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Non-Perturbative Quantum Effects in Strong Electromagnetic Fields

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with

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PR ST Accel. Beams 12, 111301 (2009)
PRD 83, 053008 (2011)
PRL 108, 240406 (2012)
PRA 87, 042106 (2013)

Bolshiye Koty @ Baikal, July 15-19, 2013

Motivation:

EM interaction is described by Lagrangian

$$\mathcal{L}_{int}^{\text{em}} = -e\left(\bar{\psi}_e \gamma \cdot A\, \psi_e\right)$$

In case of interaction of an electron with single photon(s) $|A| \sim |\varepsilon_\lambda| = 1$

the theory has a small parameter $\alpha = \frac{e^2}{4\pi} = \frac{1}{137}$ *which means that EM field does not disturb the plane wave solution* ψ_e^{PW} *of Dirac equation*

$$\psi_e(x) = \psi_e^{PW}(x) = \frac{u_p}{\sqrt{2E_p}} \mathrm{e}^{-ip\cdot x},$$

+ *Perturbation technique for* $e - A$ *interaction (Feynman rules)*

→ *"Linear Electrodynamics"* (conventionally) ***Multi-photon events with N photons are treated as multi-step events and suppresses as*** α^N

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*When an electron is moving in a strong background EM field with **LARGE*** $|A|$

$$\psi_e(x) \neq \psi_e(x)^{PW}$$

$e - A$ *interaction becomes essentially non-linear* **→ *"Non - linear Electrodynamics"***

Multi-photon events are not suppressed : charge particle may interact with N photons simultaneously. Such events play important role in dynamics

Laser pulse may be consider as a source of strong background field

$$|\vec{A}|^2 \sim |\vec{E}|^2 \sim I$$

EM field *Pulse intensity*

$$|\vec{E}|(\frac{\mathrm{V}}{\mathrm{cm}}) = 13.7\sqrt{I(\frac{\mathrm{W}}{\mathrm{cm}^2})}$$

- *UK [***VULCAN, HiPER** *(@Central Laser Facility (CLF))]*
- *EC [***Extreme Laser Infrastructure** *(ELI)]*
- *France [***APPOLON** *(@Institute de Lumiere Extreme (ILE))]*
- ⋮
- *US [* **TPL** *(Texas),(***BELLA***) Berkley, …..]*
- ⋮
- *Japan (ILE, …)*
- *Russia (***PEARL** *(Nizhnij Nivgorod))*

A.Di Piazza *et al*.,
Rev. Mod. Phys. 84, 1177 (2012)

Laser Intensities $I \sim 10^{21} - 10^{25}\ \mathrm{W/cm^2}$

Pulse duration $\tau = 5,\ 10,\ 20, \ldots\ \mathrm{fs}$

τ_0 *(one oscillation)* = 3 ~ 8 fs

optical laser

Wanted:

theory must describe different quantum processes in intensive and very short EM pulses

Outline

- ***Introduction to Breit-Wheeler (BW) process***
  - *Perturbative BW process* $\gamma'\gamma \to e^+e^-$
  - *Volkov solution of Dirac equation in Strong EM fields*
  - *BW process in an infinitely long pulse with* $N_{\text{oscillations}} >>> 1$
  - *SLAC E-144 experiment*

  Ritus *et al.*, '64~'70

- ***BW process in short EM pulses*** $N_{\text{oscillations}} < 10$
  - *General formalism*
  - *Envelope functions*
- ***Ultra short pulses*** $N_{\text{oscillations}} < 1$
- ***Short pulses*** $N_{\text{oscillations}} = 2 \sim 10$
  - *Small field intensity*
  - *Large field intensity*

- ***Compton scattering in strong EM fields***
  - *Infinitely long pulse*
  - *Short pulses* $N_{\text{oscillations}} = 0.5 \sim 10$

- ***Neutrino pair emission in strong EM fields***

- ***Conclusion***

Perturbative Breit-Wheeler (BW) process

DECEMBER 15, 1934 PHYSICAL REVIEW VOLUME 46

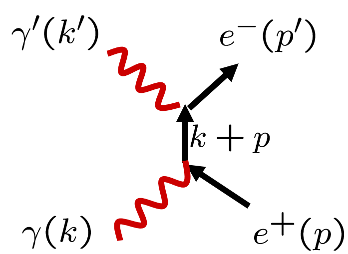
Collision of Two Light Quanta

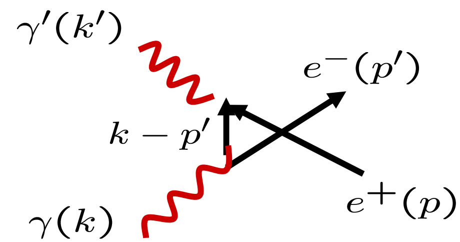
G. BREIT* AND JOHN A. WHEELER,** *Department of Physics, New York University*

(Received October 23, 1934)

The recombination of free electrons and free positrons and its connection with the Compton effect have been treated by Dirac before the experimental discovery of the positron. In the present note are given analogous calculations for the production of positron electron pairs as a result of the collision of two light quanta. The angular distribution of the ejected pairs is calculated for different polarizations, and formulas are given for the angular distribution of photons due to recombination. The results are applied to the collision of high energy photons of cosmic radiation with the temperature radiation of interstellar space. The effect on the absorption of such quanta is found to be negligibly small.

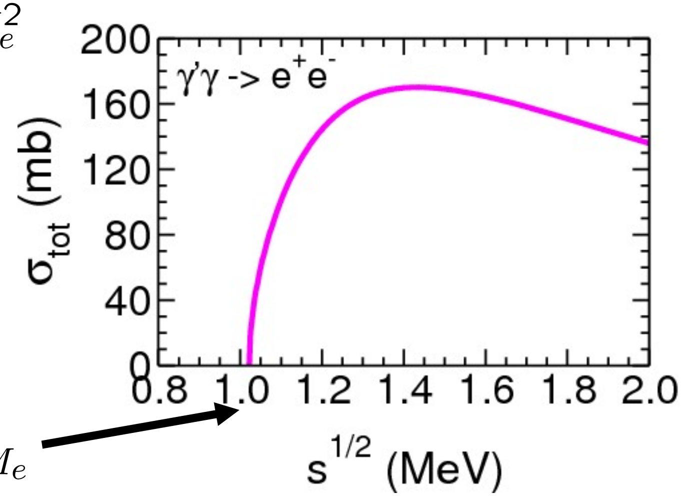
$$d\sigma^{BW} = \frac{1}{32\pi s}\frac{|\vec{p}|}{|\vec{k}|}|T^{BW}|^2\, d\cos\theta_{\widehat{kp}}$$





$$\sigma_{\rm tot}^{BW} = \frac{e^4}{4\pi s\gamma^4}\left\{(2\gamma^4+2\gamma^2-1)\operatorname{arc\,sh}(\sqrt{1-\gamma^2}) - \gamma(1+\gamma^2)\sqrt{1-\gamma2}\right\}$$

$$\gamma^2 = 1/(1-v^2) = E_p^2/M_e^2$$



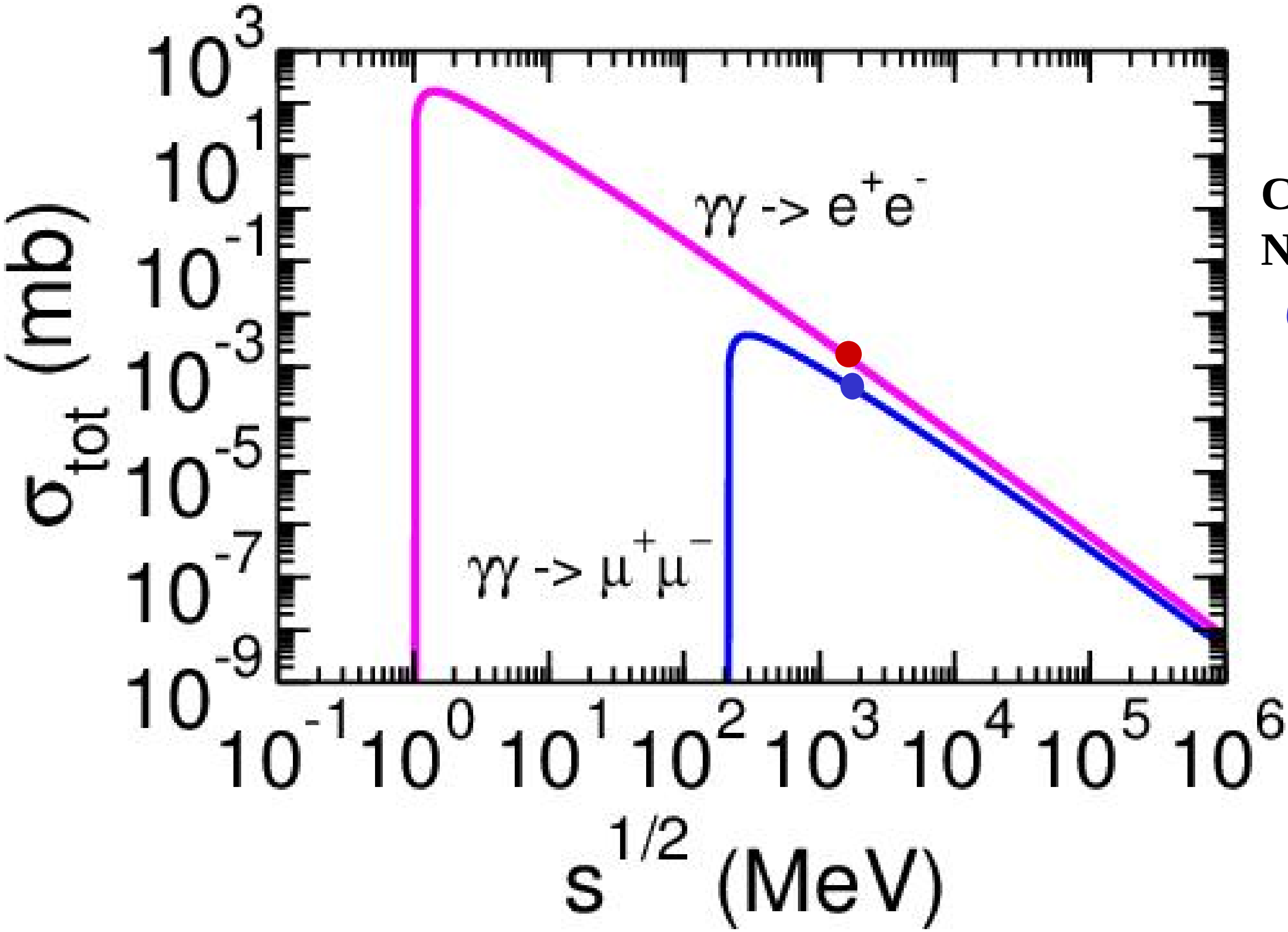
$$s_{\rm thr.}^{1/2} = 2M_e$$

$$\omega'_{\rm thr} = M_e^2/\omega$$

$$\omega'_{\rm thr} \simeq 250\text{GeV} \,@\, \omega = 1\,\text{eV}$$

$$\omega'_{\rm thr} \simeq 250\text{keV} \,@\, \omega = 1\,\text{MeV}$$

BW process for electron and muon pairs in GeV region



Courau *et al.*
NPB271 (1986)
(Orsay, DCI)

Modification of BW process in a strong EM field (expectation)

Increase of the rate of pair production $W^{e^+e^-}$ **with field intensity**

The emergence of multi-photon effects

Subthreshold e^+e^- **pair production**

Electron in a strong electromagnetic field

D.M. Volkov, Z. Phys. 94, 250 (1935)

Über eine Klasse von Lösungen der Diracschen Gleichung.

A class of solutions of the Dirac equation

1. Der Fall eines sinusoidalen Feldes. — 2. Lösung für den Fall, daß das äußere Feld aus polarisierten, in einer bestimmten Richtung fortschreitenden Wellen besteht, die ein *abzählbares* Spektrum nach Frequenz und Anfangsphasen haben.

LL v.4, §40 (BLP)

$$(i\nabla\!\!\!/ - eA\!\!\!/ + m)\cdot(i\nabla\!\!\!/ - eA\!\!\!/ - m)\psi = 0,$$

$$[(i\nabla - eA)^2 - m^2 - i\frac{1}{2}F_{\mu\nu}\sigma^{\mu\nu}]\psi = 0,$$

Second order Dirac equation

where $F_{\mu\nu} = \partial_\nu A_\mu - \partial_\mu A_\nu$ *is e.m field tensor*

Special case: $A = A(\phi)$ *with* $\phi = k\cdot x = \omega t - kz$ → *plane wave*

solution of Dirac equation is sought in the form

$$\psi = \mathrm{e}^{-ipx}\,F(\phi), \quad \phi = kx$$

2 conditions:

transversality $\partial_\mu A^\mu = k_\mu A^{\mu\prime} = 0$

and $\partial^\mu F = k^\mu F', \quad \partial_\mu\partial^\mu F = k^2 F'' = 0,$ *since* $k^2 = 0$

$$2i(kp)F' + [-2e(pA) + e^2A^2 - ie(\gamma k)(\gamma A')]F = 0$$

$$F = \exp\left(-i\int_o^{kx}\left[\frac{e}{(kp)}(pA(\phi')) - \frac{e^2A^2(\phi')}{2(kp)}\right]d\phi' + \frac{e(\gamma k)(\gamma A)}{2(kp)}\right)\frac{u}{\sqrt{2p_0}}$$

Solution:

S
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$$\psi_p = \left[1 + \frac{e(\gamma \cdot k)(\gamma \cdot A)}{2(k \cdot p)}\right] \frac{u_p}{\sqrt{2E_p}} \mathrm{e}^{-ip\cdot x} \cdot \mathrm{e}^{iS'(\phi)}$$

spinor modification ***phase factor***

with $$S'(\phi) = -\int_0^{kx} \left[\frac{e(p \cdot A)}{(k \cdot p)} - \frac{e^2 A^2}{2(k \cdot p)}\right] d\phi'$$

when $\vec{A} \to 0$ ***or*** $(a_x, a_y \to 0)$

$$\psi_p \to \psi_p^{PW} = \frac{u_p}{\sqrt{2E_p}} \mathrm{e}^{-ip\cdot x}$$

Dirac solution for free electron

Properties of Volkov's solution

✦ Effective "quasi momentum"

$$\langle \psi^*(\widehat{p}^\mu - eA^\mu)\psi\rangle = q^\mu - eA^\mu$$

$$q^\mu \equiv p^\mu - \frac{e^2\langle A^2\rangle}{2(k\cdot p)}k^\mu = p^\mu + \frac{e^2a^2}{2(k\cdot p)}k^\mu = p^\mu + \frac{\xi^2 m_e^2}{2(k\cdot p)}k^\mu$$

$$\langle A^2\rangle = -\frac{1}{2}(a_x^2 + a_y^2) = -a^2$$ **with** $$\xi^2 = \frac{e^2a^2}{m_e^2} = \frac{e^2E^2}{m_e^2\omega^2}$$

reduced field intensity

✦ Effective electron mass

$$q^2 = m_*^2 \equiv m_e^2\left(1 - \frac{e^2\langle A^2\rangle}{m_e^2}\right) = m_e^2\left(1 + \xi^2\right)$$

$$\boxed{m_{e*}^2 = m_e^2(1+\xi^2)} \longrightarrow m_*^2 > m_e^2$$

"quasi-momentum" and effective mass define momentum-energy conservation in processes with electrons (in infinite pulse !)

Dynamical variables for BW process

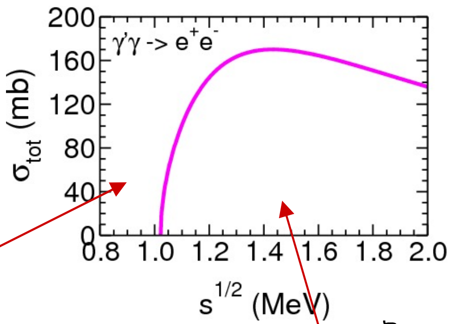
reduced EM field intensity

$$\xi^2 = \frac{e^2 a^2}{M_e^2}$$

threshold parameter

$$\zeta = \frac{4M_e^2}{s}$$

short pulse / vacuum case



$$\zeta > 1$$

$$\zeta < 1$$

sub-threshold

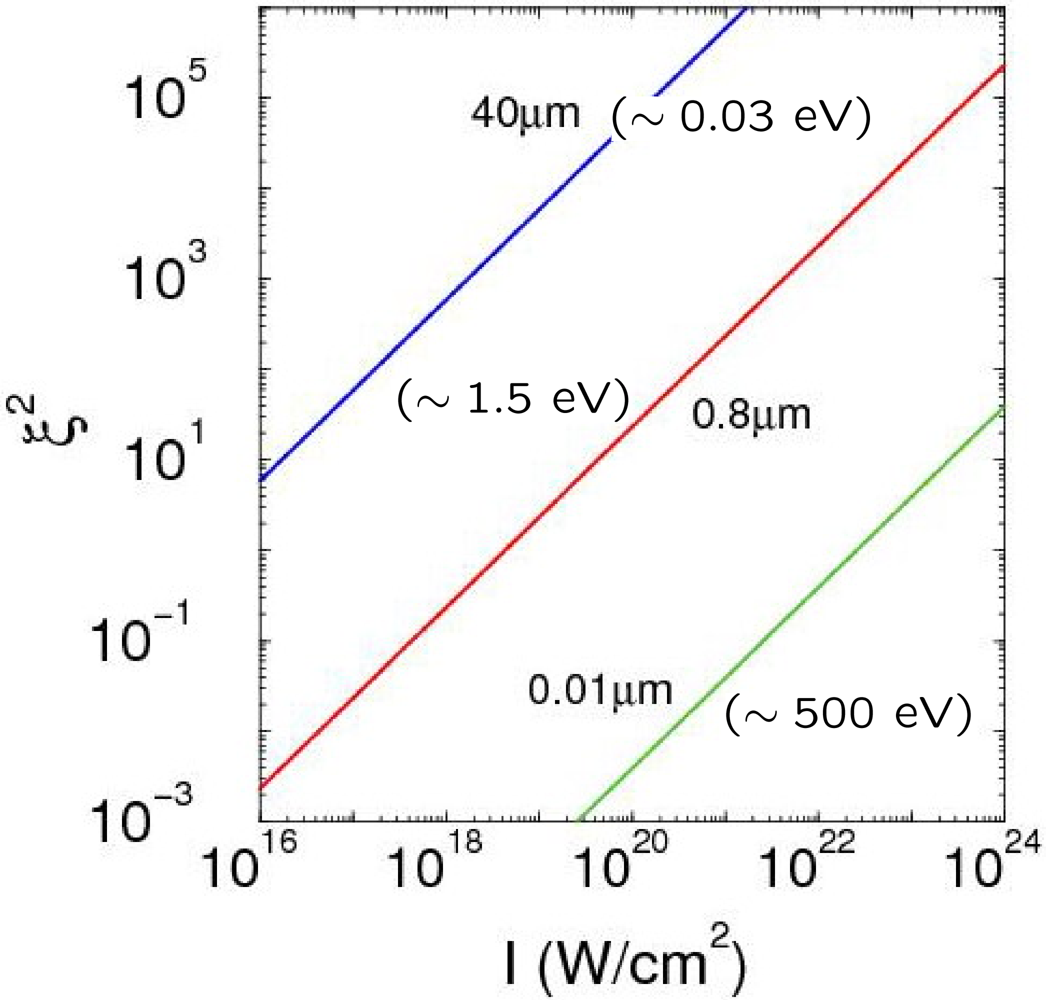
above threshold

infinite pulse

$$\zeta_{IPA} = \frac{s_{\mathrm{thr}}}{s} = \frac{4M_*^2}{s} = \frac{4M_e^2(1+\xi^2)}{s} = \zeta(1+\xi^2)$$

Infinite Pulse Approximation

Dependance of reduced field strengh ξ^2 on laser pulse intensity I at different wavelength λ



$$\xi^2 = \frac{4\pi\alpha(\hbar c)^3}{M_e^2 c^4 \omega^2} \frac{I}{c}$$

$$\xi^2 = 0.56(\omega(\mathrm{eV}))^{-2} 10^{-18} \mathrm{I}/(\mathrm{W/cm}^2)$$

Early studies:

N.~D.~Sengupta, Bull. Calcutta Math. Soc. {\bf 44}, 175, (1952).
A.~I.~Nikishov and V.~I.~Ritus, Sov.\ Phys.\ JETP {(1964-79)
N.~B.~Narozhnyi, A.~I.~Nikishov, and V.~I.~Ritus,
Sov.\ Phys.\ JETP {\bf 20}, 622 (1965)
I.~I.~Goldman, Phys. Lett. {\bf 8}, 103 (1964)
L.~S.~Brown and T. W. B. Klibbe, Phys. Rev. A {\bf 133}, 705 (1964).
H.~R.~Reiss, J. Math. Phys. {\bf 3}, 59 (1962).

⋮

$\gamma' A \to e^+ e^-$

$eA \to e'\gamma'$

$eA \to e'\nu_e\bar{\nu}_e$

$\pi^- A \to e\bar{\nu}_e$

⋮

(i) *Infinite field (pulse)* $N_{\text{oscillations}} >>> 1$

+

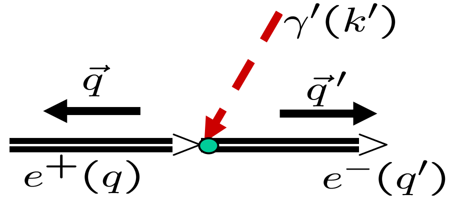
(ii) $\xi \ll 1$

(iii) $\xi^2 \to \infty; \quad \xi \sim \frac{1}{\omega_\gamma}$

$\omega_\gamma \to 0$

$$\begin{cases} \xi \cdot (k \cdot k') \to \text{constant} \\ \xi \cdot (k \cdot p) \to \text{constant} \end{cases}$$

Breit-Wheeler process for infinitely long pulse



$$S_{fi} = -ie \int \bar{\psi}_{e^-}(\gamma \cdot \varepsilon'(\gamma'))\psi_{e^+} \mathrm{e}^{-ik'\cdot x} \frac{d^4x}{\sqrt{2\omega'}} \ ,$$

$$\frac{\bar{u}_{p'}}{\sqrt{2q_0'}} \mathrm{e}^{iq'\cdot x} \left[1 + \frac{e(\gamma\cdot A)(\gamma\cdot k)}{2(k\cdot p')}\right] \mathrm{e}^{-i\int\limits_0^{k\cdot x} \frac{e(p'\cdot A)}{(k\cdot p')} d\phi'} \qquad \left[1 - \frac{e(\gamma\cdot k)(\gamma\cdot A)}{2(k\cdot p)}\right] \mathrm{e}^{-i\int\limits_0^{kx} \frac{e(p\cdot A)}{(k\cdot p)} d\phi'} \frac{v_p}{\sqrt{2q_0}} \mathrm{e}^{iq\cdot x}$$

$$S_{fi} = \frac{-ie}{\sqrt{2q_0 2q_0' 2\omega'}} \int M_{fi}(kx) \mathrm{e}^{-i(k'-q-q')x} d^4x \qquad \neq (2\pi)^4 \delta^4(k+k'-q-q') \cdot M$$

$$M_{fi}(kx) = \sum_{n=-\infty}^{\infty} \mathrm{e}^{-in\,k\cdot x} M_{fi}(n)$$

Fourier series
for functions defined in all space

$$S_{fi} = \frac{-ie}{\sqrt{2q_0 2q_0' 2\omega'}} \sum_{n=n_{\min}}^{\infty} M_{fi}(n)\,(2\pi)^4\,\delta^4(k'+nk-q-q')$$

$$M_{fi}(\phi) = \bar{u}_{p'}\left[\widehat{M}^{(0)} + \widehat{M}^{(1)}\cos\phi + \widehat{M}^{(2)}\sin\phi\right] v_p \mathrm{e}^{-iz\sin(\phi-\phi_0)}, \quad \phi = k\cdot x,$$

$$M_{fi}(n) = \bar{u}_{p'}\left[\widehat{M}^{(0)}B_n^{(0)} + \widehat{M}^{(1)}B^{(1)} + \widehat{M}^{(2)}B^{(2)}\right] v_p$$

$$\widehat{M}^{(0)} = \not{\varepsilon}' + \frac{e^2a^2(\varepsilon'\cdot k)}{2(k\cdot p)(k\cdot p')}, \qquad \widehat{M}^{(1,2)} = e\frac{\not{\varepsilon}'\not{k}\not{a}_{1,2}}{2(k\cdot p)} + e\frac{\not{a}_{1,2}\not{k}\not{\varepsilon}'}{2(k\cdot p)}$$

Breit-Wheeler process for infinitely long pulse (continuation)

the functions $B_n^{0,1,2}$ are expressed via the Bessel functions of the first kind

$$M_{fi}(n) = \bar{u}_{p'}\left[\widehat{M}^{(0)}B_n^{(0)} + \widehat{M}^{(1)}B^{(1)} + \widehat{M}^{(2)}B^{(2)}\right] v_p$$

$$B_n^{(0)} = \frac{1}{2\pi}\int_{-\pi}^{\pi} d\phi\, \mathrm{e}^{i(n\phi - z\sin(\phi-\phi_0))} = J_n(z)\,\mathrm{e}^{in\phi_0}$$

$$B_n^{(1)} = \frac{1}{2\pi}\int_{-\pi}^{\pi} d\phi \cos\phi\, \mathrm{e}^{i(n\phi - z\sin(\phi-\phi_0))} = \frac{1}{2}\left(J_{n+1}(z)\,\mathrm{e}^{i(n+1)\phi_0} + J_{n-1}(z)\,\mathrm{e}^{i(n-1)\phi_0}\right)$$

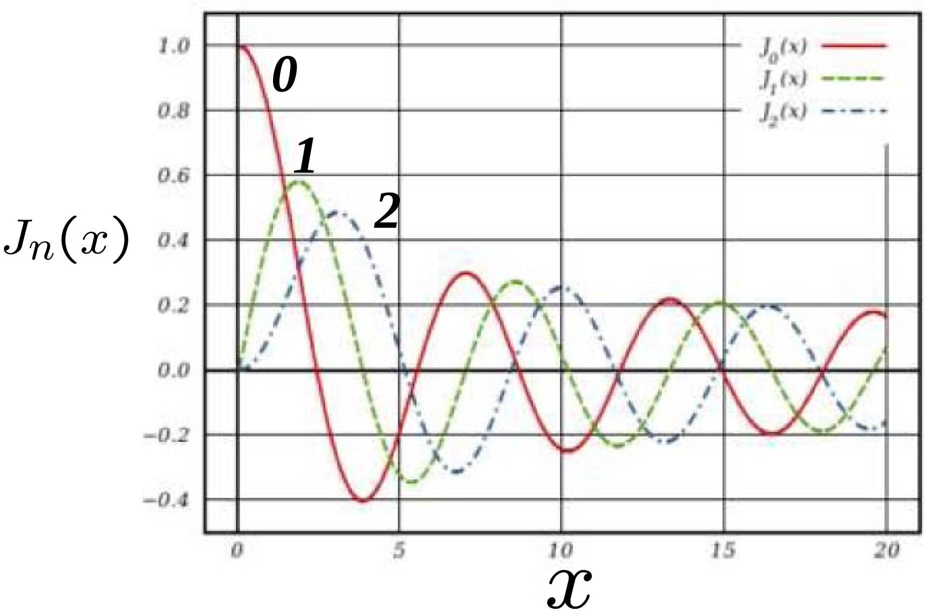
$$B_n^{(2)} = \frac{1}{2\pi}\int_{-\pi}^{\pi} d\phi \sin\phi\, \mathrm{e}^{i(n\phi - z\sin(\phi-\phi_0))} = \frac{1}{2i}\left(J_{n+1}(z)\,\mathrm{e}^{i(n+1)\phi_0} - J_{n-1}(z)\,\mathrm{e}^{i(n-1)\phi_0}\right) .$$

$$J_n(z) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \mathrm{e}^{i(n\phi - z\sin(\phi))}\, d\phi$$

$$z = \frac{2n\xi}{\sqrt{1+\xi^2}}\sqrt{\frac{u}{u_n}\left(1 - \frac{u}{u_n}\right)} ,$$

$$u = \frac{(k\cdot k')^2}{4(k\cdot p)(k\cdot p')} = \frac{1}{1 - v^2\cos^2\theta},\quad v = \frac{|\vec{q}|}{q_0}$$

$$u_n = \frac{n}{n_{\min}} = \frac{n}{\zeta(1+\xi^2)} ,$$



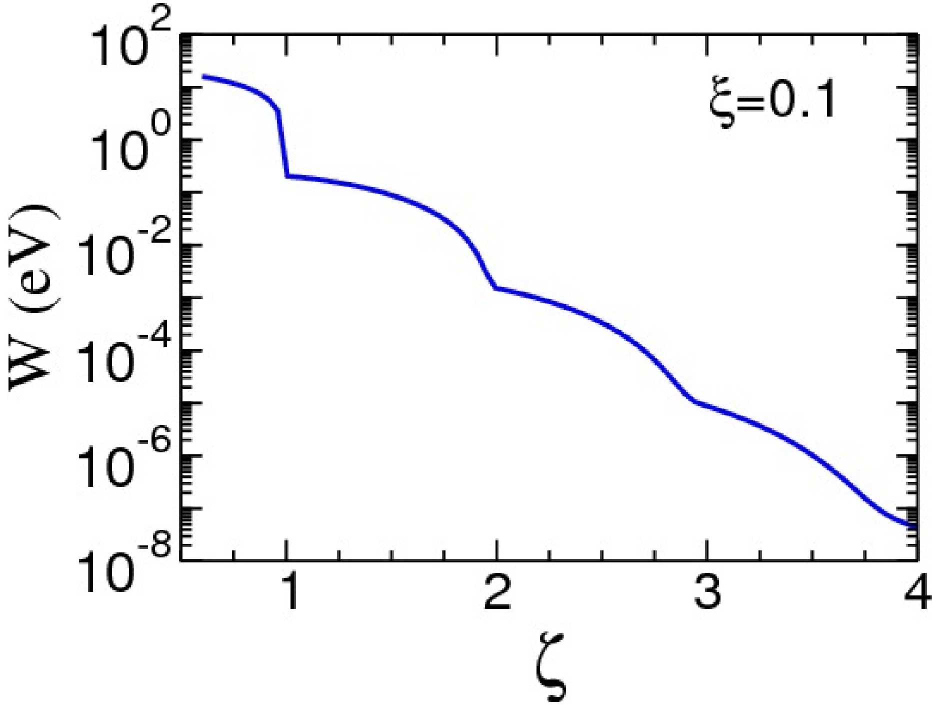
Breit-Wheeler process for infinitely long pulse (continuation)

The total rate is an infinite sum of "partial harmonics"

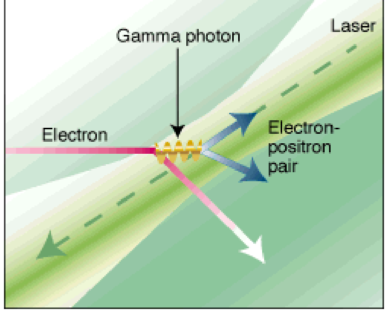
$$W = \sum_{n=n_{\text{min}}}^{\infty} W^{(n)} = \frac{\alpha M_e^2}{4\omega'} \sum_{n=n_{\text{min}}}^{\infty} w^{(n)} \frac{du}{u^{3/2}\sqrt{u-1}} \; ,$$

with $$n_{\text{min}} = I(\zeta(1+\xi^2))$$

and $$w^{(n)} = 2J_n^2(z) + \xi^2(2u-1)\left(J_{n+1}^2(z) + J_{n-1}^2(z) - 2J_n^2(z)\right)$$



SLAC (E-144) experiment *D. Burke et al.,PRL 79 (1997)*



$$\gamma' + L \to e^+ e^-$$ ***Generalized Breit-Wheeler process***

BW process (kinematics)

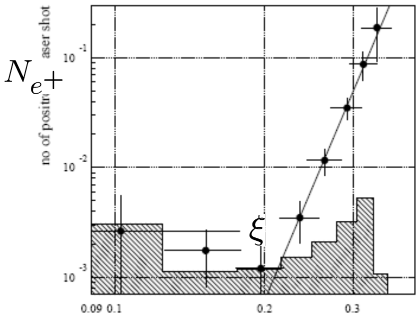
$$s_{thr}(\gamma'\gamma) = (k+k')^2 = 4\omega\omega' = 4M_e^2 \qquad \omega'_{thr} = \frac{M_e^2}{\omega} \simeq \frac{0.26\cdot 10^{12}(eV^2)}{2.35\,eV\,(SLAC)} \simeq 111 \text{ GeV}$$

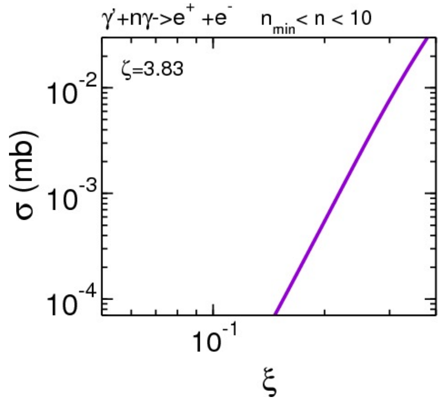
$$\omega'_{Bremsst} \simeq 29 \text{ GeV } (SLAC) \rightarrow \zeta = \frac{\omega'_{thr}}{\omega'_{Bremmst}} \simeq 3.83$$ ***with*** $0.1 < \xi < 0.36$

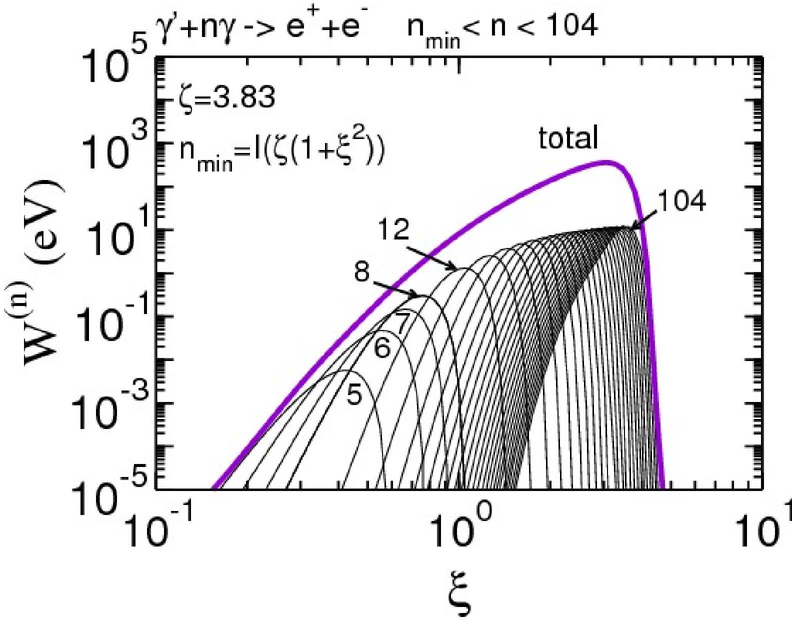
$$\gamma' + n\gamma \to e^+e^- \quad \to n_{\min} = I(\zeta(1+\xi^2)) = 5$$ ***essentially multi-photon process***

$$\xi^2 = 0.56(\omega(\text{eV}))^{-2}10^{-18}\text{I}/(\text{W/cm}^2) \;\; (\omega = 2.35 \text{ eV})$$

$$I \sim 2\times 10^{18} \text{ W/cm}^2 \rightarrow \xi^2 \sim 0.1$$







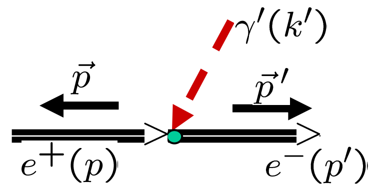
Breit-Wheeler process in a short e.m. pulse

Short circularly polarized pulse:

$$A = (0, \vec{A}_\gamma) \;\; \vec{A}_\gamma = f(kx)\,[\vec{a}_x \cos(kx) + \vec{a}_y \sin(kx)]\,, \qquad |\vec{a}_x| = |\vec{a}_y| = a \qquad \text{and} \qquad \vec{a}_x\vec{a}_y = 0$$

where $f(kx)$ *is "envelope" function*

$$S_{fi} = -ie \int \bar{\psi}_{e^-}(\gamma \cdot \varepsilon'(\gamma'))\psi_{e^+} \mathrm{e}^{-ik'\cdot x} \frac{d^4x}{\sqrt{2\omega'}}\ ,$$



$$\frac{\bar{u}_{p'}}{\sqrt{2p'_0}}\mathrm{e}^{ip'\cdot x}\left[1 + \frac{e(\gamma\cdot A)(\gamma\cdot k)}{2(k\cdot p')}\right]\mathrm{e}^{\,i\int\limits_0^{k\cdot x}\left[\frac{e(p'\cdot A)}{(k\cdot p')} - \frac{e^2A^2}{(2k\cdot p')}\right]d\phi'}$$

$$\left[1 - \frac{e(\gamma\cdot k)(\gamma\cdot A)}{2(k\cdot p)}\right]\mathrm{e}^{-i\int\limits_0^{kx}\left[\frac{e(p\cdot A)}{(k\cdot p)} + \frac{e^2A^2}{(2k\cdot p)}\right]d\phi'}\frac{v_p}{\sqrt{2p_0}}\mathrm{e}^{ip\cdot x}$$

$$S_{fi} = \frac{-ie}{\sqrt{2q_0 2q'_0 2\omega'}} \int M_{fi}(kx)\mathrm{e}^{-i(k'-q-q')x} d^4x$$

$$M_{fi}(kx) = \int\limits_{-\infty}^{\infty} dl \mathrm{e}^{-il\,k\cdot x} M_{fi}(l)$$

Fourier Integral
for functions limited in space

$$S_{fi} = \frac{-ie}{\sqrt{2p_0 2p'_0 2\omega'}} \int\limits_{\zeta}^{\infty} dl\, M_{fi}(l)(2\pi)^4\delta^4(k' + lk - p - p'),$$

Transition matrix and production probability

$$M_{fi}(l) = \sum_{i=0}^{3} M^{(i)}\, C^{(i)}(l) \;,$$

where

$$C^{(0)}(l) = \frac{1}{2\pi l} \int_{-\infty}^{\infty} d\phi \left(z \cos(\phi - \phi_0)\, f(\phi) - \xi^2 \zeta u\, f^2(\phi) \right) \mathrm{e}^{il\phi - i\mathcal{P}(\phi)}$$

$$C^{(1)}(l) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\phi f^2(\phi)\, \mathrm{e}^{il\phi - i\mathcal{P}(\phi)} \;,$$

$$C^{(2)}(l) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\phi f(\phi) \cos\phi\, \mathrm{e}^{il\phi - i\mathcal{P}(\phi)} \;,$$

$$C^{(3)}(l) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\phi f(\phi) \sin\phi\, \mathrm{e}^{il\phi - i\mathcal{P}(\phi)} \;,$$

$$M^{(i)} = \bar{u}_{p'}\, \widehat{M}^{(i)}\, v_p$$

with

$$\widehat{M}^{(0)} = \not\epsilon' \;, \qquad \widehat{M}^{(1)} = \frac{e^2(\not a_1 \not k \not\epsilon' \not k \not a_1 + \not a_2 \not k \not\epsilon' \not k \not a_2)}{4(k\cdot p)(k\cdot p')} \;,$$

$$\widehat{M}^{(2)} = \frac{e\not a_1 \not k \not\epsilon'}{2(k\cdot p')} + \frac{e\not\epsilon' \not k \not a_1}{2(k\cdot p)} \;, \qquad \widehat{M}^{(3)} = \frac{e\not a_2 \not k \not\epsilon'}{2(k\cdot p)} + \frac{e\not\epsilon' \not k \not a_2}{2(k\cdot p')} \;,$$

with

$$\mathcal{P}(\phi) = z \int_{-\infty}^{\phi} d\phi' \cos(\phi' - \phi_0) f(\phi') - \xi^2 \zeta u \int_{-\infty}^{\phi} d\phi'\, f^2(\phi') \;. \qquad \phi = k\cdot x$$

$$z = 2l\xi \sqrt{\frac{u}{u_l}\left(1 - \frac{u}{u_l}\right)} \;, \qquad u_l \equiv l/\zeta$$

Production probability

$$\frac{dW}{d\phi_p\, du} = \frac{\alpha M_e \zeta^{1/2}}{16\pi N_0} \frac{1}{u^{3/2}\sqrt{u-1}} \int_{\zeta}^{\infty} dl\;\; w(l)$$

with

$$\frac{1}{2} w(l) = (2u_l + 1)|C^{(0)}(l)|^2 + \xi^2(2u-1)(|C^{(2)}(l)|^2 + |C^{(3)}(l)|^2)$$
$$+\; \mathrm{Re}\, C^{(0)}(l) \left(\xi^2 C^{(1)}(l) - \frac{2z}{\zeta}(\alpha_1 C^{(2)}(l) + \alpha_2 C^{(3)}(l)) \right)^* \;.$$

Comparison between infinite and finite pulses

$$M_{fi}(n) = \sum_{i=0}^{2} M^{(i)} B_n^{(i)} \ ,$$

$$B_n^{(i)} = B\left[J_{n\pm1},\ J_n\right]$$

$$J_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} d\phi\, \mathrm{e}^{il\phi - iz\sin(\phi-\phi_e)}$$

$$\phi = k\cdot x$$

$$W = \sum_n [\text{one dimentional integral}]$$

$$M_{fi}(l) = \sum_{i=0}^{3} M^{(i)} C^{(i)}(l) \ ,$$

$$C^{(i)}(l) = C\left[\mathcal{Y}(l\pm 1),\ (\mathcal{Y}(l)\right]$$

$$\mathcal{Y}(l) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\phi\, \mathrm{e}^{il\phi - i\mathcal{P}(\phi)}\, f(\phi)$$

with

$$\mathcal{P}(\phi) \ = \ z\int_{-\infty}^{\phi} d\phi'\, \cos(\phi'-\phi_e) f(\phi') - \ \xi^2\zeta u \int_{-\infty}^{\phi} d\phi'\, f^2(\phi') \ .$$

$$W = [\text{five dimentional integral}]$$

Envelope functions

One parameter functions

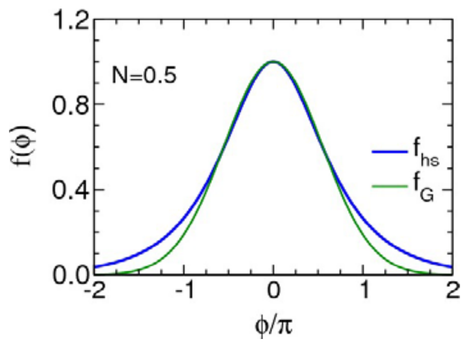
$$f_{\mathrm{hs}}(\phi) = \frac{1}{\cosh\frac{\phi}{\Delta}}\ , \qquad f_{\mathrm{G}}(\phi) = \exp\left[-\frac{\phi^2}{2\tau_{\mathrm{G}}^2}\right]\ .$$

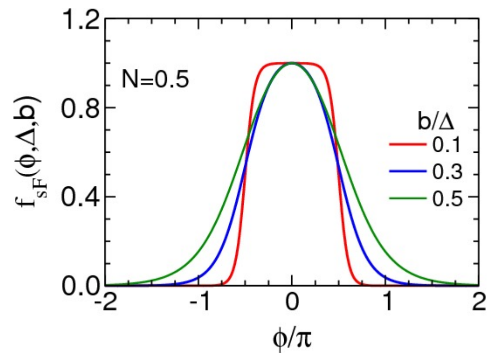
hyperbolic secant (hs) ***Gaussian (G)***

Two parameter function: ***"summarized Fermi (sF)"*** shape

$$f_{\mathrm{sF}}(\phi) = \frac{\cosh\frac{\tau_{\mathrm{sF}}}{b} + 1}{\cosh\frac{\tau_{\mathrm{sF}}}{b} + \cosh\frac{\phi}{b}} \qquad f(0) = 1$$

Lukyanov et al.,'70~'80





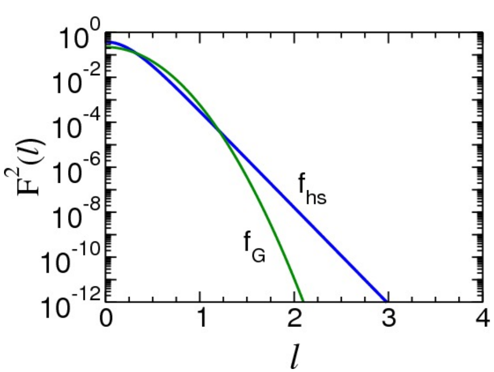
$\tau_{sF} = \Delta$

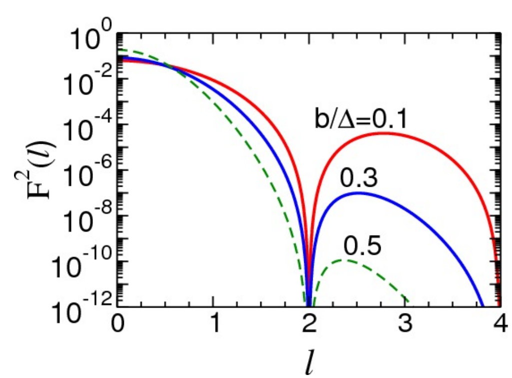
$\frac{b}{\Delta}$

$\Delta = 2\pi N$

Fourier transforms

$$F(l) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\phi\, \mathrm{e}^{il\phi} f(\phi);$$





Ultra-Short Pulse (N<1)

small field intensity $\xi^2 \ll 1,\ \zeta\xi \ll 1,\ z \ll 1$

$$\mathcal{Y}(l) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\phi\, e^{il\phi} f(\phi)\, g(\phi) = \int_{-\infty}^{\infty} dq\, F(l-q)\, G(q)\ ,$$

$$F(p) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\phi\, e^{ip\phi} f(\phi); \qquad G(q) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\phi\, e^{iq\phi} g(\phi)$$

$$g(\phi) = e^{-i\mathcal{P}(\phi)}; \qquad G(q) \simeq \delta(q-q_0), \qquad q_0 \simeq \langle \mathcal{P}'_\phi \rangle \simeq \xi\zeta \ll 1$$

$$\boxed{\mathcal{Y}(l) \simeq F(l)}$$

Production probability: $W = \alpha M_e \zeta^{1/2} \xi^2 \int_{\zeta}^{\infty} dl\, \Phi(l)\, F^2(l-1)$

$$\Phi(l) = \frac{1}{2}\left\{\left(1+\frac{\zeta}{l}-\frac{\zeta^2}{2l^2}\right)\ln\frac{1+v}{1-v} - v\left(1+\frac{\zeta}{l}\right)\right\}\ .$$

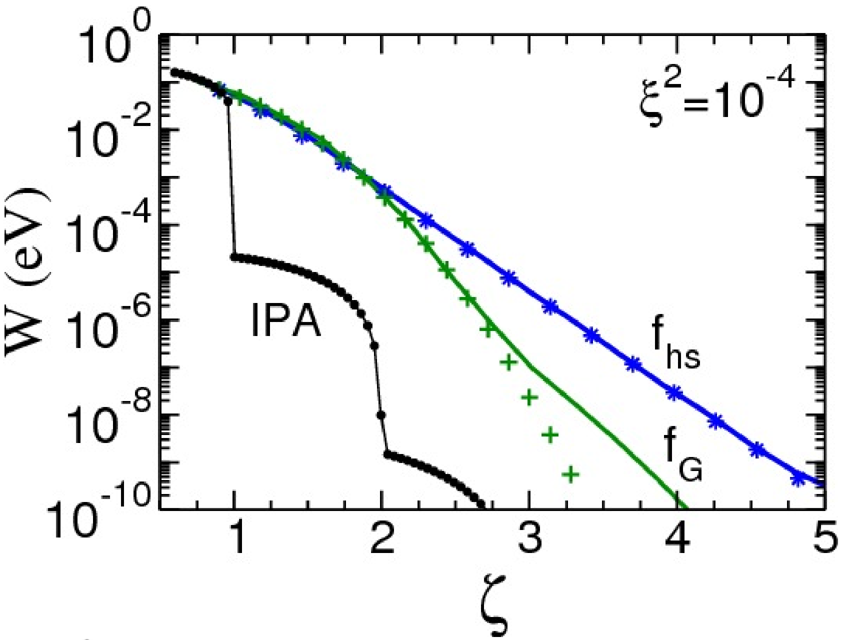
$$F_{\mathrm{hs}}(l) = \frac{\Delta}{2\cosh\frac{1}{2}\pi\Delta l}\ ,$$

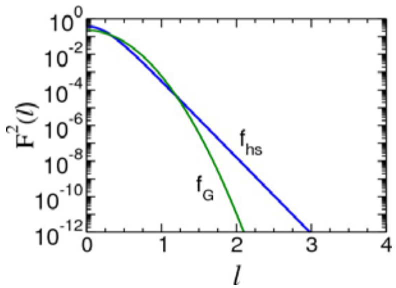
$$F_{\mathrm{G}}(l) = \frac{\tau_{\mathrm{G}}}{\sqrt{2\pi}}\exp\left[-\frac{1}{2}\tau_{\mathrm{G}}^2 l^2\right]\ ,$$

$$F_{\mathrm{sF}}(l) = \frac{1+\exp\left[-\frac{\Delta}{b}\right]}{1-\exp\left[-\frac{\Delta}{b}\right]}\,\frac{b\sin\Delta\cdot l}{\sinh\pi b l}\ .$$

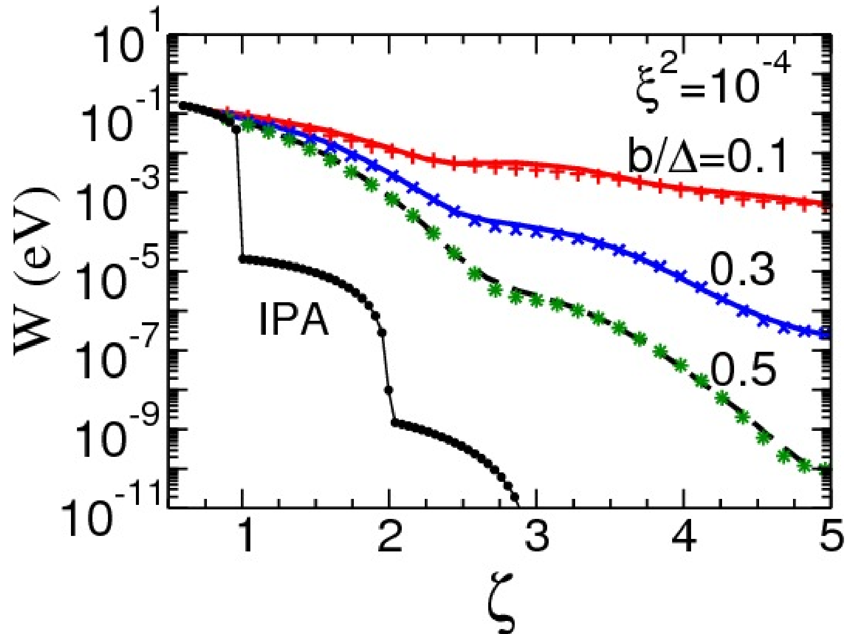
Production probability (rate) as a function of sub-threshold parameter

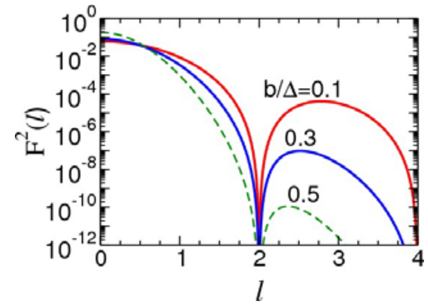
One-parameter





Two-parameters





$$F_{\mathrm{sF}}(l) = \frac{1+\exp\left[-\frac{\Delta}{b}\right]}{1-\exp\left[-\frac{\Delta}{b}\right]} \, \frac{b \sin \Delta \cdot l}{\sinh \pi b l} \, .$$

Sub-cycle pulse may be consider as a power amplifier

Anisotropy of e^- *emission at finite field intensities* $\xi^2 \sim 1$

$$\mathcal{Y}(l) \sim \int d\phi \, \mathrm{e}^{il\phi - iz \int_{-\infty}^{\phi} d\phi' f(\phi') \cos(\phi' - \phi_e)} f(\phi)$$

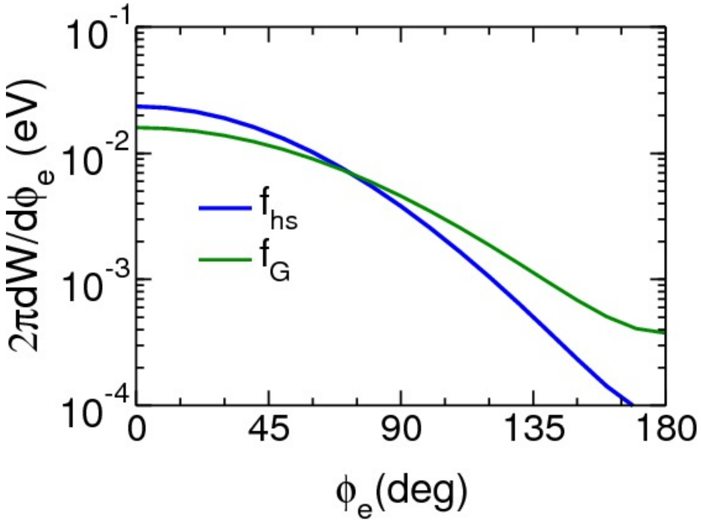
$$= \int d\phi \, \mathrm{e}^{il\phi - iz\left(\cos\phi_{e^-} \int_{-\infty}^{\phi} d\phi' f(\phi') \cos\phi' + \sin\phi_{e^-} \int_{-\infty}^{\phi} d\phi' f(\phi') \sin\phi'\right)} f(\phi) \ .$$

For the one-parameter envelope functions

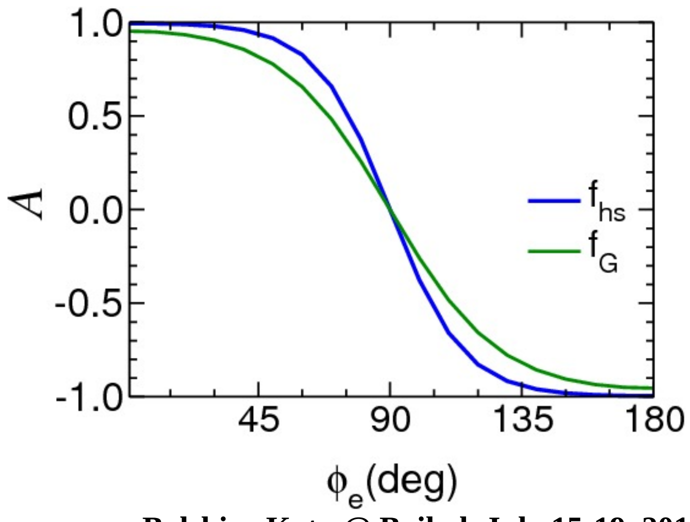
$$\cos\phi_{e^-} \int_{-\infty}^{\phi} d\phi' f(\phi') \cos\phi' \gg \sin\phi_{e^-} \int_{-\infty}^{\phi} d\phi' f(\phi') \sin\phi'.$$

$$\mathcal{A} = \frac{dW(\phi_e) - dW(\phi_e + \pi)}{dW(\phi_e) + dW(\phi_e + \pi)} \ .$$

Cross section



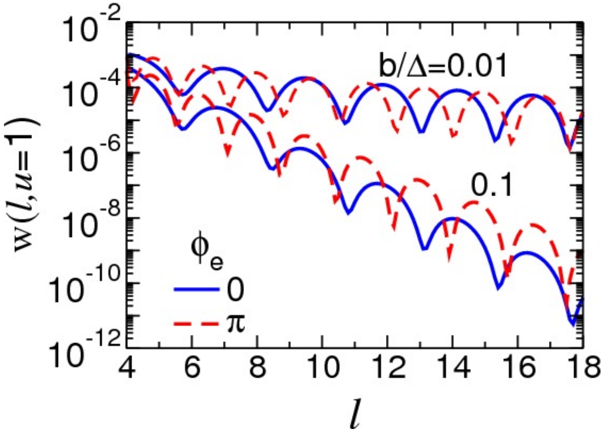
Anisotropy

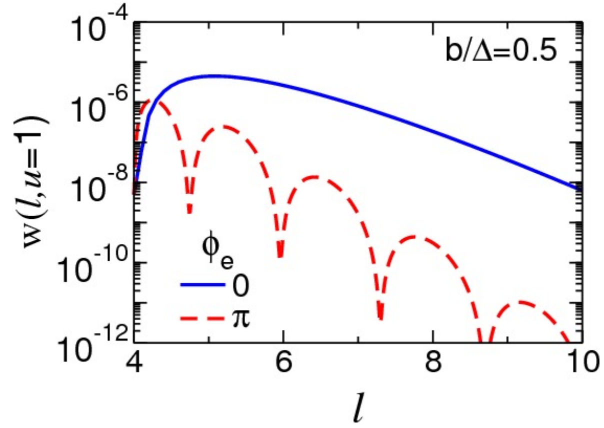


Two parameters envelope functions

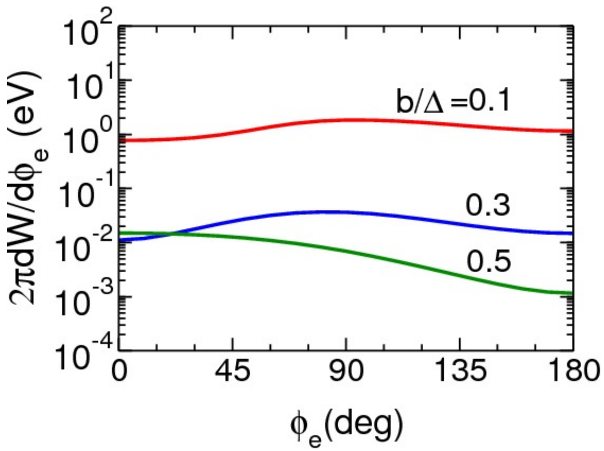
$$W \sim \int_{\zeta}^{\infty} dl\, w(l)$$

Partial probability

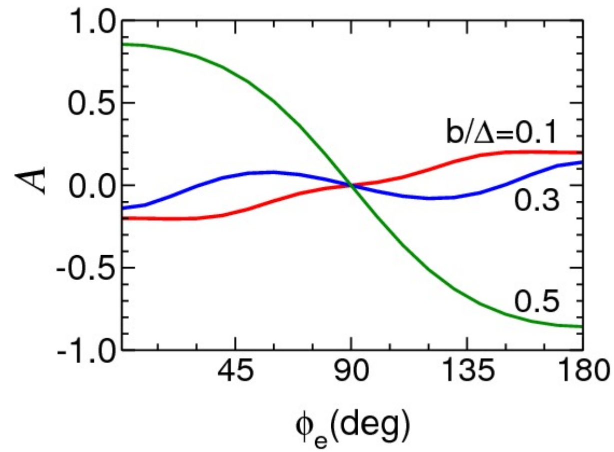




Production rate



Anisotropy



Conclusion:

Anisotropy of electrons at finite field intensities is sensitive to the shape of the pulse envelop

Short Pulse (N=2~10)

$$\mathcal{Y}(l) = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(\phi) d\phi \, e^{il\phi - i\mathcal{P}(\phi)} \quad \textit{with} \quad \mathcal{P}(\phi) = z \int_{-\infty}^{\phi} d\phi' \cos(\phi' - \phi_e) f(\phi') - \xi^2 \zeta u \int_{-\infty}^{\phi} d\phi' \, f^2(\phi') \; .$$

$$\mathcal{P}(\phi) \equiv \mathcal{P}_0(\phi) - \xi^2 \zeta u \int_{-\infty}^{\phi} d\phi' \, f^2(\phi'), \qquad \mathcal{P}_0(\phi) = z \left(\sin(\phi - \phi_e) f(\phi) + \mathcal{O}\left(\frac{1}{\Delta}\right)\right)$$

$$\mathcal{O}\left(\frac{1}{\Delta}\right) = -\frac{1}{\Delta} \int_{-\infty}^{\phi} d\phi' \sin(\phi' - \phi_e) f'(\phi') \; .$$

$$\begin{aligned}
C^{(1)}(l) &= X_l(z) \, e^{i(l)\phi_e} \; , \\
C^{(2)}(l) &= \frac{1}{2} \left(Y_{l+1} e^{i(l+1)\phi_e} + Y_{l-1} e^{i(l-1)\phi_e} \right) \; , \\
C^{(3)}(l) &= \frac{1}{2i} \left(Y_{l+1} e^{i(l+1)\phi_e} - Y_{l-1} e^{i(l-1)\phi_e} \right) \; , \\
C^{(0)}(l) &= \tilde{Y}_l(z) e^{i(l)\phi_e}, \qquad \tilde{Y}_l(z) = \frac{z}{2l} \left(Y_{l+1}(z) + Y_{l-1}(z) \right) - \xi^2 \frac{u}{u_l} X_l(z) \; .
\end{aligned}$$

$$\begin{aligned}
Y_l(z) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} d\psi \, \tilde{f}(\psi + \phi_e) \, e^{il\psi - iz \sin\psi \, f(\psi)} \; , \\
X_l(z) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} d\psi \, \tilde{f^2}(\psi + \phi_e) \, e^{il\psi - iz \sin\psi \, f(\psi)} \; , \\
\tilde{f}(\phi) &= f(\phi) \exp[i\xi^2 \zeta \, u \, r(\phi)] \; , \qquad \tilde{f^2}(\phi) = f^2(\phi) \exp[i\xi^2 \zeta \, u \, r(\phi)] \; , \\
r(\phi) &= \int_{-\infty}^{\phi} d\phi' \, f^2(\phi')
\end{aligned}$$

Infinite pulse:

$$dW = \frac{\alpha M_e \zeta^{1/2}}{8} \frac{du}{u^{3/2}\sqrt{u-1}} \sum_{n_{\min}}^{\infty} dl \;\; w(l)$$

$$w^{(n)} = 2J_n^2(z) + \xi^2(2u-1)\left(J_{n+1}^2(z) + J_{n-1}^2(z) - 2J_n^2(z)\right)$$

Finite (short) pulse

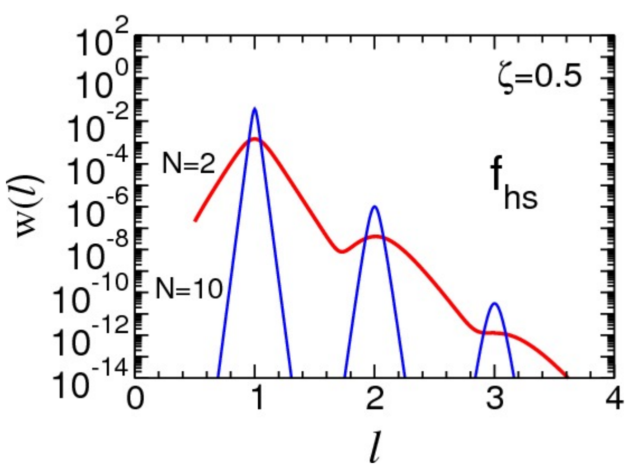
$$dW = \frac{\alpha M_e \zeta^{1/2}}{8N} \frac{du}{u^{3/2}\sqrt{u-1}} \int_{\zeta}^{\infty} dl \;\; w(l)$$

$$w(l) = 2\widetilde{Y}_l^2(z) + \xi^2(2u-1)\left(Y_{l-1}^2(z) + Y_{l+1}^2(z) - 2\widetilde{Y}_l(z)X_l^*(z)\right) \;\; ,$$

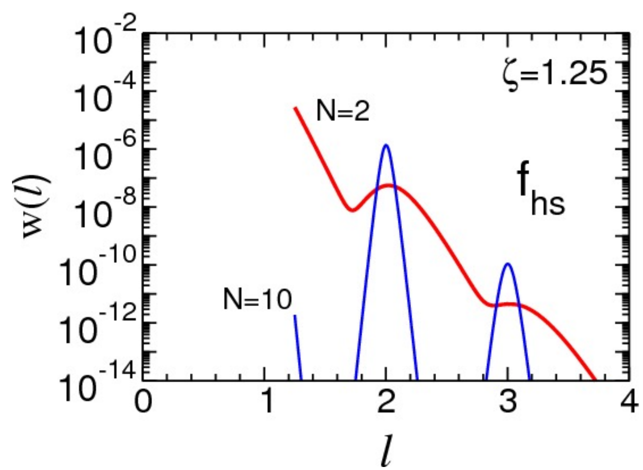
Partial probabilities

at small field intensity $\xi^2 \ll 1$ **for** $l = n + \epsilon, \;\; n = I(l), \;\; |\epsilon| < 1$

$$Y_{n+\epsilon} \simeq \frac{z^n}{2^n n!} \mathrm{e}^{-i\epsilon\phi_0} F^{(n+1)}(\epsilon) \;\; , \qquad X_{n+\epsilon} \simeq \frac{z^n}{2^n n!} \mathrm{e}^{-i\epsilon\phi_0} F^{(n+2)}(\epsilon) \;\; ,$$ **with** $$F^{(n)}(\epsilon) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\phi f^n(\phi) \mathrm{e}^{i\epsilon\phi}$$



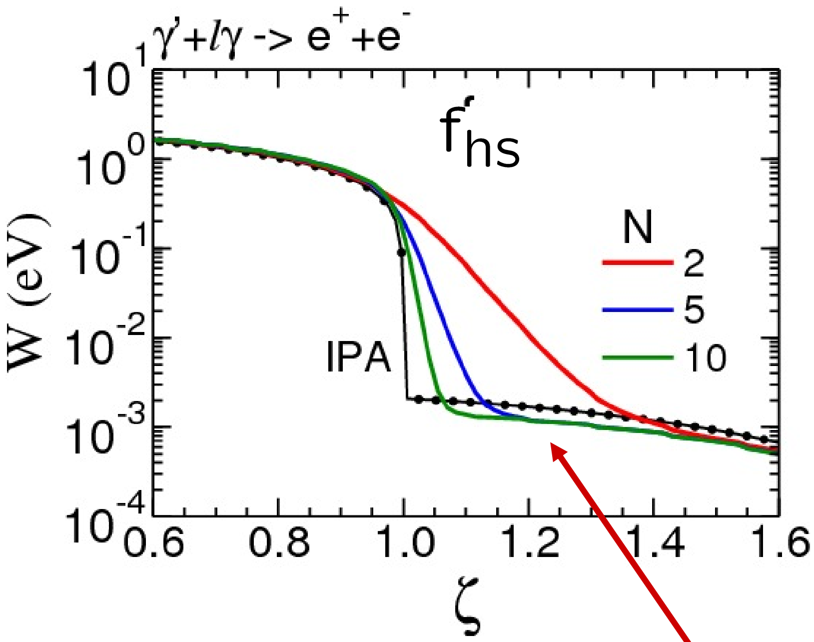
above threshold

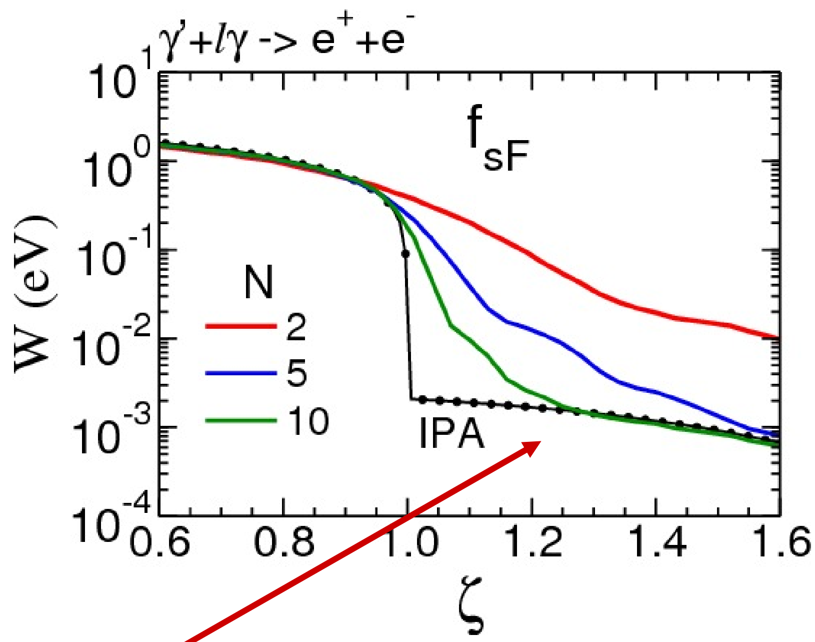


below threshold

Total probability as a function of sub-threshold parameter ζ

$$\xi^2 = 10^{-3}$$

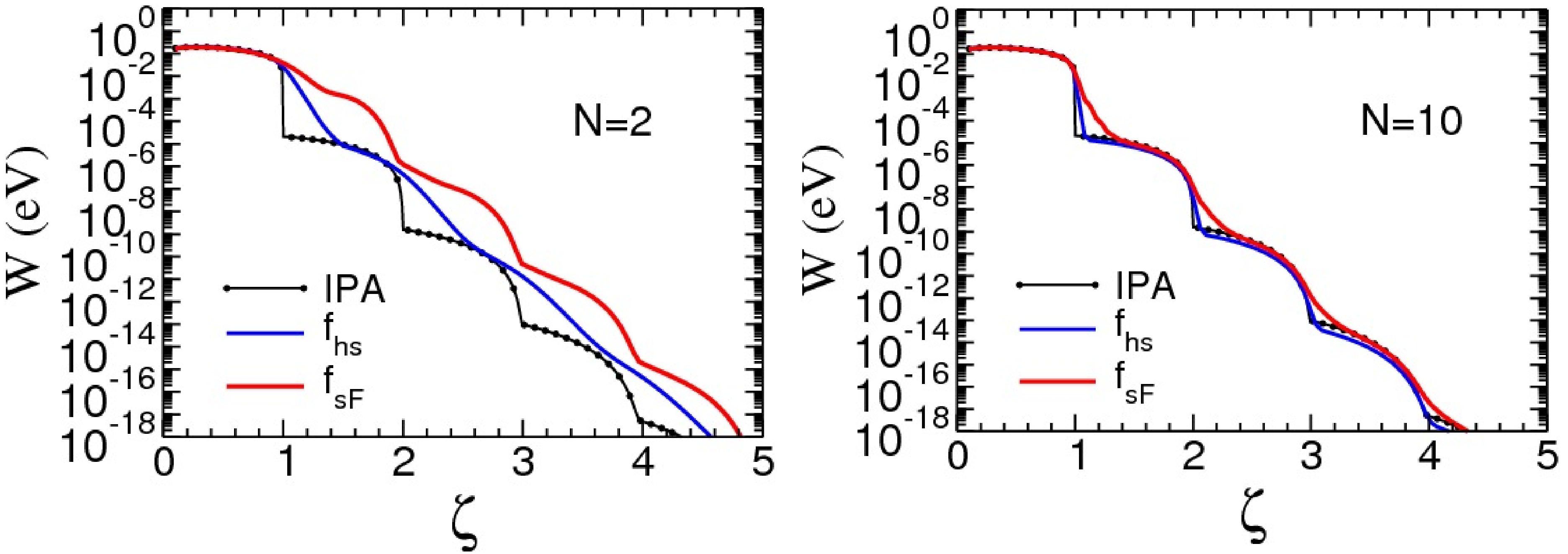




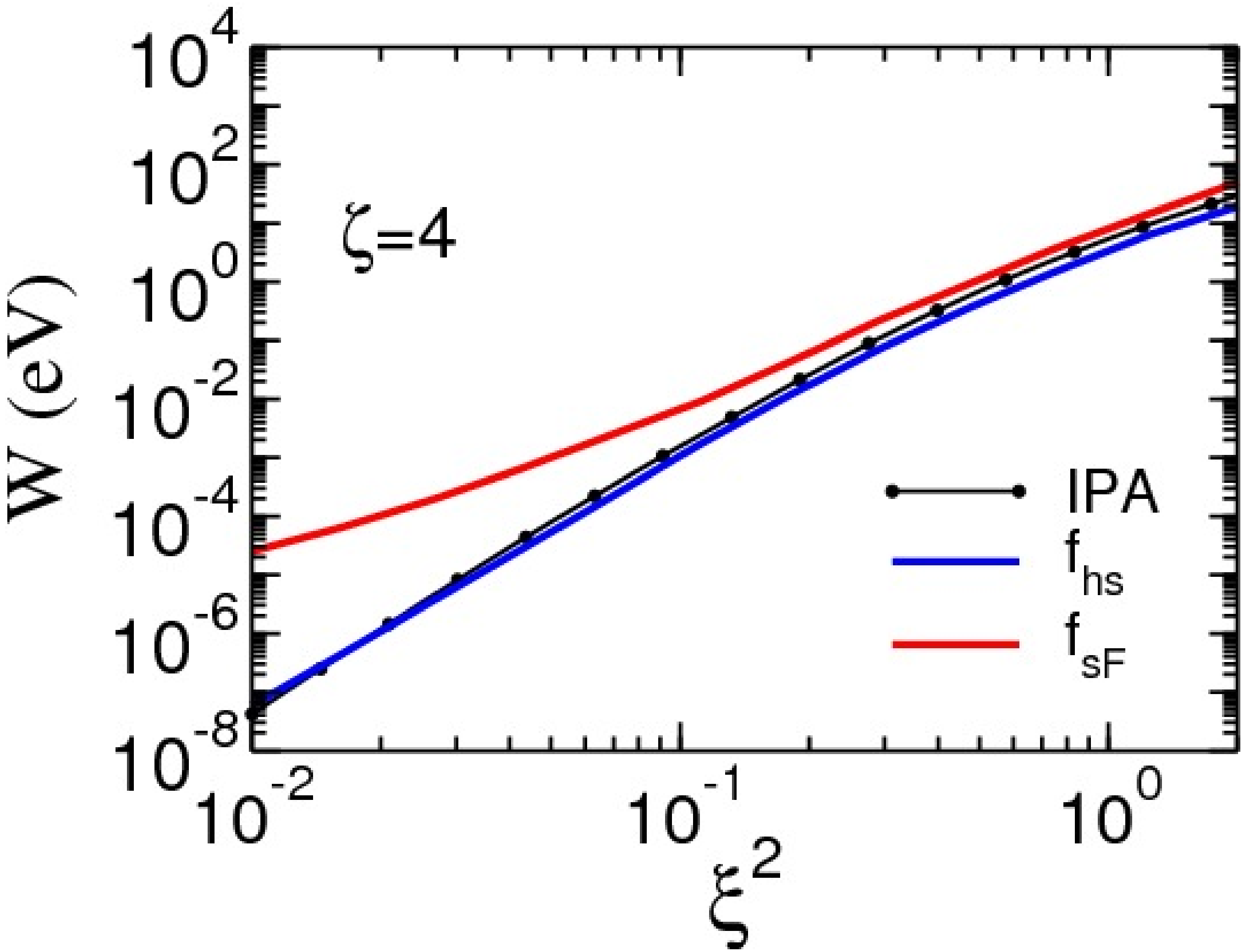
Sub-threshold regions

Total probability in wider region of sub-threshold parameter ζ

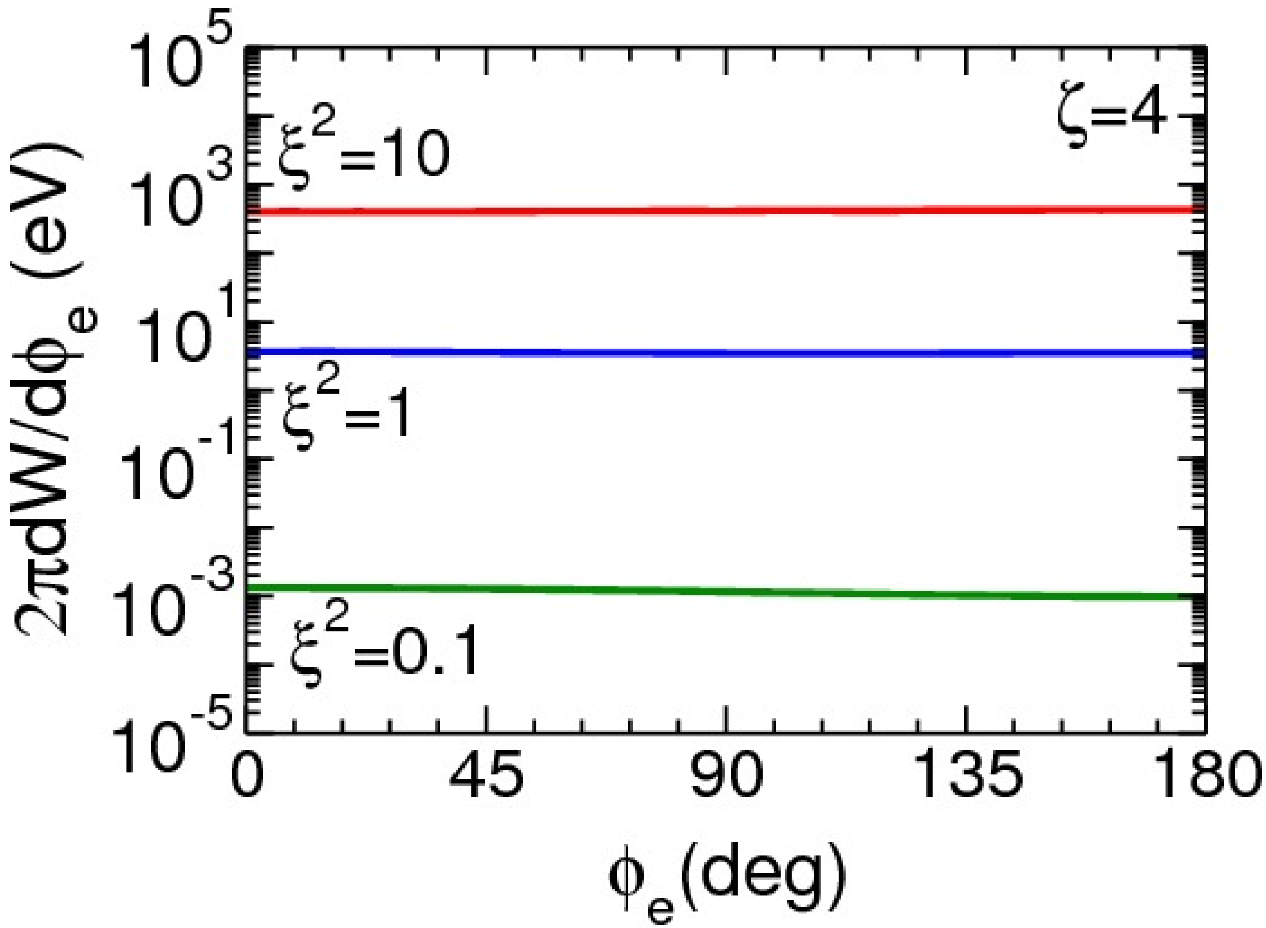
$$\xi^2 = 10^{-3}$$



Dependence of the total probability of pair production rate on the field intensity ξ at fixed sub-threshold parameter ζ



Azimuthal angle dependence (electron anisotropy) at fixed ξ and ζ



(N=2)

$$A \simeq 0$$

Production probability at large field intensity $(\xi^2 \gg 1)$

At large $\xi^2,\ \xi^2 \gg 1$ *the basic functions* Y_l *and* X_l *can be expressed as following*

$$Y_l = \int\limits_{-\infty}^{\infty} dq\, F^{(1)}(q)\, G(l-q)\ , \qquad X_l = \int\limits_{-\infty}^{\infty} dq\, F^{(2)}(q)\, G(l-q)\ ,$$

where $F^{(1)}(q)$ *and* $F^{(2)}(q)$ *are Fourier transforms of* $f(\phi)$ *and* $f^2(\phi)$, *respectively;*

$$G(l) = \frac{1}{2\pi} \int\limits_{-\infty}^{\infty} d\phi\, \mathrm{e}^{i\left(l\phi - z\sin\phi + \xi^2\zeta u\phi\right)}\ .$$

At large ξ^2, l *and* z *dominant contribution comes from region with* $\phi \simeq 0,\ f(\phi) \simeq f(0) \simeq 1$

$$I_{YY} = \int\limits_{\zeta}^{\infty} dl\, Y_l^2 = \int dq\, dq' F^{(1)}(q)\, F^{(1)}(q') \underbrace{\int\limits_{\zeta}^{\infty} dl G(l-q) G(l-q')}_{\delta(q-q')G^2(l-q)}\ .$$

$$I_{YY} = \frac{1}{2\pi} \int\limits_{-\infty}^{\infty} d\phi f^2(\phi) \int\limits_{\zeta}^{\infty} dl G^2(l) = N \int\limits_{\zeta}^{\infty} dl G^2(l)\ .$$

change of variable

$$l \to \hat{l} = l + \xi^2 \zeta u$$

$$G(l) = J_{\hat{l}}(z),\ \hat{l} \gg 1,\ z \gg 1$$

results in

$$W = \frac{1}{2}\alpha M_e \zeta^{1/2} \int_{l_0}^{\infty} d\tilde{l} \int_{1}^{u_{\tilde{l}}} \frac{du}{u^{3/2}\sqrt{u-1}} \left\{ J_{\tilde{l}}^2(z) + \xi^2(2u-1)\left[(\frac{\tilde{l}^2}{z^2} - 1) J_{\tilde{l}}^2(z) + J'^2_{\tilde{l}}(z) \right] \right\} .$$

$$l_0 = \zeta(1+\xi^2),\ \ z^2 = \frac{4\xi^2 l_0^2}{1+\xi^2}\left(u u_{\tilde{l}} - u^2\right),\ \ u_{\hat{l}} = \frac{\hat{l}}{\zeta}$$

Watson's representation for Bessel functions at large $\tilde{l}, z,\ \tilde{l} > z$

$$J_{\tilde{l}}(z) = \frac{1}{2\pi \tilde{l} \tanh \alpha} \exp[-\tilde{l}(\alpha - \tanh \alpha)],\ \cosh \alpha = \frac{\tilde{l}}{z}$$

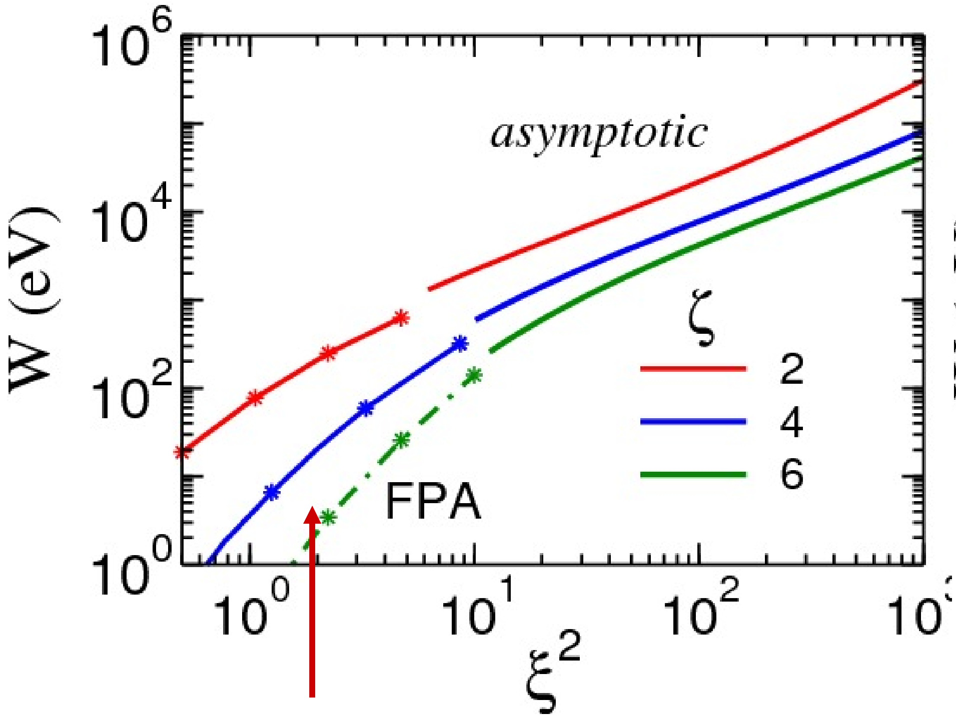
+ saddle point approximation

lead to expression:

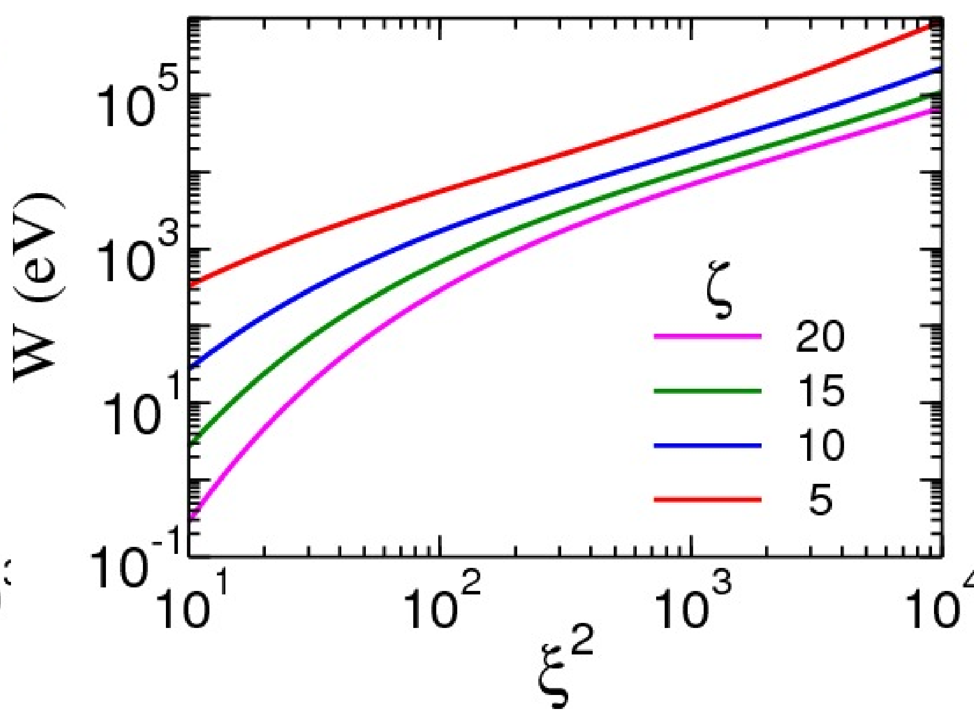
$$W = \frac{3}{8}\sqrt{\frac{3}{2}} \frac{\alpha M_e \xi}{\zeta^{1/2}} d \ \exp\left[-\frac{4\zeta}{3\xi}(1 - \frac{1}{15\xi^2}) \right],\ d = 1 + \frac{\xi}{6\zeta}\left(1 + \frac{\xi}{8\zeta}\right) .$$

Ritus result: $d = 1 \quad (\frac{\xi}{\zeta} \ll 1)$

Total probability (rate) at large field intensity $\xi^2 \gg 1$

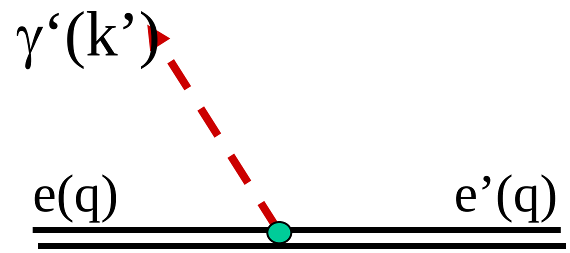


numerical



Compton Scattering in Strong EM Field

Compton Scattering in strong EM fields

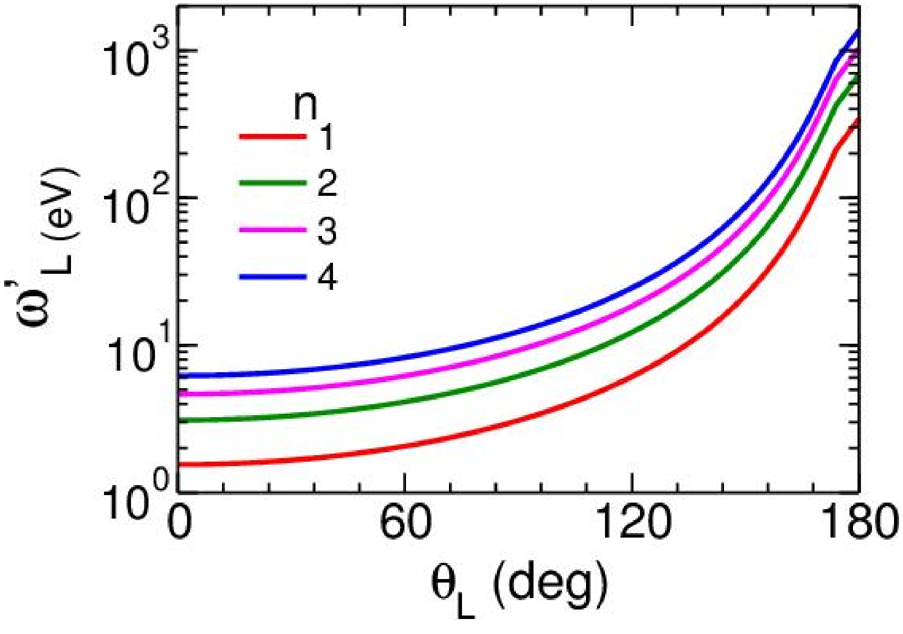


$$S_{fi} = -ie \int \psi_f^* (\gamma \cdot \varepsilon_f^*) \psi_i \mathrm{e}^{ik'x} \frac{d^4x}{\sqrt{2\omega'}}$$

Volkov solutions

Peculiarities of Klein-Nishina (Compton) scattering

- ***Reaction*** $e + L \to e' + \gamma'$ ***is always above threshold.***
- ***Scattering angle*** θ'_L ***and energy of outgoing photon*** ω' ***are related to each other by conservation lows***
- ***In multi-photon processes*** $p + nk \to p' + k'$ ***dependence*** $\omega'(\theta_L)$ ***is defined by the number of interacting photons***

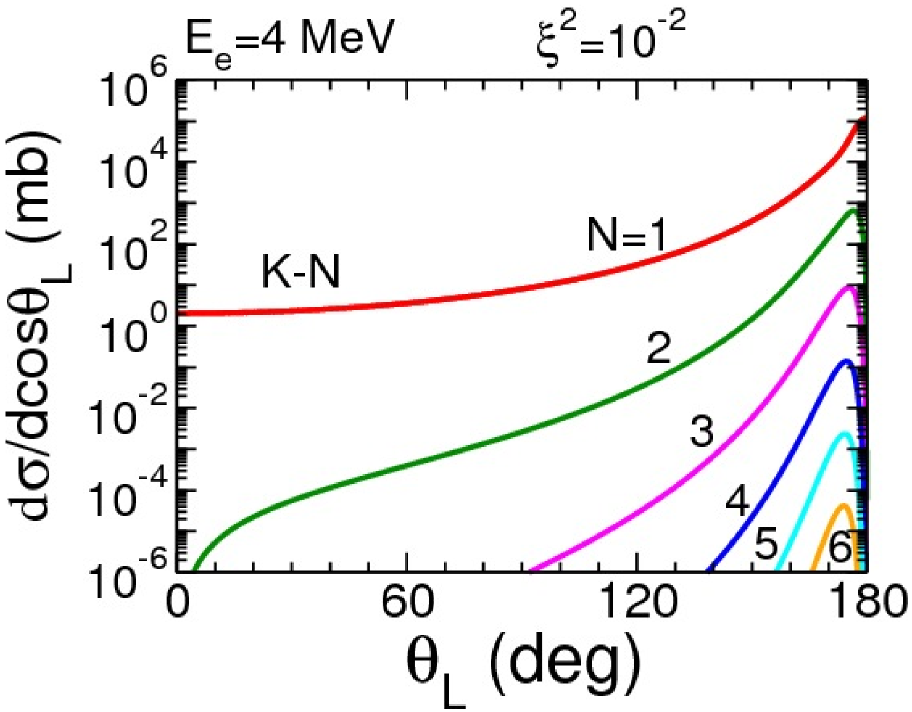


Differential cross section (infinite pulse)

$$\frac{d\sigma}{d\cos\theta_L} = \frac{\alpha^2}{\xi^2}\frac{s+M_e^2}{s-M_e^2}\sum_{n=1}^{\infty} w_n \frac{\omega'_L\, d\cos\theta_L}{E_L + n\omega_L + n\omega_L(1-\cos\theta_L)}$$

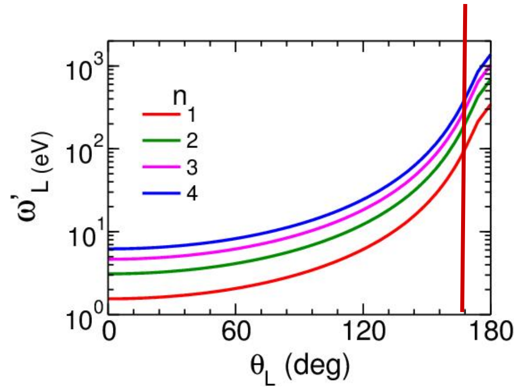
$$w_n = -4J_n^2(z) + \xi^2(2+\frac{u^2}{1+u})[J_{n-1}^2(z) + J_{n+1}^2(z) - 2J_n^2(z)]$$

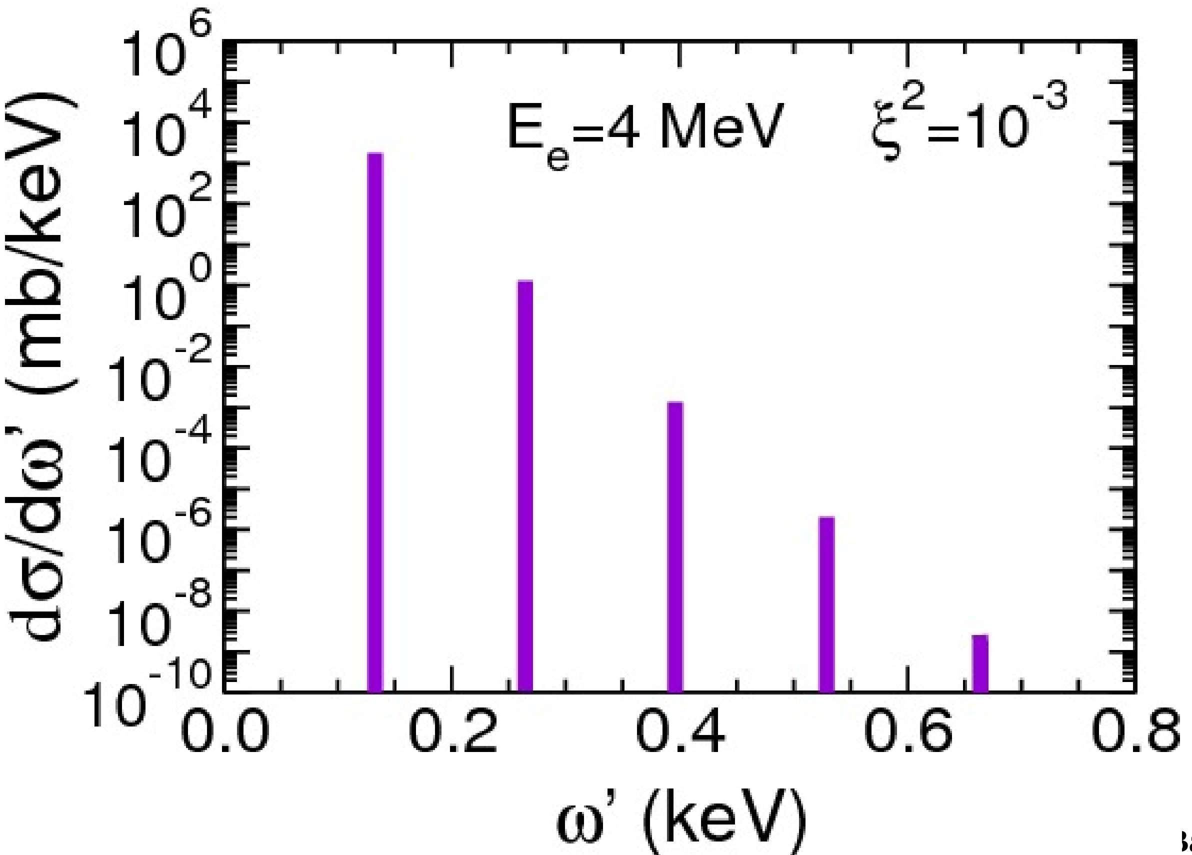
$$u = \frac{k\cdot k'}{k\cdot p'}, \qquad z^2 = \frac{4n^2\xi^2}{1+\xi^2}\,\frac{u}{u_n}(1-\frac{u}{u_n}), \qquad u_n = 2n\,\frac{p\cdot k}{M_e^2(1+\xi^2)},$$



Differential cross section at fixed angle

$$\frac{d\sigma}{d\omega'} = \frac{2\pi\alpha^2 M_e^2}{\xi^2(s - M_e^2)} \sum_{n=1}^{\infty} \frac{1}{p_e - n\omega} w_n$$





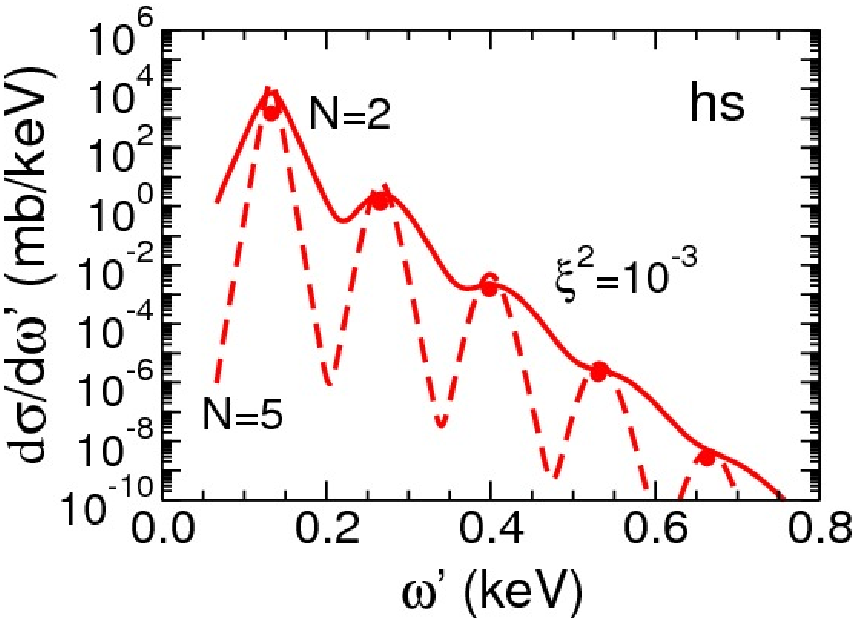
$\theta_L = 170^o$

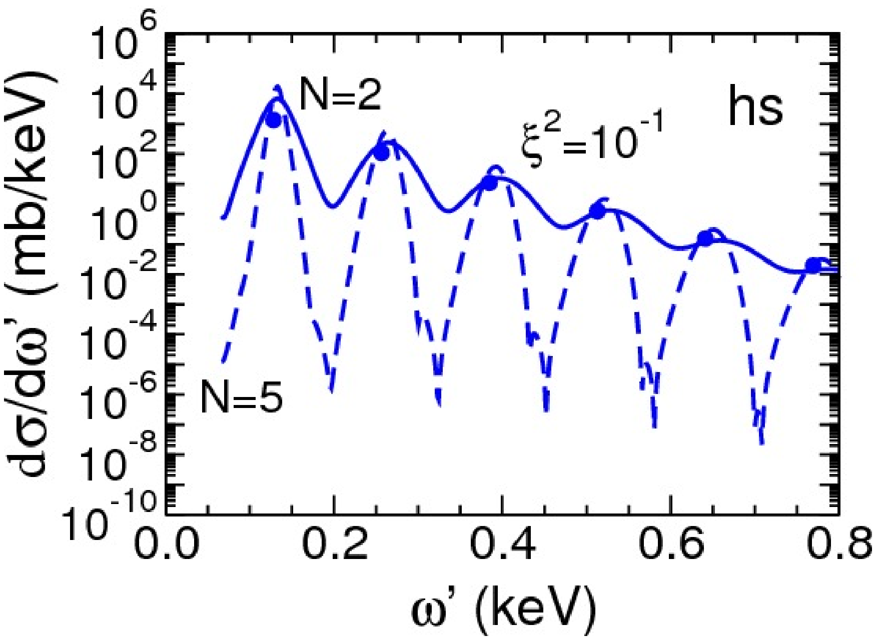
Short pulse

$$\frac{d\sigma}{d\omega'} = \frac{2\pi\alpha^2 M_e^2}{\xi^2(s - M_e^2)} \int\limits_{l_0<1}^{\infty} dl \frac{1}{p_e - l\omega} w_l$$

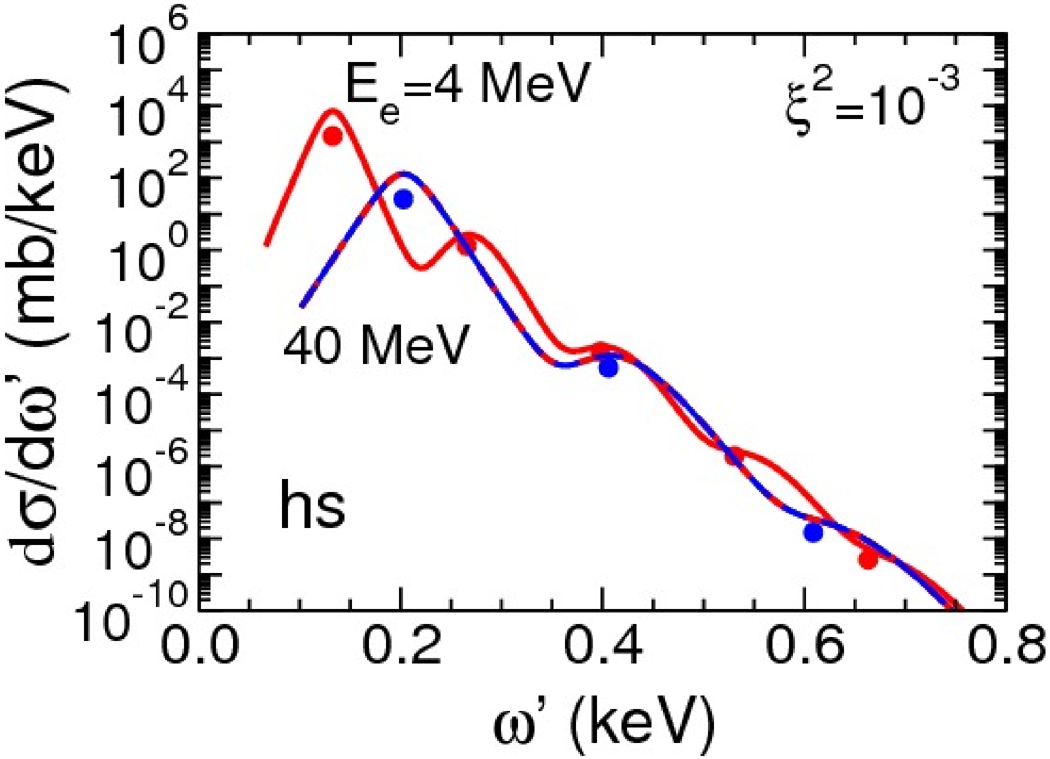
$$w_n = -4\tilde{Y}_l^2(z) + \xi^2(2 + \frac{u^2}{1+u})[Y_{l-1}^2(z) + Y_{l+1}^2(z) - 2\mathrm{Re}\,\tilde{Y}_l(z)X_l^*(z)]$$

$$u = \frac{k\cdot k'}{k\cdot p'}, \qquad z^2 = 4l^2\xi^2\,\frac{u}{u_l}(1 - \frac{u}{u_l}), \quad u_l = 2l\,\frac{p\cdot k}{M_e^2},$$



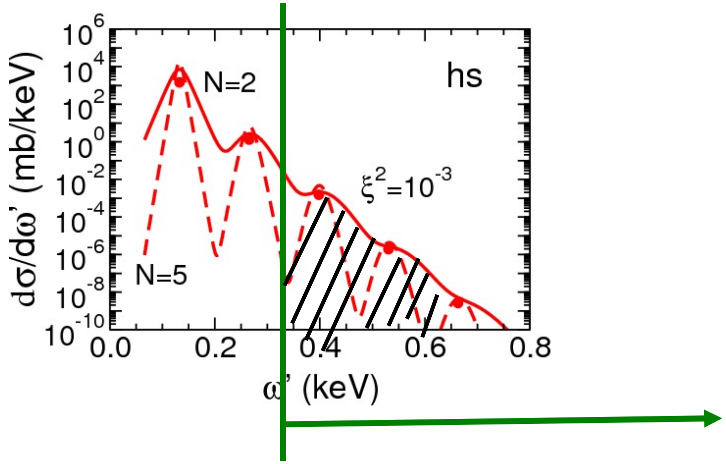


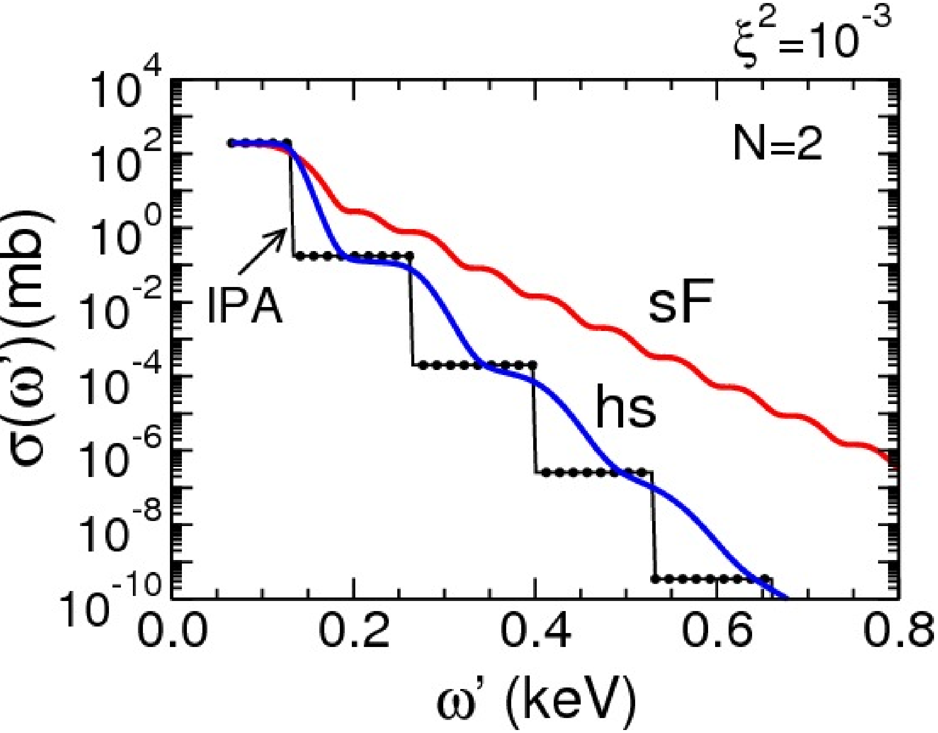
Dependence on the electron energy

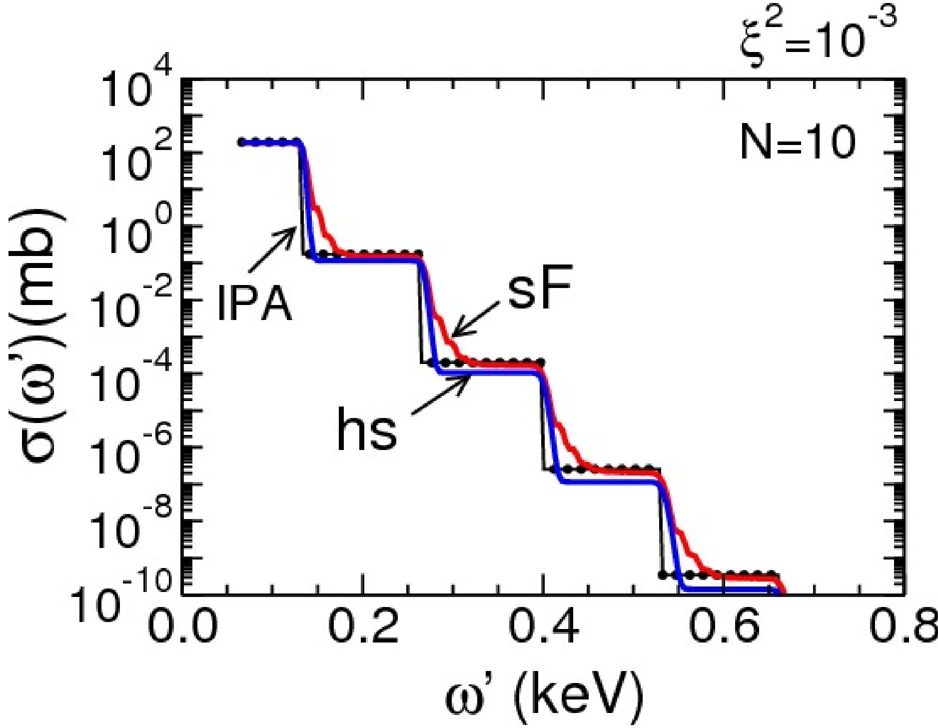


Integrated cross section

$$\sigma(\omega') = \int_{\omega'}^{\infty} d\bar{\omega}\, \frac{d\sigma}{d\bar{\omega}}$$



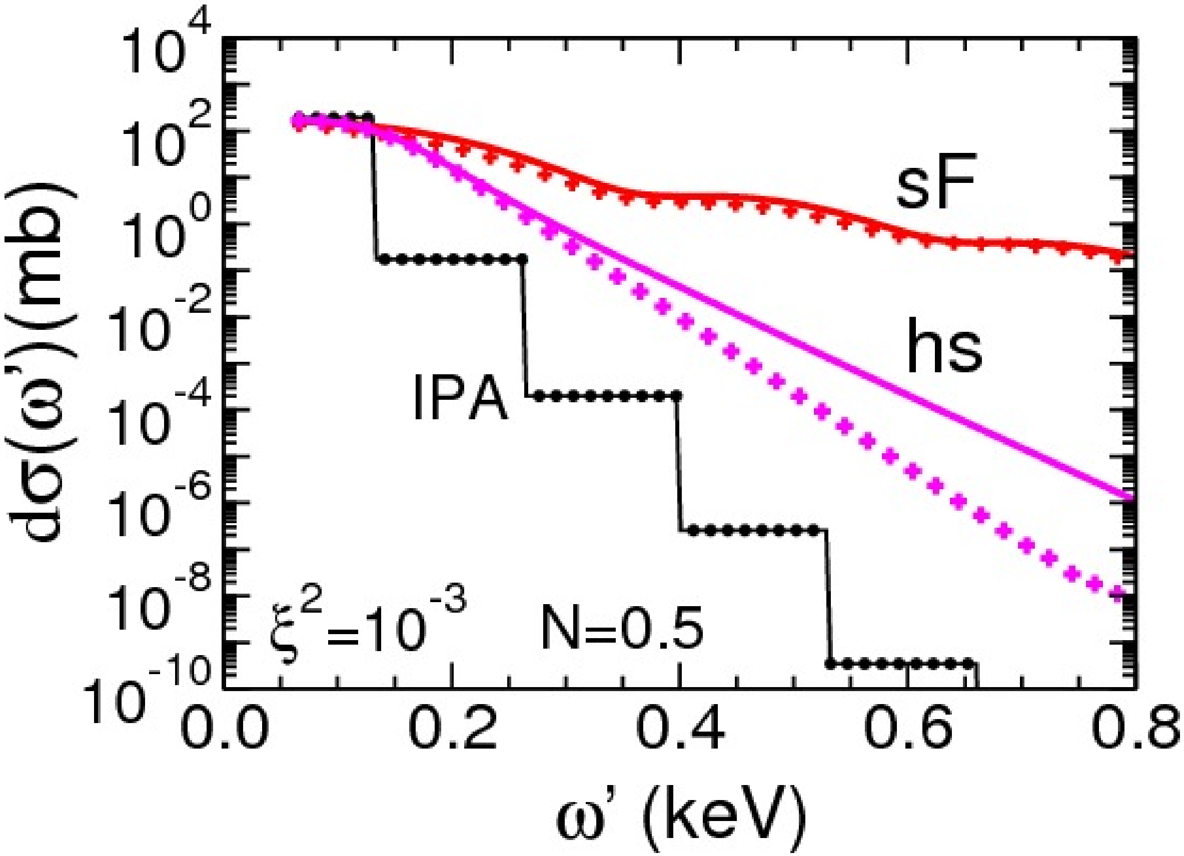




Ultra-short pulse

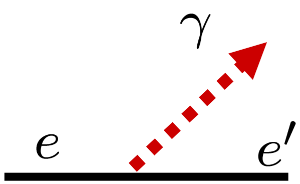
$$\mathcal{Y}(l) \simeq F(l)$$

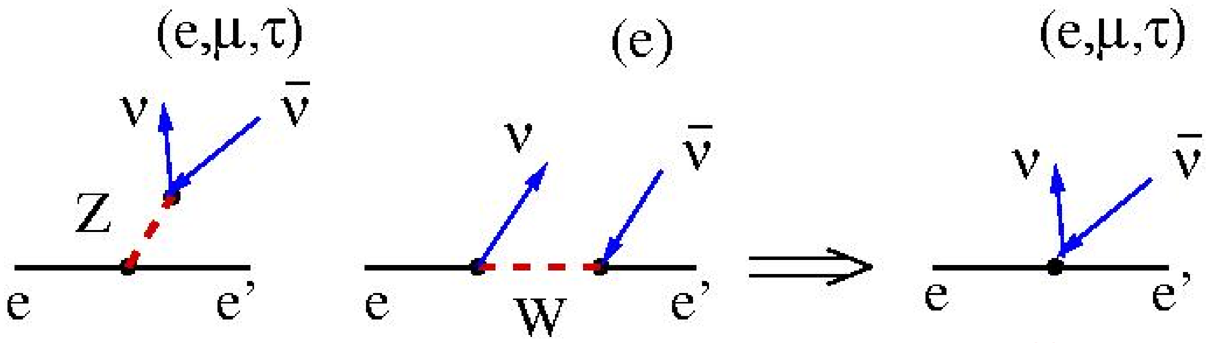
$$l = \omega' \frac{E + p\cos\theta'}{\omega(E + p - \omega'(1 - \cos\theta'))} \simeq \omega' \frac{E + p\cos\theta'}{\omega(E + p)}$$



Neutrino Emission
in a Strong Electromagnetic Field

Elementary $e \to e + \nu\bar{\nu}$ vertices





in local limit with all $p_i << M_B$

effective interactions

$$\mathcal{L}_{\text{eff}}^{(i)} = \frac{G_F}{\sqrt{2}} \left[\bar{u}^e \gamma^\alpha (C_V^{(i)} - C_A^{(i)} \gamma_5) u^e \right] \cdot L_\alpha^{\nu(i)},$$

$$L_\alpha^{\nu(i)} = [\bar{u}_{\nu_i} \gamma_\alpha (1 - \gamma_5) v_{\nu_i}] \quad ,$$

where

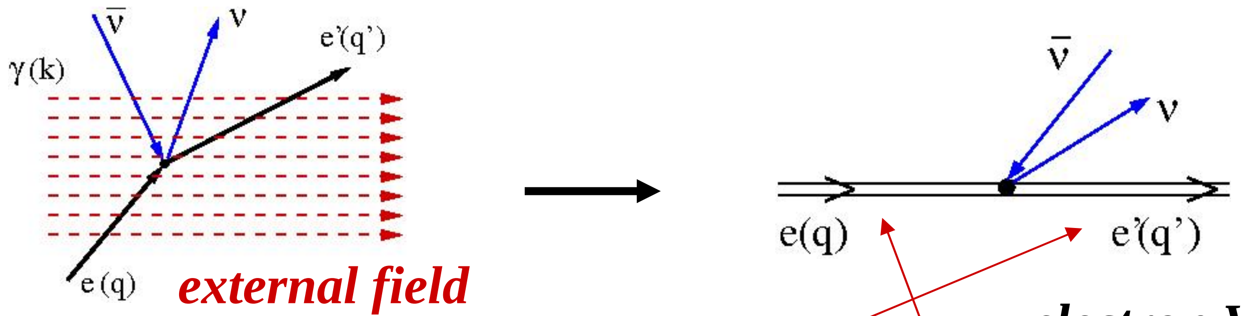
$$C_V^{(e)} = \frac{1}{2} + 2\sin^2\theta_W \ , \qquad C_V^{(\mu,\tau)} = -\frac{1}{2} + 2\sin^2\theta_W \ ,$$

$$C_A^{(e)} = \frac{1}{2} \ , \qquad C_A^{(\mu,\tau)} = -\frac{1}{2} \ ,$$

and

$$\sin^2\theta_W \simeq 0.23$$

Neutrino emission in external EM field



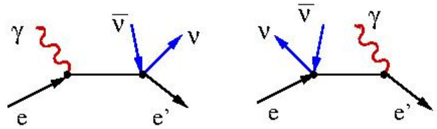
electron Volkov states

$$S_{fi}^{(\mu)} = \frac{G_F}{\sqrt{2}} L_\alpha^{\nu(i)} \otimes \int [\bar{\psi}_f(x)\gamma_\alpha(C_V^{(\mu)} - C_A^{(\mu)}\gamma_5)\psi_i(x)] \mathrm{e}^{i(k_\nu + k_{\bar\nu})x} \frac{d^4x}{\sqrt{2E_\nu 2E_{\bar\nu}}}$$

$$\frac{\bar{u}_{p'}}{\sqrt{2E'_p}} \mathrm{e}^{-ip'\cdot x}\left[1 + \frac{e(\gamma\cdot A)(\gamma\cdot k)}{2(k\cdot p')}\right] \mathrm{e}^{iS'(k\cdot x, p'_e)} \qquad \left[1 + \frac{e(\gamma\cdot k)(\gamma\cdot A)}{2(k\cdot p)}\right] \mathrm{e}^{iS'(k\cdot x, p_e)} \frac{u_p}{\sqrt{2E_p}} \mathrm{e}^{-ip\cdot x}$$

$$\to \int M(kx) \mathrm{e}^{-i(q-q'-k_\nu-k_{\bar\nu})x} d^4x \neq (2\pi)^4 \delta^4(q + k - q' - k_\nu - k_{\bar\nu}) \cdot M$$

$$M_{fi}(kx) = \sum_{n=-\infty}^{\infty} \mathrm{e}^{-in\,k\cdot x} M_{fi}(n)$$



Infinite Pulse Approximation

$$\gamma + e \to e' + \nu\bar\nu$$

Emission probability

$$dW = \sum_{n \geq 1}^{\infty} dW^{(n)}$$

invariant mass of $\nu\bar{\nu}$ pair

$$dW^{(n)} = R^{(n)} \frac{du\, dM_Q^2}{(1+u)^2}$$

with $u = \frac{k \cdot Q}{k \cdot p'}$

$$R^{(n)} = F_V^{(n)} C_V^2 + F_A^{(n)} C_A^2 + 2\lambda F_I^{(n)} C_V C_A$$

$$F_X^{(n)} = F_X^{(n)}(J_n(z),\ J_{n\pm 1}(z), \xi, u, M_Q^2)$$

$$z = \frac{2n\xi}{\sqrt{1+\xi^2}} \sqrt{\frac{u}{u_n}\left(1 - \frac{u}{u_n}\right) - \frac{1+u}{u_n} \frac{M_Q^2}{(1+\xi^2)M_e^2}}\ ,$$

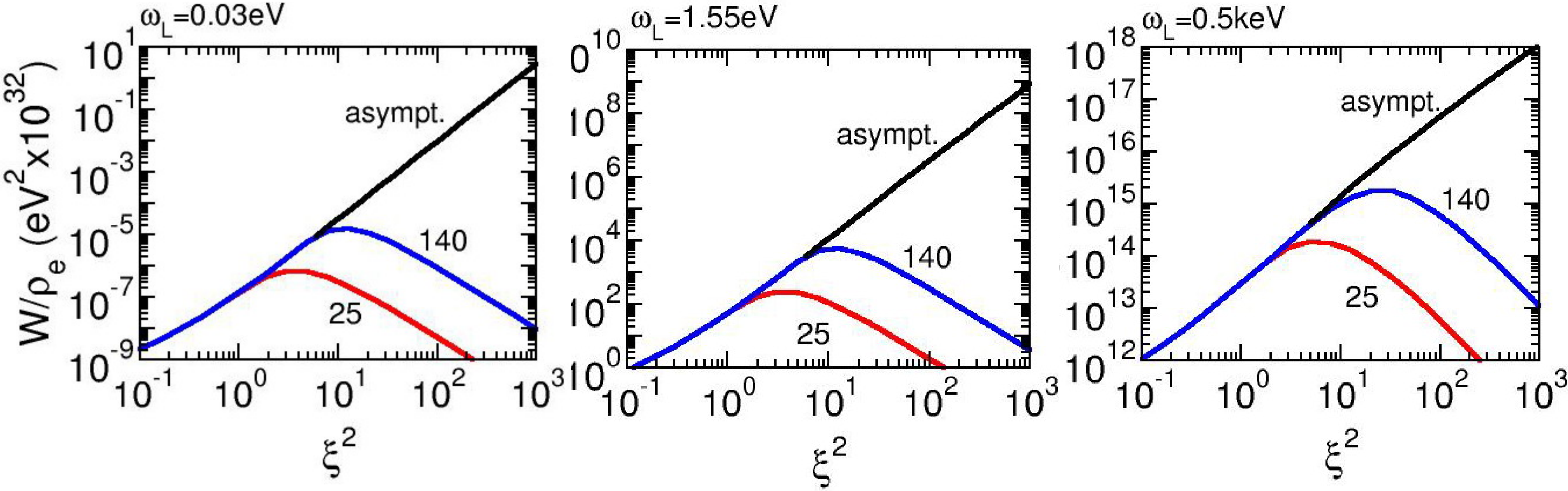
$$u_n = \frac{2n(k \cdot p)}{M_e^2(1+\xi^2)}$$

Large field intensity $\xi^2 \gg 1$

Method of asymptotic summation $\sum_n \rightarrow \int_{n_{\min}(\xi)}^{\infty} dn$

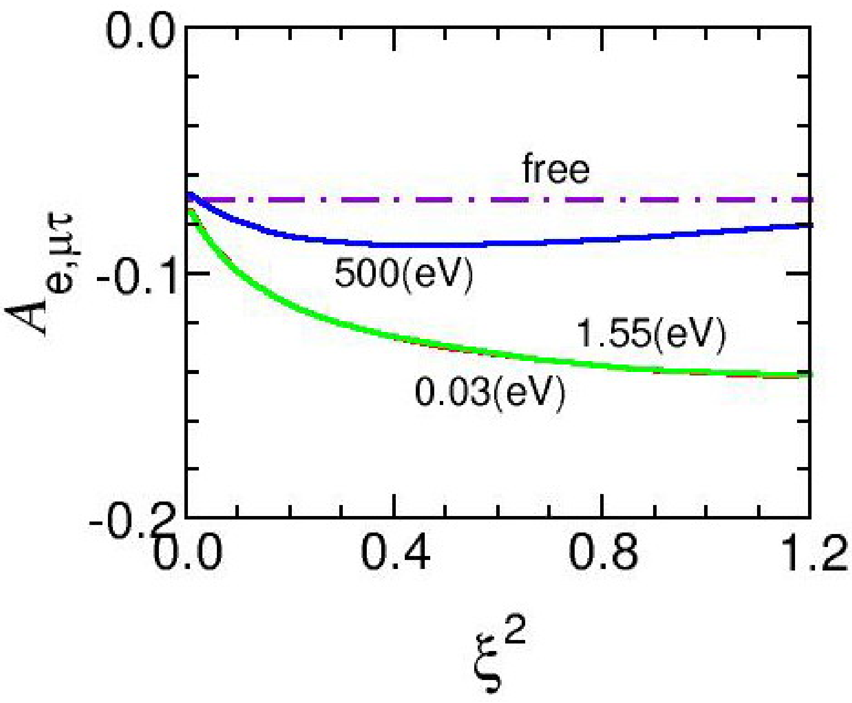
$n_{\min} \sim \xi^3$

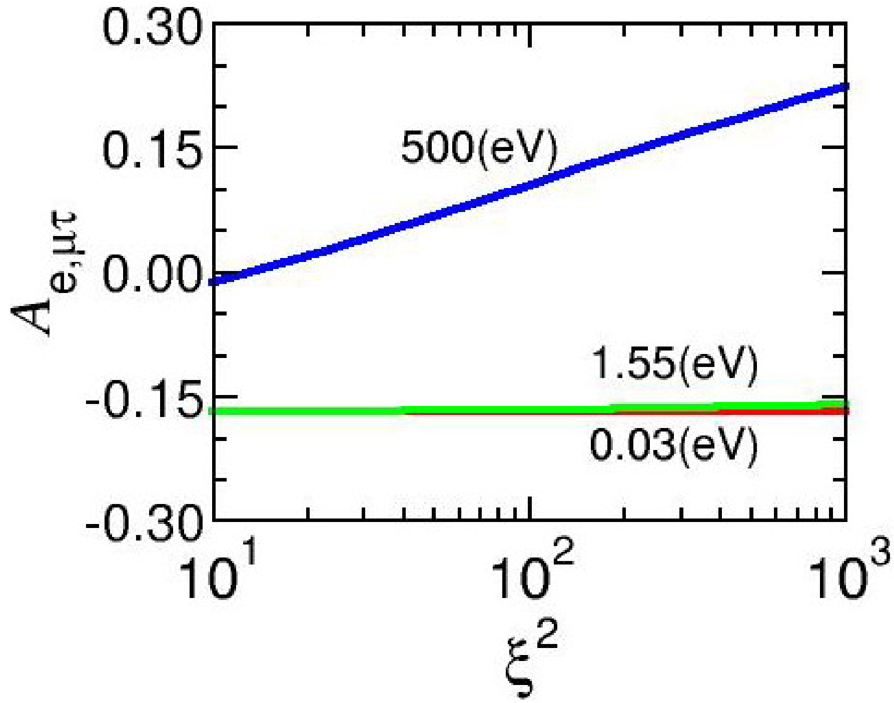
$$J_n(z) \rightarrow \Phi(y); \qquad y = y(\xi, kp, n, M_Q^2)$$



Asymmetry of production of ν_e *and* $\nu_\mu + \nu_\tau$

$$\mathcal{A}_{(e,\mu\tau)} = \frac{W_{(e)} - W_{(\mu+\tau)}}{W_{(e)} + W_{(\mu+\tau)}}.$$



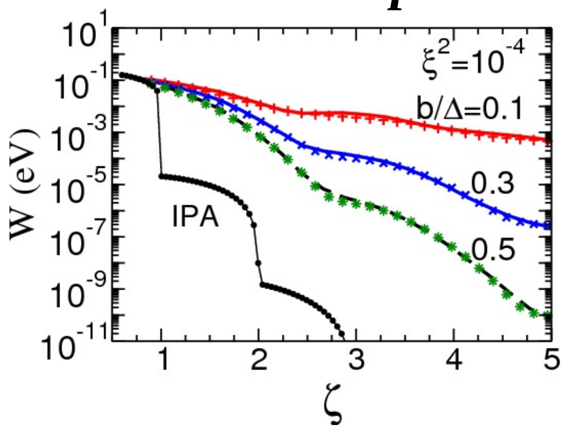


Conclusion

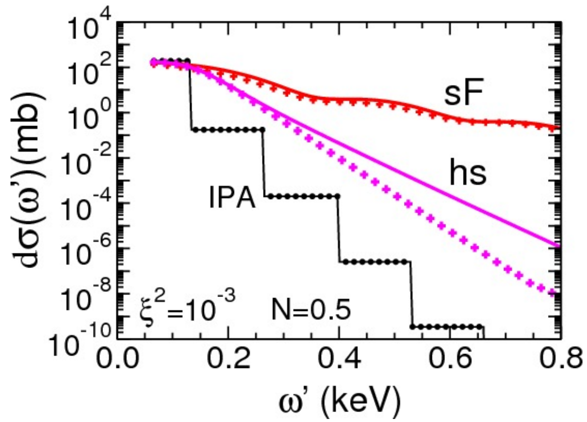
We considerably extended existing models for description of quantum effects in strong EM field in a wide region of kinematical and dynamical variables like initial energy, field intensity ξ *, sub-threshold parameter* $\zeta > 1$ *and so on, and predicted new observables like flavor asymmetry in neutrino pair production, or anisotropy relative to the beam polarization in* e^+e^- *production which may be studied experimentally*

But most interesting in the present time is our finding that short pulses "generates" high momentum components which results in a great amplifier effect by many orders of magnitude depending on field intensity and the bam shape !

Breit-Wheeler process



Compton scattering process



THE END

Thank you very much for attention !