Production of superheavy and exotic nuclei in fusion reactions

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   - superheavy nuclei;
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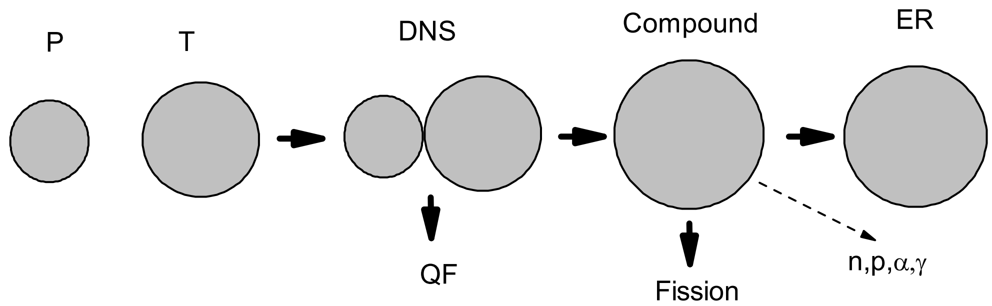
Fusion-evaporation reactions



The evaporation residue cross-section

$$\sigma_{ER}(E_{c.m.}) = \sum_{J=0} \sigma_c(E_{c.m.}, J) P_{CN}(E_{c.m.}, J) W_{sur}(E_{c.m.}, J)$$

depends on the capture cross section σ_c describing the transition of the colliding nuclei over the entrance (Coulomb) barrier, on the probability P_{CN} of the compound nucleus formation after the capture, and on the survival probability W_{sur} of the excited compound nucleus against fission.

At small angular momenta,

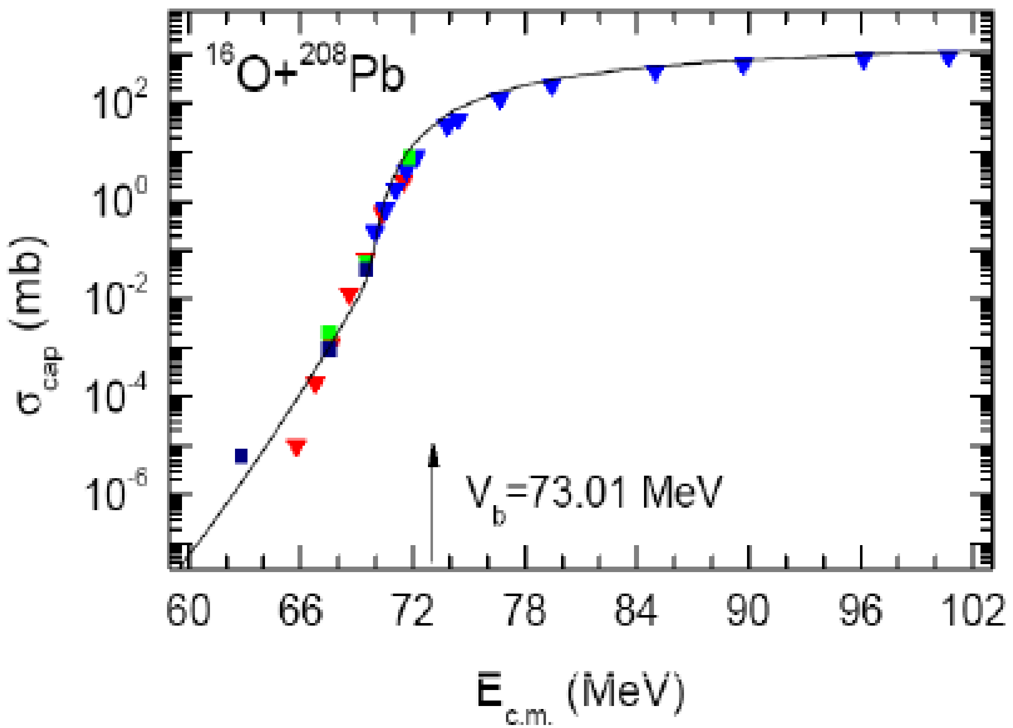
$$\sigma_{ER}(E_{c.m.}) \approx \sigma_c(E_{c.m.}) P_{CN}(E_{c.m.}) W_{sur}(E_{c.m.}),$$

where $\sigma_c(E_{c.m.}) = \frac{\pi\hbar^2}{2\mu E_{c.m.}} (J_{max} + 1)^2 P_c(E_{c.m.})$.

<u>Capture probability</u>

a) Parabolic approximation

$$P_c(E_{c.m.}) = \frac{1}{1+\exp[2\pi(V_b - E_{c.m.})/(\hbar\omega)]}$$



b) Quantum diffusion approach based on the formalism of reduced density matrix:

V.V. Sargsyan et al., EPJA 45 (2010) 125; 47 (2011) 38

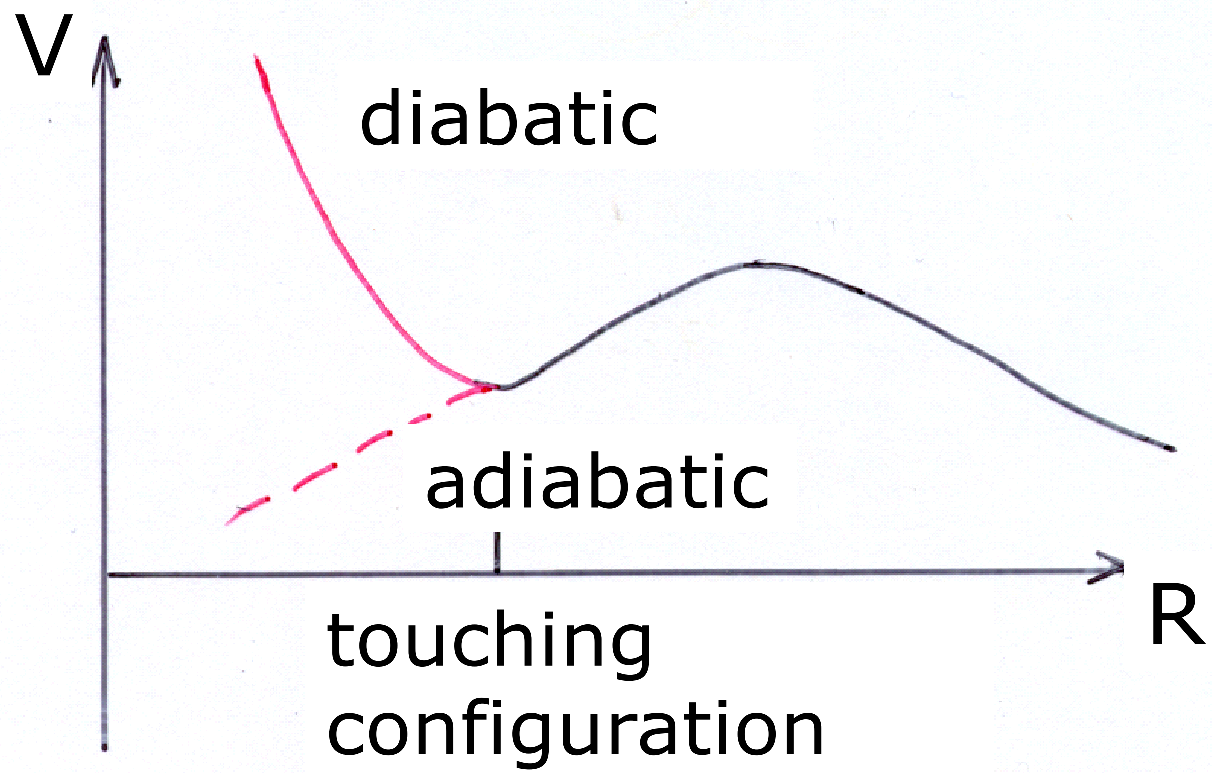
Dynamics of fusion. Collective coordinates

Heavy and superheavy nuclei can be produced by fusion reactions with heavy ions.

Two main collective coordinates are used for the description of the fusion process:

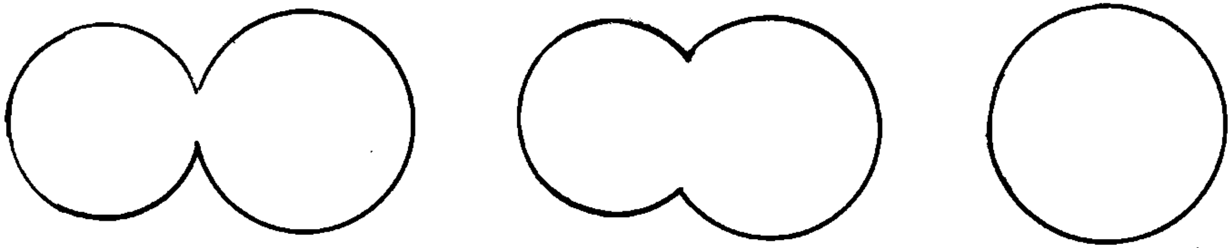
- Relative internuclear distance **R**
- Mass asymmetry coordinate $\eta=(A_1-A_2)/(A_1+A_2)$ for transfer of nucleon between nuclei

Description of fusion dynamics depends strongly whether adiabatic or diabatic potential energy surfaces are assumed.



a) Models using adiabatic potentials

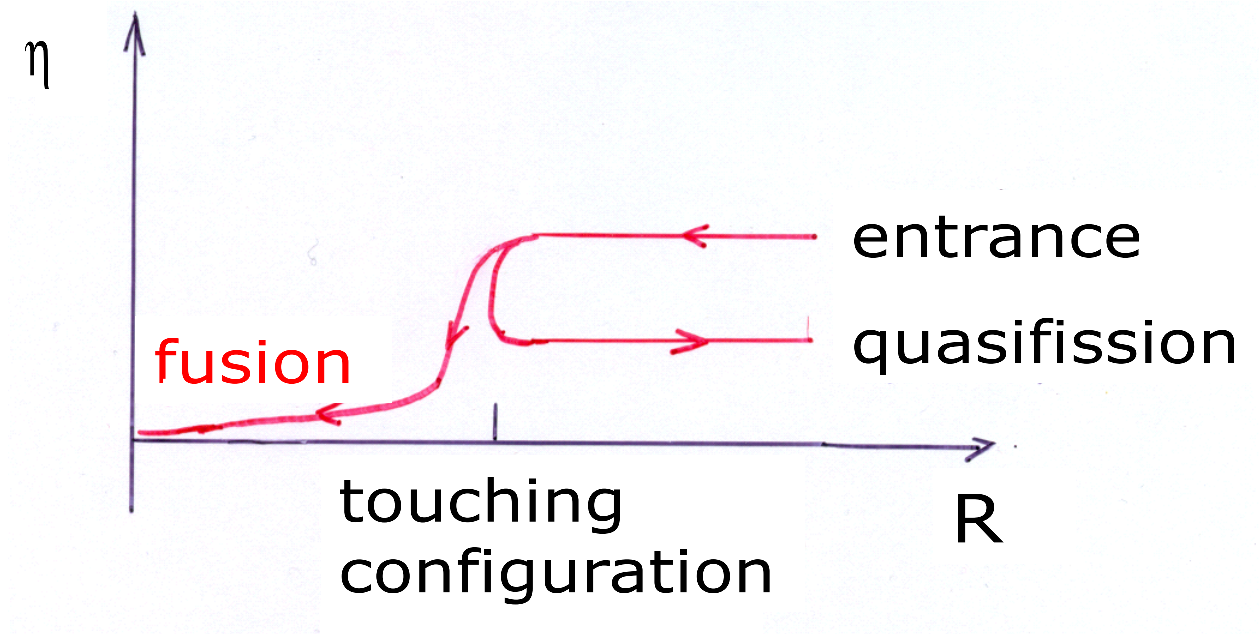
Minimization of potential energy, essentially adiabatic potential in the internuclear distance, nuclei melt together.



$R \rightarrow 0$

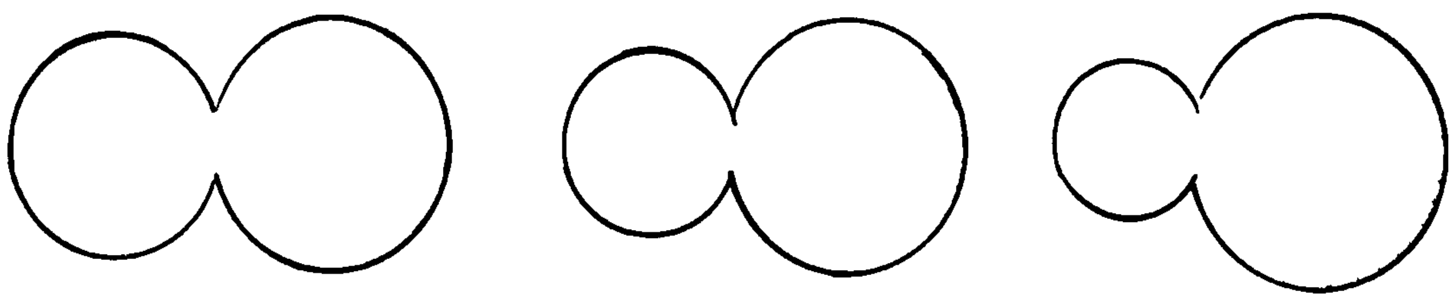
Large probabilities of fusion for producing nuclei with similar projectile and target nuclei.

Adiabatic potential.



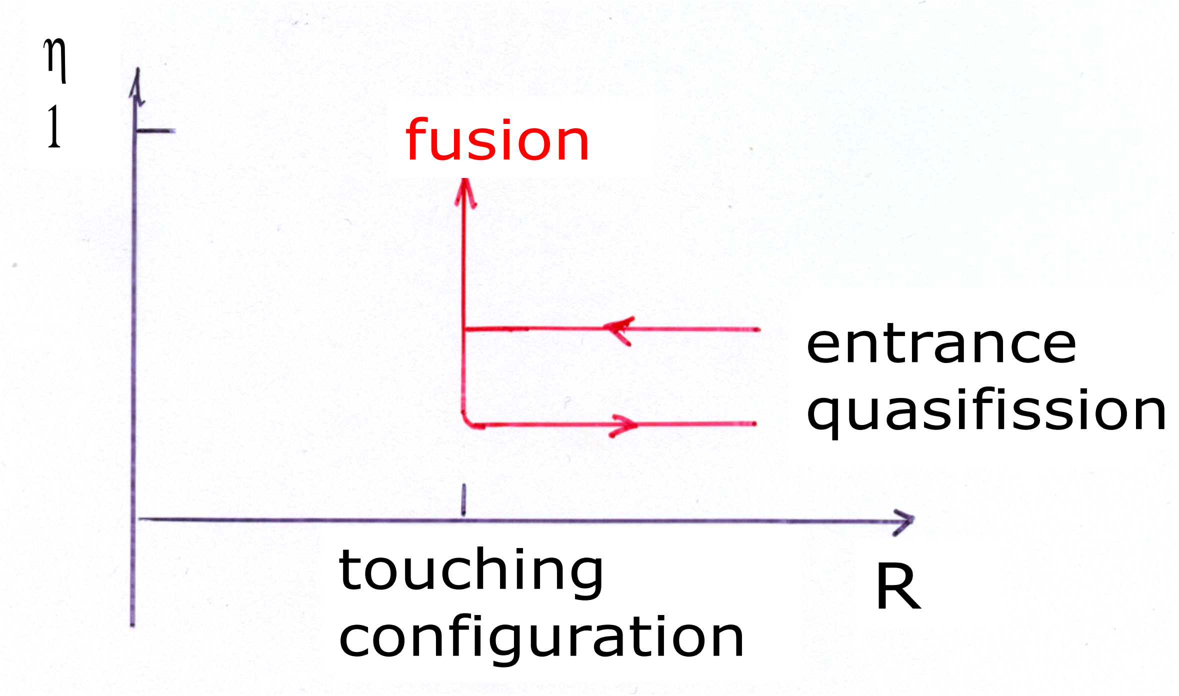
b) Dinuclear system (DNS) concept

Fusion by transfer of nucleons between the nuclei (idea of V. Volkov, also von Oertzen), mainly dynamics in mass asymmetry degree of freedom, use of diabatic potentials, e.g. calculated with the diabatic two-center shell model.

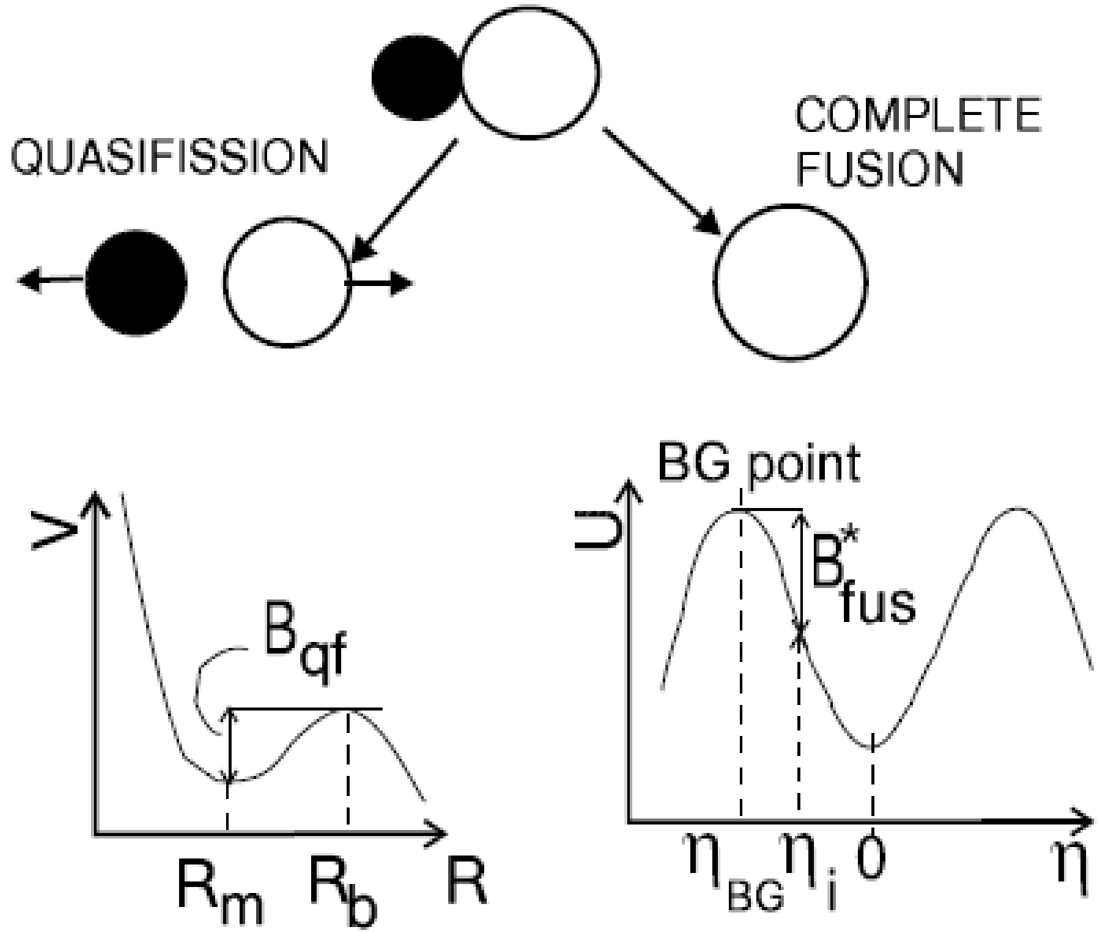


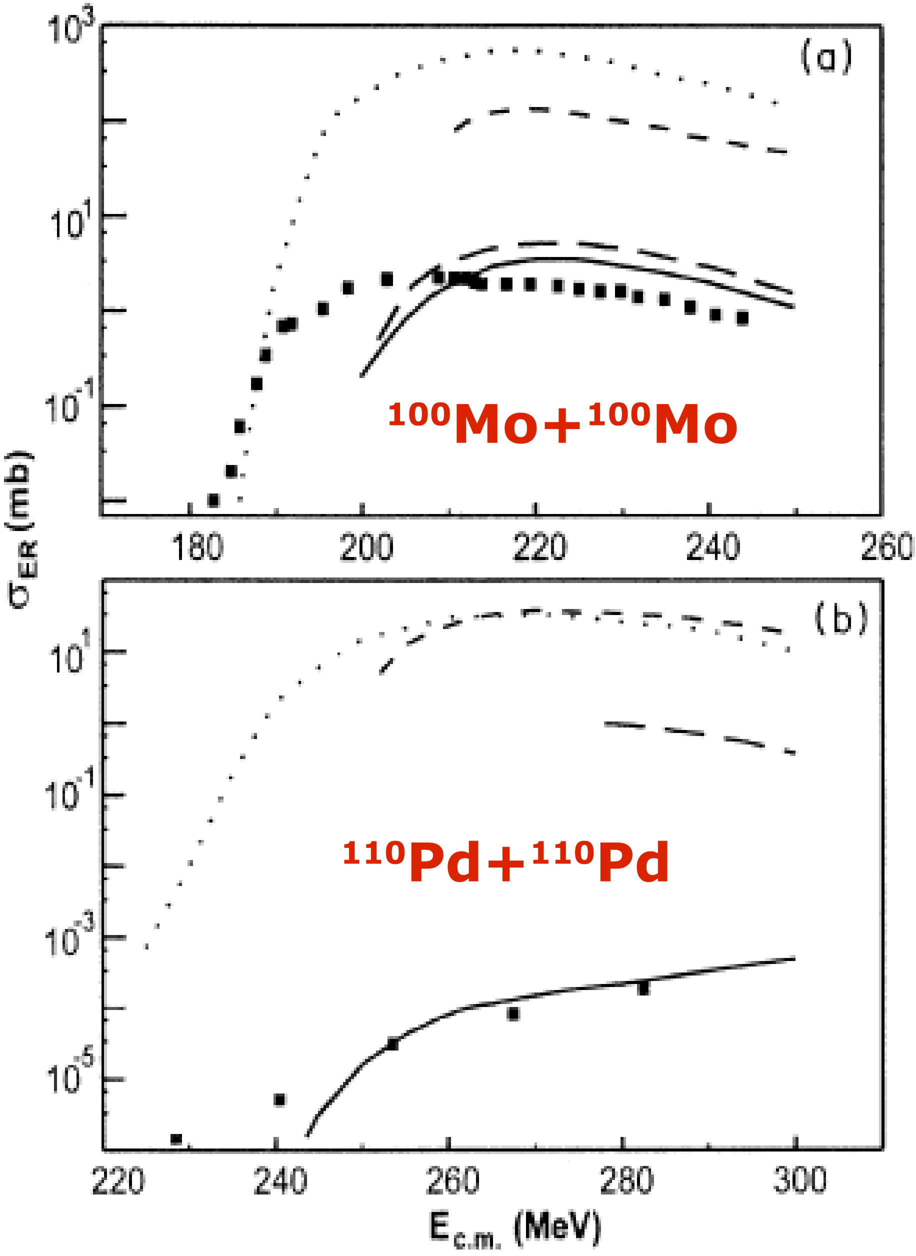
$\eta \rightarrow 1$

Diabatic potential.



PROCESSES IN THE INITIAL DNS





Dots – experiment.

Solid line – DNS model.

Dashed line – MDM (adiabatic) model.

Shot dashed line – surface friction model.

Dotted line – optical model.

DNS model. Master equation.

$$
\begin{aligned}
\frac{d}{dt}P_{Z,N}(t) &= \Delta^{(-,0)}_{Z+1,N}P_{Z+1,N}(t) + \Delta^{(+,0)}_{Z-1,N}P_{Z-1,N}(t) \\
&+ \Delta^{(0,-)}_{Z,N+1}P_{Z,N+1}(t) + \Delta^{(0,+)}_{Z,N-1}P_{Z,N-1}(t) \\
&- \left(\Delta^{(-,0)}_{Z,N} + \Delta^{(+,0)}_{Z,N} + \Delta^{(0,-)}_{Z,N} + \Delta^{(0,+)}_{Z,N}\right) P_{Z,N}(t) \\
&- (\Lambda^{qf}_{Z,N} + \Lambda^{fis}_{Z,N})P_{Z,N}(t)
\end{aligned}
$$

Rates Δ depend on single-particle energies and temperature related to excitation energy.

Only one-nucleon transitions are assumed.

$\Lambda^{qf}{}_{Z,N}$: rate for quasifission

$\Lambda^{fis}{}_{Z,N}$: rate for fission of heavy nucleus

The charge and mass yields for quasifission can be expressed

$$Y_{Z,N}(t_0) = \Lambda^{qf}_{Z,N} \int_0^{t_0} P_{Z,N}(t)dt$$

The time t_0 of reaction is determined by solving the normalization condition

$$\sum_{Z,N} Y_{Z,N}(t_0) + P_{CN} \approx 1$$

$$P_{CN} = \sum_{Z<Z_{BG},N<N_{BG}} P_{Z,N}(t_0)$$

Survival probability

The survival probability under the evaporation of a certain sequence s of x particles is considered as

$$W_{sur}^{s}(E^*_{CN}) \approx P_s(E^*_{CN}) \prod_{i_s=1}^{x} \frac{\Gamma_i(E^*_{i_s})}{\Gamma_t(E^*_{i_s})}$$

Here, i_s, P_s, and $E^*_{i_s}$ are the index of the evaporation step, the probability of realization of the channel s at the initial excitation energy E^*_{CN} of the compound nucleus, $E^*_{i_s}$ is the mean value of excitation energy at the step i_s.

In the case of the emission of x neutrons:

$$W_{sur}(E^*_{CN}) \approx P_{xn}(E^*_{CN}) \prod_{i=1}^{x} \frac{\Gamma_n(E^*_i)}{\Gamma_n(E^*_i)+\Gamma_f(E^*_i)} \approx P_{xn}(E^*_{CN}) \prod_{i=1}^{x} \frac{\Gamma_n(E^*_i)}{\Gamma_f(E^*_i)}.$$

The dependence of P_{xn} on E^*_{CN} is Gaussian-like function with the maximum at $E^*_{CN} = \sum_{k=1}^{x} B_k + x\overline{T}$, $B_n(k)$ is the separation energy of the k^{th} evaporated neutron, $\overline{T}$ the average nuclear temperature.

In the case of the emission of one neutron:

$$W_{sur}(E^*_{CN}) \approx P_{1n}(E^*_{CN}) \frac{\Gamma_n(E^*_{CN})}{\Gamma_f(E^*_{CN})}.$$

Survival probability

The decay width of channel i is given in terms of the probability R_{CN_i} of this process as

$\Gamma_i = \frac{R_{CN_i}}{2\pi\rho(E^*_{CN},J)}$.

The probability of evaporation of particle j (neutron, proton, α-particle)

$$R_{CN_j}(E^*_{CN},J) = \sum_{J_d} \int\limits_0^{E^*_{CN}-B_j} d\epsilon \rho_d(E^*_{CN}-B_j-\epsilon,J_d) \sum_{S=|J_d-s|}^{J_d+s} \sum_{l=|J-S|}^{J+S} T_{jl}(\epsilon)$$

can be calculated by using the separation energy B_j of particle j with spin s and the level density $\rho_d(E^*_{CN}-B_j-\epsilon,J_d)$ of the daughter nucleus. The transmission coefficient $T_{jl}(\epsilon)$ through the barrier is calculated by using an optical model potential.

The fission probability in the case of an one-hump barrier of height $B_f(E^*_{CN})$ and curvature $\hbar\omega$ is given as

$$R_{CN_f}(E^*_{CN},J) = \int\limits_0^{E^*_{CN}-B_f(E^*_{CN})} \frac{\rho_f(E^*_{CN}-B_f(E^*_{CN})-\epsilon)d\epsilon}{1+\exp[2\pi(\epsilon+B_f(E^*_{CN})-E^*_{CN})/(\hbar\omega)]}.$$

Calculation of level density. Fermi-gas model.

$$\rho(E^*, J) = \frac{2J+1}{24\sqrt{2}\sigma^3 a^{1/4}(E^*-\delta)^{5/4}} \exp\left\{2\sqrt{a(E^*-\delta)} - \frac{(J+1/2)^2}{2\sigma^2}\right\},$$

where $\sigma^2 = 6\overline{m^2}\sqrt{a(E^*-\delta)}/\pi^2$, δ is the pairing correction. The level density parameter a is proportional to the density of single-particle states at the Fermi surface. The average projection of the angular momentum of these states is estimated as $\overline{m^2} \approx 0.24 A^{2/3}$.

The damping of shell effects with the increasing of the excitation energy is taken into account in fission barrier.

$$B_f(E^*) = B_f^{LD} + B_f^M(E^* = 0)\exp[-E^*/E_D].$$

The main parameters here are a, a_f/a_n, E_D.

For small excitation energies, $E^* - \delta < U_x$ it is better to use

$$\rho(E^*, J) = \frac{1}{T_0}\exp\left(\frac{E^* - U_0}{T_0}\right)\frac{\exp\{-(J+1/2)^2/(2\sigma^2)\}}{2\sqrt{2\pi}\sigma^3},$$

which corresponds to the model with constant temperature T. $U_x = 2.2$ MeV.

Model with the collective enhancement of level density

$$\rho(E^*,J) = K_{vib}(E^*)K_{rot}(E^*)\frac{2J+1}{24\sqrt{2}\sigma^3_{eff}(a(A,E^*-E_c)[E^*-E_c]^5)^{1/4}} \times$$
$$\exp\left\{2\sqrt{a(A,E^*-E_c)[E^*-E_c]} - \frac{(J+1/2)^2}{2\sigma^2_{eff}}\right\},$$

where the condensation energy E_c decreases the ground state energy of the Fermi-gas by 1–3 MeV due to the correlation interaction.

$$\sigma^2_{eff} = \begin{cases} \Im_\perp^{2/3}\Im_\parallel^{1/3}\sqrt{(E^*-E_c)/a} \text{ for axial deformed nuclei,} \\ \Im_\parallel\sqrt{(E^*-E_c)/a} \text{ for spherical nuclei.} \end{cases}$$

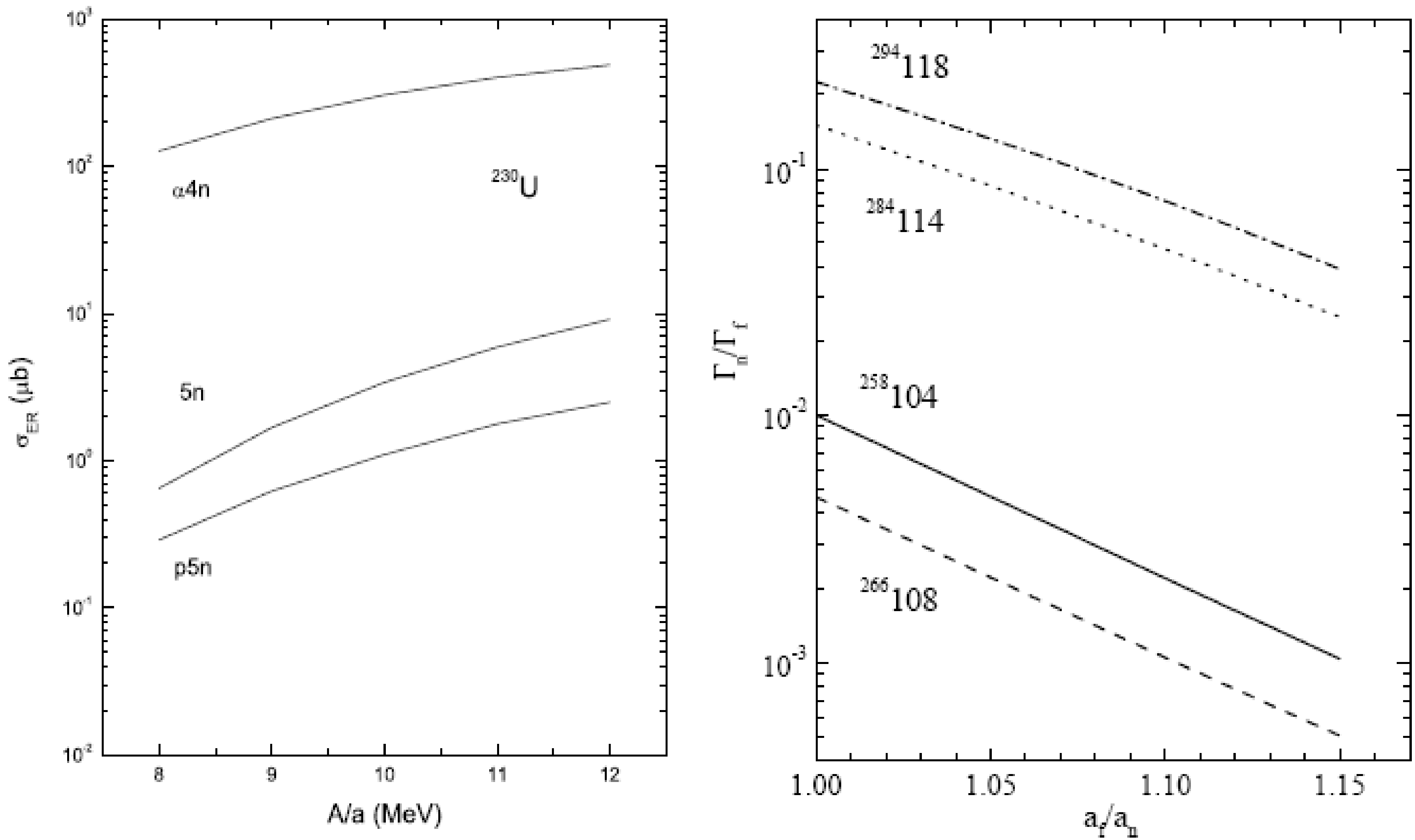
The damping of shell effects with the increasing of the excitation energy is taken into account in level density parameter

$$a(A,E^*-E_c) = \tilde{a}(A)\left[1 + \frac{1-\exp\{-(E^*-E_c)/E'_D\}}{E^*-E_c}\delta W\right],$$

which depends on the microscopic correction δW and excitation energy E^* .

The fission barrier: $B_f = B_f(E^*=0)$.

The main parameters here are $\tilde{a}(A)$, E'_D.

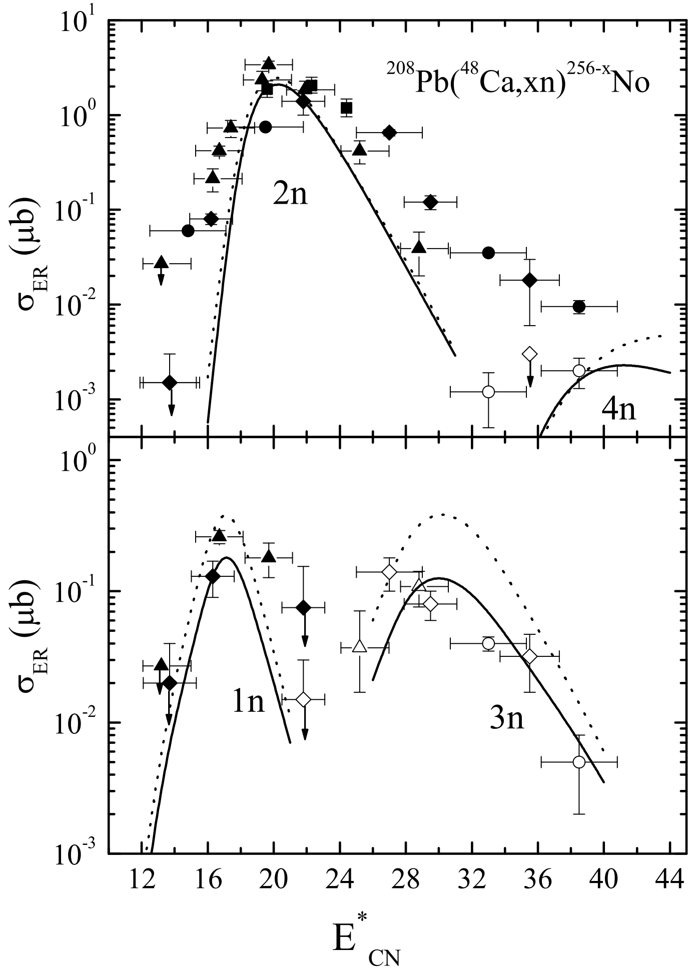


The variation of parameters can be very influent to the results of calculation!

Survival probability. Scheme of calculation.

- Choice of theoretical model which predictions of nuclear properties (binding energies, fission barriers, microscopic corrections, etc) for the nuclei in the considered region will be used in the calculation.
- Determination and verification of the parameters of statistical model by comparison of the calculated values for some nuclei from considered region with the existing experimental data.
- Set of these parameters for the whole region.
- The description of the other experimental data and predictions for the cross sections of the reactions which have not been investigated experimentally yet.

Superheavy nuclei. Pb-based reactions.



The predictions of microscopic-macroscopic model by A. Sobiczewski *et al.* are used: Acta Phys. Pol. **B34** (2003) 2153; Acta Phys. Pol. **B34** (2003) 2073.

Set of parameters:

1) Fermi-gas model:

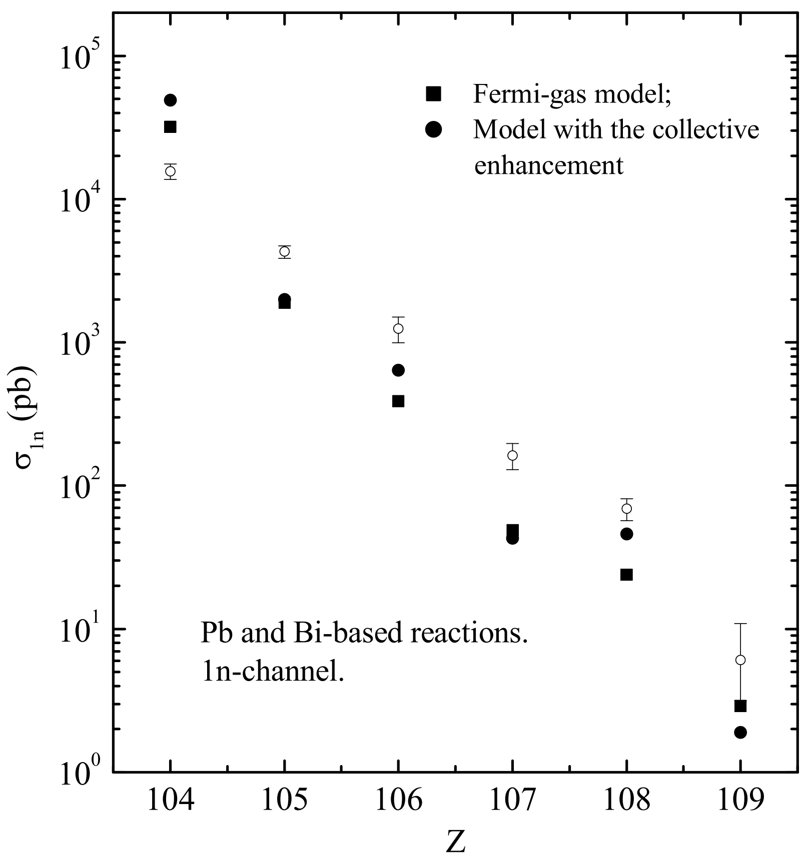
$a_n = A/10$, $a_f = 1.02 a_n$,
$E_D = 0.4 A^{4/3}/a$.

2) Model with the collective enhancement of level density:

$\tilde{a}(A) = 0.114A + 0.162A^{2/3}$,
$E'_D = 0.4A^{4/3}/\tilde{a}$

Determination of parameters – from the analysis of the reactions ^{48}Ca+^{204,206,208}Pb.

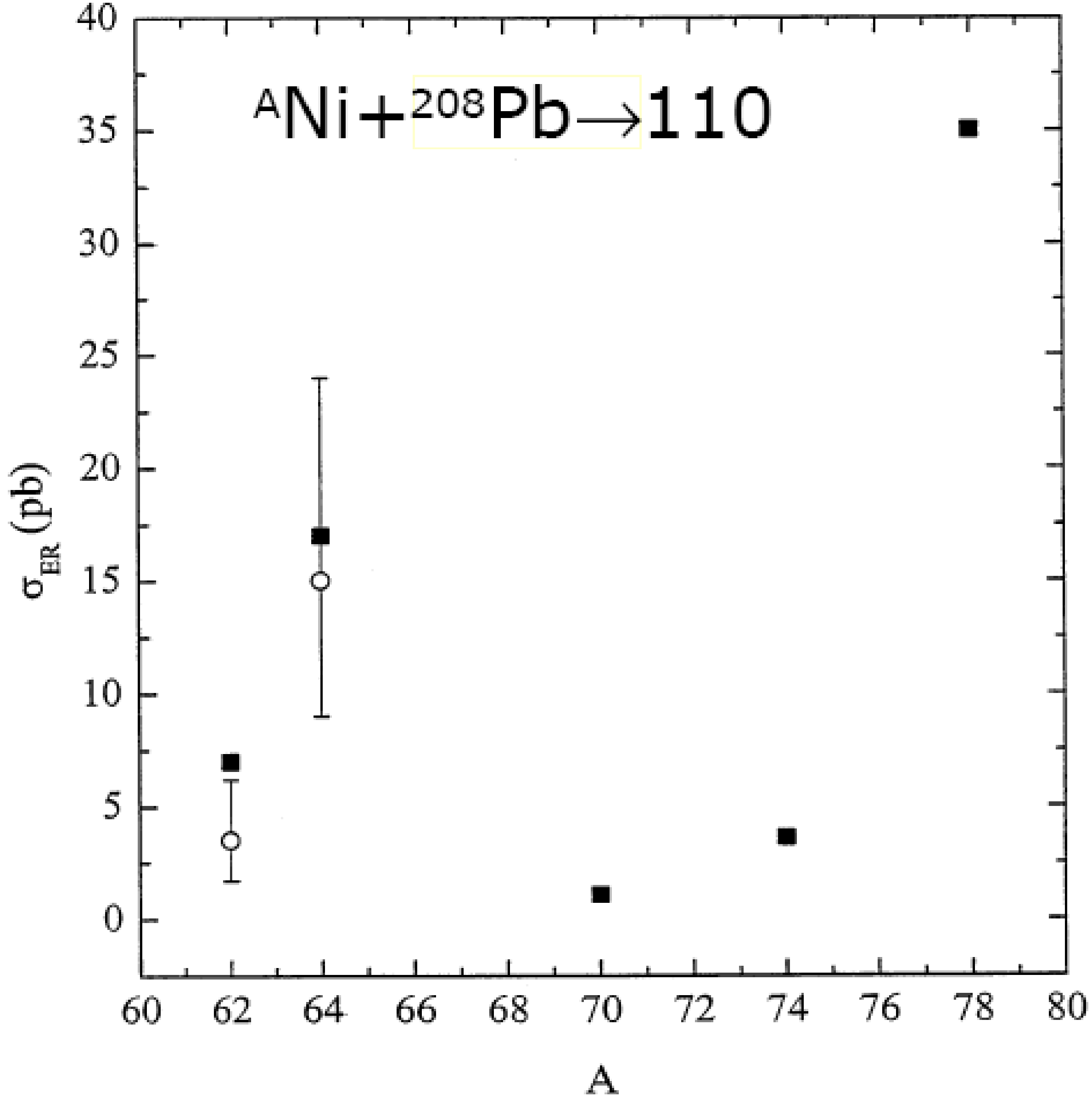
Superheavy nuclei. Pb-based reactions.



A. S. Zubov, G. G. Adamian, N. V. Antonenko, S. P. Ivanova, W. Scheid, Eur. Phys. J. A23 (2005) 249

Superheavy nuclei. Pb-based reactions.

Reactions	E^*_{CN} (MeV)	σ^{th}_{ER} (fermi)	σ^{th}_{ER} (coll)	σ^{exp}_{ER}
^{48}Ca+^{209}Bi$\to^{255}$103+$2n$	20	0.5 μb	0.15 μb	
^{48}Ca+^{209}Bi$\to^{254}$103+$3n$	30.5	25 nb	14 nb	
^{50}Ti+^{208}Pb$\to^{256}$104+$2n$	21.5	44 nb	44 nb	$18.5^{+1.42}_{-1.42}$ nb
^{50}Ti+^{208}Pb$\to^{255}$104+$3n$	29.5	2.3 nb	4.5 nb	$0.993^{+0.21}_{-0.21}$ nb
^{50}Ti+^{209}Bi$\to^{257}$105+$2n$	21.9	1.7 nb	0.6 nb	$2.4^{+0.3}_{-0.3}$ nb
^{50}Ti+^{209}Bi$\to^{256}$105+$3n$	31	150 pb	70 pb	190^{+40}_{-40} pb
^{54}Cr+^{208}Pb$\to^{260}$106+$2n$	22	0.27 nb	0.16 nb	$0.5^{+0.069}_{-0.069}$ nb
^{54}Cr+^{208}Pb$\to^{259}$106+$3n$	32	27 pb	41 pb	10^{+23}_{-8} pb
^{54}Cr+^{209}Bi$\to^{261}$107+$2n$	22	14.5 pb	3 pb	
^{54}Cr+^{209}Bi$\to^{260}$107+$3n$	32	3.2 pb	0.8 pb	
^{58}Fe+^{208}Pb$\to^{264}$108+$2n$	20.5	4.7 pb	5.1 pb	$4.54^{+5.7}_{-2.9}$ pb
^{58}Fe+^{208}Pb$\to^{263}$108+$3n$	32	0.96 pb	1.5 pb	
^{58}Fe+^{209}Bi$\to^{265}$109+$2n$	22	5 pb	1.2 pb	



G.G. Adamian, N.V. Antonenko, W. Scheid, PRC69 (2004) 011601(R).

Production of neutron-deficient actinides in the reactions

$^{24,25,26}Mg + {}^{204,206,207,208}Pb \longrightarrow Pu + xn$

$^{28,30}Si + {}^{204,206,207,208}Pb \longrightarrow Cm + xn$ and αxn

$^{27}Al + {}^{204,206}Pb \longrightarrow Am + xn$

$^{40,44,48}Ca + {}^{184,186}W \longrightarrow Pu + xn$

$^{40,44}Ca + {}^{190,192}Os \longrightarrow Cm + xn$ and αxn

$^{28}Si + {}^{192}Pt$, $^{44}Ca + {}^{176}Hf \longrightarrow U + xn$

$^{34}S + {}^{204,206}Pb \longrightarrow Cf + xn$

Fusion probabilities are calculated with the master equations.

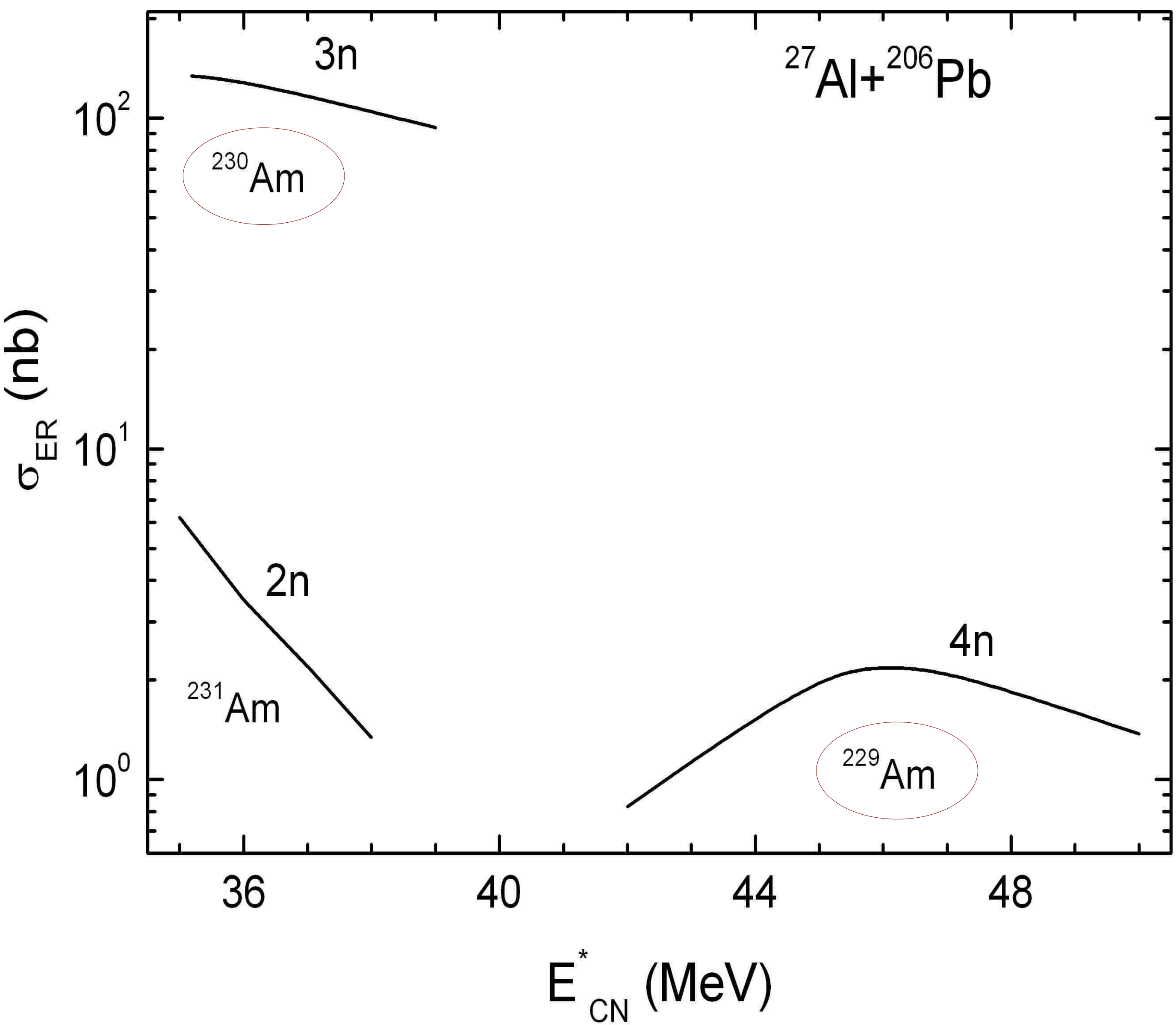
Parameters to calculate W_{sur}

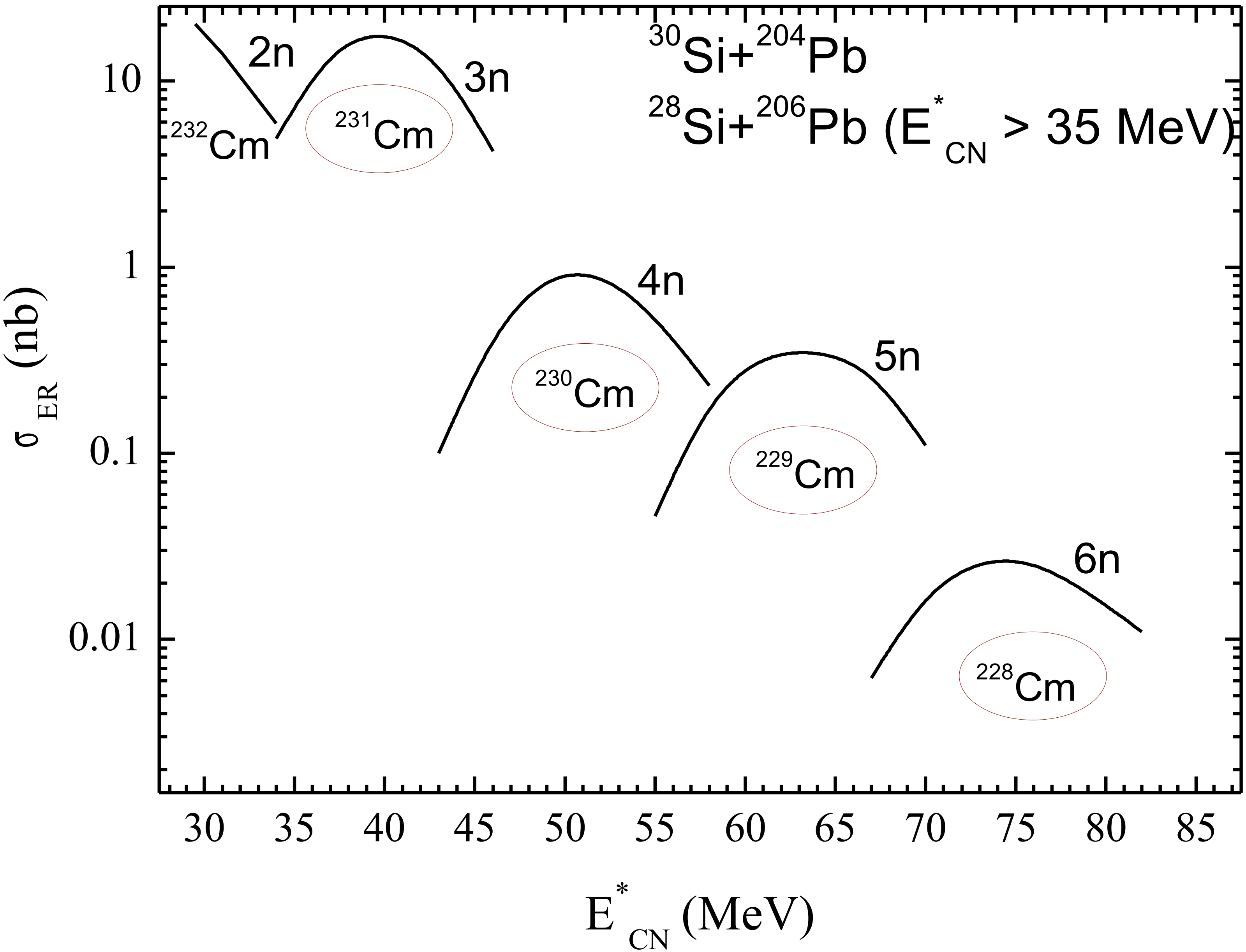
For N>128 (N<125), we take a=A/10 (a=A/8) in all evaporation channels.

For the near magic nuclei with N=125-128, $a=a_n=a_\alpha=A/10$ and $a_p=A/9.7$.

$a_f=1.03a$ ($a_f=1.02a$) for $Z<95$ ($Z\geq95$)

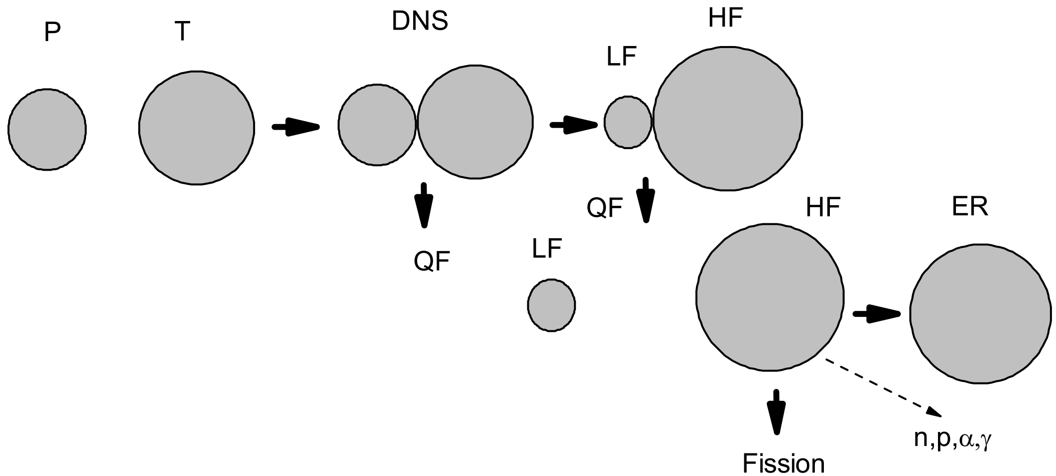
Reaction	$E_{\rm c.m.}$ (MeV)	Channel	$\sigma_{ER}^{th.}$	$\sigma_{ER}^{exp.}$
^{86}Kr+^{138}Ba	213.3	$\underline{n} + \alpha n + 2\alpha n$	4 nb	20^{+15}_{-12} nb
	218.6	$n + \underline{\alpha n} + 2\alpha n$	15 nb	8^{+10}_{-6} nb
	225.3	$n + \alpha n + \underline{2\alpha n}$	65 nb	50^{+42}_{-32} nb
	225.3	$2n + \underline{\alpha 2n} + 2\alpha 2n$	23 nb	19^{+38}_{-19} nb
	232.3	$2n + \underline{\alpha 2n} + 2\alpha 2n$	140 nb	140^{+120}_{-90} nb
	237.4	$3n + \underline{\alpha 3n}$	213 nb	180^{+130}_{-100} nb
^{86}Kr+^{134}Ba	220	$2n + \underline{\alpha 2n}$	4 nb	2^{+5}_{-2} nb
	220.9	$np + \underline{\alpha np}$	1 nb	6^{+12}_{-6} nb
	227	$np + \underline{\alpha np}$	4 nb	6^{+13}_{-6} nb
	229	$np + \underline{\alpha np}$	7 nb	
^{28}Si+^{204}Pb	141	$4n$	3.5 nb	$1.7^{+2.2}_{-1.3}$ nb
	141	$\alpha 3n$	16 nb	6^{+4}_{-3} nb
	143	$3np$	6 nb	6^{+4}_{-3} nb


3n
^{27}Al+^{206}Pb
10^2
^{230}Am
σ_{ER} (nb)
10^1
2n
4n
^{231}Am
10^0
^{229}Am
36
40
44
48
E^*_{CN} (MeV)



^{30}Si+^{204}Pb
^{28}Si+^{206}Pb (E^*_{CN} > 35 MeV)
2n
3n
4n
5n
6n
^{232}Cm
^{231}Cm
^{230}Cm
^{229}Cm
^{228}Cm
10
1
0.1
0.01
30
35
40
45
50
55
60
65
70
75
80
85
σ_{ER} (nb)
E^*_{CN} (MeV)

Incomplete fusion reactions



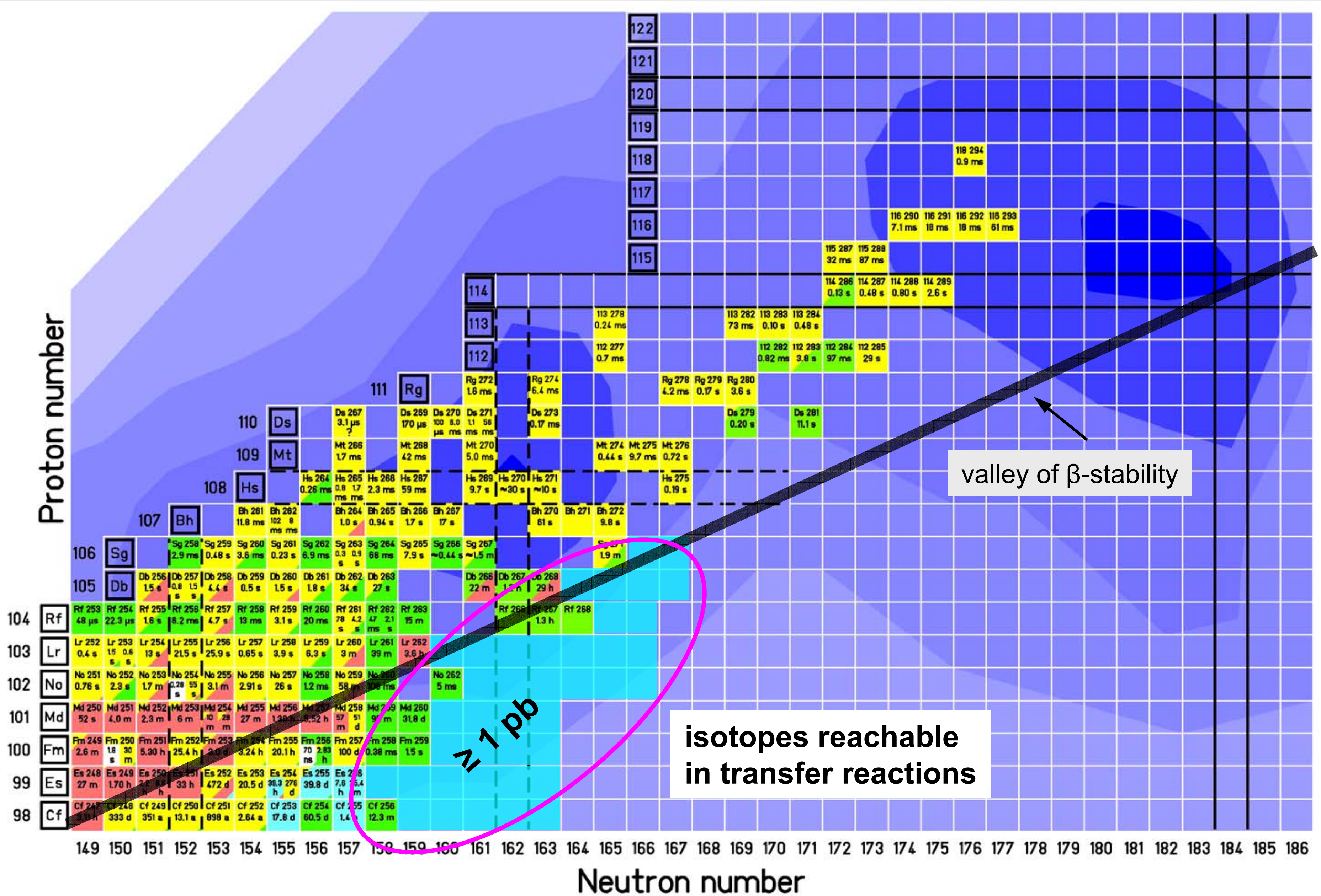
G. G. Adamian, N. V. Antonenko, A. S. Zubov, Phys. Rev. C71 (2005) 034603

The evaporation residue cross-section

$\sigma_{ER}(Z, N - x) = \sigma_{cap} Y_{Z,N} W_{sur}(xn),$

where Z and N are charge and mass of the heavy fragment.

We desrcibe the production of nuclei with $101 \le Z \le 108$ in reactions ^{48}Ca+^{238}U, ^{243}Am, ^{244,246,248}Cm. The produced ER have mass numbers between those for superheavies produced in the cold and hot fusion reactions.


Proton number
Neutron number
98 Cf
99 Es
100 Fm
101 Md
102 No
103 Lr
104 Rf
105 Db
106 Sg
107 Bh
108 Hs
109 Mt
110 Ds
111 Rg
112
113
114
115
116
117
118
119
120
121
122
149 150 151 152 153 154 155 156 157 158 159 160 161 162 163 164 165 166 167 168 169 170 171 172 173 174 175 176 177 178 179 180 181 182 183 184 185 186
valley of β-stability
≥ 1 pb
isotopes reachable
in transfer reactions

$^{48}Ca+^{244}Cm$ ▲

E_{cm}=207 MeV

$^{48}Ca+^{246}Cm$ ●

E_{cm}=205.5 MeV

$^{48}Ca+^{248}Cm$ ■

E_{cm}=204 MeV