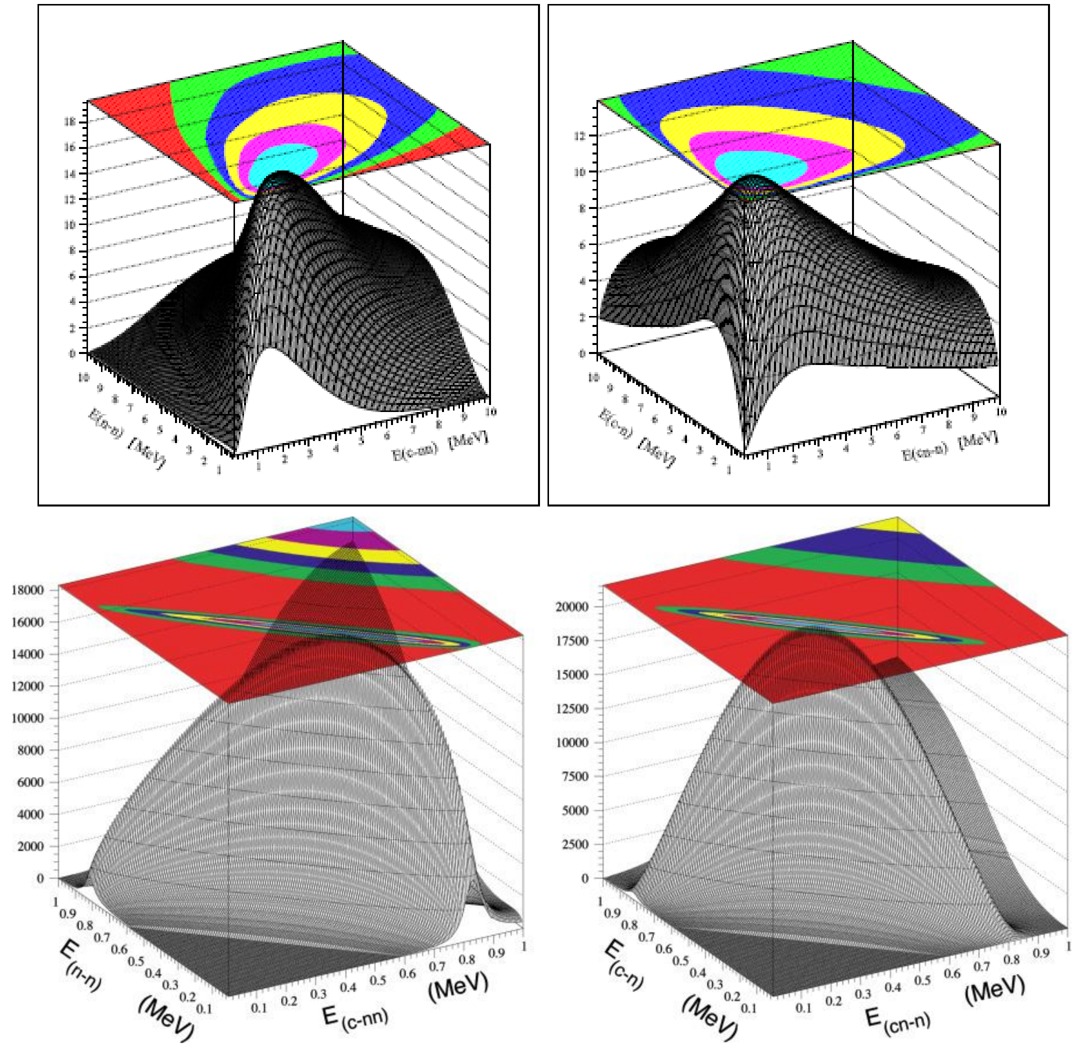
$$P_J(\varepsilon_x, \varepsilon_y) = \sqrt{\varepsilon_x\, \varepsilon_y}\ \sum_{K\gamma\gamma'} \Big| \sum_{K'} \left(\delta_{K\gamma,K'\gamma'} - S^J_{K\gamma,K'\gamma'} \right) \psi^{l'_x l'_y}_{K'}(\alpha_\kappa)\ \Big|^2$$

The three-body energy correlations in ^{6}He continua ($J^\pi = 2^+$)

T-system **Y-system**

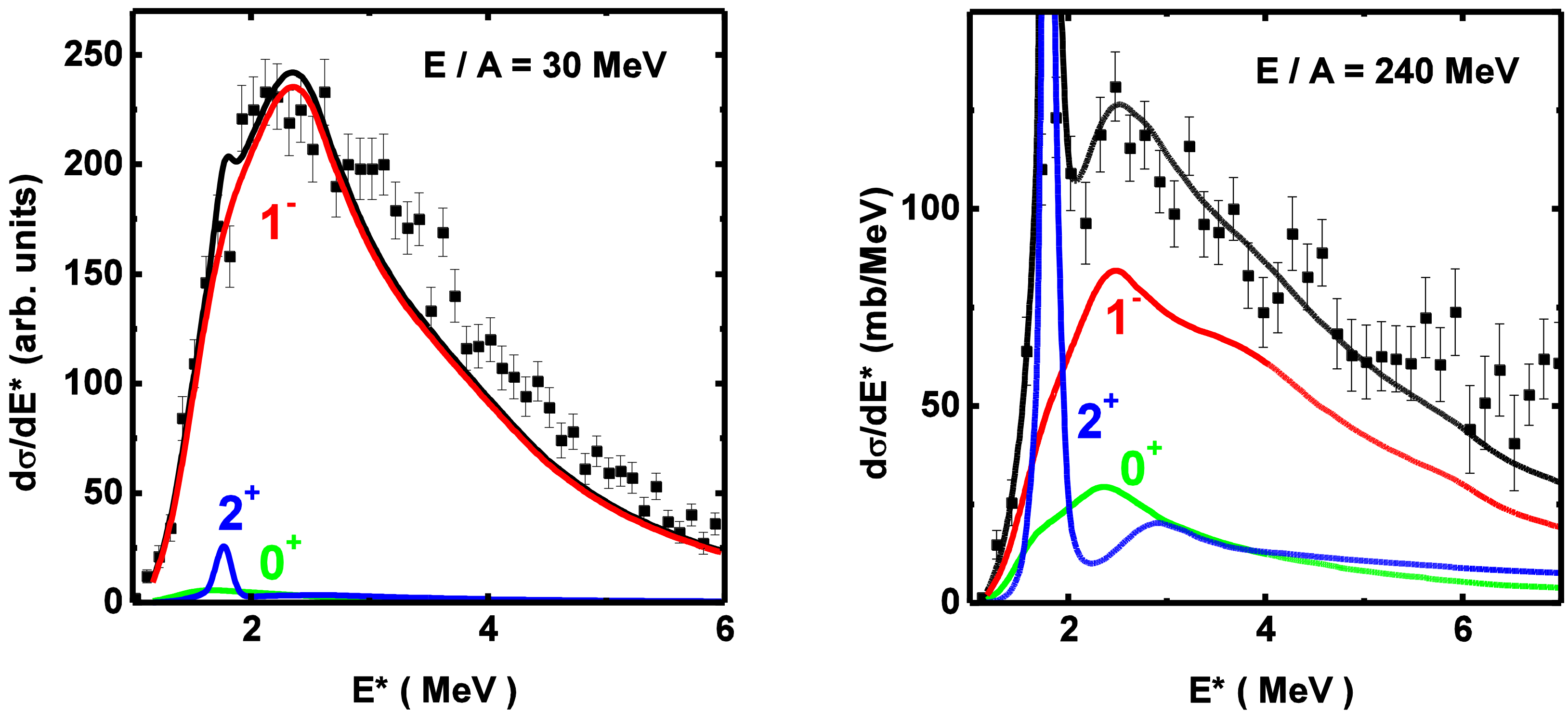
no FSI (plane wave)

full FSI



$$P_J(\varepsilon_x, \varepsilon_y) = \sqrt{\varepsilon_x\, \varepsilon_y} \sum_{K\gamma\gamma'} \left| \sum_{K'} \left(\delta_{K\gamma,K'\gamma'} - S^J_{K\gamma,K'\gamma'} \right) \psi^{l'_x l'_y}_{K'}(\alpha_\kappa) \right|^2$$

$^{6}He + ^{208}Pb$ --- > $^{4}He + n + n + X$

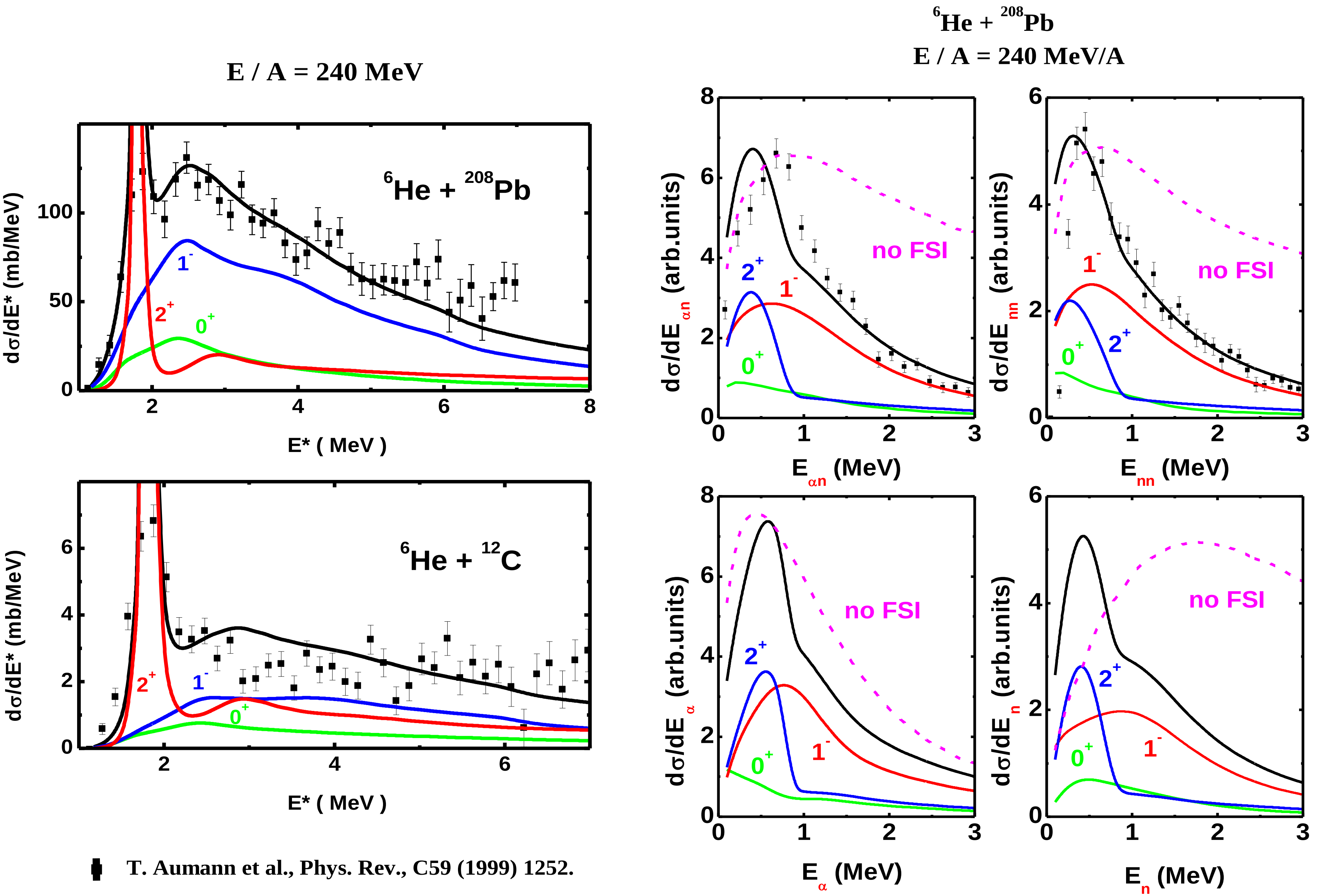


Experimental data :

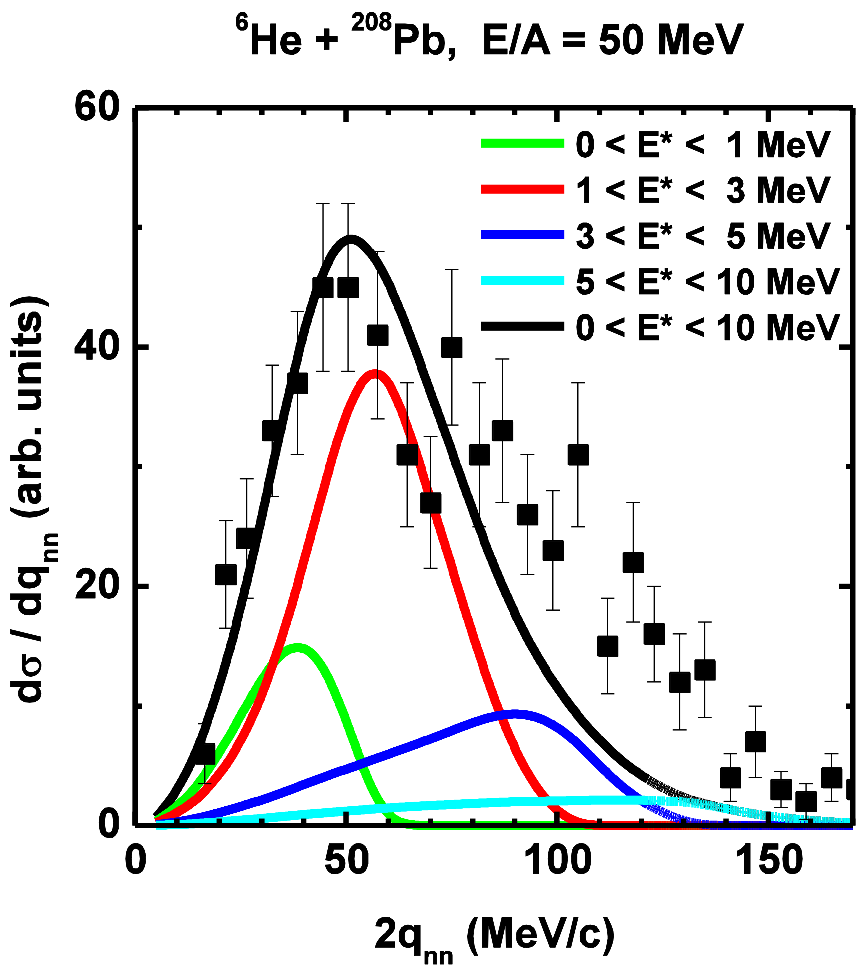
E / A = 240 MeV , T. Aumann et al, Phys. Rev. C59 (1999) 1252

E / A = 30 MeV , N.A. Orr, arXiv: 0803.0886 [nucl-ex]

Halo scattering on nuclei

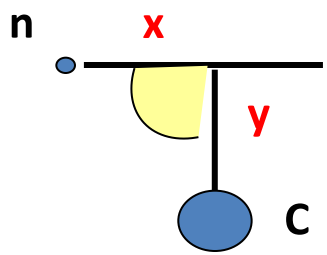


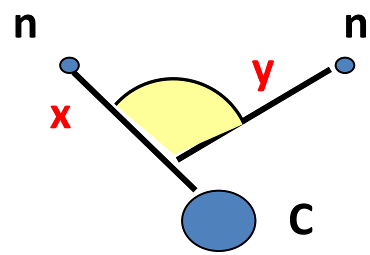
■ T. Aumann et al., Phys. Rev., C59 (1999) 1252.



Experimental data :

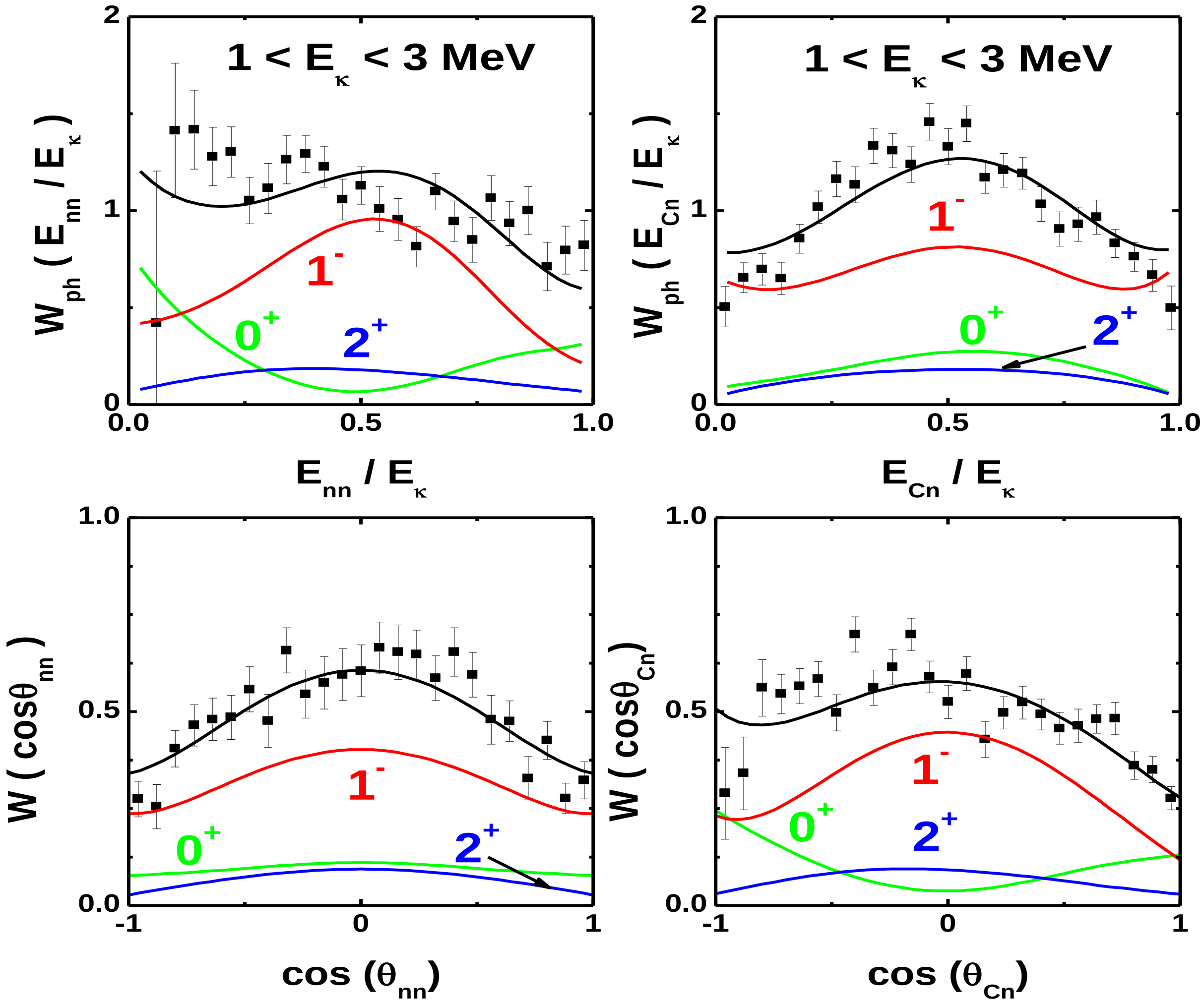
E / A = 50 MeV , F.M. Marques et al, Phys. Lett. B476 (2000) 219





$^{6}He + {}^{208}Pb$

E/A = 240 MeV

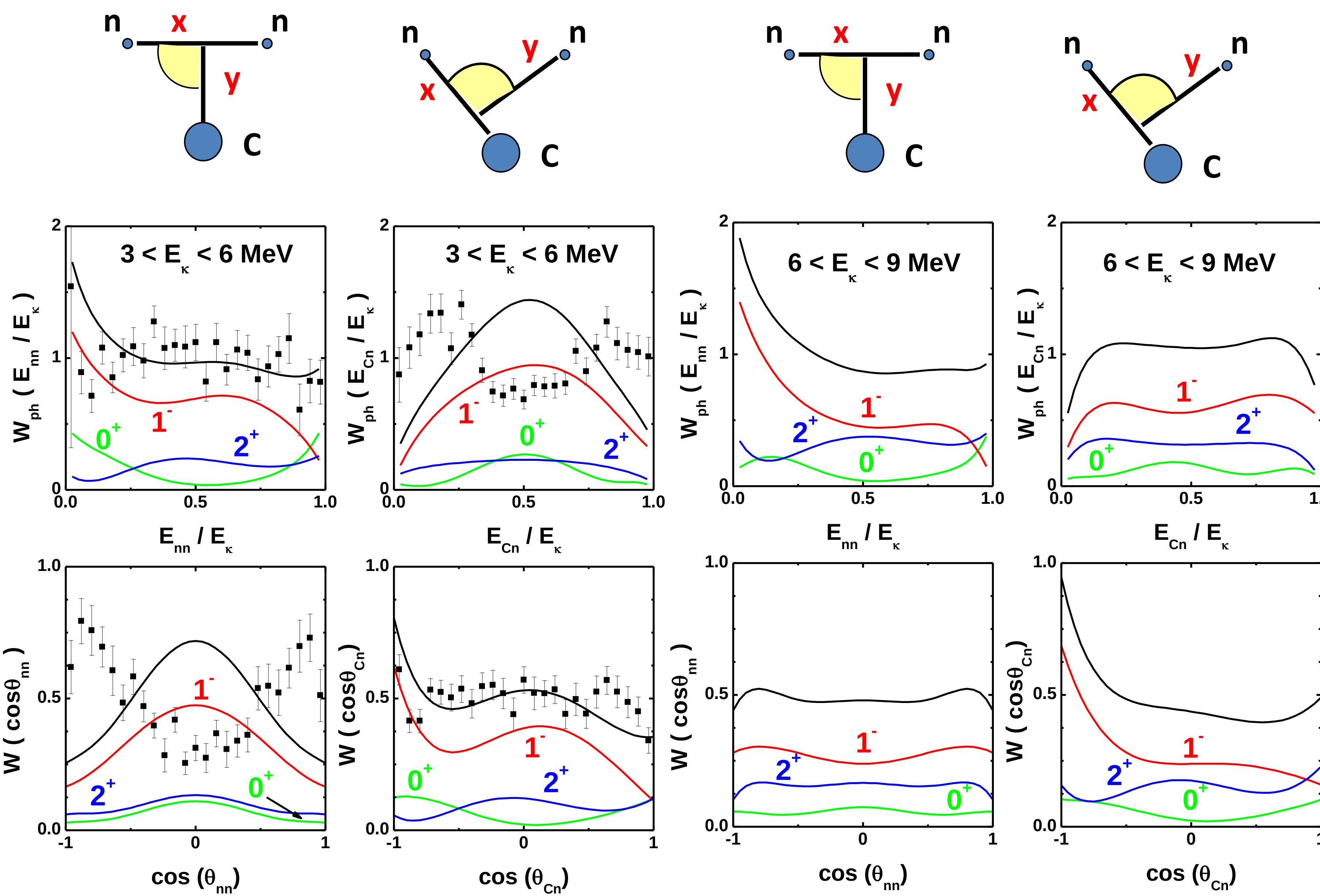


$$W(\epsilon) = \frac{1}{\sqrt{\epsilon(1-\epsilon)}} \frac{d^2\sigma}{d\epsilon dE_\kappa}$$

For single mode

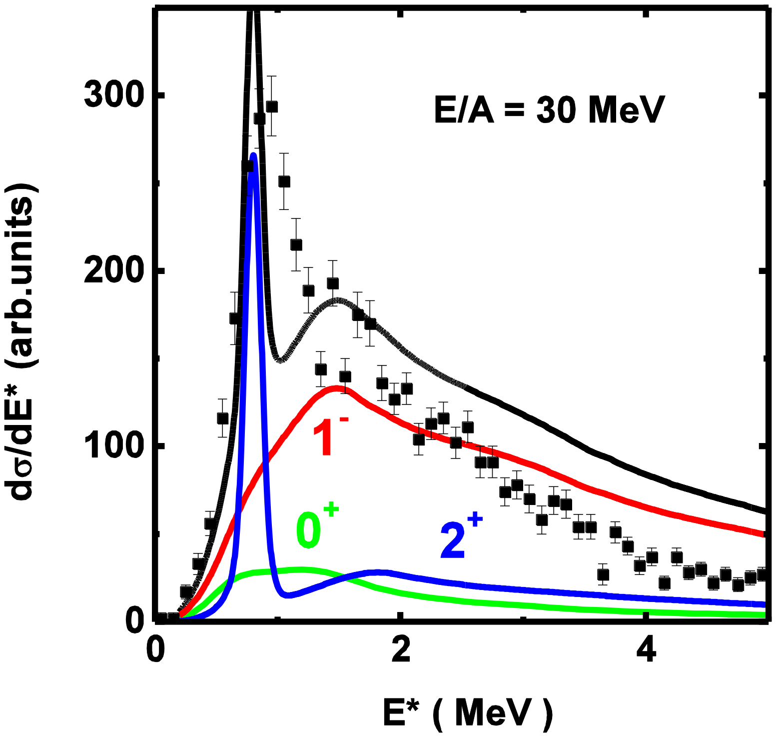
$$W(\epsilon) \sim \epsilon^{l_x} (1-\epsilon)^{l_y}$$

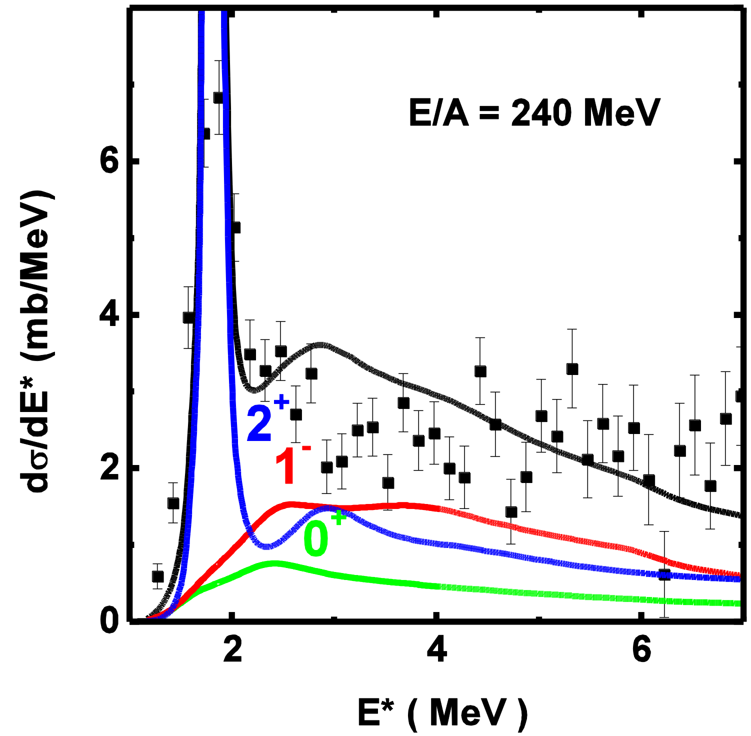
■ *L.V. Chulkov et al.*, Nucl. Phys. A759, 23 (2005)



n x n y C
n x y n C
3 < E_κ < 6 MeV
6 < E_κ < 9 MeV
W_ph (E_nn / E_κ)
W_ph (E_Cn / E_κ)
E_nn / E_κ
E_Cn / E_κ
W (cosθ_nn)
W (cosθ_Cn)
cos (θ_nn)
cos (θ_Cn)
1-
0+
2+

$^{6}He + ^{12}C$ --- > $^{4}He + n + n + X$



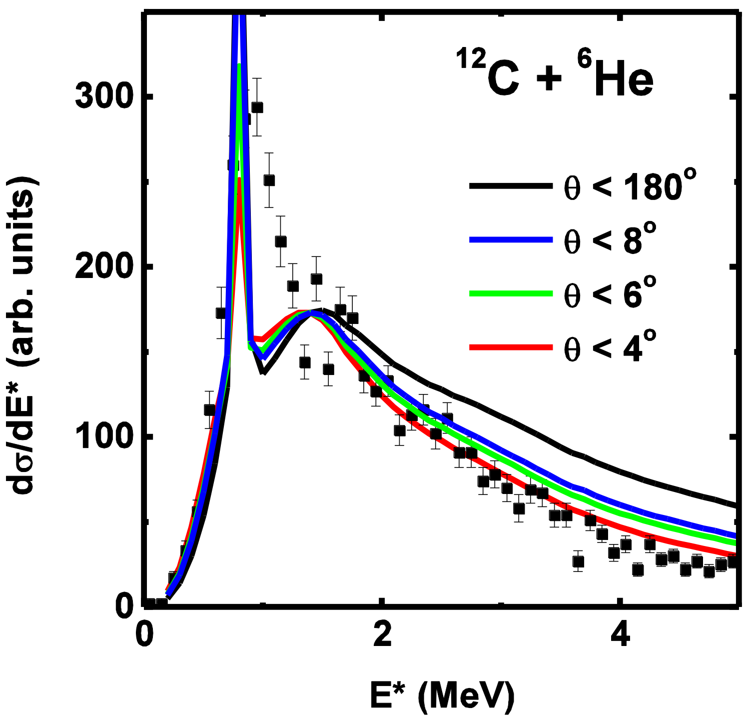


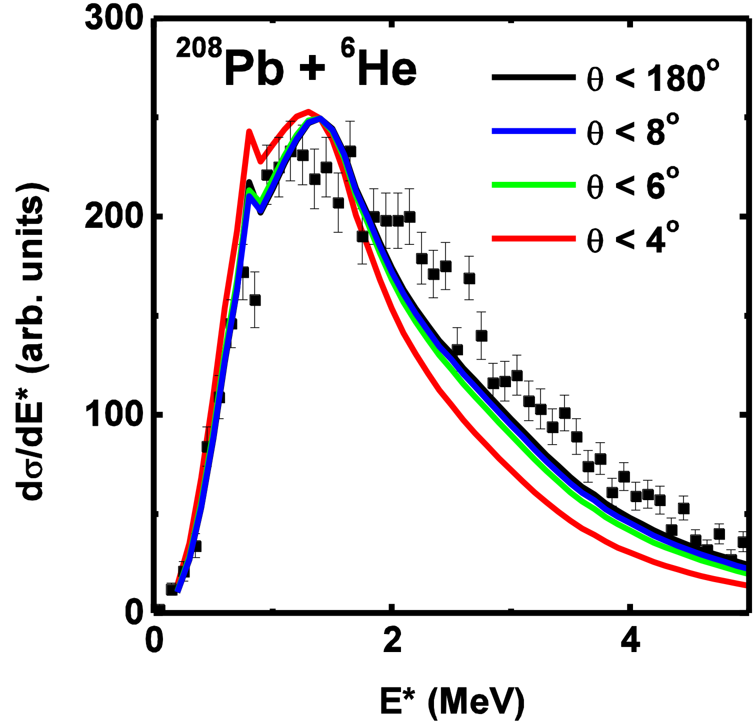
Experimental data :

E / A = 240 MeV , T. Aumann et al, Phys. Rev. C59 (1999) 1252

E / A = 30 MeV , N.A. Orr, arXiv: 0803.0886 [nucl-ex]

$^{6}He + A \; \text{---}> \; ^{4}He + n + n + X$





$$\frac{d\sigma}{dE^*} = 2\pi \int_0^{\theta_{max}} d\theta \sin\theta \frac{d\sigma}{d\Omega dE^*}$$

Experimental data :

E / A = 30 MeV , N.A. Orr, arXiv: 0803.0886 [nucl-ex]

CONCLUSIONS

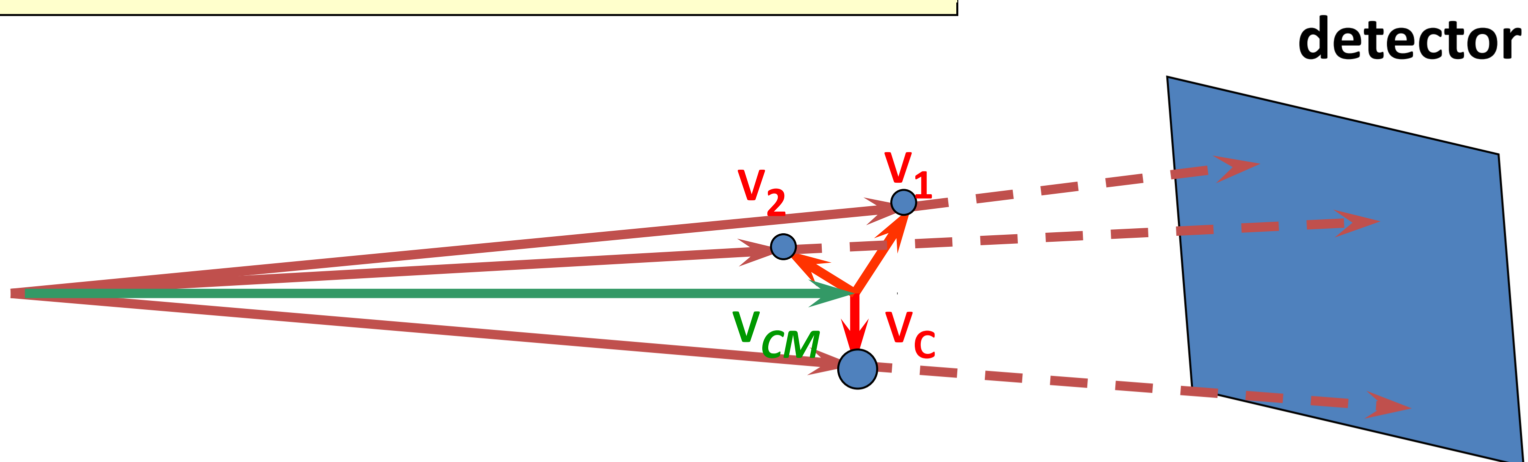
- The task of continuum spectroscopy is to define the dominant excitation modes (multipolarities) and their quantum numbers (elementary modes).

- The way to achieve this task is to explore the world of various correlations in fragment motions.

- This demands kinematically complete experiments and theoretical understanding of underlying reaction dynamics and nuclear structure.

Theoretical calculations ⟷ **?** ⟷ **Experimental data**

assume **4π** -measurements of fragments



$$E_{\kappa} = \frac{m_1 V_1^2}{2} + \frac{m_2 V_2^2}{2} + \frac{m_C V_C^2}{2} \qquad E_{\kappa} \ll E_{CM}$$

Inverse kinematics: ⟶ ***forward*** **focusing**

Fragment *correlations* are accessible via different cross sections

$$\epsilon = \varepsilon_x / E_\kappa \,, \quad E_\kappa = \varepsilon_x + \varepsilon_y \,, \quad 0 \le \epsilon \le 1$$

$$\frac{d\sigma}{dE_\kappa} = \sum_{\delta l_x l_y L K} |A_{\delta K}^{l_x l_y L}(E_\kappa)|^2$$

$$\frac{d^2\sigma}{d\epsilon\, dE_\kappa} = \sqrt{\epsilon(1-\epsilon)} \sum_{\delta l_x l_y L K K'} \frac{1}{2} \imath^{-(K-K')} \psi_K^{l_x l_y}(\alpha_\kappa) \psi_{K'}^{l_x l_y}(\alpha_\kappa) \times \sum_{\delta} A_{\delta K}^{l_x l_y L}(E_\kappa) A_{\delta K'}^{l_x l_y L}(E_\kappa)^*$$

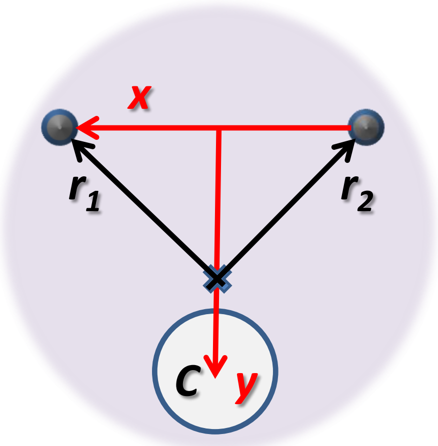
$$\frac{d^3\sigma}{d\cos\theta_{xy}\, d\epsilon\, dE_\kappa} = \sqrt{\epsilon(1-\epsilon)} \sum_{\lambda} \frac{1}{4} P_\lambda(\cos\theta_{xy}) \sum_{L l_x l_y K l'_x l'_y K'} C_{l'_x l'_x L}^{l_x l_x \lambda} \times \imath^{-(K-K')} \psi_K^{l_x l_y}(\alpha_\kappa) \psi_{K'}^{l'_x l'_y}(\alpha_\kappa) \sum_{\delta} A_{\delta\, K}^{l_x l_y L}(E_\kappa) A_{\delta\, K'}^{l'_x l'_y L}(E_\kappa)^*$$

$$\frac{d^8\sigma}{d\hat{p}_f d\hat{\mathrm{k}}_{\mathrm{x}} d\hat{\mathrm{k}}_{\mathrm{y}} d\varepsilon_{\mathrm{y}} d\mathrm{E}_\kappa} = \sqrt{\epsilon(1-\epsilon)} \sum_{\nu_i} \Big| \sum_{\delta K l_x l_y L} \imath^{-K} \psi_K^{l_x l_y}(\alpha_\kappa)\, A_{\delta K}^{l_x l_y L}(E_\kappa) \left[Y_{l_x}(\hat{\mathrm{k}}_{\mathrm{x}}) \otimes Y_{l_y}(\hat{\mathrm{k}}_{\mathrm{y}}) \right]_L \Big|^2$$

S. N. Ershov, J. S. Vaagen and M. V. Zhukov : Phys. Rev. C86, 034331 (2012)

" Binding energy constraint on matter radius and soft dipole excitations of ^{22}C "

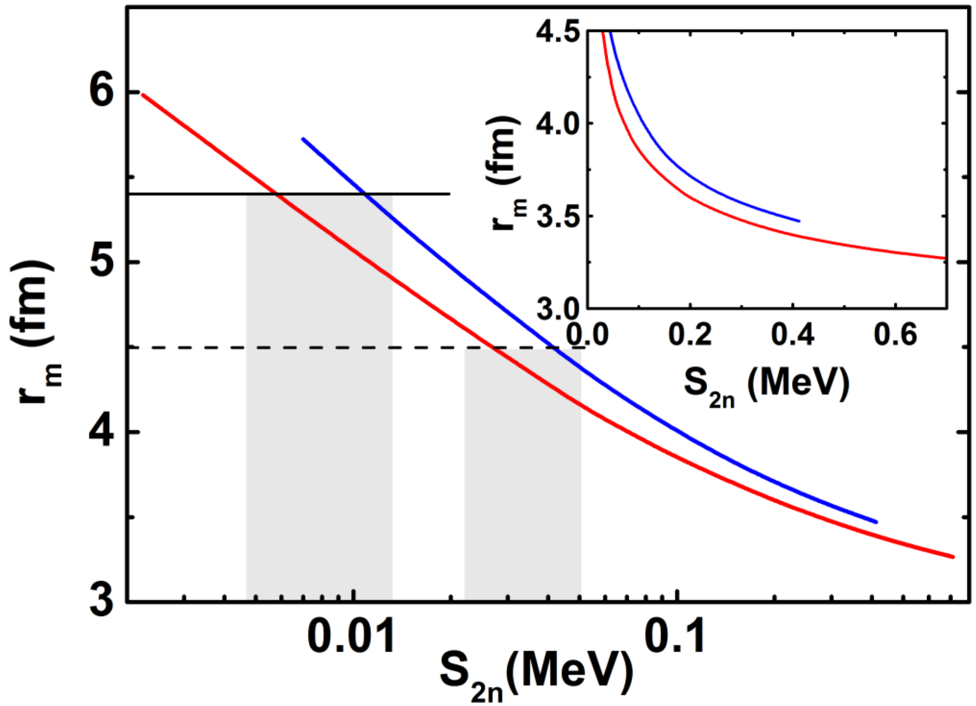
^{22}C is now the heaviest observed Borromean nucleus



the nuclear three-body cluster model

$$\Psi_{JM}(\vec{r}_1,\ldots,\vec{r}_A) \simeq \phi_C(\vec{r}_1,\ldots,\vec{r}_{A_C})\ \Phi_{JM}(\vec{x},\vec{y})$$

$$\langle r_m^2\rangle_A = \frac{A-2}{A}\langle r_m^2\rangle_C + \frac{1}{A}\langle\rho^2\rangle$$



$$\langle r^2\rangle_C^{1/2} = 2.98\ \text{fm}$$

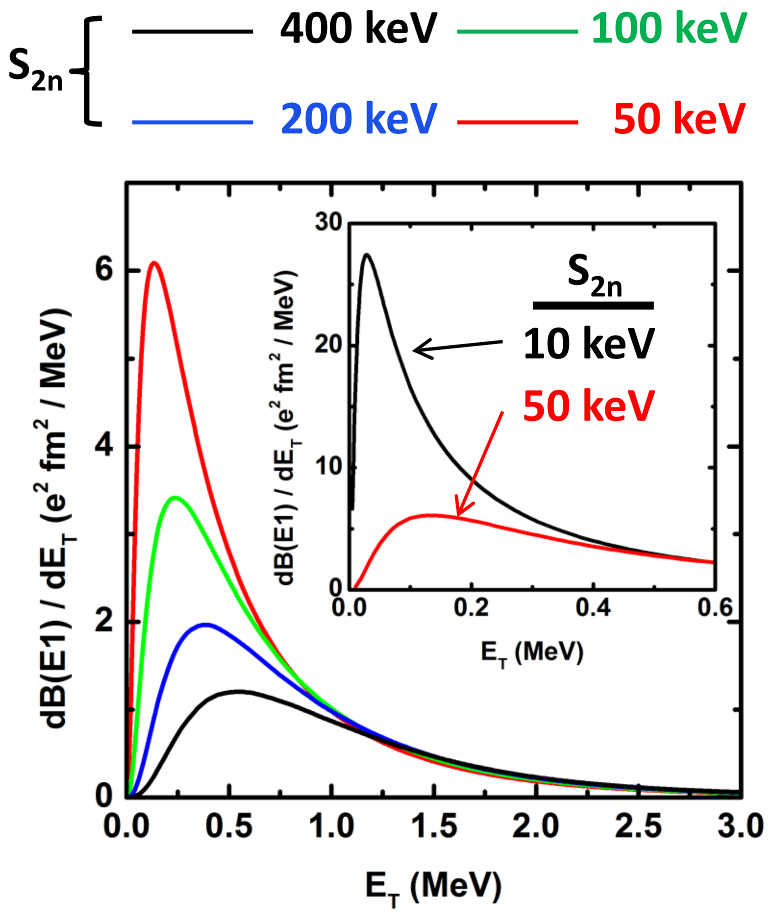
$$\langle r_m^2\rangle_A^{1/2} = 5.4\ \text{fm}$$

⇩

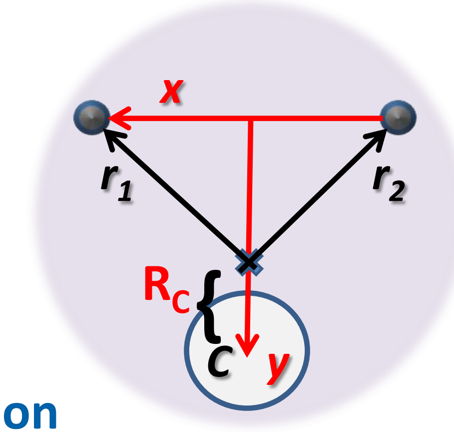
$$\langle\rho^2\rangle^{1/2} = 21.5\ \text{fm}$$

K. Tanaka et al, PRL 104, 062701 (2010)

$$\langle r^2\rangle^{\frac{1}{2}} = 5.4 \pm 0.9\ \text{fm}$$



Soft dipole mode



Electromagnetic dissociation cross sections

$$\frac{d\sigma_{EMD}}{dE_T} = \frac{N_{E1}(E_T + S_{2n})}{\hbar c} \frac{16\pi^3}{9} \frac{dB(E1)}{dE_T}$$

$N_{E1}(E_T)$ is the spectrum of virtual photons

$^{22}C + {}^{208}Pb$

S_{2n} (MeV)	0.40	0.20	0.10	0.05	0.01
E/A (MeV)	σ_{EMD}(b)	σ_{EMD}(b)	σ_{EMD}(b)	σ_{EMD}(b)	σ_{EMD}(b)
240	1.1	1.6	2.5	3.6	7.4
140	1.4	2.2	3.4	4.9	10.4
40	3.0	4.8	7.7	11.7	26.0