The 9th APCTP-BLTP JINR Joint Workshop in Kazakhstan
Modern Problems in Nuclear and Elementary Particle Physics

Observational Constrains on the Mass-Radius Relations of Neutron Stars

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in collaboration with

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ALMATY, KAZAKHSTAN
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Motivation

Investigation of the motion of test particles in the field of rotating and deformed objects are relevant to several astrophysical phenomena:

- in particular to the observed high frequency, kilohertz Quasi Periodic Oscillations (kHz QPOs) in the X-ray luminosity from black hole and neutron star sources;
- it is believed that kHz QPO data may be used to test the strong field regime of Einstein's general relativity, and the physics of super-dense matter of which neutron stars are made of.

External Hartle-Thorne Solution

$$ds^2 = -\left(1-\frac{2M}{r}\right)\left[1+2k_1P_2(\cos\theta)+2\left(1-\frac{2M}{r}\right)^{-1}\frac{J^2}{r^4}(2\cos^2\theta-1)\right]dt^2$$
$$+\left(1-\frac{2M}{r}\right)^{-1}\left[1-2k_2P_2(\cos\theta)-2\left(1-\frac{2M}{r}\right)^{-1}\frac{J^2}{r^4}\right]dr^2$$
$$+r^2[1-2k_3P_2(\cos\theta)](d\theta^2+\sin^2\theta d\phi^2)-4\frac{J}{r}\sin^2\theta dt d\phi$$

$$k_1 = \frac{J^2}{Mr^3}\left(1+\frac{M}{r}\right)-\frac{5}{8}\frac{Q-J^2/M}{M^3}Q_2^2\left(\frac{r}{M}-1\right), \qquad k_2 = k_1-\frac{6J^2}{r^4},$$
$$k_3 = k_1+\frac{J^2}{r^4}-\frac{5}{4}\frac{Q-J^2/M}{M^2 r}\left(1-\frac{2M}{r}\right)^{-1/2}Q_2^1\left(\frac{r}{M}-1\right), \qquad P_2(x) = \frac{1}{2}(3x^2-1),$$
$$Q_2^1(x) = (x^2-1)^{1/2}\left[\frac{3x}{2}\ln\frac{x+1}{x-1}-\frac{3x^2-2}{x^2-1}\right], \qquad Q_2^2(x) = (x^2-1)\left[\frac{3}{2}\ln\frac{x+1}{x-1}-\frac{3x^3-5x}{(x^2-1)^2}\right].$$

- *Hartle, J. B., ApJ 150, 1005 (1967)*
- *Hartle, J. B. & Thorne, K. S., ApJ, 153, 807 (1968)*

Limiting cases

- Q=0, J=0, SCHW;
- Q=0, J≠0, neglecting terms ~ J^2, LT;
- Q≠0, J≠0, HT

or Kerr solution in the Boyer-Lindquist coordinates using the following substitution and coordinate transformations:

$$J = -Ma, \quad Q = J^2/M,$$

and

$$\begin{aligned}
t &= t\,, \\
r &= R + \frac{a^2}{2R}\left[\left(1+\frac{2M}{R}\right)\left(1-\frac{M}{R}\right) - \cos^2\Theta\left(1-\frac{2M}{R}\right)\left(1+\frac{3M}{R}\right)\right], \\
\theta &= \Theta + \frac{a^2}{2R^2}\left(1+\frac{2M}{R}\right)\sin\Theta\cos\Theta\,, \\
\phi &= \phi.
\end{aligned}$$

The Orbital Angular Velocity

$$\boxed{\zeta = U^\phi / U^t}$$

$$\zeta(r) = \frac{-g_{t\phi,r} \pm \sqrt{(g_{t\phi,r})^2 - g_{tt,r}\, g_{\phi\phi,r}}}{g_{\phi\phi,r}}$$

$$\zeta(r) = \pm\zeta_0(r)\left[1 \mp F_1(r)\frac{J}{M^2} + F_2(r)\frac{J^2}{M^4} + F_3(r)\frac{Q}{M^3}\right]$$

$$\begin{aligned}
\zeta_0(r) &= \frac{M^{1/2}}{r^{3/2}} \\
F_1(r) &= \frac{M^{3/2}}{r^{3/2}} \\
F_2(r) &= \frac{48M^7 - 80M^6 r + 4M^5 r^2 + 42M^4 r^3 - 40M^3 r^4 - 10M^2 r^5 - 15M r^6 + 15r^7}{16M^2 r^4 (r - 2M)} - F(r) \\
F_3(r) &= -\frac{5(6M^4 - 8M^3 r - 2M^2 r^2 - 3M r^3 + 3r^4)}{16M^2 r (r - 2M)} + F(r) \\
F(r) &= \frac{15(r^3 - 2M^3)}{32M^3} \ln\frac{r}{r - 2M}
\end{aligned}$$

The Specific Angular Momentum Per Unit Energy

$$\boxed{l = -U_\phi / U_t}$$

$$l = -\frac{g_{t\phi} + \zeta g_{\phi\phi}}{g_{tt} + \zeta g_{t\phi}},$$

$$l = \pm l_0 \left[1 \mp G_1(r) \frac{J}{M^2} + G_2(r) \frac{J^2}{M^4} + G_3(r) \frac{Q}{M^3} \right],$$

$$\begin{aligned}
l_0 &= \frac{M^{1/2} r^{3/2}}{r - 2M} \\
G_1(r) &= \frac{M^{3/2}(3r - 4M)}{r^{3/2}(r - 2M)} \\
G_2(r) &= \frac{96M^8 - 112M^7 r - 8M^6 r^2 + 72M^5 r^3 - 18M^4 r^4 - 220M^3 r^5 + 260M^2 r^6 - 105M r^7 + 15r^8}{16M^2 r^4 (r - 2M)^2} - G(r) \\
G_3(r) &= \frac{5(6M^4 - 22M^2 r^2 + 15M r^3 - 3r^4)}{16M^2 r(r - 2M)} + G(r) \\
G(r) &= \frac{15(2M^3 + 4M^2 r - 4M r^2 + r^3)}{32M^3} \ln\left(\frac{r}{r - 2M}\right)
\end{aligned}$$

The Specic Energy Per Unit Mass

$$\boxed{\varepsilon = -U_t}$$

$$\varepsilon = \frac{g_{tt} + \zeta g_{t\phi}}{\sqrt{-g_{tt} - 2\zeta g_{t\phi} - \zeta^2 g_{\phi\phi}}},$$

$$\varepsilon = \varepsilon_0 \left[1 \mp H_1(r) \frac{J}{M^2} + H_2(r) \frac{J^2}{M^4} + H_3(r) \frac{Q}{M^3} \right]$$

$$\varepsilon_0 = \frac{r - 2M}{r^{1/2}(r - 3M)^{1/2}}$$

$$H_1(r) = \frac{M^{5/2}}{r^{1/2}(r - 2M)(r - 3M)}$$

$$H_2(r) = -\frac{144M^8 - 144M^7 r - 28M^6 r^2 + 122M^5 r^3 + 184M^4 r^4 - 685M^3 r^5 + 610M^2 r^6 - 225M r^7 + 30r^8}{16M r^4 (r - 2M)(r - 3M)^2} + H(r)$$

$$H_3(r) = \frac{5(r - M)(6M^3 + 20M^2 r - 21M r^2 + 6r^3)}{16M r (r - 2M)(r - 3M)} - H(r)$$

$$H(r) = \frac{15r(8M^2 - 7Mr + 2r^2)}{32M^2(r - 3M)} \ln\left(\frac{r}{r - 2M}\right)$$

Radius of marginally stable, marginally bound and photon orbit.

$$\varepsilon = -U_t = 1 \longrightarrow r_{mb} \qquad r_{mb} = 4M\left[1 \pm \frac{a}{2M} - \frac{a^2}{16M^2}\right],$$

$$P^\alpha P_\alpha = 0 \longrightarrow r_{ph} \qquad r_{ph} = 3M\left[1 \mp \frac{2\sqrt{3}}{9}\frac{a}{M} - \frac{2a^2}{27M^2}\right],$$

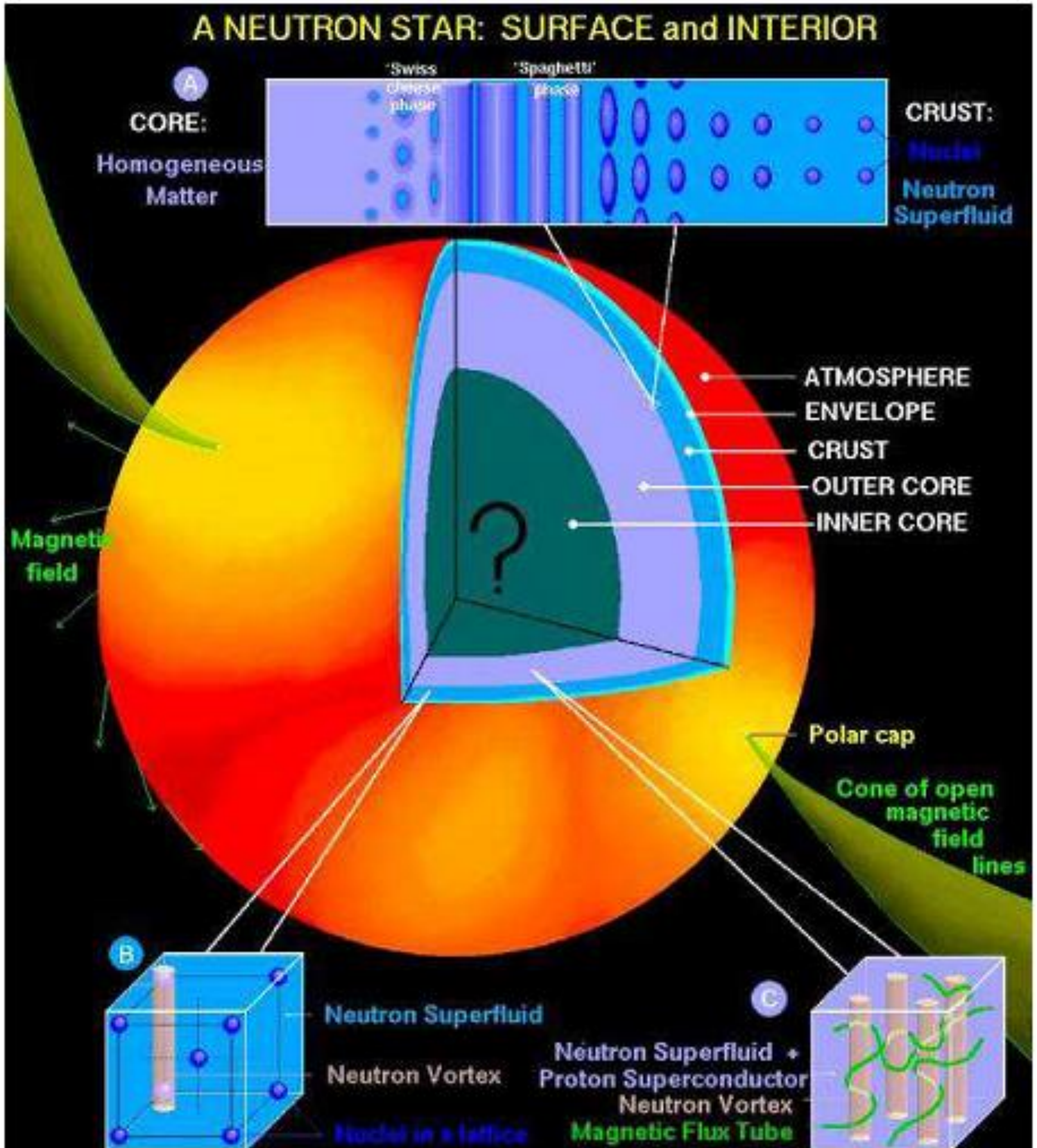
$$dl/dr = 0 \longrightarrow r_{ms} \qquad r_{ms} = 6M\left[1 \mp \frac{2}{3}\sqrt{\frac{2}{3}}\frac{a}{M} - \frac{7a^2}{108M^2}\right].$$

J. Bardeen et. al. (1972)

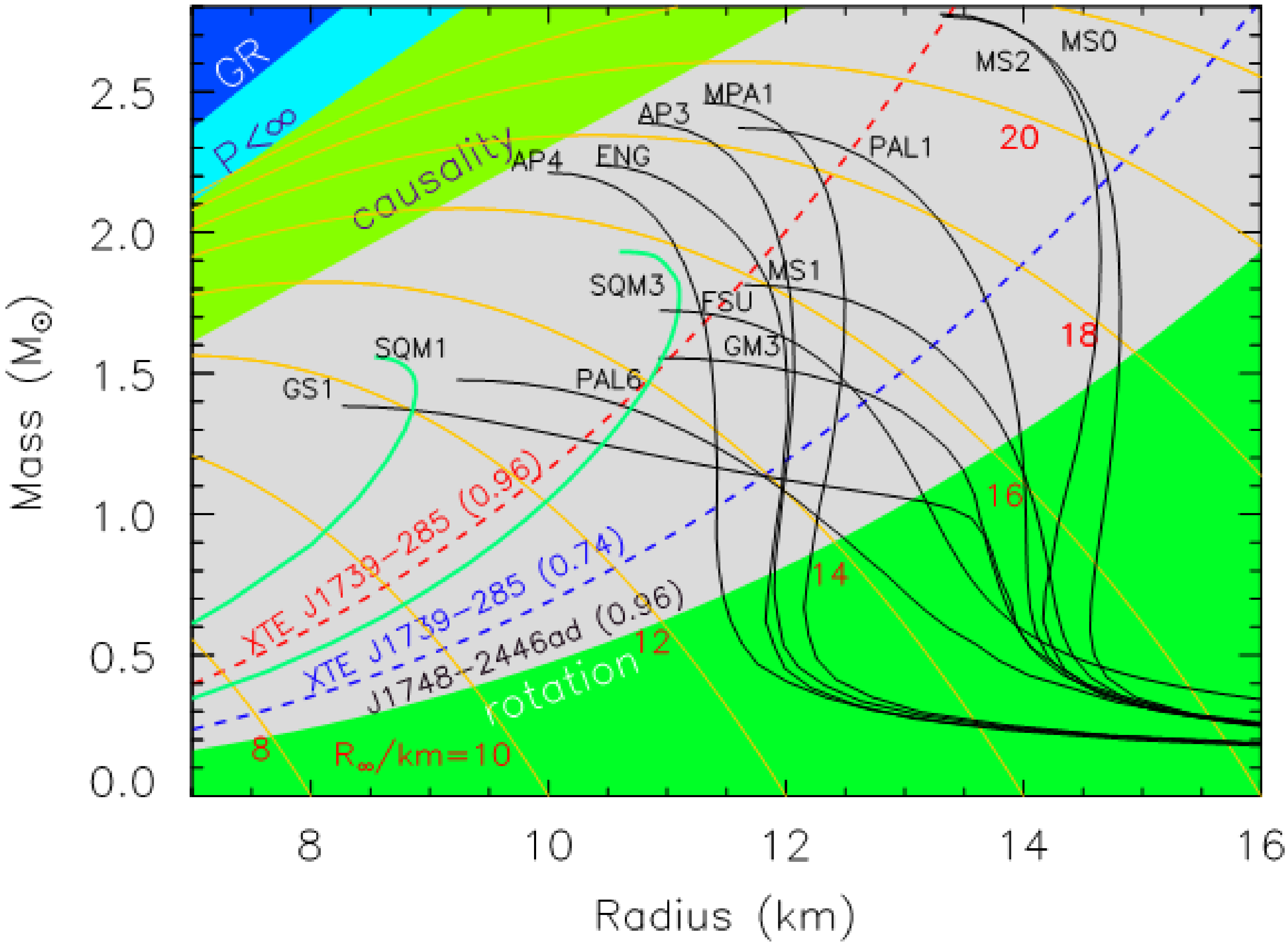
$$r_{mb} = 4M\left[1 \mp \frac{1}{2}\frac{J}{M^2} + \left(\frac{8033}{256} - 45\ln 2\right)\frac{J^2}{M^4} + \left(-\frac{1005}{32} + 45\ln 2\right)\frac{Q}{M^3}\right]$$

$$r_{ph} = 3M\left[1 \pm \frac{2\sqrt{3}}{9}\frac{J}{M^2} + \left(\frac{1751}{324} - \frac{75}{16}\ln 3\right)\frac{J^2}{M^4} + \left(-\frac{65}{12} + \frac{75}{16}\ln 3\right)\frac{Q}{M^3}\right]$$

$$r_{ms} = 6M\left[1 \pm \frac{2}{3}\sqrt{\frac{2}{3}}\frac{J}{M^2} + \left(-\frac{251903}{2592} + 240\ln\frac{3}{2}\right)\frac{J^2}{M^4} + \left(\frac{9325}{96} - 240\ln\frac{3}{2}\right)\frac{Q}{M^3}\right]$$



Current observations of neutron star masses suggest the existence of stars with a mass of order 2 solar masses. Such stars require a **stiff equation of state** for the neutron matter that makes up most of the star to be able to balance the attraction of gravity—and rules out the presence of **exotic forms of matter (pion-kaon condensates, quark matter or core solids)** in the core of any neutron star because at any mass its central density is less than the transition densities for these exotic phases.



J.M. Lattimer, M. Prakash / Physics Reports 442 (2007) 109–165

Neutron Star Model

Belvedere, R.; Pugliese, D.; Rueda, J.A.; Ruffini, R; Xue, Sh.
“Neutron star equilibrium configurations within a fully relativistic theory with strong, weak, electromagnetic, and gravitational interactions”
Nuclear Physics A, Volume 883, p. 1-24. 2012

Belvedere, R.; Boshkayev, K.; Rueda, Jorge A.; Ruffini, R.
“Uniformly rotating neutron stars in the global and local charge neutrality cases”
Nuclear Physics A, Volume 921, p. 33-59. 2014

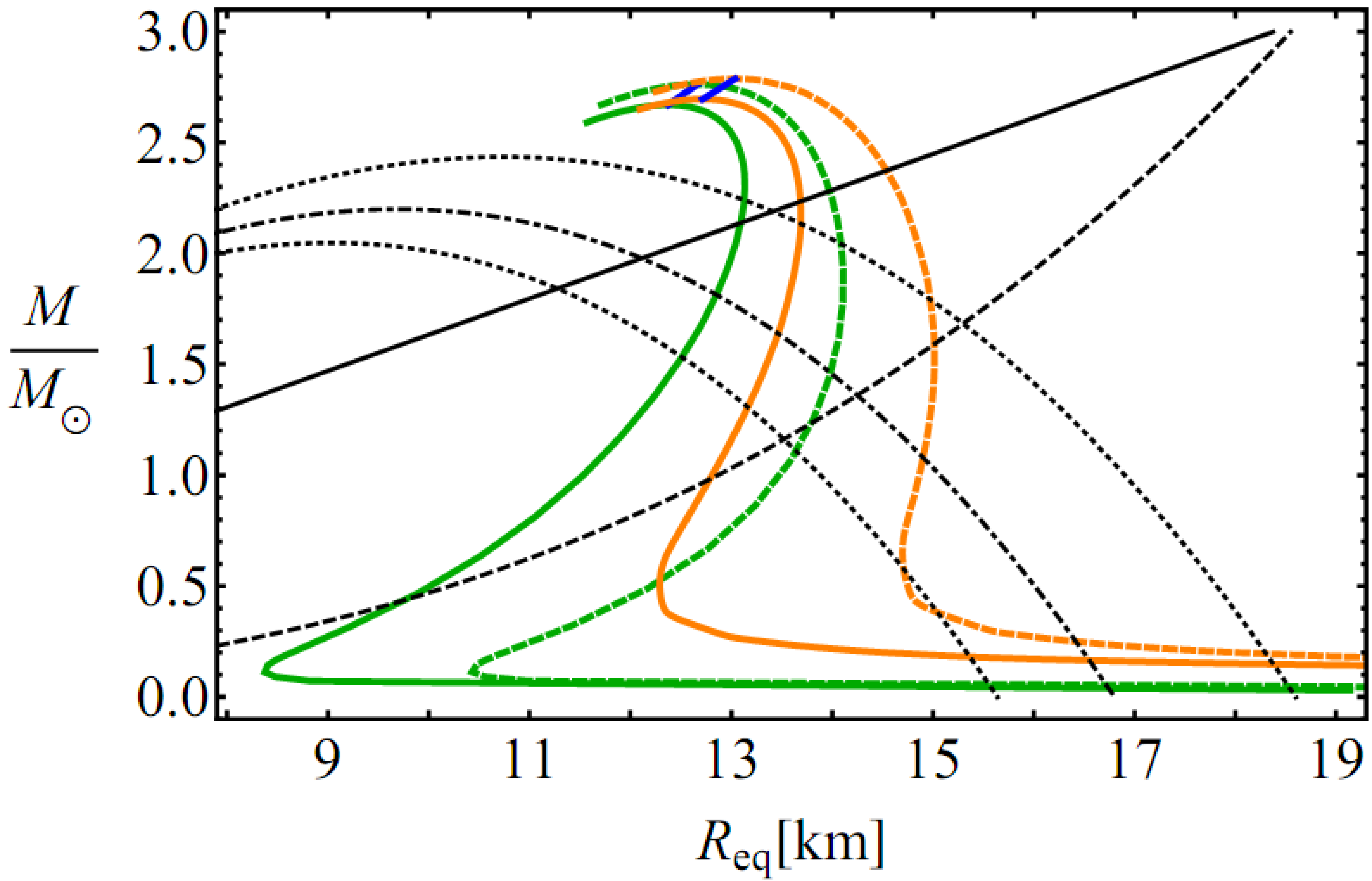
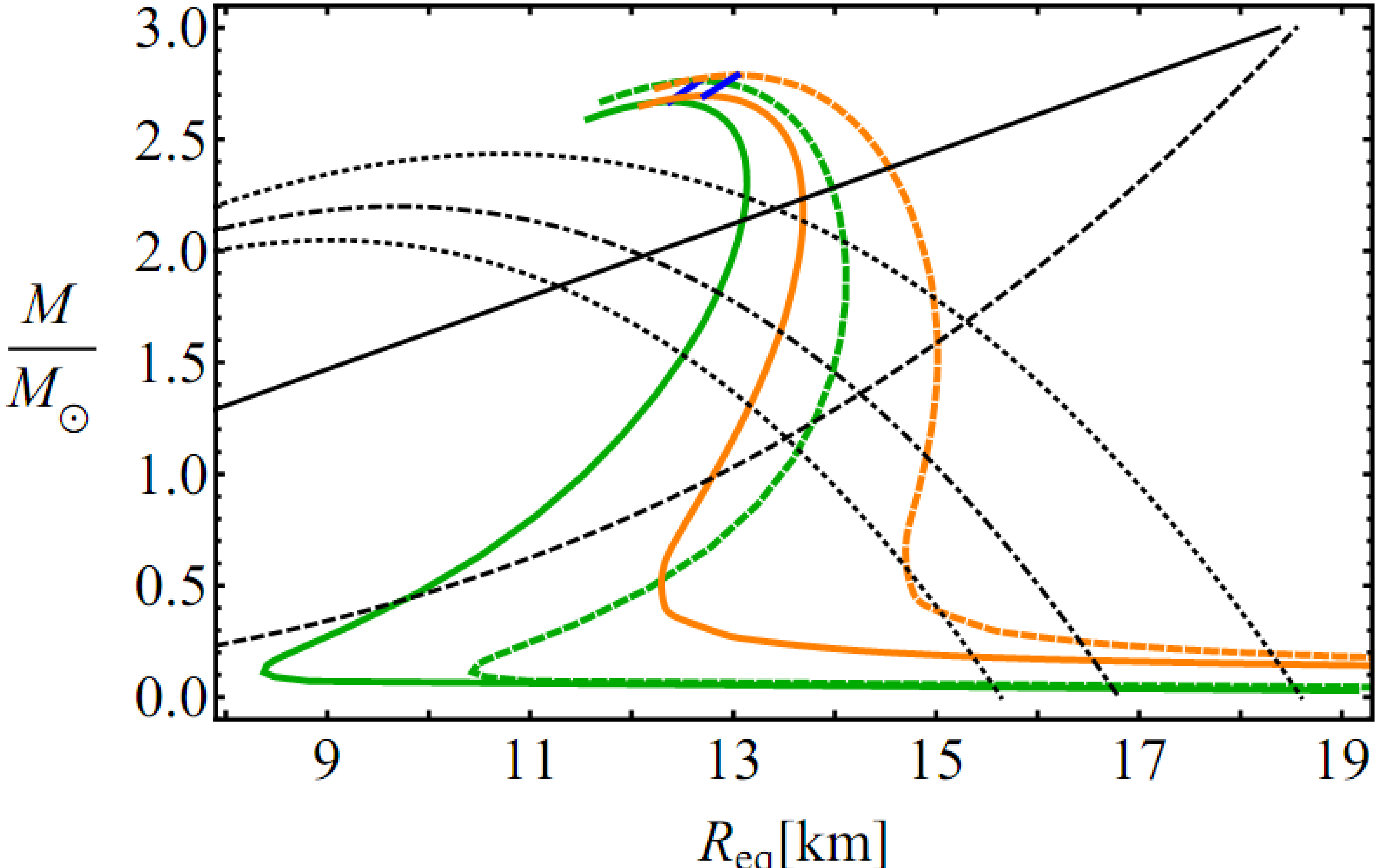


Figure: Constraints on the mass-radius relation given by J.E. Trümper (2011) and the theoretical mass-radius relation presented in this work. Globally neutral configurations are in green and locally neutral ones are in orange. The solid lines correspond to the static case and dashed lines correspond to the Keplerian sequence. We use here the NL3 nuclear model.

Belvedere et. al. Nuclear Physics A, Volume 883, p. 1-24. (2012)
Belvedere et. al. Nuclear Physics A, Volume 921, p. 33-59. (2014)



Constraints on the mass-radius relation given by J. E. Trumper in and the theoretical mass-radius in our work **the solid line is the upper limit of the surface gravity of XTE J1814-338**, **the dotted-dashed curve corresponds to the lower limit to the radius of RX J1856-3754, the dashed line is the constraint imposed by the fastest spinning pulsar PSR J1748-2246ad, and the dotted curves are the 90% confidence level contours of constant R_∞ of the neutron star in the low-mass X-ray binary X7**. Any mass-radius relation should pass through the area delimited by the solid, the dashed and the dotted lines and, in addition, it must have a maximum mass larger than the mass of PSR J1614-2230, $M = 1.97 \pm 0.04 M_\odot$.

QPOs

In X-ray astronomy, **quasi-periodic oscillation** (**QPO**) is the manner in which the X-ray light from an astronomical object flickers about certain frequencies. In these situations, the X-rays are emitted near the inner edge of an accretion disk in which gas swirls onto a compact object such as a white dwarf, neutron star, or black hole.

QPOs were first identified in white dwarf systems and then in neutron star systems.

van der Klis et al. 1985, Nature, 316, 225
Middleditch and Priedhorsky 1986, Astrophysical Journal 306, 230

The Relativistic Precession Model (RPM)

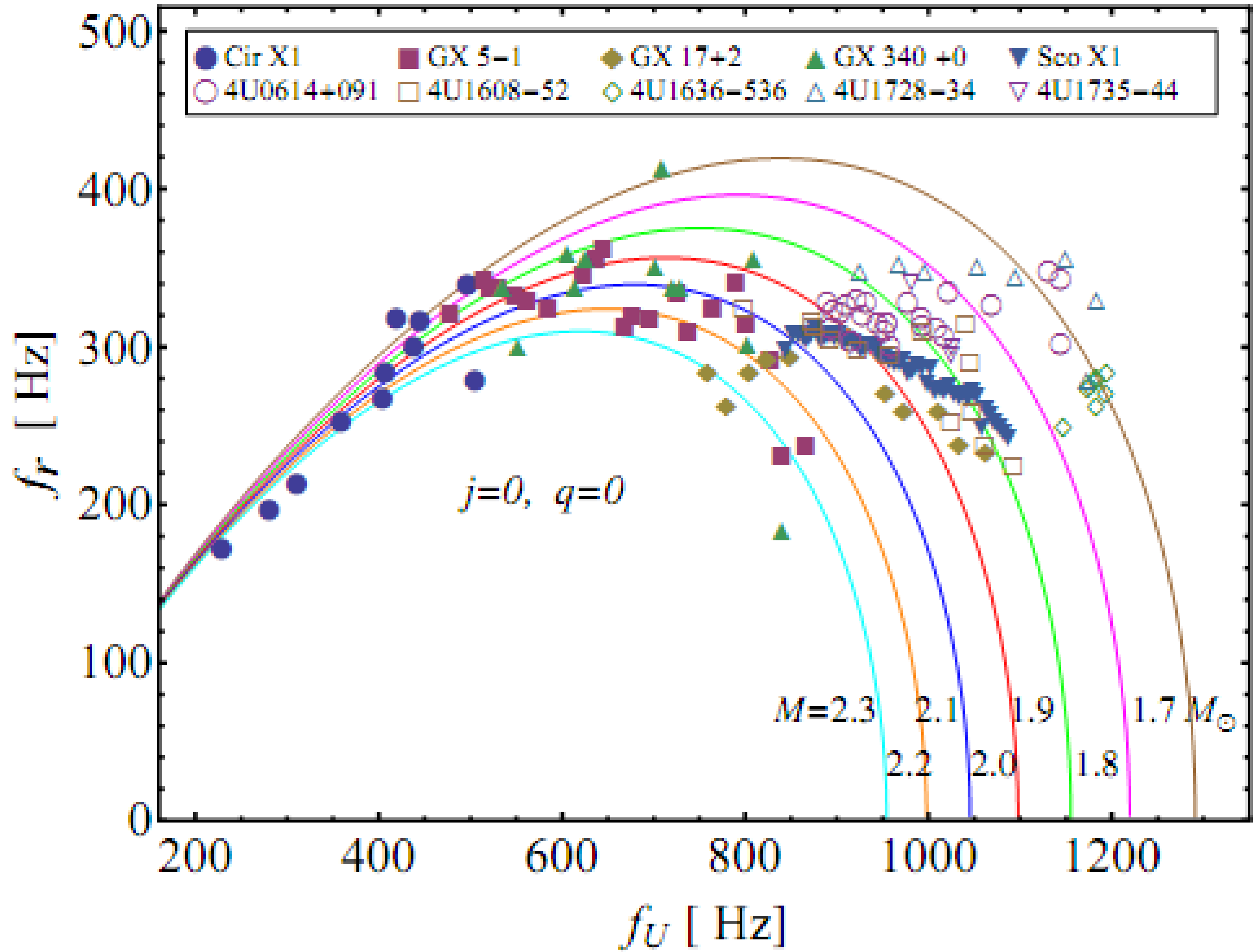
The RPM has been proposed in a series of papers by Stella and Vietri. ***It explains the kHz QPOs as a direct manifestation of modes of relativistic epicyclic motion of blobs arising at various radii r in the inner parts of the accretion disk.*** *The model identifies the lower and upper kHz QPOs with the periastron precession* f_{per} *and Keplerian* f_K *frequency.*

$$f_L = f_{per} = f_\phi - f_r$$

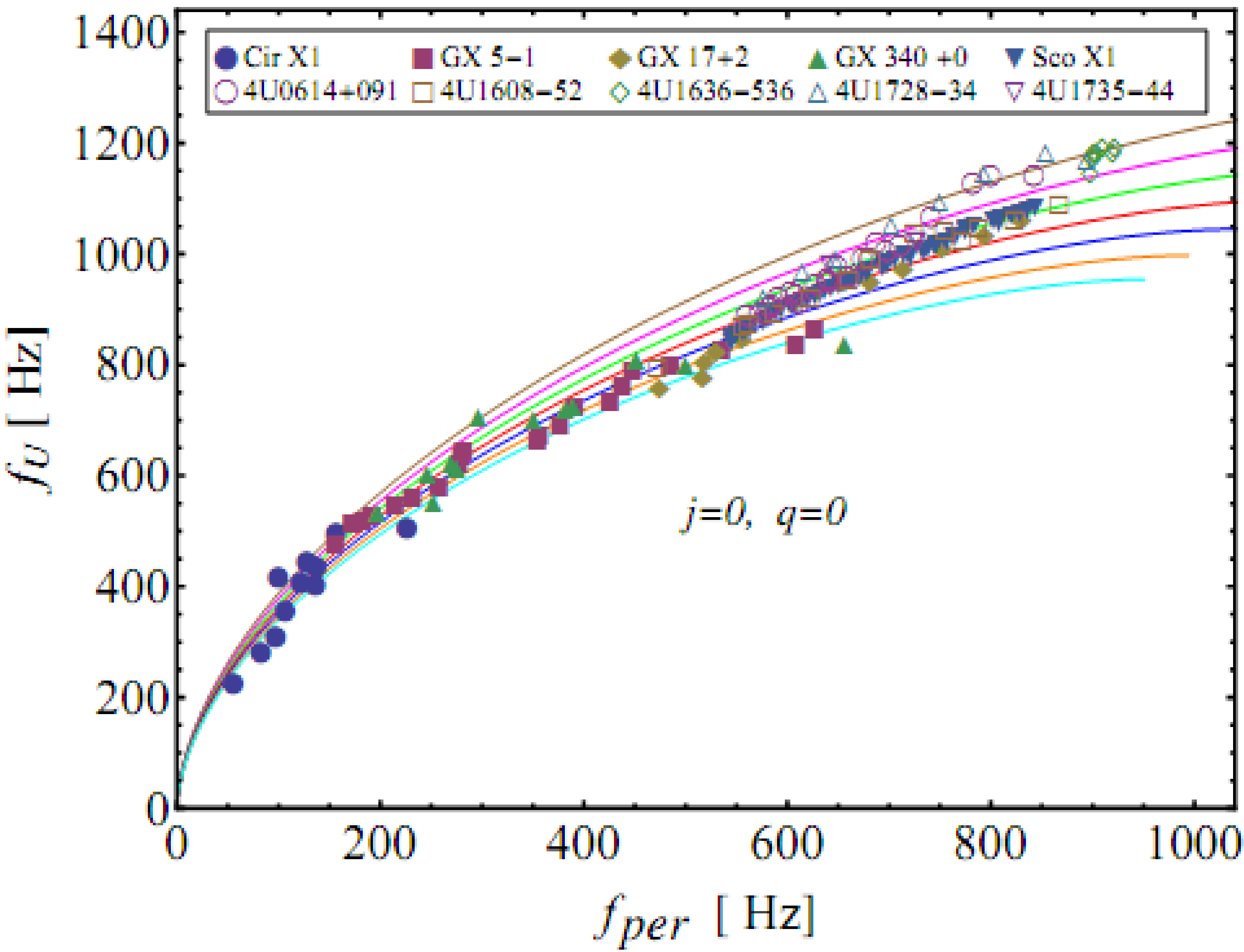
$$f_U = f_\phi = f_K$$

$$\Delta f = f_U - f_L = f_r$$

Stella, L., Vietri, M., 1999, Phys. Rev. Lett. , 82, 17.



Cir X1
GX 5−1
GX 17+2
GX 340 +0
Sco X1
4U0614+091
4U1608−52
4U1636−536
4U1728−34
4U1735−44
j=0, q=0
M=2.3
2.2
2.1
2.0
1.9
1.8
1.7 $M_\odot$
f_r [Hz]
f_U [Hz]



Cir X1
GX 5−1
GX 17+2
GX 340 +0
Sco X1
4U0614+091
4U1608−52
4U1636−536
4U1728−34
4U1735−44
j=0, q=0
f_U [Hz]
f_{per} [Hz]
0
200
400
600
800
1000
1200
1400

Fundamental frequencies

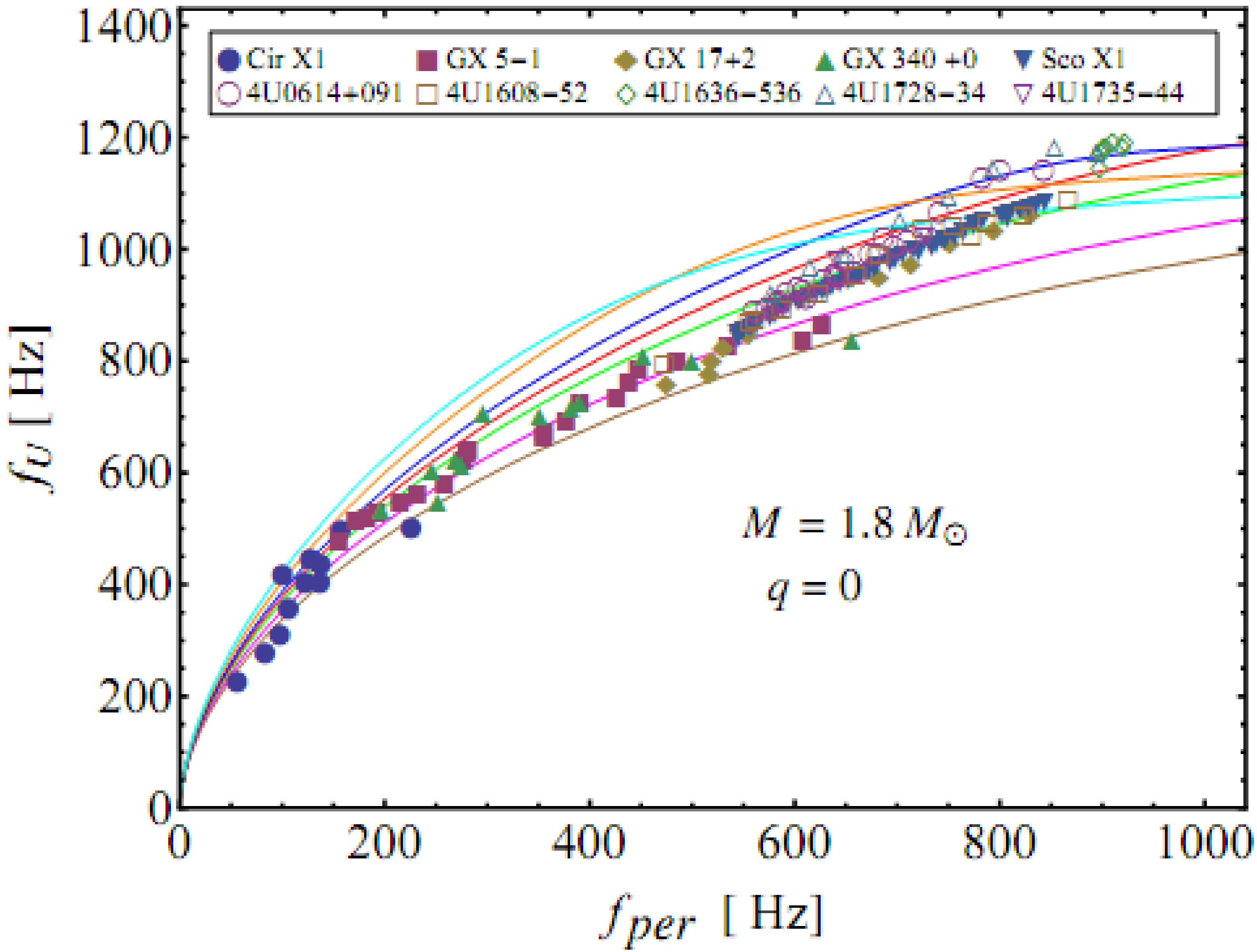
$$\omega_K^2(u) = \omega_{K0}^2(u)\left[1 \mp jF_1(u) + j^2F_2(u) + qF_3(u)\right]$$

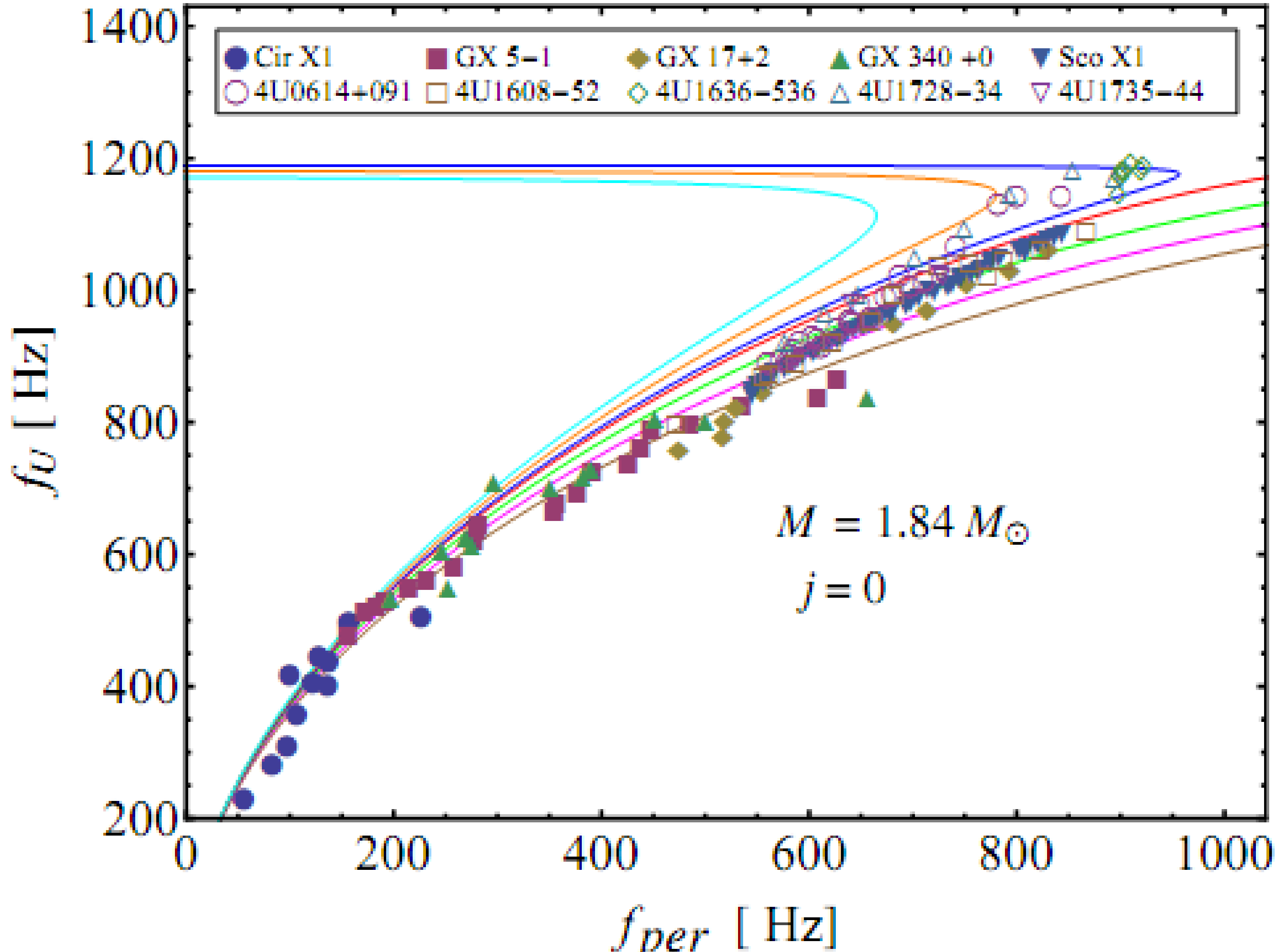
$$\omega_r^2(u) = \omega_{r0}^2(u)\left[1 \pm jX_1(u) + j^2X_2(u) + qX_3(u)\right],$$

$$\omega_\theta^2(u) = \omega_{\theta 0}^2(u)\left[1 \mp jY_1(u) + j^2Y_2(u) + qY_3(u)\right],$$

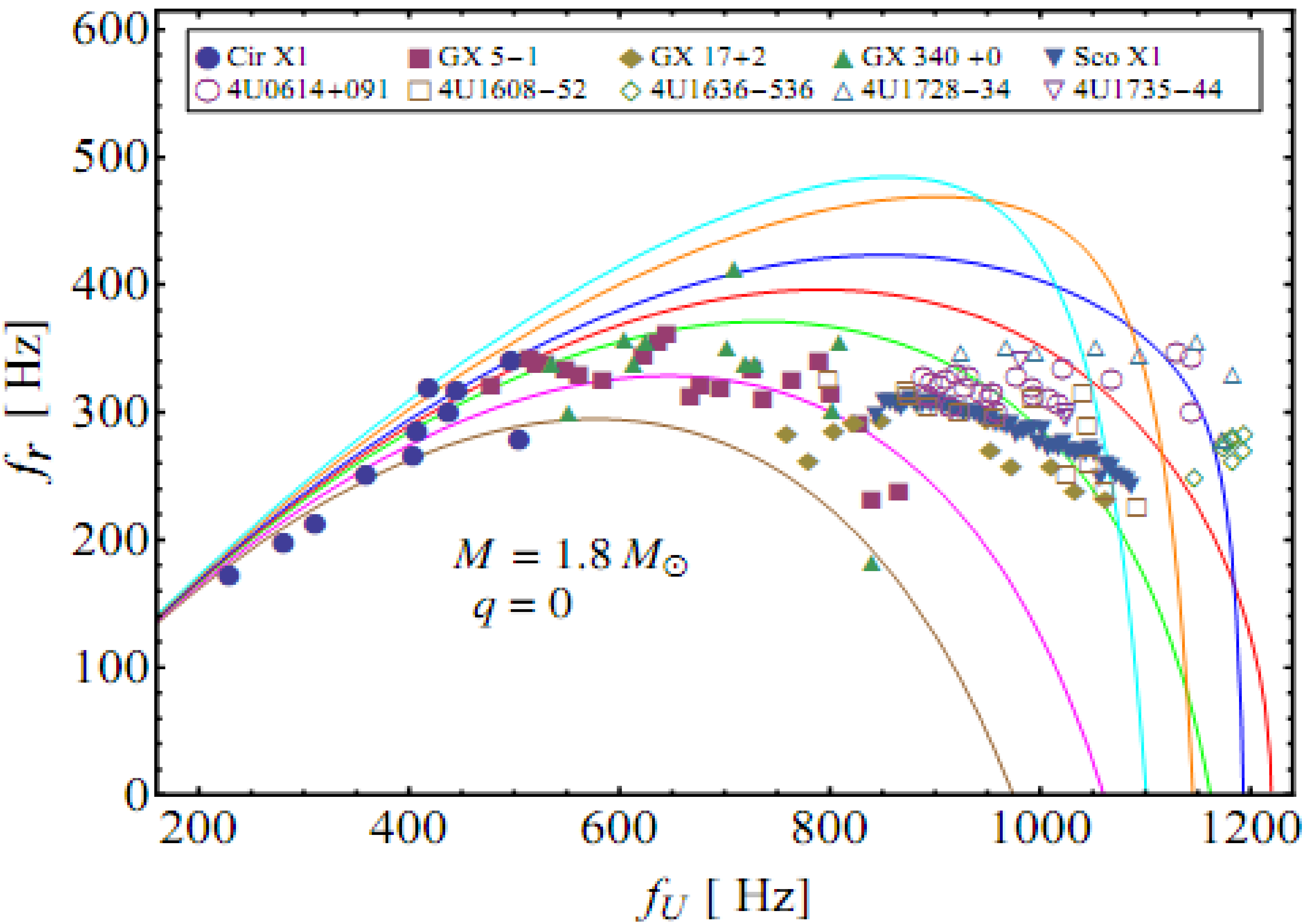
$$u = M/r \qquad j = J/M^2 \text{ and } q = Q/M^3$$

$$f_\phi(u) = \omega_K(u)/(2\pi), \qquad f_r(u) = \omega_r(u)/(2\pi), \qquad f_\theta(u) = \omega_\theta(u)/(2\pi).$$

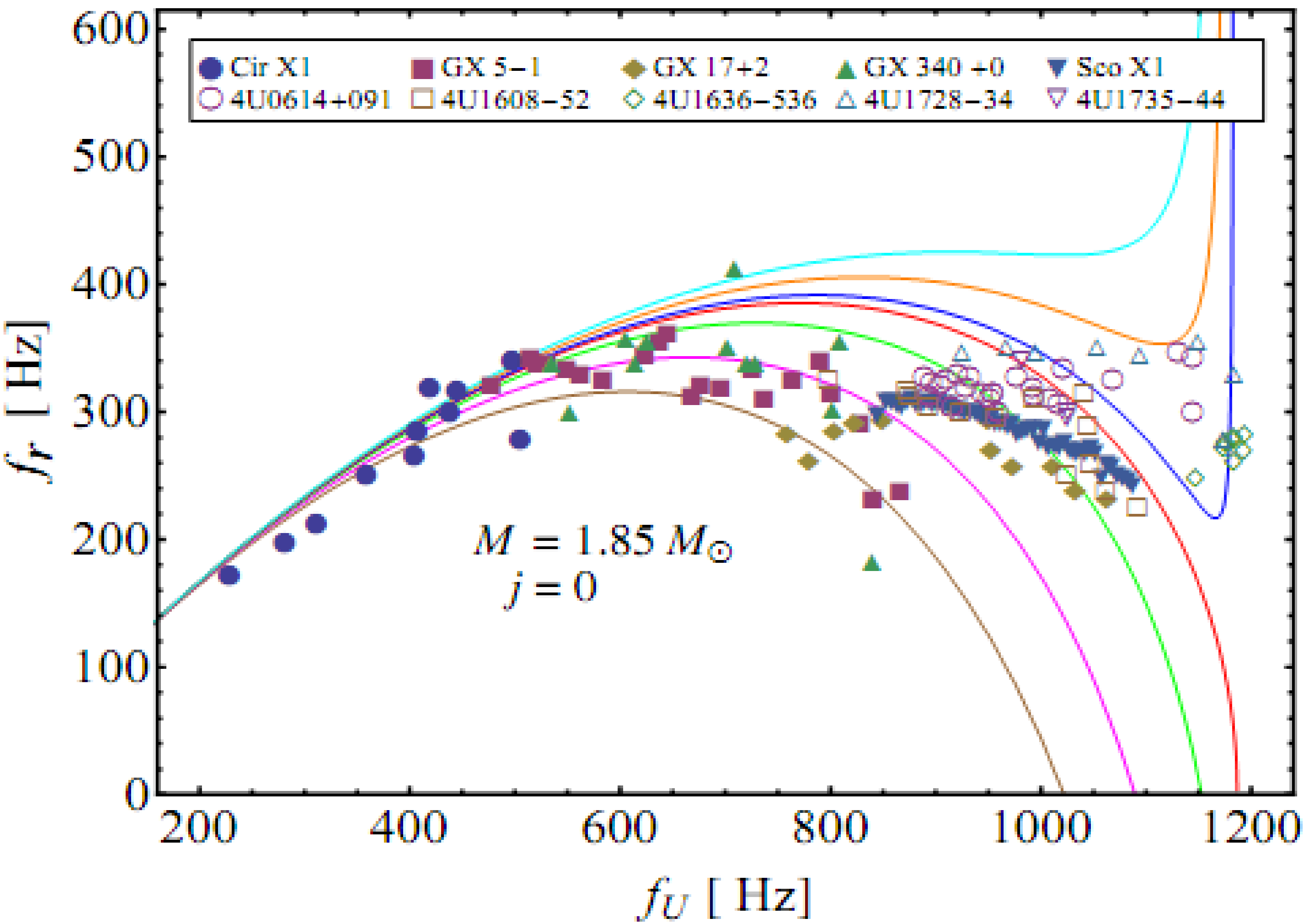

Cir X1
GX 5−1
GX 17+2
GX 340 +0
Sco X1
4U0614+091
4U1608−52
4U1636−536
4U1728−34
4U1735−44
M = 1.8 M☉
q = 0
f_U [Hz]
f_per [Hz]
0
200
400
600
800
1000
1200
1400



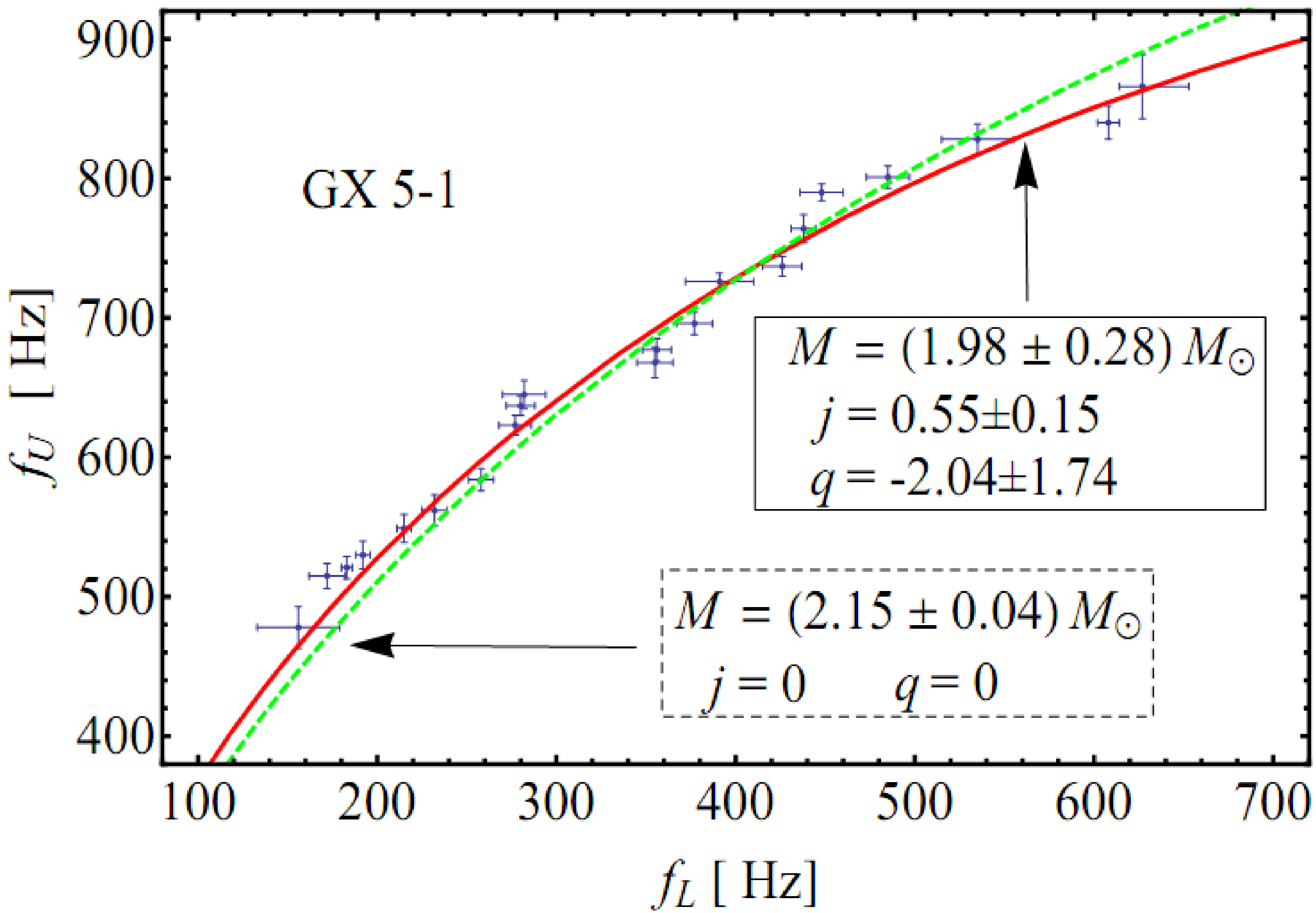
Cir X1
GX 5−1
GX 17+2
GX 340 +0
Sco X1
4U0614+091
4U1608−52
4U1636−536
4U1728−34
4U1735−44
$M = 1.84\, M_{\odot}$
$j = 0$
f_U [Hz]
f_{per} [Hz]

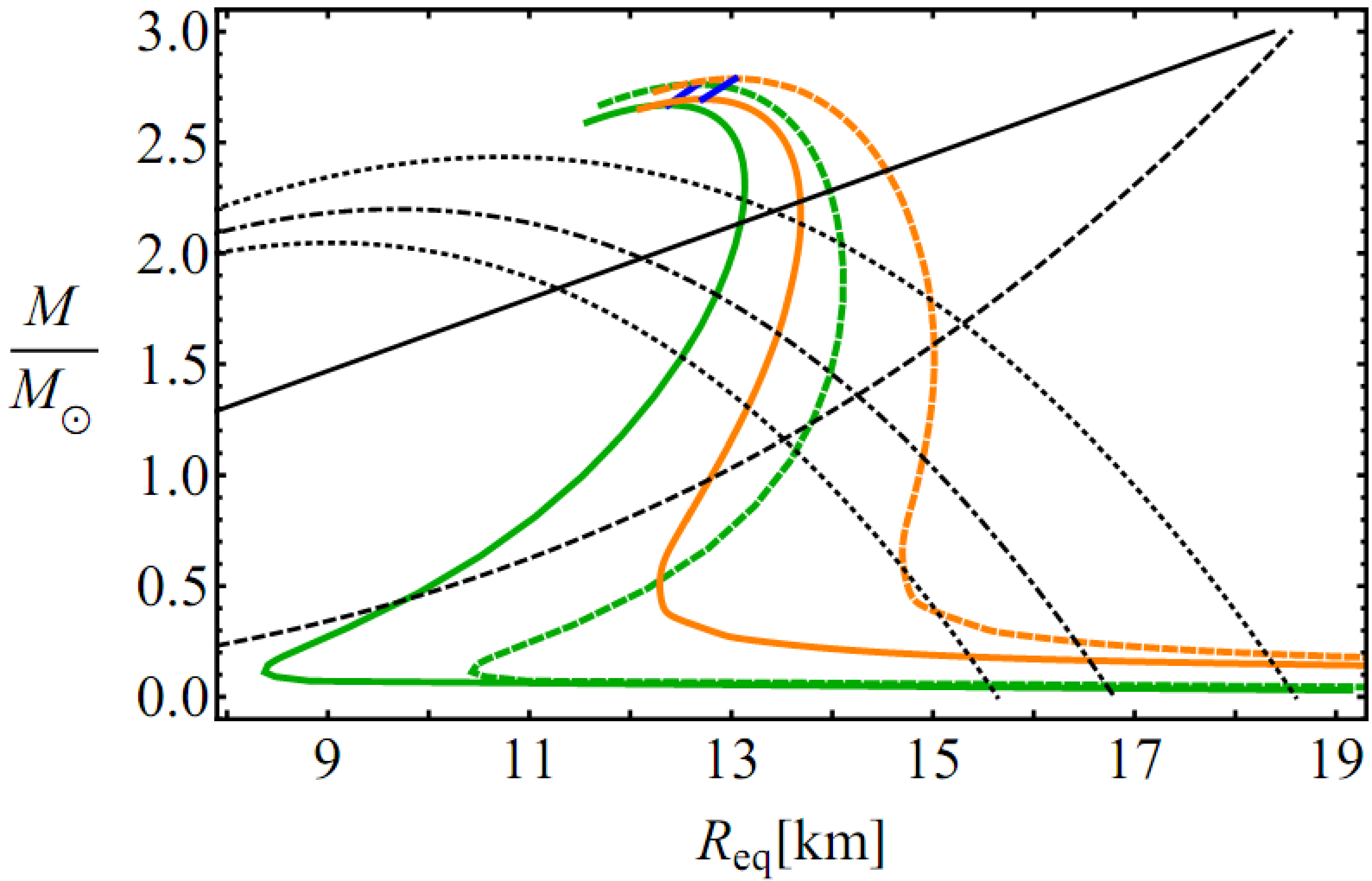


Cir X1
GX 5-1
GX 17+2
GX 340 +0
Sco X1
4U0614+091
4U1608-52
4U1636-536
4U1728-34
4U1735-44
$M = 1.8\,M_\odot$
$q = 0$
f_r [Hz]
f_U [Hz]


Cir X1
GX 5−1
GX 17+2
GX 340 +0
Sco X1
4U0614+091
4U1608−52
4U1636−536
4U1728−34
4U1735−44
M = 1.85 M⊙
j = 0
f_r [Hz]
f_U [Hz]
600
500
400
300
200
100
0
200
400
600
800
1000
1200

Data Fitting





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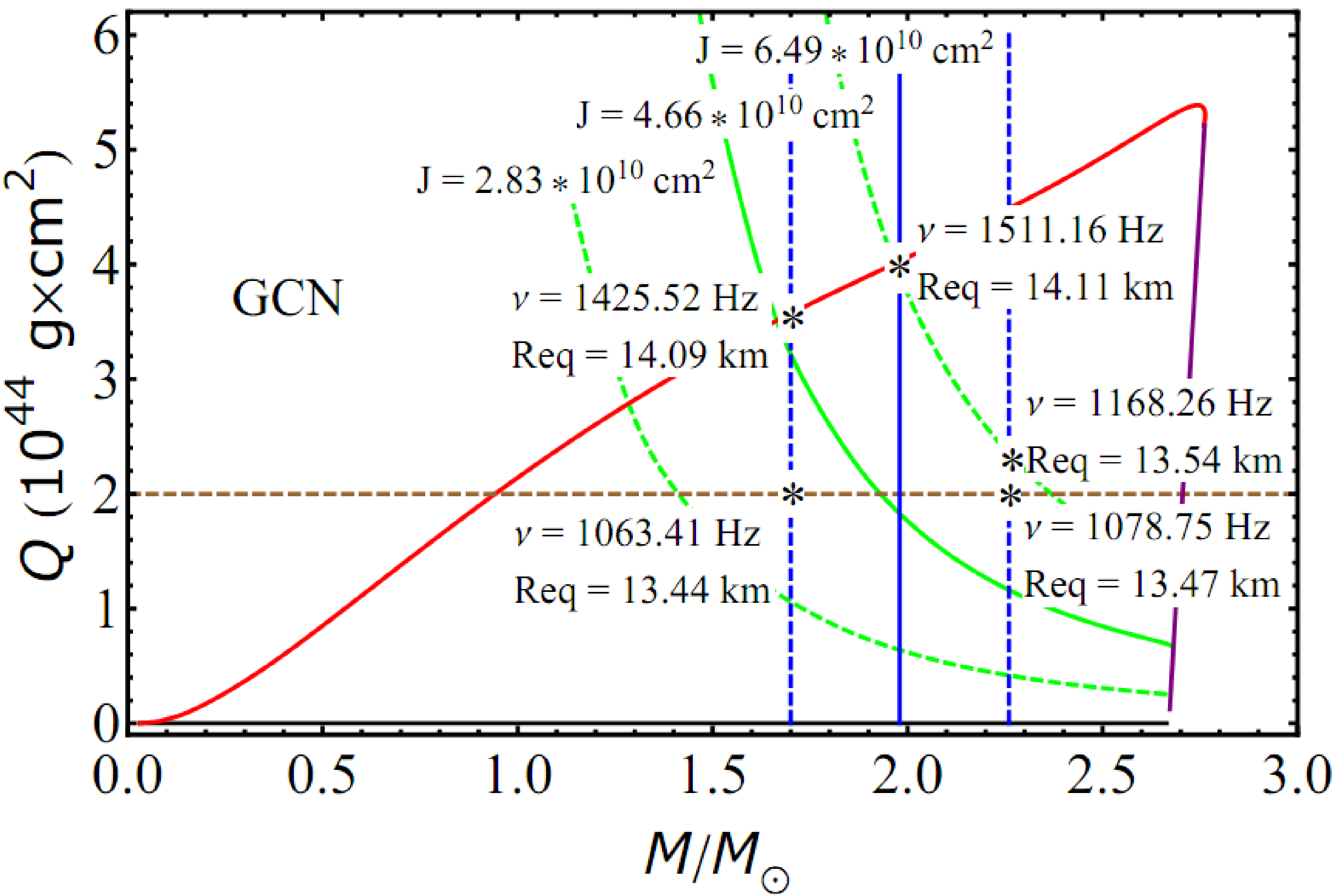
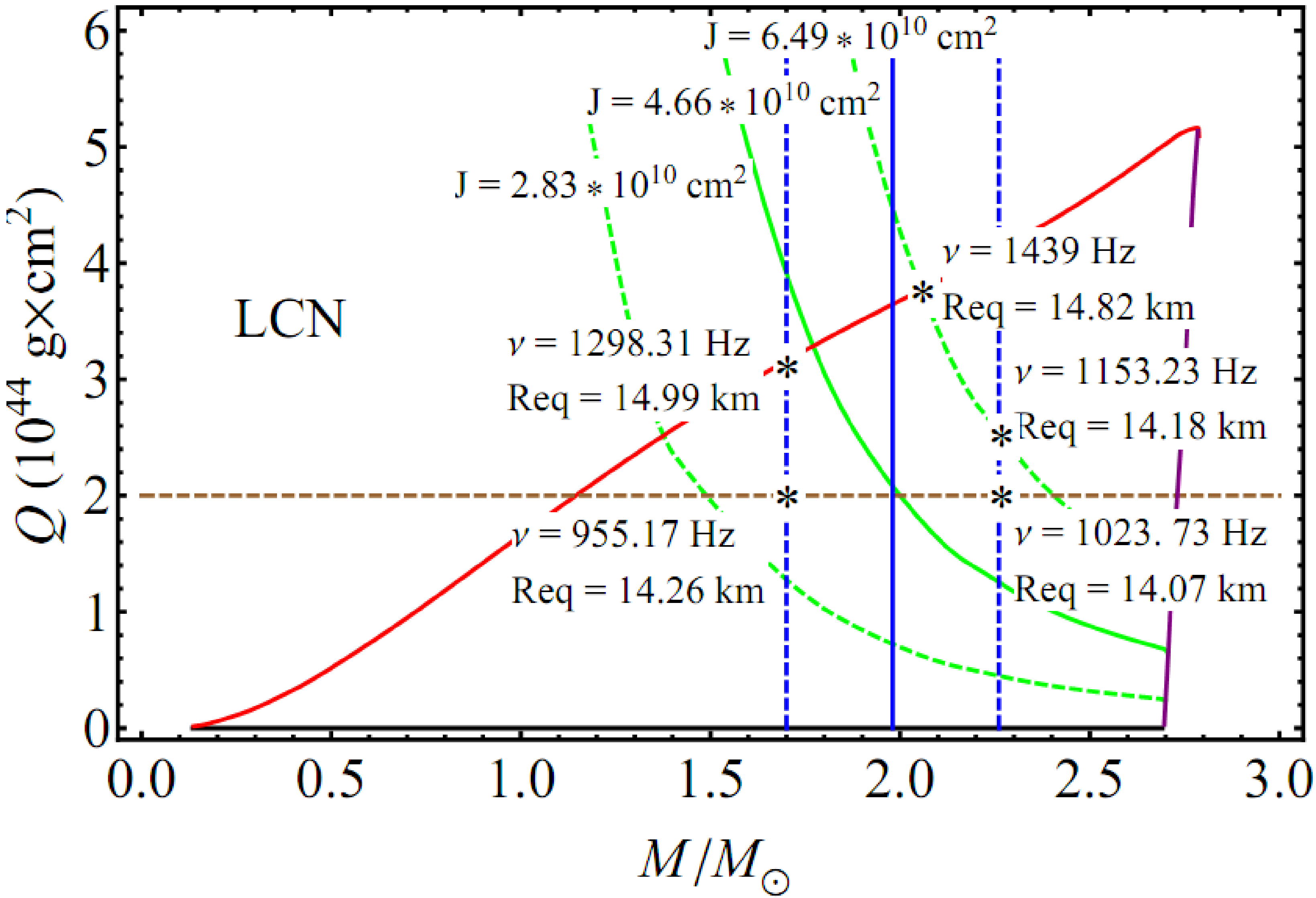
Results for GCN case



Results for LCN case



Conclusion and future prospects

- We considered equatorial circular geodesics and investigated the influence of quadrupole moment on the motion of a test particle.
- We derived the fundamental frequencies of the test particles in the Hartle-Thorne spacetime at the equatorial plane for circular orbits.
- On the basis of the RPM using the QPOs data of GX 5-1 (LMXB) and from the fitting we extracted the mass, angular momentum and quadrupole moment of the source with error bars (from observation).
- From the neutron star model of Belvedere et. al. (2012, 2014) we derived the rest parameters of the source such as radius, angular velocity (frequency) etc. of the neutron star (from theory).
- We will consider the RPM for the elliptical and inclined orbits.
- Different models for different sources.

Work in progress…

Thank you
for your kind attention!