Light hypernuclei in Halo/Cluster EFT

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in collaboration with

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Outline

- Efimov effect and limit cycle in three-body systems
- ${}_{\Lambda\Lambda}^{4}$H as $\Lambda\Lambda d$ system in halo EFT
- ${}_{\Lambda\Lambda}^{6}$He as $\Lambda\Lambda\alpha$ system in cluster EFT
- the $nn\Lambda$ bound state in pionless EFT
- Summary

Efimov effect

- Three-boson system in unitary limit
  Large two-body scattering length makes the system insensitive to details in short range physics.
- Efimov effect
  Infinitely many, arbitrarily-shallow three-body bound states (whose energies $B^{(n)}$) are accumulated at the threshold.

$$B^{(n)} = \left(e^{-2\pi/s_0}\right)^{n-n*} \kappa_*^2/m \,,$$

where $s_0 \simeq 1.00624$ and $e^{\pi/s_0} \simeq 22.7$.

Limit cycle

- Three-body systems in the asymptotic limit
  If the interaction in the integral equations is singular, the system exhibits the cyclic singularities, so called limit cycle.
- The Efimov-like bound states, which are associated with the limit cycle, emerge in the system in the asymptotic limit.

- Effective Field Theories (EFTs)
  - Model independent approach
  - Separation scale
  - Counting rules
  - Parameters should be fixed by experiments
  - For the study of three-body systems

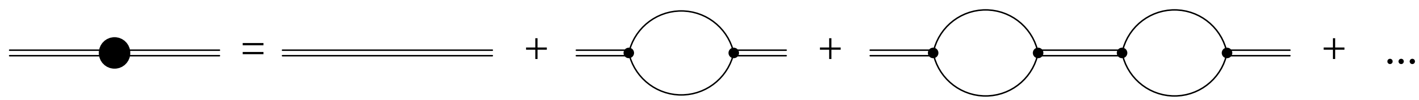
    the unitary limit or the asymptotic limit is chosen as our theoretical first approximation.

${}^{4}_{\Lambda\Lambda}$H at LO

- $\Lambda\Lambda d$ ($S=1$)
- ${}^{3}_{\Lambda}$H, $B_\Lambda = 0.13$ MeV
- d, $B_2 = 2.22$ MeV
- $S=1$ of ${}^{4}_{\Lambda\Lambda}$H shows the limit cycle and the three-body interaction is fixed by using the results of the potential models.
- Three parameters at LO, $a_{\Lambda\Lambda}$, $\gamma_{\Lambda d}$, $g(\Lambda_c)$.

Two-body part: $\Lambda\Lambda$ *in* $^1S_0$ *state*

- Dressed dibaryon propagator

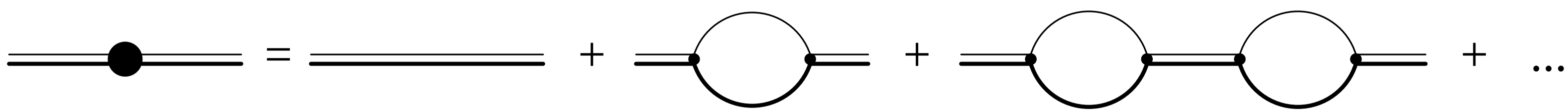


- Renormalized dressed dibaryon propagator

$$D_s(p_0,\vec{p}) = \frac{4\pi}{m_\Lambda y_s^2}\frac{1}{\frac{1}{a_{\Lambda\Lambda}} - \sqrt{-m_\Lambda p_0 + \frac{1}{4}\vec{p}^2 - i\epsilon}}\,.$$

Two-body part: Λd *in* ${}^3_\Lambda$*H channel*

- Dressed ${}^3_\Lambda$H propagator



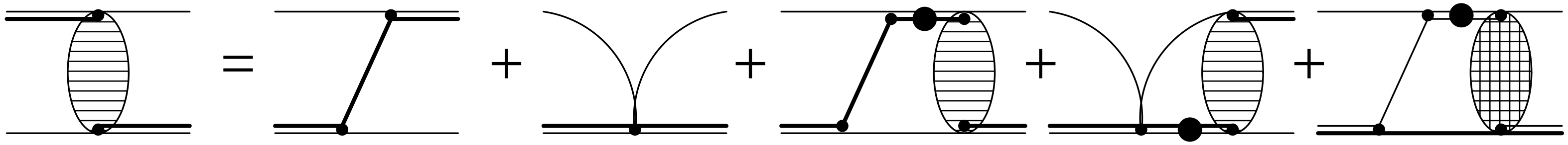
- Renormalized dressed ${}^3_\Lambda$H propagator

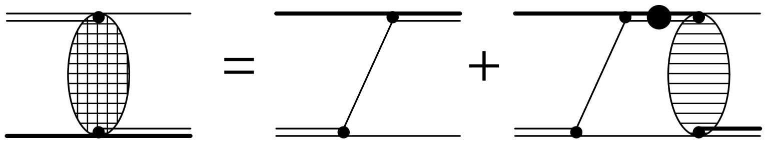
$$D_t(p_0,\vec{p}) = \frac{2\pi}{\mu_{\Lambda d} y_t^2} \frac{1}{\gamma_{\Lambda d} - \sqrt{-2\mu_{\Lambda d}\left(p_0 - \frac{1}{2(m_\Lambda + m_d)}\vec{p}^2\right)}}\,,$$

with

$$\gamma_{\Lambda d} = \sqrt{2\mu_{\Lambda d} B_\Lambda}\,.$$

Three-body part: $S = 1$ channel



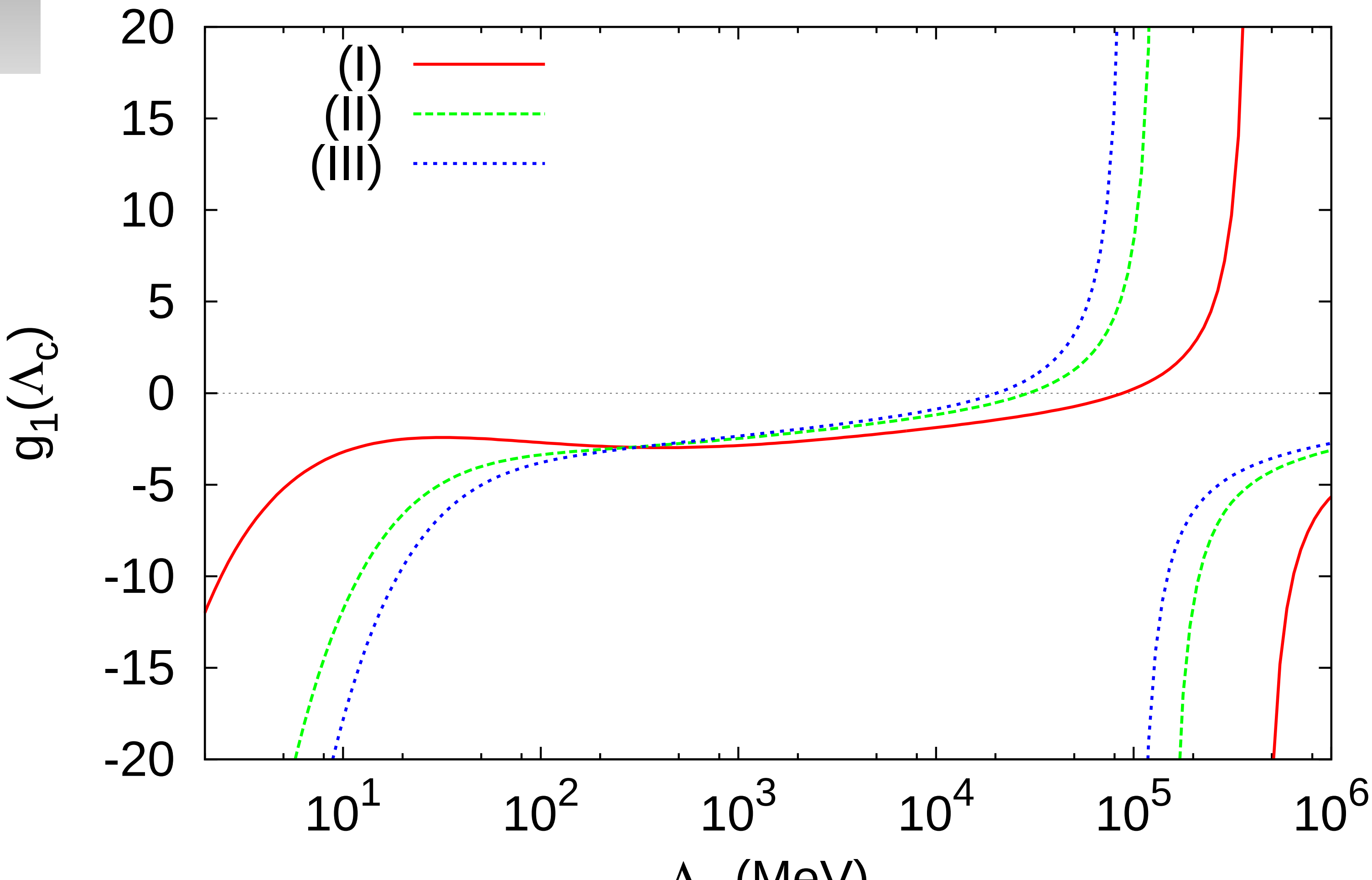


$$
\begin{aligned}
a(p,k;E) &= K_{(a)}(p,k;E) - \frac{g_1(\Lambda_c)}{\Lambda_c^2} \\
&\quad - \frac{1}{2\pi^2}\int_0^{\Lambda_c} dl l^2 \left[K_{(a)}(p,l;E) - \frac{g_1(\Lambda_c)}{\Lambda_c^2}\right] D_t\left(E - \frac{1}{2m_\Lambda}l^2, \vec{l}\right) a(l,k;E) \\
&\quad - \frac{1}{2\pi^2}\int_0^{\Lambda_c} dl l^2 K_{(b1)}(p,l;E) D_s\left(E - \frac{1}{2m_d}l^2, \vec{l}\right) b(l,k;E)\,, \\
b(p,k;E) &= K_{(b2)}(p,k;E) \\
&\quad - \frac{1}{2\pi^2}\int_0^{\Lambda_c} dl l^2 K_{(b2)}(p,l;E) D_t\left(E - \frac{1}{2m_\Lambda}l^2, \vec{l}\right) a(l,k;E)\,,
\end{aligned}
$$

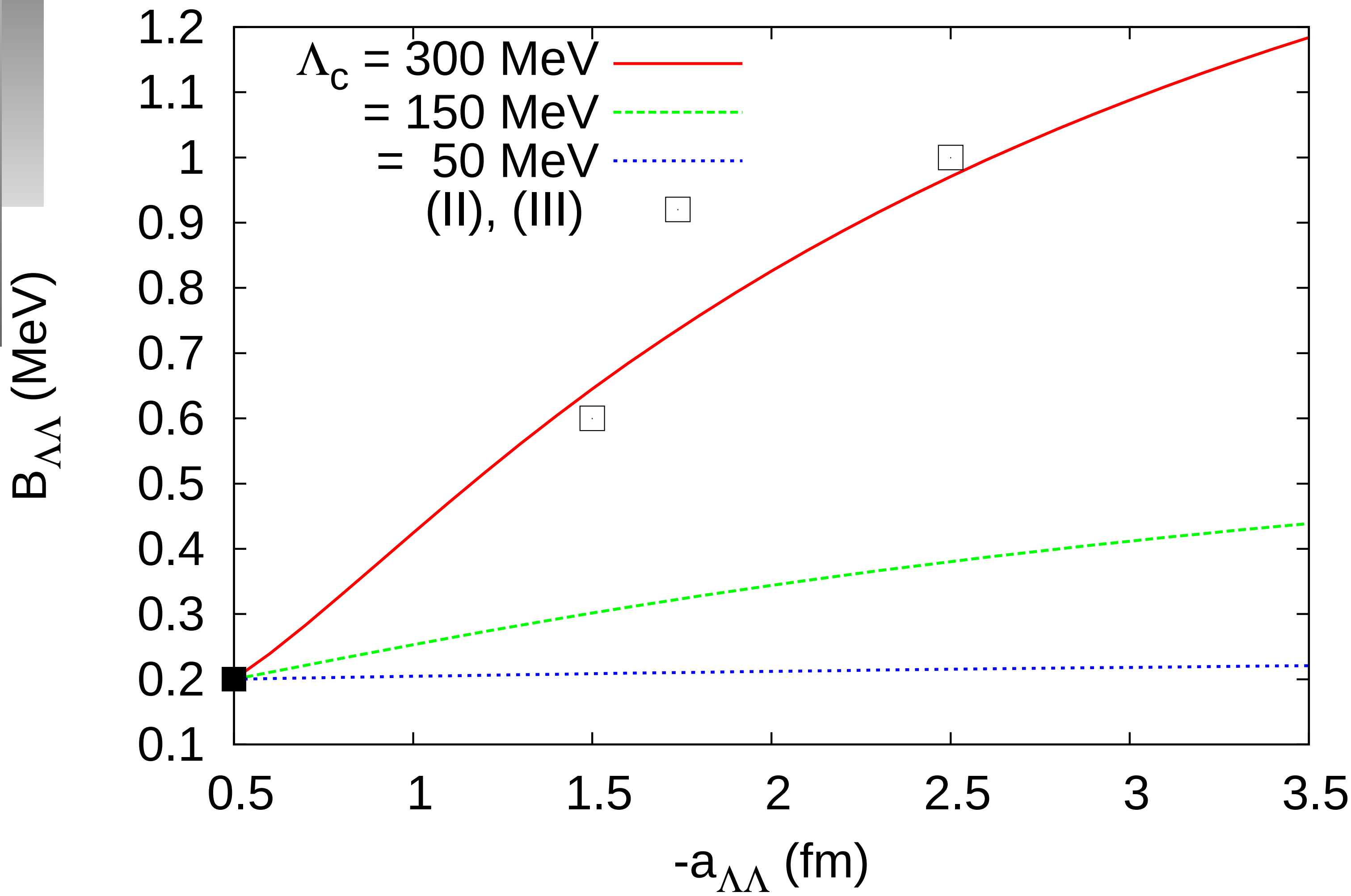
Numerical results: $S = 1$ ***channel***

- With $g_1(\Lambda_c)$,

$(B_{\Lambda\Lambda}, a_{\Lambda\Lambda})$ = (I) $(0.2$ MeV, -0.5 fm), (II) $(0.6, -1.5)$, (III) $(1.0, -2.5)$.



Numerical results: $S = 1$ ***channel***

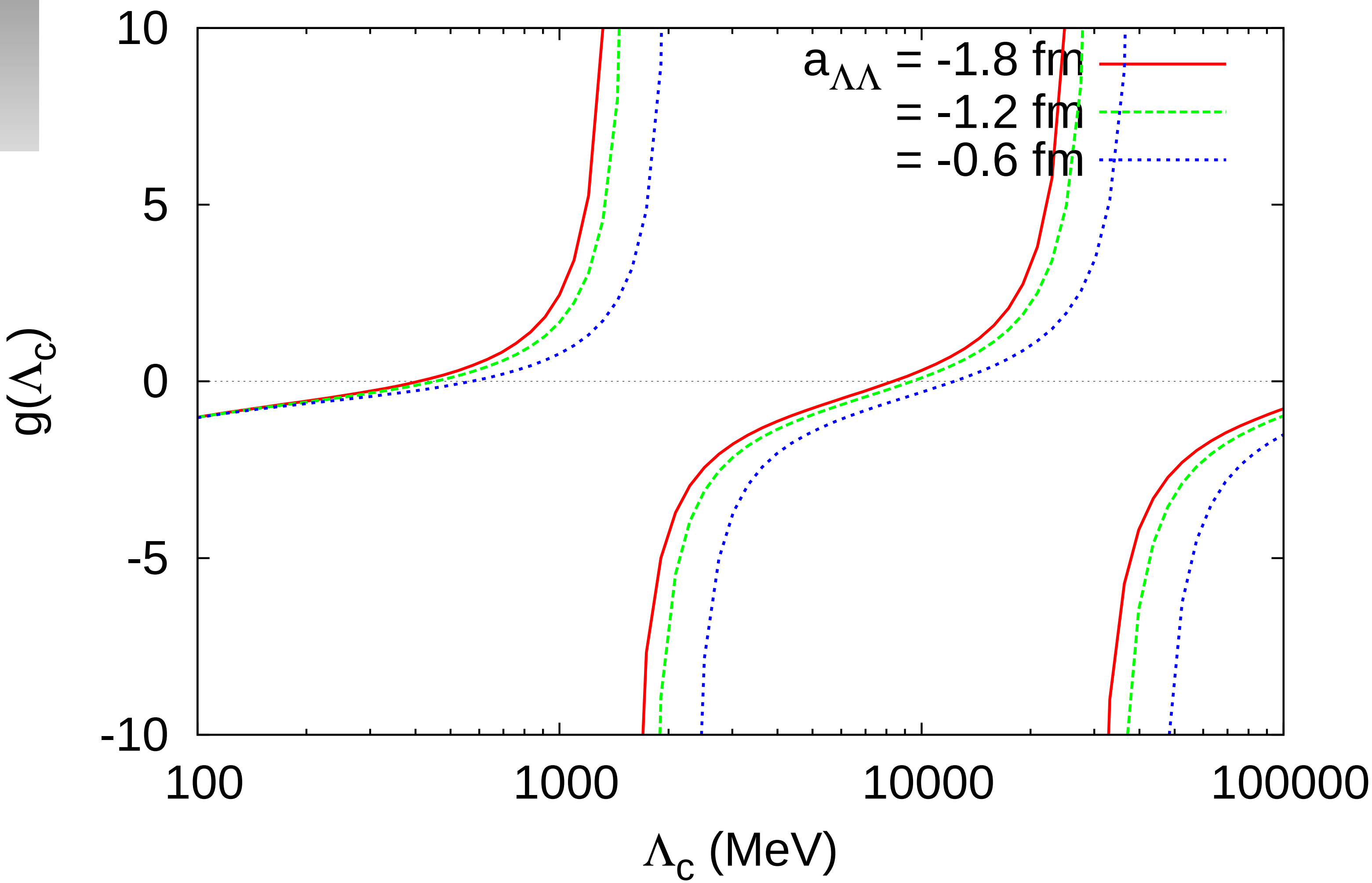


${}^{6}_{\Lambda\Lambda}$*He at LO*

- $\Lambda\Lambda\alpha$ ($S = 0$)
- First excited energy of α, $B_1 \simeq 20$ MeV
- ${}^{5}_{\Lambda}$He, $B_\Lambda \simeq 3$ MeV
- The limit cycle appears and the three-body interaction is fixed by using the Nagara event, $B_{\Lambda\Lambda} \simeq 6.93$ MeV
- Three parameters at LO: $a_{\Lambda\Lambda}$, $\gamma_{\Lambda\alpha}$, $g(\Lambda_c)$.

Numerical results:

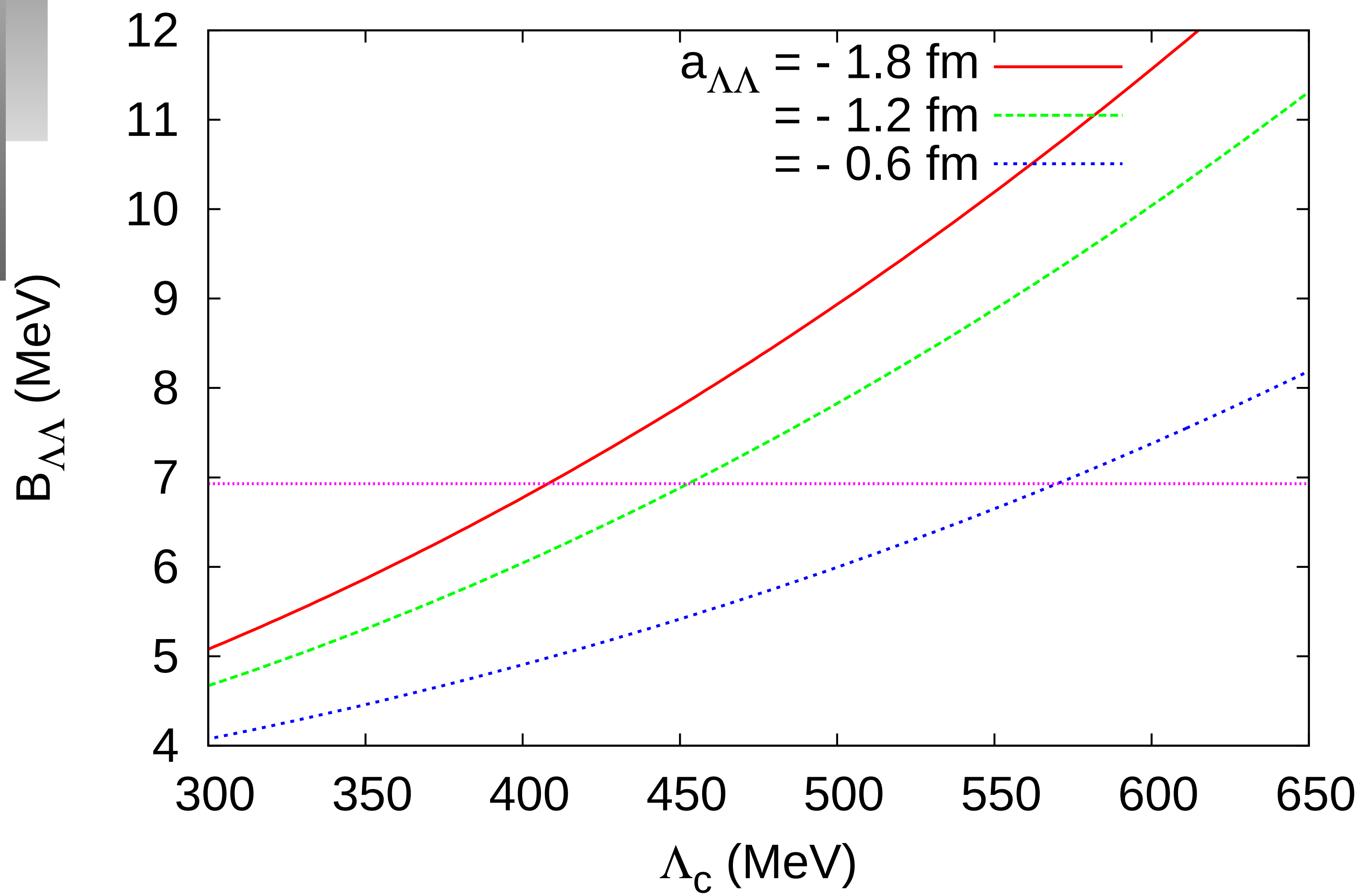
- With $g_1(\Lambda_c)$ (Input: $B_{\Lambda\Lambda}$ =6.93MeV)



$$\Lambda_n = \Lambda_0 \exp(n\pi/s_0)\,, \quad s_0 \simeq 1.05\,, \quad \exp(\pi/s_0) \simeq 19.9\,.$$

Numerical results:

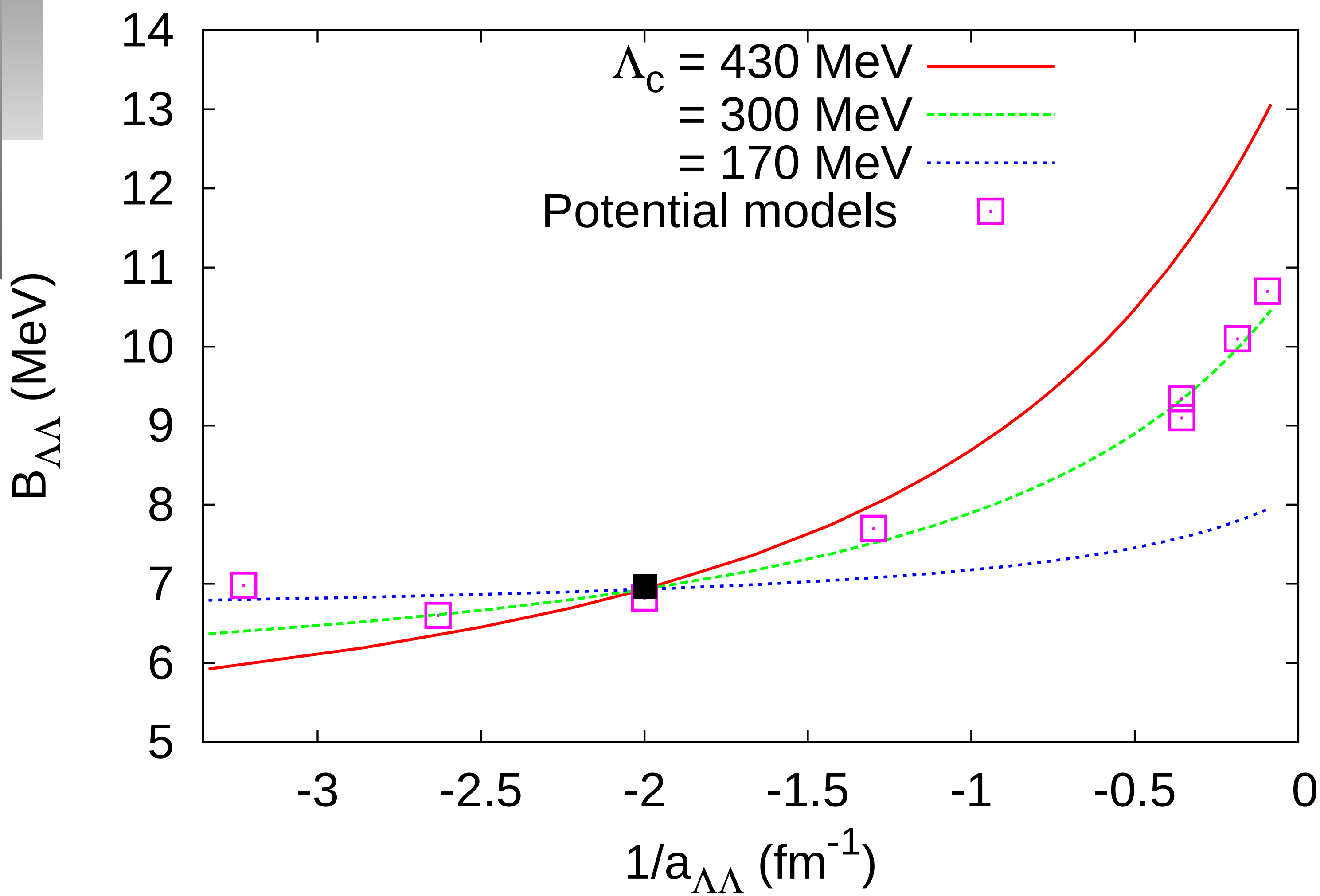
- Without $g_1(\Lambda_c)$



- $r_c = \Lambda_c^{-1} \simeq 0.35$ to 0.5 fm

Numerical results:

- With $g_1(\Lambda_c)$ (Input: $B_{\Lambda\Lambda}$ =6.93MeV, $a_{\Lambda\Lambda} = -0.5$fm)



[Filikhin and Gal, NPA707,491(2002)]

Does $nn\Lambda$ make a bound state ?

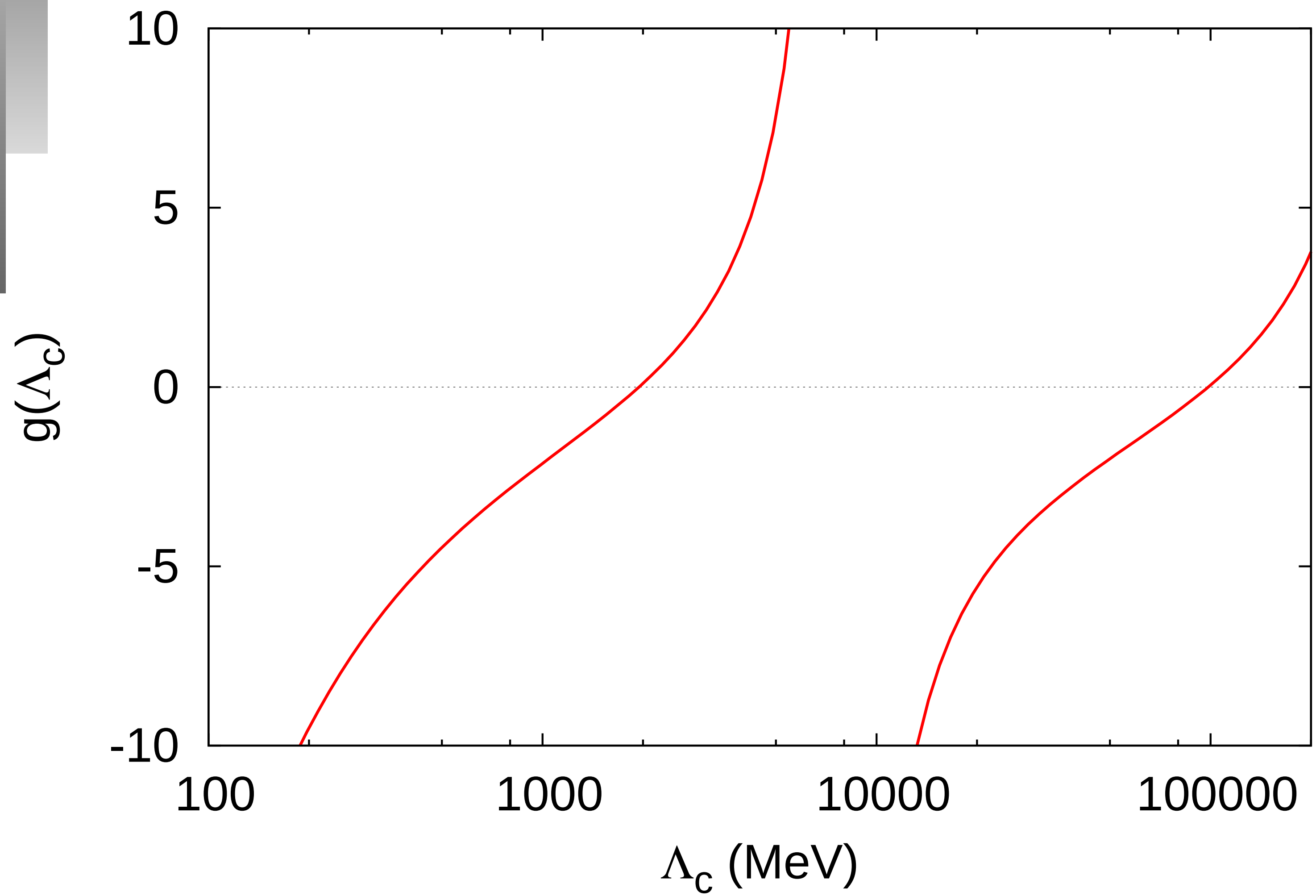
- HypHI collaboration
- Lattice QCD in flavor SU(3) limit
- Potential model calculations including the ΛN-ΣN mixing

$nn\Lambda$ ***at LO***

- $nn\Lambda$ ($S = 1/2$, $I = 1$)
- m_π and $\Delta = m(\Sigma N) - m(\Lambda N) \simeq 80$ MeV
- $B_3 \sim 0$
- Limit cycle appears in the asymptotic limit
- Four parameters at LO: a_{nn}, $a_{s(\Lambda n)}$, $a_{t(\Lambda n)}$, $g(\Lambda_c)$.

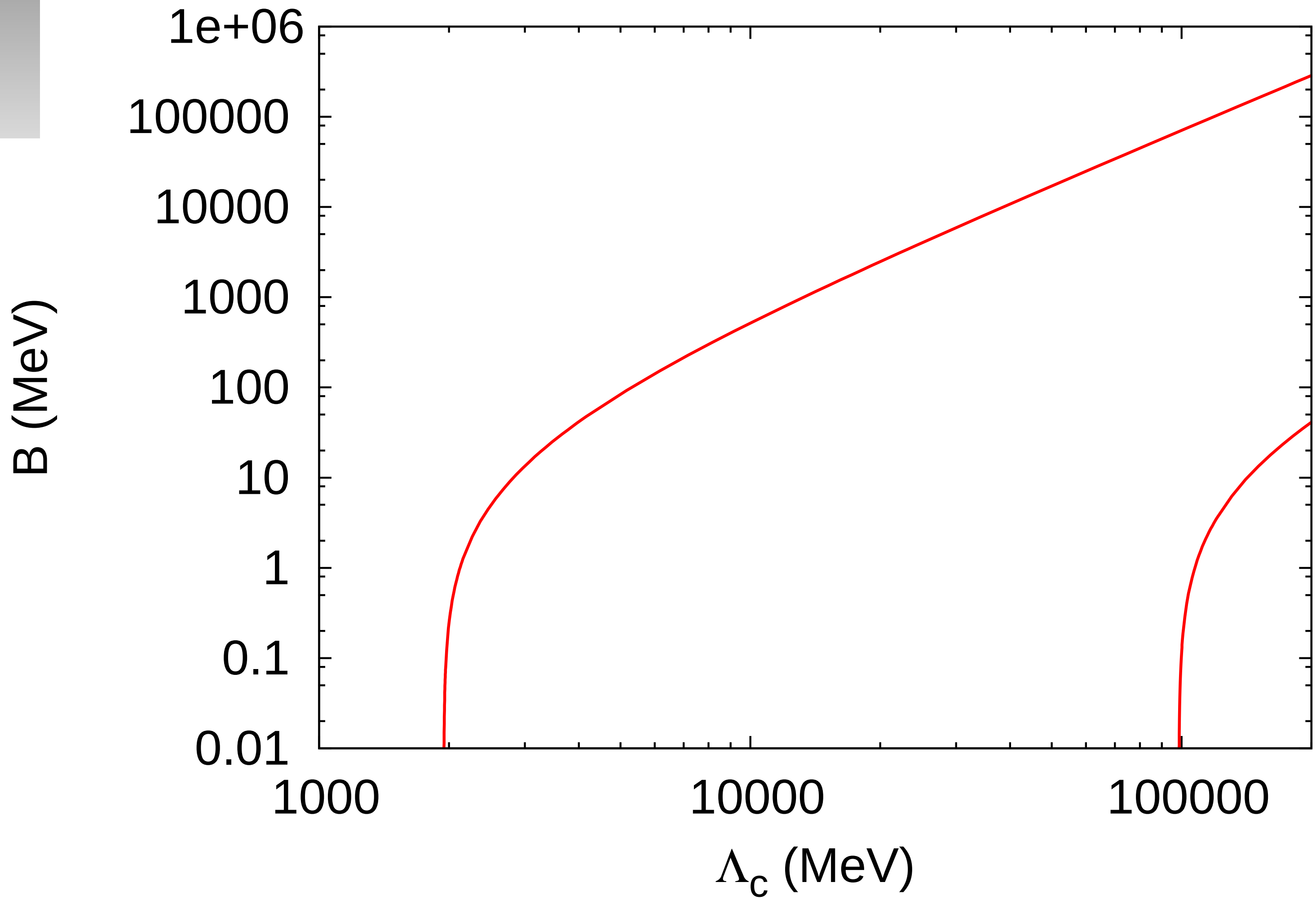
Numerical results:

- With $g_1(\Lambda_c)$ (Input: $B = 0$ MeV)



Numerical results:

- Without $g_1(\Lambda_c)$



$nn\Lambda$ *at LO*

- Uncertainties in the present calculation:
  - No exp. data for the $n\Lambda$ scattering
  - No exp. data for the three-body force
  - Higher order corrections not included
- The bound state of the $nn\Lambda$ system is formed at $\Lambda_c \simeq 2$ GeV $>> \Lambda_H \sim m_\pi$.
- Three-body bound states in other systems without the three-body contact interaction can be formed at $\Lambda_c \simeq 500$ MeV.
- Thus the $nn\Lambda$ bound state found at LO may be unrealistic.

Summary

- Halo/Cluster EFTs to describe the light hypernuclei at LO are constructed.
- Those three-body systems described by means of EFTs at LO exhibit the limit cycle which implies the formation of bound states in the asymptotic limit.
- For more conclusive results, we need to have the exp. data and include higher order corrections.