WHAT CAN WE VERIFY BY THE BOOMERANG EFFECT ON THE NUCLEAR LIFETIME ?

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Historical Attempts

Chemical

1953 K. Bainbridge at al.

- $[\lambda(KTcO_4)-\lambda(Tc_2S_7)]/\lambda(Tc_2S_7)=$ $(2.70\pm0.10)\times10^{-3}$

1980 H.Mazaki et al.

$[\lambda(TcO_4)-\lambda(Tc_2S_7)]/\lambda(Tc_2S_7)=$ $(3.18\pm0.70)\times10^{-3}$

$[\lambda(TcS_7)-\lambda(Tc_2S_7)]/\lambda(Tc_2S_7)=$ $(5.6\pm0.7)\times10^{-4}$

High Pressure 10 GPa

1952 K. Bainbridge et al.

$[\lambda(10\ GPa)-\lambda(0\ Pa)]/\lambda(0\ Pa)=$ $(2.3\pm0.5)\times10^{-4}$

1980 H.Mazaki et al.

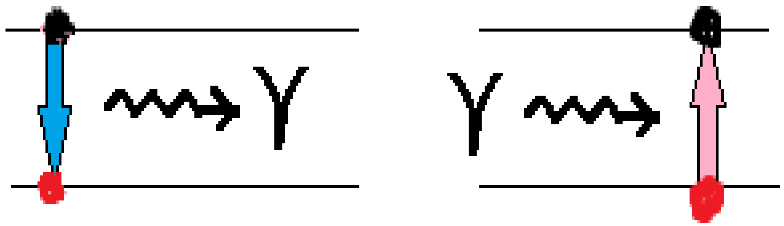
$[\lambda(10\ GPa)-\lambda(0\ Pa)]/\lambda(0\ Pa)=$ $(4.6\pm2.3)\times10^{-4}$

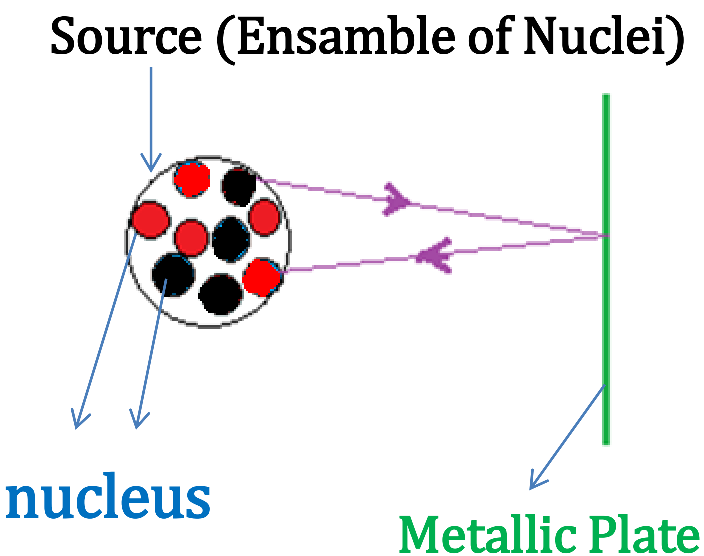
Low Temoerature

1958 D. Byers et al.

$[\lambda(4.2\ K)-\lambda(293\ K)]/\lambda(293\ K)=$ $(1.3\pm0.4)\times10^{-4}$

γ-ray emission & absorption





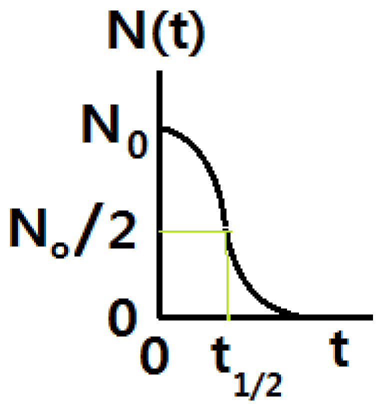
Number of excited nucleus increases partially because some of emitted γ-ray are reabsorbed by the source nuclei.
Therefore, decay curve is altered a little bit.

$$\frac{dN(t)}{dt} = -\lambda N(t)$$

$$N(t) = N_o \exp(-\lambda t)$$

N(t) : Number of nuclei in excited state.

N_o : Number of nuclei at t=0.

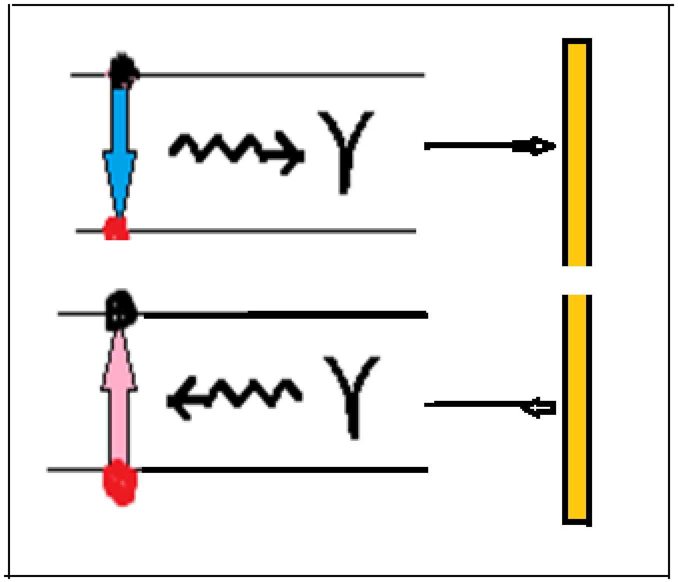


$$N(\tau_{½}) = N_o/2$$

$\tau½$: Half- life

$\lambda = \ln 2/\tau½$: Decay constant

In case of a single nucleus



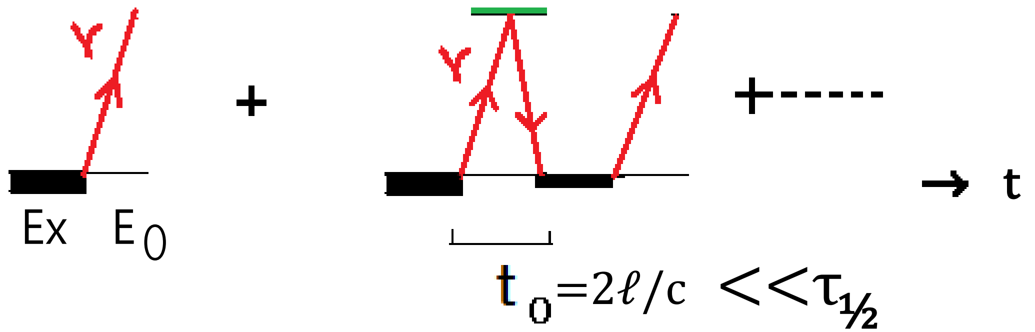
γ-ray can be absorbed after a lifetime has passed.

Therefore, the nuclear lifetime is not modified at all, because γ-ray emission-absorption processes do not make any contribution to build-up nuclear energy levels.

This is the "classical" picture.

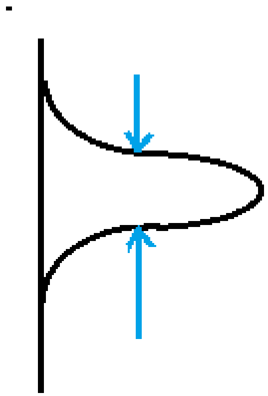
γ-emission

reabsorption of the returned γ



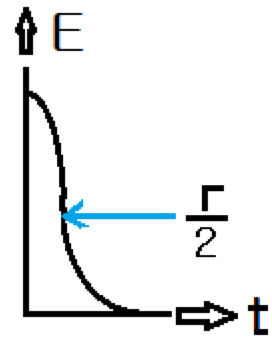
in the probability amplitude

Breit-Wigner Formula for Nuclear Energy Level



$$|\phi(E)|^2 = \frac{1}{2\pi}\frac{\hbar^2}{(E-E_e)^2+(\frac{\Gamma}{2})^2}$$

$$\phi(E) = \frac{1}{\sqrt{2\pi}}\frac{i\hbar}{(E-E_e)+i\frac{\Gamma}{2}}$$



Fourier Transform

$$\psi(t) = \exp[-\frac{i}{\hbar}(E_e - i\frac{\Gamma}{2})t]$$
$$= \exp(-i\kappa t),$$

$$\kappa = \frac{E_e}{\hbar} - i\frac{\lambda}{2}$$

$$\lambda = \frac{\Gamma}{\hbar}$$

$$|\psi(t)|^2 = \exp(-\lambda t)$$

The single nucleus is described by the **probability amplitude ψ(t).**

$$\psi(t) = \exp[-\frac{i}{\hbar}(E_{\varepsilon} - i\frac{\Gamma}{2})t]$$
$$= \exp(-i\kappa t),$$

$$\kappa = \frac{E_{\varepsilon}}{\hbar} - i\frac{\lambda}{2}$$

$$|\psi(t)|^2 = \exp(-\lambda t)$$

$d\psi = -i\kappa\psi(t)\ dt$

ℓ : distance between the source and the reflector

$t_0 = \ell/c$: time for the γ to make a round trip.

For the time interval $0 \le t \le (t_0 + \varepsilon)$ $(\varepsilon \to 0)$,

$$\psi(t_0) = \psi(0)\exp(-i\kappa^{(0)} t_0)$$

where

$$-i\kappa^{(0)} = -i\frac{E_{\varepsilon}}{\hbar} - \frac{\lambda^{(0)}}{2}$$

For $t_0 \le t \le (2t_0 - \epsilon)$

$d\psi(t) = -i\kappa^{(1)}\psi(t)\, dt$ $\lambda^{(1)} = \lambda^{(o)}(1-q)$

q : coefficient for elastic scattering and reabsorption

$0 \le q \le 1$

$$\psi(2t_0) = \psi(t_0)\exp(-i\kappa^{(1)} t_0)$$
$$= \psi(0)\exp[-i(\kappa^{(0)} + \kappa^{(1)})t_0]$$

$$-i\kappa^{(1)} = -i\frac{E_e}{\hbar} - \frac{\lambda^{(1)}}{2}$$

Verification of the above equation

- **In analogy to the rate equations used in Laser physics,**
- **$dP_e(t)/dt=-\lambda\ P_e(t)+\Sigma\ dn(t)/dt$,**
- **$dP_g(t)/dt=\lambda\ P_e(t)-\ \Sigma\ dn(t)/dt$,**
- **$dn(t)/dt=\lambda\ P_e(t)-\Sigma\ dn(t)/dt$,**

- **Initial conditions:** $P_e(0)=1$, $P_g(0)=0$, $n(0)=0$.

- **Solutions are**
- $P_e(t)=\exp[-\ \lambda t/(1+\Sigma)]$, $P_g(t)=n(t)=1-\ P_e(t)$.
- **$dP_e(t)/dt=-[\lambda/(1+\Sigma)]\ P_e(t) \fallingdotseq -\ \lambda(1-\Sigma)\ P_e(t)$.**

For $mt_0 \leq t \leq ((m+1)t_0 - \varepsilon)$

$d\psi = -i\kappa^{(m)}\psi(t)\, dt$

$\psi((m+1)t_0) = \psi(mt_0)\exp(-i\kappa^{(m)}t_0)$

$$\psi((m+1)t_0) = \psi(0)\exp(-i\sum_{s=0}^{m}\kappa^{(s)}t_0)$$

$$-i\sum_{s=0}^{m}\kappa^{(s)} = -i\frac{E_\varepsilon}{\hbar}(m+1) - \frac{\lambda^{(0)}}{2}\sum_{s=0}^{m}(1-q)^s$$

$$\lambda^{(m)} = \lambda^{(0)}(1-q)^m$$

$$-i\sum_{s=0}^{m}\kappa^{(s)} = -i\frac{E_e}{\hbar}(m+1) - \frac{\lambda^{(0)}}{2}\sum_{s=0}^{m}(1-q)^s$$

$$\psi((m+1)t_0) = \psi(0)\exp(-i\sum_{s=0}^{m}\kappa^{(s)}t_0)$$
$$= \psi(0)\exp[-i\frac{E_e}{\hbar}(m+1)\,t_0 - \frac{\lambda^{(0)}}{2}\sum_{s=0}^{m}(1-q)^s t_0]$$

$(m+1)t_o=t$ and $\sum_{s=0}^{m}(1-q)^s = \frac{1-(1-q)^{m+1}}{q}$

$$\psi(t) = \psi(0)\exp[-i(\frac{E_e}{\hbar} - i\frac{\lambda^{(0)}}{2}\frac{1-(1-q)^{m+1}}{(m+1)q})t]$$

$$\psi(t) = \psi(0)\exp[-i(\frac{E_e}{\hbar} - i\frac{\lambda^{(0)}}{2}\frac{1-(1-q)^{m+1}}{(m+1)q})t]$$

$$\lambda^{(0)}\frac{1-(1-q)^{m+1}}{(m+1)q} \equiv \tilde{\lambda}$$

and set $\psi(0) = 1$

$$\psi(t) = \exp(-i\frac{E_e}{\hbar}t)\exp(-\frac{\tilde{\lambda}}{2}t)$$

Fourier transform

$$\phi(E) = \frac{1}{\sqrt{2\pi}}\frac{i\hbar}{(E-E_e)+i\frac{\hbar\tilde{\lambda}}{2}}$$

$$\hbar\tilde{\lambda} = \tilde{\Gamma}$$

$t=(m+1)t_o$ and $t_o=2\ell/c$

$\therefore \quad (m+1)=(2\ell/c)t$

For $t=\tilde{\tau}_{1/2}$, $(m+1)=(2\ell/c)\,\tilde{\tau}_{1/2}$

$$|\psi(\tilde{\tau}_{1/2})|^2 = \frac{1}{2}$$

$$\tilde{\tau}_{1/2} = \frac{\ln 2}{\tilde{\lambda}}$$

$$\frac{1-(1-q)^{m+1}}{(m+1)q}\tilde{\tau}_{1/2} = \tau_{1/2}$$

$$\frac{\ln 2}{\lambda^{(0)}} = \tau_{1/2}$$

$\tilde{\tau}_{1/2} \rightarrow \tau_{1/2}$ **for m➔0**

$$(m+1) = \frac{c}{2l}\tilde{\tau}_{1/2}$$

$$(1-q)^{\frac{c}{2l}\tilde{\tau}_{1/2}} = 1-(\frac{c}{2l}\tau_{1/2})q$$

$$\tilde{\tau}_{1/2} = (\frac{2l}{c})\frac{\ln[1-(\frac{c\tau_{1/2}}{2l})q]}{\ln(1-q)}$$

$$\tilde{\tau}_{1/2} = \left(\frac{2l}{c}\right)\frac{\ln\left[1-\left(\frac{c\tau_{1/2}}{2l}\right)q\right]}{\ln(1-q)}$$

$$q = \int (\zeta n d f \sigma_{\chi}) \rho \, dS$$

$$\zeta = \frac{\sigma_{\gamma}^{N} f_1}{\sigma_{tot}}$$

f and f_1: Debye-Waller factor

dS: surface element of reflector

ρ: surface density of the γ-ray on the surface of reflector

n: number of atoms per cm^3

d: thickness of reflector

σ_{χ}: γ backward scattering cross-section

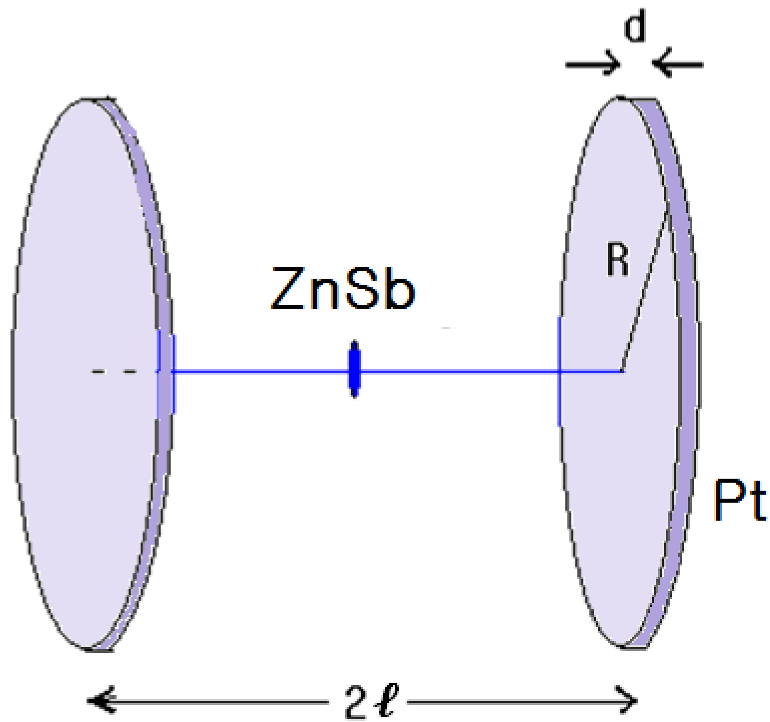
$$\sigma_{tot} = f_1(\sigma_\gamma^N + \sigma_{coh}^N + \sigma_{coh}^A) + \sigma_{pe}^N + \sigma_{incoh}^N + \sigma_{pe}^A + \sigma_{incoh}^A$$

$$q = \int \zeta \, n d f \, \sigma_{\pi} \, \rho \, dS$$

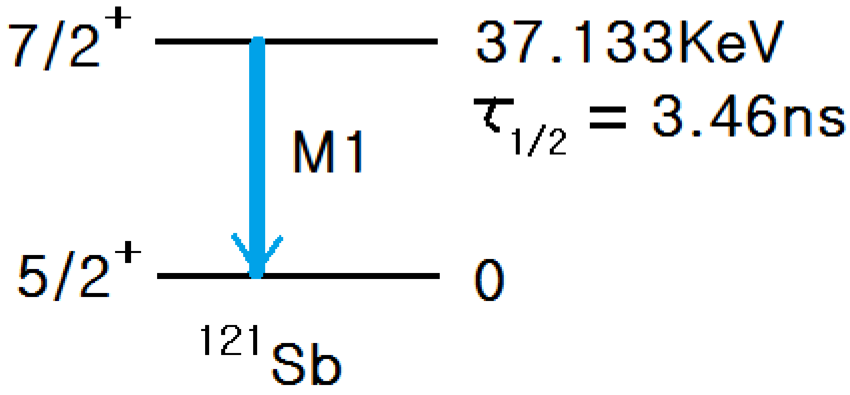
n : number of atoms per unit surface
d : thickness of reflector
f :Debye-Waller factor
σ_{π} : gamma-backward scattering cross-section
$\rho \, dS$: partial number of the photon passing through the surface element

$$\zeta = \frac{f_1 \sigma_{\gamma}^{N}}{\sigma_{tot}}$$

$$\sigma_{tot} = f_1(\sigma_{\gamma}^{N} + \sigma_{coh}^{N} + \sigma_{coh}^{A}) + \sigma_{pe}^{N} + \sigma_{incoh}^{N} + \sigma_{pe}^{A} + \sigma_{incoh}^{A}$$



R=1.4mm, ℓ =5mm, d=0.5mm


7/2+
37.133KeV
τ1/2 = 3.46ns
M1
5/2+
0
121Sb

Result at T=4.2K

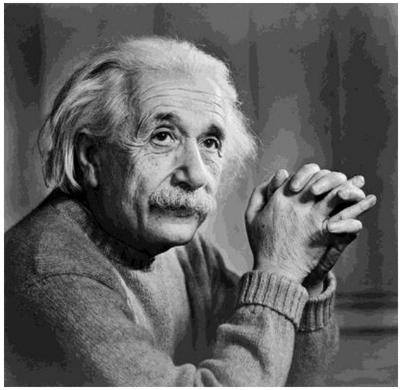
$$\bar{\tau}_{1/2} = 3.57\text{ns}$$

$$\Delta\tau_{1/2} = 2.52\%$$

Conclusion

- Probability amplitudes play essential roles in microscopic world

  - the key point of Quantum Mechanics.



- Einstein's statements :

  - God does not throw a dice.

  - One should see the real entity hidden behind the phenomena.