On the fluctuation-dissipation theorem for soft fermionic excitations in a hot QCD plasma

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We present the comprehensive derivation of the fluctuation-dissipation theorem for soft fermion fluctuations in a hot non-Abelian plasma being in a thermal equilibrium. A distinctive characteristic of the problem under consideration, is the fact that an external perturbation described by a Hamiltonian $\hat{H}_t^1$, changes the number of particles and antiparticles in the system, i.e., in other words, an operator of the total number of particles $\hat{N}$ (more precisely, the difference between the number of particles and antiparticles) does not commute with the $\hat{H}_t^1$ (but it commutes with a Hamiltonian of the many-particle system $\hat{H}_0$).

Such a case has already been discussed in the general statement in textbook materials, but as far as we know, any specific physical situation, where this circumstance would play crucial role, has not been considered. As a consequence of this fact, a concrete expression for the fluctuation-dissipation theorem in which this noncommutativity could be manifested by obvious fashion, has not been given anywhere.

Callen-Velton fluctuation-dissipation theorem (1951). Boson case

Let $\hat{F}_a(t,\mathbf{x})$, $a=1,2,\ldots$ be quantities describing behaviour of the system under certain external forces. The latter are described by the functions $f_a(t,\mathbf{x})$. The interaction energy operator has the form

$$\hat{H}^1_t = -\sum_a \int d\mathbf{x}\, f_a(t,\mathbf{x})\hat{F}_a(t,\mathbf{x}), \quad \hat{H}^1_t\Big|_{t=-\infty} = 0.$$

Let us denote the mean values of the quantities $\hat{F}_a(t,\mathbf{x})$, which are linear functionals of the forces $f_a(t,\mathbf{x})$, by $\bar{F}_a(t,\mathbf{x})$. In terms of Fourier components we write

$$\bar{F}_a(\omega,\mathbf{k}) = \sum_b \kappa_{ab}(\omega,\mathbf{k}) f_b(\omega,\mathbf{k}),$$

where $\kappa_{ab}(\omega,\mathbf{k})$ are the generalized (complex) susceptibilities of the system under consideration relating with $\hat{F}_a$ by the Kubo formula:

$$\kappa_{ab}(\omega,\mathbf{k}) = \frac{i}{\hbar}\int\limits_0^{\infty} dt\, \mathrm{e}^{i\omega t}\int\limits_{-\infty}^{+\infty} d\mathbf{x}\, \mathrm{e}^{-i\mathbf{k}\cdot\mathbf{x}}\big\langle\big[\hat{F}_a(t,\mathbf{x}),\hat{F}_b(0,\mathbf{0})\big]\big\rangle.$$

Callen-Velton fluctuation-dissipation theorem (1951). The FDT-relation

Further we introduce a correlation function of the fluctuating quantities $\hat{F}_a$:

$$\Phi_{ab} \equiv \frac{1}{2}\left\langle \left\{ \hat{F}_a(t, \mathbf{x}), \hat{F}_b(0, \mathbf{0}) \right\} \right\rangle,$$

the Fourier-image of which is denoted by $(F_a F_b)_{\omega \mathbf{k}}$. Then the fluctuation-dissipation theorem states

$$(F_a F_b)_{\omega \mathbf{k}} = -\frac{i\hbar}{2}\left(\kappa_{ab}(\omega, \mathbf{k}) - \kappa^*_{ba}(\omega, \mathbf{k})\right) \coth\left(\frac{\hbar\omega}{2k_B T}\right).$$

The FDT-theorem relates characteristics of the dissipative process to spectral density of equilibrium fluctuations in the system.

Thus, it expresses nonequilibrium properties through equilibrium ones.

In the case of electromagnetic field in a medium the interaction energy is

$$\hat{H}^1_t = -\frac{1}{c}\sum_{\mu=0}^{3}\int d\mathbf{x}\, j^{\mu}_{\text{ext}}(t, \mathbf{x}) \hat{A}_\mu(t, \mathbf{x}).$$

Identification is $f_a(t, \mathbf{x}) \to j_\mu(t, \mathbf{x})$, $\hat{F}_a(t, \mathbf{x}) \to \hat{A}_\mu(t, \mathbf{x})$, $\kappa_{ab}(\omega, \mathbf{k}) \to -\frac{D^{(R)}_{\mu\nu}(\omega, \mathbf{k})}{\hbar c^2}$.

FDT for soft Bose-fluctuations

Let us introduce a spectral density for soft bosonic fluctuation by means of the relation

$$\left\langle A_\mu^{*a}(k) A_\nu^b(k') \right\rangle = (A_\mu^{*a} A_\nu^b)_{\omega\mathbf{k}}\, \delta^{(4)}(k-k'), \quad k = (\omega, \mathbf{k}). \tag{1}$$

The spectrum of soft gluonic modes in an equilibrium plasma can be defined through the (quantum) fluctuation-dissipation theorem:

$$(A_\mu^{*a} A_\nu^b)_{\omega\mathbf{k}} = -\frac{1}{(2\pi)^4}\, i\delta^{ab} \left(\frac{\Theta_B(\omega,T)}{\omega}\right) \left\{ \mathcal{D}_{\mu\nu}^{(R)}(k) - (\mathcal{D}_{\nu\mu}^{(R)}(k))^* \right\}, \tag{2}$$

where

$$\Theta_B(\omega,T) = \frac{\hbar\omega}{2} \coth\left(\frac{\hbar\omega}{2k_B T}\right)$$

is the mean energy of a quantum boson oscillator.

FDT for soft Bose-fluctuations. *Semiclassical limit*

In the semiclassical limit the photon (gluon) retarded Green's function has the form:

$$ {}^*\mathcal{D}^{(R)}_{\mu\nu}(k) = -P_{\mu\nu}(k)\,{}^*\Delta^t(k) - Q_{\mu\nu}(k)\,{}^*\Delta^l(k) + \,\ldots . $$

In the covariant gauge the longitudinal projector $Q_{\mu\nu}(k)$ in the rest frame of a heat bath is equal to

$$ Q_{\mu\nu}(k) = -\frac{1}{k^2}\begin{pmatrix} \mathbf{k}^2 & \omega\mathbf{k} \\ \omega\mathbf{k} & \omega^2\,\dfrac{\mathbf{k}\otimes\mathbf{k}}{\mathbf{k}^2} \end{pmatrix};\; P^2 = P,\; Q^2 = Q,\; PQ = QP = 0. $$

In the limit $\hbar \rightarrow 0$ the FDT-relation (2) reduces to

$$ (A^{*a}_{\mu}A^{b}_{\nu})_{\omega\mathbf{k}} = \frac{1}{(2\pi)^4}\,\delta^{ab}\left(\frac{2k_BT}{\omega}\right)\mathrm{Im}({}^*\mathcal{D}^{(R)}_{\mu\nu}(k)) = \frac{1}{(2\pi)^4}\,\delta^{ab}\left(\frac{2k_BT}{\omega}\right)\times $$
$$ \times\Big\{P_{\mu\nu}(k)\,\mathrm{Im}({}^*\Delta^{-1\,t}(k))^*\,|{}^*\Delta^t(k)|^2 + Q_{\mu\nu}(k)\,\mathrm{Im}({}^*\Delta^{-1\,l}(k))^*\,|{}^*\Delta^l(k)|^2\Big\}. \quad (3) $$

FDT for soft Bose-fluctuations. *Classical model*

Let us calculate the spectral density for bosonic fluctuations in the hot non-Abelian plasma within the classical model suggested by Ulrich Heinz (1983) and compare the obtained expression for spectral density with a similar expression resulting from the FDT for fluctuations of a gauge field.

According to this classical model, the soft gauge field A^a_μ induced by a hard test particle (which is centered at the position $\mathbf{x}_0$) in the momentum representa- tion is

$$A^a_\mu(k) = -\,{}^*\mathcal{D}^{(R)}_{\mu\nu}(k)j^{a\nu}(k;\mathbf{x}_0), \quad k = (\omega, \mathbf{k}), \tag{4}$$

where in turn,

$$j^{a\nu}(k;\mathbf{x}_0) = \frac{g}{(2\pi)^3}\, v^\nu Q^a \delta(v\cdot k)\, \mathrm{e}^{-i\mathbf{k}\cdot\mathbf{x}_0},\ v = (1, \mathbf{v}) \tag{5}$$

is the current of the hard color-charged particle. Here, $\mathbf{v}$ and Q^a are the velocity and classical color charge of the hard particle, respectively.

The correlation function of soft Bose-fluctuations within the framework of the classical model can be written as follows

$$\big\langle A^{*a}_{\mu}(k)A^{b}_{\nu}(k')\big\rangle = 2\,\frac{g^2}{(2\pi)^6}\sum_{\zeta=\mathrm{Q},\,\bar{\mathrm{Q}},\,\mathrm{G}}\int dQ\,Q^aQ^b\!\int d\mathbf{x}_0\,\mathrm{e}^{i(\mathbf{k}-\mathbf{k}')\cdot\,\mathbf{x}_0} \tag{6}$$

$$\times\int \mathbf{p}^2 f^{(\zeta)}_{|\mathbf{p}|}\,\frac{d|\,\mathbf{p}|}{2\pi^2}\,\Big({}^{*}\mathcal{D}^{(R)}_{\mu\mu'}(k)\Big)^{*}\,\Big({}^{*}\mathcal{D}^{(R)}_{\nu\nu'}(k')\Big)\int\frac{d\Omega_{\mathbf{v}}}{4\pi}\,v^{\mu'}v^{\nu'}\,\delta(v\cdot k)\delta(v\cdot k').$$

It was shown that the bosonic spectral density from the above expression exactly reproduces spectral density (3) if in the averaging procedure we replace the *classical statistical factor* in (6) by the *quantum* one by the rule:

$$\int \mathbf{p}^2 f^{(\zeta)}_{|\mathbf{p}|}\,\frac{d|\mathbf{p}|}{2\pi^2}\;\Rightarrow\;\begin{cases}\displaystyle\int \mathbf{p}^2 f^{(\mathrm{G})}_{|\mathbf{p}|}\Big(1+f^{(\mathrm{G})}_{|\mathbf{p}|}\Big)\frac{d|\mathbf{p}|}{2\pi^2}, & \zeta=\mathrm{G},\\[2ex] \displaystyle N_f\!\int \mathbf{p}^2 f^{(\mathrm{Q},\,\bar{\mathrm{Q}})}_{|\mathbf{p}|}\Big(1-f^{(\mathrm{Q},\,\bar{\mathrm{Q}})}_{|\mathbf{p}|}\Big)\frac{d|\mathbf{p}|}{2\pi^2}, & \zeta=\mathrm{Q},\ \bar{\mathrm{Q}},\end{cases}$$

FDT for soft Bose-fluctuations. *Color charge average*

and we choose the quark and gluon quadratic Casimirs in the average with respect to the color charges in the following form

$$\int dQ\, Q^a Q^b = \left(\frac{C_2^{(\zeta)}}{d_A}\right)\delta^{ab},$$

$$C_2^{(\mathsf{Q},\bar{\mathsf{Q}})} \equiv \mathrm{tr}(t^a t^a) = T_F d_A, \quad C_2^{(\mathsf{G})} \equiv \mathrm{tr}(T^a T^a) = C_A d_A,$$

where $d_A = N_c^2 - 1$ is dimension of the gauge group $SU(N_c)$; $T_F \equiv 1/2$ and $C_A \equiv N_c$ are the group invariants.

The formula of averaging over the direction of velocity is

$$\int \frac{d\Omega_{\mathbf{v}}}{4\pi}\, v^\mu v^\nu \delta(v\cdot k) = a_t P^{\mu\nu}(k) + a_l\, Q^{\mu\nu}(k) + \ldots,$$

where

$$a_l \equiv Q^{\mu\nu}(k)\int \frac{d\Omega_{\mathbf{v}}}{4\pi}\, v_\mu v_\nu\, \delta(v\cdot k) = -\frac{k^2}{|\mathbf{k}|^3}\,\theta(\mathbf{k}^2-\omega^2)$$

and the coefficient function a_t can be found similarly.

FDT for soft Fermi-fluctuations. *Pseudoclassical model*

Soft spinor field ψ^i_α induced by a test spin-1/2 particle is

$$\psi^i_\alpha(q) = -\,{}^*S^{(R)}_{\alpha\beta}(q)\,\eta^i_\beta(q;\mathbf{x}_0), \quad q = (\omega, \mathbf{q}), \tag{7}$$

where a color source η^i_β has the form

$$\eta^i_\beta(q;\mathbf{x}_0) = \frac{g}{(2\pi)^3}\,\theta^i\chi_\beta\,\delta(v\cdot q)\,\mathrm{e}^{-i\mathbf{q}\cdot\mathbf{x}_0} \tag{8}$$

and, respectively, the propagator for the ψ-field is

$${}^*S^{(R)}(q) = h_+(\hat{\mathbf{q}})\,{}^*\triangle_+(q) + h_-(\hat{\mathbf{q}})\,{}^*\triangle_-(q),$$

where the matrix functions $h_\pm(\hat{\mathbf{q}}) = (\gamma^0 \mp \hat{\mathbf{q}}\cdot\boldsymbol{\gamma})/2$ with $\hat{\mathbf{q}} \equiv \mathbf{q}/|\mathbf{q}|$, are the spinor projectors onto eigenstates of helicity, and

$${}^*\triangle_\pm(q) = -\frac{1}{q^0 \mp [\,|\mathbf{q}| + \delta\Sigma_\pm(q)]}.$$

Evaluation of correlation function of soft Fermi-excitations within the limits of pseudoclassical model is written in the form

$$\langle \psi^i_\alpha(q)\bar{\psi}^j_\beta(-q')\rangle = -\frac{g^2}{(2\pi)^6}\sum_{\zeta=\mathrm{Q},\,\bar{\mathrm{Q}}}\int d\theta d\theta^\dagger\,\theta^{\dagger i}\theta^j \int \mathbf{p}^2\Big[\,f^{(\zeta)}_{|\mathbf{p}|} + f^{(\mathrm{G})}_{|\mathbf{p}|}\,\Big]\frac{d|\,\mathbf{p}|}{2\pi^2}\,\times$$

$$\times\int d\mathbf{x}_0\,\mathrm{e}^{i(\mathbf{q}-\mathbf{q}')\cdot\,\mathbf{x}_0}\int\frac{d\Omega_{\mathbf{v}}}{4\pi}\,({}^*S(q)\chi)_\alpha\,\big(\bar{\chi}\,{}^*S(-q')\big)_\beta\,\delta(v\cdot q)\,\delta(v\cdot q').$$

The Grassmann color charge integration measure is

$$d\theta d\theta^\dagger \equiv \left(\prod_{i=1}^{N_c} d\theta^i d\theta^{\dagger i}\right) f(\theta^\dagger\theta), \quad \theta^\dagger\theta \equiv \sum_{i=1}^{N_c}\theta^{\dagger i}\theta^i,$$

$$f(\theta^\dagger\theta) = \frac{1}{N_c!}\left\{(C^{(\zeta)}_\theta)^{N_c} + (C^{(\zeta)}_\theta)^{N_c-1}\theta^\dagger\theta + \ldots + C^{(\zeta)}_\theta(\theta^\dagger\theta)^{N_c-1} + (\theta^\dagger\theta)^{N_c}\right\}.$$

In particular, $\int d\theta d\theta^\dagger\,\theta^{\dagger i}\theta^j = (C^{(\zeta)}_\theta/N_c)\delta^{ij}$.

FDT for soft Fermi-fluctuations. *Spectral density*

Within the limits of the suggested model expression for the correlation function of the spinor field fluctuations has the form

$$\left\langle \psi^i_\alpha(q)\bar{\psi}^j_\beta(-q')\right\rangle = (\psi^i_\alpha\bar{\psi}^j_\beta)_{\omega\mathbf{q}}\,\delta^{(4)}(q-q'),$$

where

$$(\psi^i_\alpha\bar{\psi}^j_\beta)_{\omega\mathbf{q}} = \delta^{ij}\,\frac{1}{2(2\pi)^3}\,\frac{\omega_0^2}{2|\mathbf{q}|}\,\theta(\mathbf{q}^2-\omega^2)\times \tag{9}$$

$$\times\left[\left(1-\frac{\omega}{|\mathbf{q}|}\right)(h_+(\hat{\mathbf{q}}))_{\alpha\beta}\,|{}^*\triangle_+(q)|^2 + \left(1+\frac{\omega}{|\mathbf{q}|}\right)(h_-(\hat{\mathbf{q}}))_{\alpha\beta}\,|{}^*\triangle_-(q)|^2\right]$$

is the spectral density for soft quark fluctuations, and

$$\omega_0^2 = -\frac{g^2}{4\pi^2}\sum_{\zeta=\mathsf{Q},\,\bar{\mathsf{Q}}}\left(\frac{C_\theta^{(\zeta)}}{N_c}\right)\int|\mathbf{p}|\left[\,f^{(\zeta)}_{|\mathbf{p}|}+f^{(\mathsf{G})}_{|\mathbf{p}|}\,\right]d|\,\mathbf{p}|$$

is the plasma frequency of the quark sector of plasma excitations and

$$C_\theta^{(\zeta)} \equiv -C_F N_c.$$

FDT for soft Fermi-fluctuations. *Exact quantum expression*

Our many-body system is described by the following Hamiltonian $\hat{H}(t) = \hat{H}_0 + \hat{H}_t^1$ with the interaction term (in the Schrödinger picture)

$$\hat{H}_t^1 = \int d\mathbf{x} \Big[\bar{\eta}^i_{\alpha,\,\rm ext}(\mathbf{x},t)\hat{\psi}^i_\alpha(\mathbf{x}) + \hat{\bar{\psi}}^i_\alpha(\mathbf{x})\eta^i_{\alpha,\,\rm ext}(\mathbf{x},t) \Big], \quad \hat{H}_t^1\Big|_{t=-\infty} = 0.$$

Here, $\bar{\eta}^i_{\alpha,\,\rm ext}(\mathbf{x},t)$ and $\eta^i_{\alpha,\,\rm ext}(\mathbf{x},t)$ play a role of the generalized Grassmann external forces. We introduce the double-time Green's functions for soft ψ-fields as follows

$$S^{(R,\,A)\,ij}_{\alpha\beta}(x-x') = \pm\,\frac{1}{i\hbar}\,\theta(\pm(t-t'))\,\mathrm{Sp}\Big(\hat{\rho}_0\Big\{\hat{\psi}^i_\alpha(\mathbf{x},t), \hat{\bar{\psi}}^j_\beta(\mathbf{x}',t')\Big\}\Big)\,. \quad (10)$$

Here, the average is taken over the Gibbs large canonical distribution for a system consisting of the medium and the radiation of soft fermionic excitations, being at thermal equilibrium with the medium

$$\hat{\rho}\Big|_{t=-\infty} \equiv \hat{\rho}_0 = \exp\{\Omega - \beta(\hat{H}_0 - \mu\hat{N})\}, \;\; \beta = 1/k_BT.$$

The correlation function for soft Fermi-fluctuations is

$$\Xi^{ij}_{\alpha\beta}(x-x') \equiv \frac{1}{2}\,\mathrm{Sp}\Big(\hat{\rho}_0\Big[\hat{\psi}^i_\alpha(\mathbf{x},t), \hat{\bar{\psi}}^j_\beta(\mathbf{x}',t')\Big]\Big)\,. \quad (11)$$

FDT for soft Fermi-fluctuations. *Exact quantum expression*

The quantum FDT-relation for soft Fermi-excitations in terms of Fourier components reads

$$(\psi^i_\alpha \bar{\psi}^j_\beta)_{\omega\mathbf{q}} = -\frac{1}{(2\pi)^4}\, i\hbar\, \delta^{ij}\, \frac{\Theta_F(\omega,\mu,T)}{(\hbar\,\omega - \mu)} \left\{ S^{(R)}_{\alpha\beta}(\mathbf{q},\omega) - S^{(A)}_{\alpha\beta}(\mathbf{q},\omega) \right\},$$

where

$$\Theta_F(\omega,\mu,T) = -\frac{1}{2}\,(\hbar\,\omega - \mu)\tanh\!\left(\frac{\hbar\omega - \mu}{2k_BT}\right)$$

is mean energy of a quantum fermionic oscillator. In the bosonic case we have

$$(A^{*a}_\mu A^b_\nu)_{\omega\mathbf{k}} = -\frac{1}{(2\pi)^4}\, i\delta^{ab} \left(\frac{\Theta_B(\omega,T)}{\omega}\right) \left\{ \mathcal{D}^{(R)}_{\mu\nu}(k) - (\mathcal{D}^{(R)}_{\nu\mu}(k))^* \right\},$$

where, in turn,

$$\Theta_B(\omega,T) = \frac{\hbar\omega}{2}\coth\!\left(\frac{\hbar\omega}{2k_BT}\right).$$

FDT for soft Fermi-fluctuations. *Semiclassical approximation*

FDT-relation for soft Fermi-fluctuations in the semiclassical limits $\hbar \to 0$

$$S^{(R,\,A)}(q) \to \frac{1}{\hbar}\,{}^{*}\!S^{(R,\,A)}(q),$$

where the retarded Green's function ${}^{*}\!S^{(R)}$ is

$${}^{*}\!S^{(R)}(q) = h_{+}(\hat{\mathbf{q}})\,{}^{*}\!\triangle_{+}(q) + h_{-}(\hat{\mathbf{q}})\,{}^{*}\!\triangle_{-}(q),$$

takes the following form

$$(\psi^{i}_{\alpha}\bar{\psi}^{j}_{\beta})_{\omega\mathbf{q}} = \frac{1}{2(2\pi)^3}\,\delta^{ij}\,\tanh\Big(\frac{1}{2}\,\beta\mu\Big)\,\frac{\omega_0^2}{2\,|\mathbf{q}|}\,\theta(\mathbf{q}^2-\omega^2)\times \tag{12}$$

$$\times\bigg\{\bigg(1-\frac{\omega}{|\mathbf{q}|}\bigg)(h_{+}(\hat{\mathbf{q}}))_{\alpha\beta}|\,{}^{*}\!\triangle_{+}(q)|^2 + \bigg(1+\frac{\omega}{|\mathbf{q}|}\bigg)(h_{-}(\hat{\mathbf{q}}))_{\alpha\beta}|\,{}^{*}\!\triangle_{-}(q)|^2\bigg\}.$$

Comparing the expression (12) with that for the spectral density (9) obtained within our simple model, we see that they differ from each other by the $\tanh(\beta\mu/2)$ factor. Since the derivation of expression (12) is more fundamental, it is natural, therefore, to assume that we have overlooked something in deriving (9).

FDT for soft Fermi-fluctuations. *Improvement of the model*

In the procedure of average when doing the calculation of the correlator $\langle \psi^i_\alpha(q)\bar{\psi}^j_\beta(-q')\rangle$ the following simple fact has not been taken into account, namely, the system under consideration is that with a varying number of particles and antiparticles. In our case a baryon number N that for massless thermal quarks and antiquarks can be presented as

$$N = N_f N_c \int \frac{d^3p}{(2\pi)^3}\left\{\frac{1}{\mathrm{e}^{\beta(\varepsilon-\mu)}+1} - \frac{1}{\mathrm{e}^{\beta(\varepsilon+\mu)}+1}\right\}, \quad \varepsilon = |\mathbf{p}|.$$

The spectral density (9) was generally calculated at some fixed N.

In the procedure of average it is necessary to introduce the weighting factor $a_N \mathrm{e}^{-\beta\mu|N|}$. The weighting factor accounts for different probabilities of different numbers of particles and antiparticles in the system. In our case the baryon number runs not only over all positive values, but also over all *negative* values by virtue of exchange of the fermion number between the hard and soft fermion subsystems of a quark-gluon plasma. Further, a careful distinction must be made between even and odd number of fermions. This is achieved by introducing the fermion number operator $(-1)^{\hat{F}}$ (the factor $(-1)^N$ in our case), which is identified with the a_N coefficient.

Taking into account the above mentioned, we get:

$$\left\langle \psi^i_\alpha(q)\bar{\psi}^j_\beta(-q')\right\rangle =$$
$$-\frac{g^2}{(2\pi)^6}\sum_{N=-\infty}^{+\infty}(-1)^N \mathrm{e}^{-\beta\mu|N|}\sum_{\zeta=\mathsf{Q},\,\bar{\mathsf{Q}}}\int d\theta d\theta^\dagger\,\theta^{\dagger i}\theta^j\int \mathbf{p}^2\left[\,f^{(\zeta)}_{|\mathbf{p}|}+f^{(\mathsf{G})}_{|\mathbf{p}|}\,\right]\frac{d|\,\mathbf{p}|}{2\pi^2}\times$$
$$\times\int d\mathbf{x}_0\,\mathrm{e}^{i(\mathbf{q}-\mathbf{q}')\cdot\,\mathbf{x}_0}\int\frac{d\Omega_{\mathbf{v}}}{4\pi}\left({}^*\!S(q)\chi\right)_\alpha\left(\bar{\chi}\,{}^*\!S(-q')\right)_\beta\delta(v\cdot q)\,\delta(v\cdot q'),$$

where the sum equals

$$\sum_{N=-\infty}^{+\infty}(-1)^N \mathrm{e}^{-\beta\mu|N|}=1+2\sum_{N=1}^{+\infty}(-1)^N\mathrm{e}^{-\beta\mu N}=\tanh\!\left(\frac{1}{2}\,\beta\mu\right),\quad \mu>0,$$

This results in an identical coincidence between the spectral densities (9) and (12).

Conclusion

We have proved two ways of deriving the fluctuation-dissipation theorem (FDT) for soft fermion excitations in a hot non-Abelian plasma being in a thermal equilibrium.

The first of them is based on the extended (pseudo)classical model in describing a quark-gluon plasma suggested by us, while the second one rests on the standard technique of calculation of the FDT for thermodynamically equilibrium systems.

We have shown that full accounting all subtleties that are common to the fermion system under consideration, results in perfect coincidence of thus obtained FDTs. This provides a rather strong argument for the validity of the pseudoclassical model suggested.

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Thanks for attention!