Contribution to anomalous magnetic moment of muon from light-by-light in nonlocal quark model

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$(g-2)_\mu$ - motivation

The general form of element of interaction of muon with external electromagnetic fields is

$$-ie\bar{u}(p')\left\{\gamma_\mu F_1(q^2) + i\sigma_{\mu\nu}\frac{q_\nu}{2m}F_2(q^2) + \gamma_5\sigma_{\mu\nu}\frac{q_\nu}{2m}F_3(q^2)\right\}u(p)e_\mu(q) \quad (1)$$

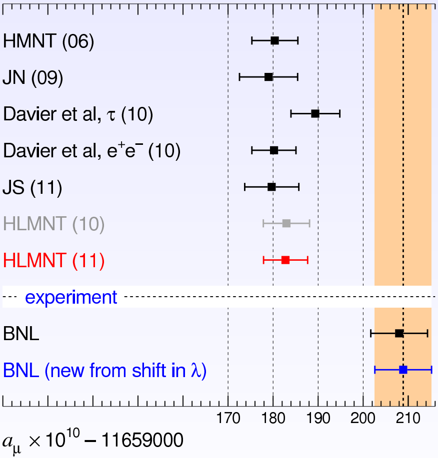
F_1 - is the electric charge distribution
F_2 - corresponds to Anomalous Magnetic Moment(AMM)

$$a = (g-2)/2 = F_2(0)$$

F_3 - is Anomalous Electric Dipole Moment

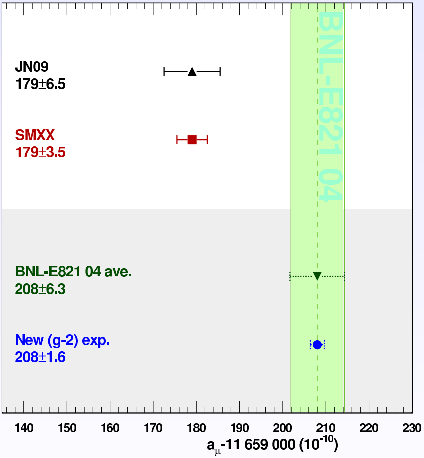
However in SM $a \neq 0$. This is due to Radiative Correction

$(g-2)_\mu$



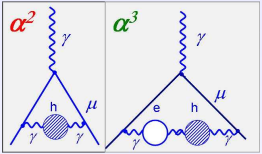
by Graziano Venanzoni, arXiv:1203.1501v1

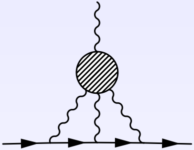
$(g-2)_\mu$



by Graziano Venanzoni, arXiv:1203.1501v1

$(g-2)_\mu$. Hadron contribution





$$a_\mu^{HVP} = (692.3 \pm 4.2) \times 10^{-10} (\text{Davier, Hoecker, Zhang}) \qquad (2)$$

- Hadronic Vacuum Polarization contributes 99% of absolute value and half of error fixed by experiment
- Light-by-light process contributes 1% and half of error. The LbL contribution is model depending.

NχQM

Calculation is performed in framework non-local chiral quark model:

$$\mathcal{L} = \bar{q}(x)(i\hat{\partial} - m_c)q(x) + \frac{G}{2}[J_S^a(x)J_S^a(x) + J_P^a(x)J_P^a(x)]$$
$$- \frac{H}{4}T_{abc}[J_S^a(x)J_S^b(x)J_S^c(x) - 3J_S^a(x)J_P^b(x)J_P^c(x)] \tag{3}$$

$$T_{abc} = \frac{1}{6}\epsilon_{ijk}\epsilon_{mnl}(\lambda_a)_{im}(\lambda_b)_{jn}(\lambda_c)_{kl},$$

where currents are:

$$J_M^a(x) = \int d^4x_1 d^4x_2\, f(x_1)f(x_2)\,\bar{q}(x-x_1)\,\Gamma_M^a q(x+x_2), \tag{4}$$

$$q(x) \to Q(x,y) = q(y)Pexp(-i\int_x^y dz^\mu A_\mu(z)) \tag{5}$$

(Anikin, Dorokhov, Tomio, Phys. Part. Nucl. 31, 509 (2000) [Fiz. Elem. Chast. Atom. Yadra 31, 1023 (2000)]
and A. Scarpettini, D. Gomez Dumm, N.N. Scoccola, Phys. Rev. D 69, 114018 (2004))

The effective action can be obtained after bosonization

$$\int D\Phi \exp\left(i\int \pm A\Phi - B\Phi\right) = \frac{1}{\acute{N}}\exp i\int d^4x \frac{A}{4B}, \qquad (6)$$

$$S = \ln\det A - \int d^4x \Big[\sigma^a S^a + \pi^a P^a + \frac{G}{2}(S^a S^a + P^a P^a) + \frac{H}{4} T_{abc}(S^a S^b S^c + S^a P^b P^c)\Big]$$

where

$$A = (\widehat{p} - m_c)\delta(p-p') + f(p)[\sigma^a(p-p') + i\gamma^5\pi^a(p-p')]\lambda^a f(p')$$

NχQM

The gap equations for quarks with dynamical masses m_d are follow from effective action

$$m_{d,u} + GS_u + \frac{H}{2} S_u S_s = 0,$$
$$m_{d,s} + GS_s + \frac{H}{2} S_u^2 = 0,$$
$$S_i = -8N_c \int \frac{d_E^4 K}{(2\pi)^4} \frac{f^2(K^2) m_i(K^2)}{D_i(K^2)},$$

where $m_i(K^2) = m_{c,i} + m_{d,i} f^2(K^2)$, $D_i(K^2) = K^2 + m_i^2(K^2)$

The Bethe-Salpeter equations for the mesons are

$$\mathbf{T}_{P,S} = \hat{\mathbf{T}}_{P,S}(K^2)\delta^4\left(P_1 + P_2 - P_3 - P_4\right)\prod_{i=1}^{4} f(P_i^2),$$
$$\hat{\mathbf{T}}_{P,S}(P^2) = \Gamma^k_{P,S}\left(\frac{1}{-\mathbf{G}^{-1} + \Pi_{P,S}(P^2)}\right)_{kl}\Gamma^l_{P,S} \tag{7}$$

where the masses are

$$\det(\mathbf{G}^{-1} - \Pi_M(-M^2)) = 0 \tag{8}$$

$$\hat{\mathbf{T}}_{P,S}(P^2) = \sum_a \frac{\bar{V}_M(P^2)\otimes V_M(P^2)}{-(P^2 + M_M^2)}, \tag{9}$$

The mixing effect should be taken into account in order to obtain the physical states

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos\theta_P & -\sin\theta_P \\ \sin\theta_P & \cos\theta_P \end{pmatrix} \begin{pmatrix} \eta_8 \\ \eta_0 \end{pmatrix},$$
$$\begin{pmatrix} \sigma \\ f_0(980) \end{pmatrix} = \begin{pmatrix} \cos\theta_S & -\sin\theta_S \\ \sin\theta_S & \cos\theta_S \end{pmatrix} \begin{pmatrix} \sigma_8 \\ \sigma_0 \end{pmatrix}. \tag{10}$$

As a results the meson fields takes the form

$$\begin{aligned} V_{a_0}\left(P^2\right) &= ig_{a_0}(P^2)\lambda_3, \\ V_{\sigma}\left(P^2\right) &= ig_{\sigma}(P^2)\left(\lambda_8\cos\theta_S(P^2) - \lambda_0\sin\theta_S(P^2)\right), \\ V_{f_0}\left(P^2\right) &= ig_{f_0}(P^2)\left(\lambda_8\sin\theta_S(p^2) + \lambda_0\cos\theta_S(P^2)\right), \end{aligned} \tag{11}$$

For including gauge fields, the quark fields should be rotated in these fields with help of path exponent:

$$q(x) \to Q(x, y) = q(x) P \exp\left(ie \int_y^x du V(u) \right) \qquad (12)$$

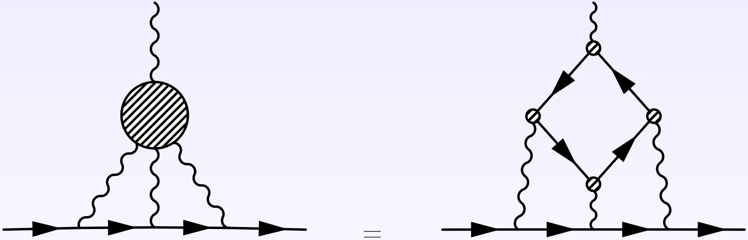
As a result we can obtain the multiphoton vertex of interaction quark-antiquark-photons (or meson-quark-antiquark-photons). For example, vertex of interaction photon and quark-antiquark is:

$$\Gamma_\mu = \gamma_\mu - (p_1 + p_2)_\mu m^{(1)}(p_1, p_2) \qquad (13)$$

$$ie\Gamma^\mu(x, y, z) = -\frac{\delta^3 S}{\delta V_\mu(x) \delta q(y) \delta \bar{q}(z)} \qquad (14)$$

[J. Terning, Phys. Rev. D 44, 887 (1991)]

Light-by-Light



Light-by-Light

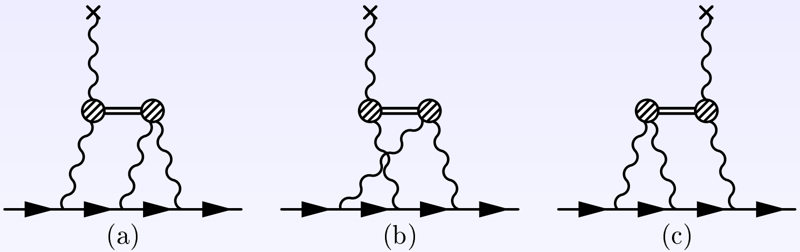


Figure : Contribution of processes scattering light-by-light with intermediate meson state.

Light-by-Light

Basic element for calculation of contribution in AMM of muon is four order tensor of vacuum polarization.

$$\Pi_{\mu\nu\lambda\rho}(q_1,q_2,q_3) = \int d^4x_1 \int d^4x_2 \int d^4x_3 e^{i(q_1x_1+q_2x_2+q_3x_3)} \times \\ \times \langle 0|T(j_\mu(x_1)j_\nu(x_2)j_\lambda(x_3)j_\rho(0))|0\rangle \quad (15)$$

AMM of muon is obtained by following projection

$$a_\mu^{\mathrm{LbL}} = \frac{1}{48m_\mu}\mathrm{Tr}\left((\hat{p}+m_\mu)[\gamma^\rho,\gamma^\sigma](\hat{p}+m_\mu)\Pi_{\rho\sigma}(p,p)\right), \quad (16)$$

where

$$\Pi_{\rho\sigma}(p',p) = -ie^6 \int \frac{d^4q_1}{(2\pi)^4} \int \frac{d^4q_2}{(2\pi)^4} \frac{1}{q_1^2q_2^2(q_1+q_2-k)^2} \times \\ \times \gamma^\mu \frac{\hat{p}'-\hat{q}_1+m_\mu}{(p'-q_1)^2-m_\mu^2}\gamma^\nu \frac{\hat{p}-\hat{q}_1-\hat{q}_2+m_\mu}{(p-q_1-q_2)^2-m_\mu^2}\gamma^\lambda \times \\ \times \frac{\partial}{\partial k^\rho}\Pi_{\mu\nu\lambda\sigma}(q_1,q_2,k-q_1-q_2), \quad (17)$$

m_μ is the muon mass, $k_\mu = (p'-p)_\mu$ and it is necessary to consider the limit $k_\mu \to 0$.

Light-by-Light

Corresponding polarization tensor can be connected with intermediate meson state as:

$$\Pi^{\mu\nu\lambda\rho}(q_1,q_2,q_3) = \tag{18}$$
$$i\frac{\Delta^{\mu\nu}(q_1+q_2,q_1,q_2)\Delta^{\lambda\rho}(q_1+q_2,q_3,q_1+q_2+q_3)}{(q_1+q_2)^2-M^2}+$$
$$+\,i\frac{\Delta^{\mu\rho}(q_2+q_3,q_1,q_1+q_2+q_3)\Delta^{\nu\lambda}(q_2+q_3,q_2,q_3)}{(q_2+q_3)^2-M^2}+$$
$$+\,i\frac{\Delta^{\mu\lambda}(q_1+q_3,q_1,q_3)\Delta^{\nu\rho}(q_1+q_3,q_2,q_1+q_2+q_3)}{(q_1+q_3)^2-M^2},$$

Light-by-Light

$$\frac{\partial}{\partial k^\rho}\Pi^{\mu\nu\lambda\sigma}(q_1,q_2,k-q_1-q_2) =$$
$$i\frac{\Delta^{\mu\nu}(q_1+q_2,q_1,q_2)}{(q_1+q_2)^2-M^2}\frac{\partial}{\partial k^\rho}\Delta^{\lambda\sigma}(q_1+q_2,-q_1-q_2,k)$$
$$+i\frac{\Delta^{\nu\lambda}(-q_1,q_2,-q_1-q_2)}{q_1^2-M^2}\frac{\partial}{\partial k^\rho}\Delta^{\mu\sigma}(-q_1,q_1,k) \tag{19}$$
$$+i\frac{\Delta^{\mu\lambda}(-q_2,q_1,-q_1-q_2)}{q_2^2-M^2}\frac{\partial}{\partial k^\rho}\Delta^{\nu\sigma}(-q_2,q_2,k)+O(k).$$

Where

$$\frac{\partial}{\partial k^\rho}\Delta_S^{\mu\nu}(-q,q,k) = \mathrm{A}_{S^*\gamma^*\gamma^*}(q^2,q^2,0)(g^{\mu\nu}q^\rho - q^\nu g^{\mu\rho})+$$
$$+\mathrm{B}'_{S^*\gamma^*\gamma^*}(q^2,q^2,0)\left(\frac{q^\mu q^\nu q^\rho}{q^2} - q^\nu g^{\mu\rho}\right)+O(k). \tag{20}$$

Light-by-Light

After averaging over the momentum the expression for calculation from processes LbL takes the form:

$$a_\mu^{\mathrm{LbL}} = -\frac{2\alpha^3}{3\pi^2}\int_0^\infty dQ_1^2\int_0^\infty dQ_2^2\int_{-1}^1 dt\sqrt{1-t^2}\,\frac{1}{Q_3^2}\times$$

$$\times\sum_{mesons}\left[2\frac{\mathcal{N}_{\mathbf{1}}^S}{Q_2^2+M_S^2}+\frac{\mathcal{N}_{\mathbf{2}}^S}{Q_3^2+M_S^2}\right], \qquad (21)$$

$$\mathcal{N}_{\mathbf{1}}^S = \sum_{\mathrm{X=A,B'}}\sum_{\mathrm{Y=A,B}} \mathrm{X}_S\left(Q_2^2;Q_2^2,0\right)\mathrm{Y}_S\left(Q_2^2;Q_1^2,Q_3^2\right)\mathrm{Ts}_{\mathbf{1}}^{\mathrm{XY}},$$

$$\mathcal{N}_{\mathbf{2}}^S = \sum_{\mathrm{X=A,B'}}\sum_{\mathrm{Y=A,B}} \mathrm{X}_S\left(Q_3^2;Q_3^2,0\right)\mathrm{Y}_S\left(Q_3^2;Q_1^2,Q_2^2\right)\mathrm{Ts}_{\mathbf{2}}^{\mathrm{XY}},$$

For pseudoscalar $B' = 0$ and respectively Ts have simple form.

Light-by-Light / meson to two photons

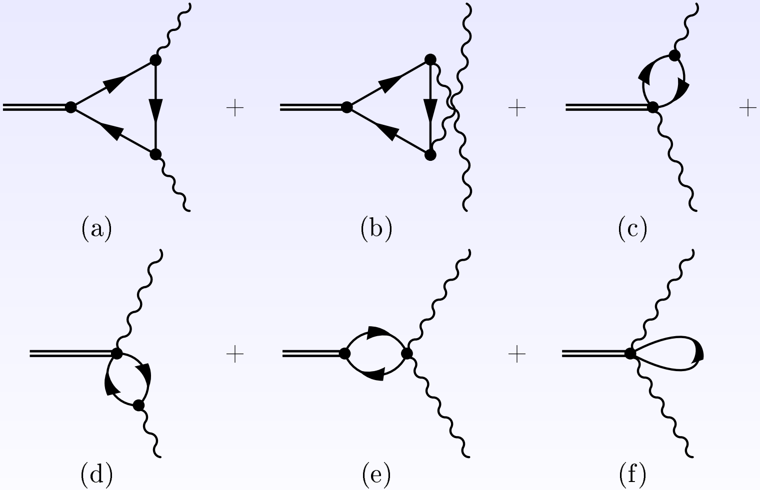


Figure : Diagrams of decay meson into two photons.

Light-by-Light / meson to two photons

$$A\left(\gamma^*\left(q_1,\epsilon_1\right)\gamma^*\left(q_2,\epsilon_2\right)\to P^*\left(p\right)\right)=-ie^2\varepsilon_{\mu\nu\rho\sigma}\epsilon_1^{\mu}\epsilon_2^{\nu}q_1^{\rho}q_2^{\sigma}\mathrm{F}_{P^*\gamma^*\gamma^*}\left(p^2;q_1^2,q_2^2\right),$$

where $q_{1,2}$ - momentum of photons , and $\epsilon_{1,2}$ is vector of polarization, $p=q_1+q_2$. For different pseudoscalar states we have:

$$\begin{aligned}
\mathrm{F}_{\pi_0^*\gamma^*\gamma^*}\left(p^2;q_1^2,q_2^2\right) &= g_\pi(p^2)F_u\left(p^2;q_1^2,q_2^2\right),\\
\mathrm{F}_{\eta^*\gamma^*\gamma^*}\left(p^2;q_1^2,q_2^2\right) &= \frac{g_\eta(p^2)}{3\sqrt{3}}\times\\
&\times\Big[\left(5F_u\left(p^2;q_1^2,q_2^2\right)-2F_s\left(p^2;q_1^2,q_2^2\right)\right)\cos\theta(p^2)-\\
&-\sqrt{2}\left(5F_u\left(p^2;q_1^2,q_2^2\right)+F_s\left(p^2;q_1^2,q_2^2\right)\right)\sin\theta(p^2)\Big],\\
\mathrm{F}_{\eta'^*\gamma^*\gamma^*}\left(p^2;q_1^2,q_2^2\right) &= \frac{g_{\eta'}(p^2)}{3\sqrt{3}}\times \qquad (22)\\
&\times\Big[\left(5F_u\left(p^2;q_1^2,q_2^2\right)-2F_s\left(p^2;q_1^2,q_2^2\right)\right)\sin\theta(p^2)+\\
&+\sqrt{2}\left(5F_u\left(p^2;q_1^2,q_2^2\right)+F_s\left(p^2;q_1^2,q_2^2\right)\right)\cos\theta(p^2)\Big],
\end{aligned}$$

Light-by-Light / meson to two photons

And similarly for different scalar meson states

$$
\begin{aligned}
\mathrm{A}_{a_0}\left(p^2; q_1^2, q_2^2\right) &= g_{a_0}(p^2) A_u\left(p^2; q_1^2, q_2^2\right), \\
\mathrm{A}_{\sigma}\left(p^2; q_1^2, q_2^2\right) &= \frac{g_\sigma(p^2)}{3\sqrt{3}} \times \\
&\times \Big[5A_u\left(p^2; q_1^2, q_2^2\right)\left(\cos\theta_S(p^2) - \sqrt{2}\sin\theta_S(p^2)\right) - \\
&- \sqrt{2}A_s\left(p^2; q_1^2, q_2^2\right)\left(\sqrt{2}\cos\theta_S(p^2) + \sin\theta_S(p^2)\right)\Big], \\
\mathrm{A}_{f_0}\left(p^2; q_1^2, q_2^2\right) &= \frac{g_{f_0}(p^2)}{3\sqrt{3}} \times \\
&\times \Big[5A_u\left(p^2; q_1^2, q_2^2\right)\left(\sin\theta_S(p^2) + \sqrt{2}\cos\theta_S(p^2)\right) - \\
&- \sqrt{2}A_s\left(p^2; q_1^2, q_2^2\right)\left(\sqrt{2}\sin\theta_S(p^2) - \cos\theta_S(p^2)\right)\Big].
\end{aligned}
\tag{23}
$$

Form-factor of pion into two photons is written as:

$$F_i\left(p^2;q_1^2,q_2^2\right) = 8\int\frac{d_E^4k}{(2\pi)^4}\frac{f(k_1^2)f(k_2^2)}{D_i(k_1^2)D_i(k_2^2)D_i(k^2)}\times$$
$$\times\left[m_i(k^2) - \mathrm{m}_i^{(1)}(k_1,k)J_1 - \mathrm{m}_i^{(1)}(k_2,k)J_2\right],$$
$$J_1 = k^2 + \frac{q_2^2(kq_1)(k_1q_1) - q_1^2(kq_2)(k_1q_2)}{q_1^2q_2^2-(q_1q_2)^2}, \tag{24}$$
$$J_2 = k^2 + \frac{q_1^2(kq_2)(k_2q_2) - q_2^2(kq_1)(k_2q_1)}{q_1^2q_2^2-(q_1q_2)^2},$$

where

$$k_1 = k + q_1; \qquad k_2 = k - q_2; \tag{25}$$

Light-by-Light / meson to two photons

In special kinematic we have

$$F_i\left(q_1^2;q_1^2,0\right)=8\int\frac{d_E^4k}{(2\pi)^4}\frac{f(k_1^2)f(k^2)}{D_i(k_1^2)D_i^2(k^2)}\times \tag{26}$$

$$\times\left[m_i(k^2)-\mathrm{m}_i^{(1)}(k_1,k)\overline{J}_1-m_i'(k^2)\overline{J}_2\right],$$

$$\overline{J}_1\left(k,q_1\right)=(kq_1)+\frac{2}{3}\left[k^2+2\frac{(kq_1)^2}{q_1^2}\right],$$

$$\overline{J}_2=\frac{4}{3}\left[k^2-\frac{(kq_1)^2}{q_1^2}\right],$$

$$F_i\left(0;0,0\right)=\frac{1}{m_{d,i}}\left[\frac{1}{4\pi^2}-8m_{c,i}\int\frac{d_E^4k}{(2\pi)^4}\frac{m_i(k^2)-2m_i'(k^2)k^2}{D_i^3(k^2)}\right], \tag{27}$$

Light-by-Light

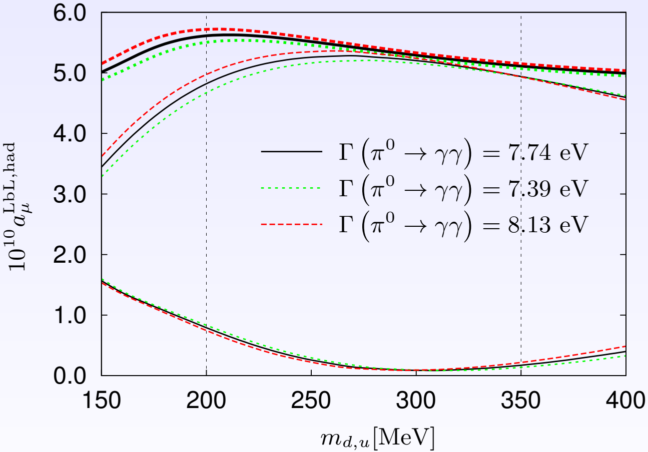
set	π^0	η	η'	$\eta+\eta'$	PS
G_I	5.05	0.55	0.27	0.82	5.87
G_{II}	5.05	0.59	0.48	1.08	6.13
G_{III}	5.05	0.53	0.18	0.71	5.76
G_{IV}	5.10	0.49	0.25	0.74	5.84

Table : Contribution pseudoscalar mesons into AMM of muon for different parametrization of model. All number times on factor 10^{-10}.

set	$a_0(980)$	σ	$f_0(980)$	S	$\pi^0+\sigma$	PS+S
G_I	0.0064	0.100	0.0035	0.110	5.15	5.98
G_{II}	0.0079	0.100	0.0038	0.110	5.15	6.24
G_{III}	0.0058	0.100	0.0034	0.109	5.15	5.87
G_{IV}	0.0060	0.115	0.0038	0.126	5.25	5.97

Table : Contribution scalar mesons into AMM of muon for different parametrization of model and general contribution with pseudoscalar mesons. All number times on factor 10^{-10}.

Light-by-Light



[DRZ, EPJC 72, 2012]

Light-by-Light

Model	π^0	η	η'	$\pi^0+\eta+\eta'$
VMD	5.74	1.34	1.19	8.27(0.64)
ENJL	5.6			8.5(1.3)
LMD+V	5.8(1.0)	1.3(0.1)	1.2(0.1)	8.3(1.2)
NJL	8.18(1.65)	0.56(0.13)	0.80(0.17)	9.55(1.66)
(LMD+V)′	7.97	1.8	1.8	11.6(1.0)
oLMDV	7.2(1.2)	1.45(0.23)	1.25(0.2)	9.9(1.6)
NχQM	6.5			
HM	6.9	2.7	1.1	10.7
DIP	6.54(0.25)			
DSE	5.75(0.69)	1.36(0.30)	0.96(0.21)	8.07(1.20)
Our results (NχQM)	5.01(0.37)	0.54(0.32)	0.30(0.18)	5.85(0.87)

Table : Contribution pseudoscalar mesons into AMM of muon for models. All number times on factor 10^{-10}.

[DRZ (EPJC 71 1702 (2011))]

Light-by-Light / Quark Box

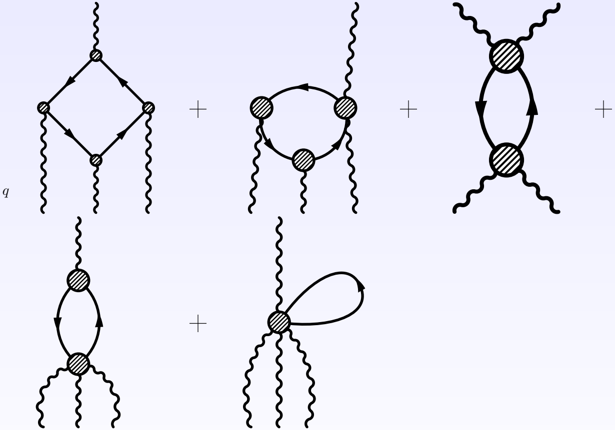
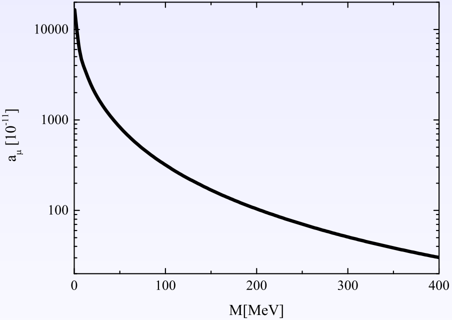


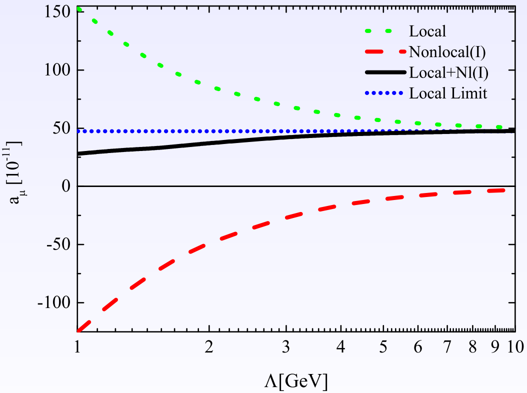
Figure : LBL box.

Light-by-Light / Quark Box



Dependence of Lepton mass

Light-by-Light / Quark Box



Light-by-Light / Quark Box

Group	Model	$a_\mu^{LBL}(\pi^0)$	a_μ^{LBL} (q-loop)
Bijnens, Prades, Pallante	ENJL	59(11)	21(3)
Hayakawa, Kinoshita	HLS	57(4)	9.7(11.1)
HK and Sanda			
Knecht and Nyffeler	LMD+V	58(10)	–
Melnikov and Vainshtein	LMD+V	77(5)	–
Dorokhov and Broniowski	NχQM	65(2)	–
Nyffeler	LMD+V	72(12)	–
Goecke, Fischer, Williams	DSE	58(10)	136(59)
Our results	NχQM	50.1(3.7)	–

Table : Results for the π^0-pole and quark-loop contribution to hadronic light-by-light scattering, in different models. All number times on factor 10^{-11}.

Conclusion

- The total contribution of pseudoscalar exchanges $a_\mu^{LbL,PS} = (5.85 \pm 0.87) \times 10^{-10}$ is approximately by factor 1.5 less than the most of previous estimates..
- The scalar mesons contribution is positive and partially cancels model dependence of the pseudoscalar contribution. The combined value for the scalar and pseudoscalar contribution is estimated as $a_\mu^{LbL,PS+S} = (6.25 \pm 0.83) \times 10^{-10}$
- We need calculate all diagrams for estimation full contribution on LBL processes

Thank you for your attention.

$(g-2)_\mu$ - motivation

We have fantastic measurable value AMM of electron (Harvard, 2008)

$$a_e^{exp} = 1159652180.73(0.28) \times 10^{-12}[0.24\text{ppb}] \quad (28)$$

$$a_e^{theor} = 1159652460.73 \times 10^{-12} \quad (29)$$

$$a_e^{SM} = a_e(QED) + +a_e(Hadron) + a_e(Weak);$$

$$a_e(QED) = \sum_{n=1}^{5} C_2 n(\frac{\alpha}{\pi})^n + ...;$$

The theoretical error is dominated by the uncertainty in the input value of the QED coupling $\alpha = e^2/(4\pi)$

$$\alpha^{-1} = 137.035999084(51)[0.37\text{ppb}] \quad (30)$$

QED is at the level of the best theory ever built to describe nature

$(g-2)_\mu$ - motivation. Lepton Anomaly

- Electron anomaly is measured extremely accurately. QED test.
- It is the best for determining α
- For a lepton L, Mass Scale contributes to a_L as $(\frac{m_L^2}{\Lambda^2})$
- Muon anomaly is measured to 0.5 parts in a million (ppm) SM test.
- Thus muon AMM leads to a $(m_\mu/m_e)^2 \sim 40000$ enhancement of the sensitivity to New Physics versus the electron AMM, the muon anomaly is sensitive to ≥ 100 GeV scale physics.

$(g-2)_\mu$. Why not tay?

- Tau due to its highest mass is the best for searching for New Physics,
- But Tau is short living particle, so the precession method is not perspective
- The best existing limits (see S. Eidelman, M. Passera, 2007)

$$-0.052 < a_\tau^{Exp} < 0.013 \tag{31}$$

- are obtained at OPAL, L3 and DELPHI (LEP, CERN) from the high energy process $e^+e^- \to e^+e^-t^+t^-$,
- While the SM estimate is

$$a_\tau^{SM} = 1.17721(5) \times 10^{-3} \tag{32}$$